

Teachers' influence on the quality of pupils' written explanations – Third-graders solving a simplified arithmagon task during a mathematics lesson

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The aim of this study is to find out whether there is a connection between teacher's request and guidance for written explanation and third-graders' achievements in solving a non-standard problem. Pupils' task was to solve a simplified arithmagon and to explain their solution. The lessons of seven teachers were recorded and their actions were examined and categorized during a problem-solving lesson. Also pupils' solutions were checked and classified. The teacher's behavior seems to have a crucial role in the quality of pupils' written explanations. The third-graders had difficulties in writing their reasoning for solving the problem.

Keywords: arithmagon, elementary education, mathematics, problem solving, written explanations

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1 Introduction

A developmental shift has taken place in research and teaching on proof and proving (Mariotti, 2006). Originally proof and proving were related to older students' learning of more advanced mathematical topics. More recently, proving and argumentation has been acknowledged as an essential part of mathematical knowledge building for all and there seems to be a general trend towards including the theme of proof in the curriculum (Mariotti 2006; Hanna & de Villiers 2008). This is the case also in Finland. Hemmi, Lepik and Viholainen (2013) have analyzed Finnish, Swedish and Estonian mathematics curricula from the perspective of how proof-related competences are built during compulsory school. The Finnish compulsory school curriculum guides teachers to consider proof-related competences from the very beginning in a systematic way (Hemmi et al., 2013). The pupils are, for example, expected to learn to explain their solutions and reasoning by concrete models, pictures, orally and in written form already in 1st and 2nd grade (FNBE 2004; 2016). Explanations about own thinking and solutions are therefore regarded as important first steps toward proof and proving competences.

This paper concentrates on third-grade pupils' explanations of their strategies in a non-standard problem. Non-standard problem is here defined as a problem that has



more than one solution and requires at least some new thinking in order to be solved. In this article we are especially interested in what kind of teachers' actions help the pupils to write down their explanations for their solution.

2 Theoretical background

Here we will deal with theoretical constructs that are central for the following empirical study: explaining and justifying own thinking and the teacher's role in fostering written explanations during a problem-solving lesson. We will observe the teachers' role during the problem-solving lesson, especially from the viewpoint of their support for pupils explaining their thinking.

2.1 Explaining and justifying own thinking

Justification is one of the main components in the mathematical reasoning process (Lannin, Ellis, Elliot & Zhiiek, 2011). Mathematical reasoning happens through making conjectures, investigating and representing findings and explaining and justifying conclusions (Martin & Kasmer, 2009). Justification can be defined as "providing mathematical arguments to support a strategy or solution" (Grønmo, Lindquist, Arora, & Mullis, 2015.) This means that a mathematical justification is a logical argument based on already understood ideas (Lannin & al, 2011).

Whereas a justification provides grounds, evidence, or reasons to convince others that a claim is true an explanation can be defined as making clear or telling why something exists or happens (Thomas, 1973). Yackel (2001) gives an example in the case of the task "How can you figure out $16+8+14$?" If a pupil responds, "I took one from the 16 and added it to the 14 to get 15 and 15; then I added the 15 and the 15 to get 30, and the other 8 to get 38," we would infer that she was explaining her solution to others. However, the challenge that "you first have to add the 16 and the 8 and then add 14 to that sum" is a request for a justification. Learning to explain own thinking provides therefore a basis for learning to justify.

Explaining own thinking is not easy. Evens and Houssard (2004) investigated how well 11-year old pupils were able to write down explanation for their thinking. They found out that many children appeared to understand the mathematics but were not able to give adequate explanations. Many children were part-way to providing a full answer, but they would have needed more details or they should have been more precise. According to Yackel (2001) the meaning of acceptable mathematical

explanation is not something that can be outlined in advance for students to “apply.” Instead, it is formed in and through the interactions of the participant in the classroom. Both explicit and implicit negotiations contribute to developing these understandings.

Teachers have an important role in learning to explain and justify own thinking. Teachers can help pupils to improve their abilities to write down explanations by questioning. Evens and Houssard (2004) conclude that teachers should assist children to express what they already know in a more precise way and encourage them to improve their answer and to build on it. Pehkonen (2000) found out in her research that pupils’ ability to justify their solution depended more on teaching group than on age because some teachers are better in supporting pupils’ explanation and justification skills. This could be due to the fact that children need to learn that their explanations and justifications need a mathematical, rather, than a social basis (see Yackel & Cobb 1996).

Learning to explain own thinking has many benefits. Explaining and justifying answers in writing develops metacognition (see Schoenfeld, 1992; Schneider & Artelt, 2010). Therefore it is important already at elementary level (FNBE 2004; 2016). Explanation of a solution also helps the pupils as well as the teacher to find a possible error in reasoning. Simon & Blume (1996) emphasize the development of mathematical justification in the connection of communication within a class. Teachers have to understand how such competence develops and they should establish a mathematics community that sees mathematical reasoning as an important part of learning (ibid.). The introduction of young children to the practice of “mathematical argumentation” has been the key objective of a number of research projects (e.g. Maher & Martino 1996) designed to create classroom norms that support the emergence of argumentation and proof making in children’s discourse.

2.2 Teacher’s role in fostering written explanations during the problem-solving lesson

Problem solving is a good way to practice explaining and justifying own thinking but components of reasoning should be a consistent, everyday part of pupils’ mathematics studies (Parrish, 2011). Stein, Engle, Smith and Hughes (2008) identify three main stages of a problem-solving lesson, and teacher role in each of the phases. First, during the *launch phase* the teacher introduces the problem and helps pupils to understand

the content of the problem. S/he introduces the problems without giving solution methods or examples. S/he must ensure that pupils understand what they are required to do and the nature of the things they are expected to produce. S/he should motivate the pupils, organise the work, set up the resources and plan the timing for the session. During this phase the teacher can highlight, if oral or written explanation is required.

In the *explore phase* (Stein et al., 2008), pupils work on the problem, often discussing it in pairs or small groups. During this phase the teacher typically supports pupils' autonomous work by encouraging them to solve the problem both using activating support (i.e. focusing on relevant ideas in pupils' thinking) or commenting support (i.e. giving positive feedback like "Good work") (Laine et al., 2018). The teacher also reminds the pupils what they are required to do. The teacher can facilitate pupils' construction of an explanation by first asking them to explain their thinking in their own words and then encouraging them to write it down (see also Evens & Houssart, 2004).

In the *discussion and summary phase*, the lesson concludes with a whole-class discussion of pupils' solutions for the problem (Stein et al., 2008). During this phase, the whole class views and discusses a variety of approaches to the problem. During this phase the teacher can discuss the different explanations given and focus pupils' attention on the elements of a good explanation.

3 Research questions

In a problem-solving lesson third graders were given a non-standard problem to be solved. In the task assignment, the pupils were also asked to write down how they had found their solution. When we were looking at pupils' responses, we found that these varied substantially across the different classes. To find a reason for this, we looked at how the teachers had requested the pupils to justify their solutions. Consequently, the following two research questions were set:

1. How do the pupils explain their solutions for the non-standard problem?
2. In what way are the teachers' actions to foster written explanations related to the explanations given by the pupils?

4 Methodology

This study is part of the Finland–Chile research project (Academy of Finland, project #135556) in which the participating teachers conducted a mathematics lesson, where they used an open problem-solving task, once a month (see more, e.g. in Laine et al., 2018). In this study we concentrate on the lessons of seven female teachers (Eva, Julia, Katie, Lily, Ruby, Sarah, and Sophie) and their 94 third-graders from the metropolitan area of Helsinki. The arithmagon task was conducted in schools in February 2011. At the time of the study the pupils were about 9 years old.

4.1 Arithmagon task

An arithmagon is a triangle where the sum of the corner numbers is given in the middle of the sides (cf. Mason, Burton, & Stacey, 1982, 160). The task sheet given to the teachers contained both the verbal definition of an arithmagon and a numerical example (Figure 1a). In the introduction an example in which the sums in the middle of the sides were to be solved (Figure 1b) was given. As the actual task the pupils had to 'solve two simplified arithmagons' (Figures 1c and 1d). Furthermore, they were asked to 'invent a method how you can always solve the corner numbers when the numbers in the middle of the sides are given and two of these are equal'. In an extension task the pupils were asked to make their own arithmagon problems with their group members. While finding a method for solving general arithmagon is too difficult for most pupils of this age, our simplified version with two same numbers is more appropriate for them.

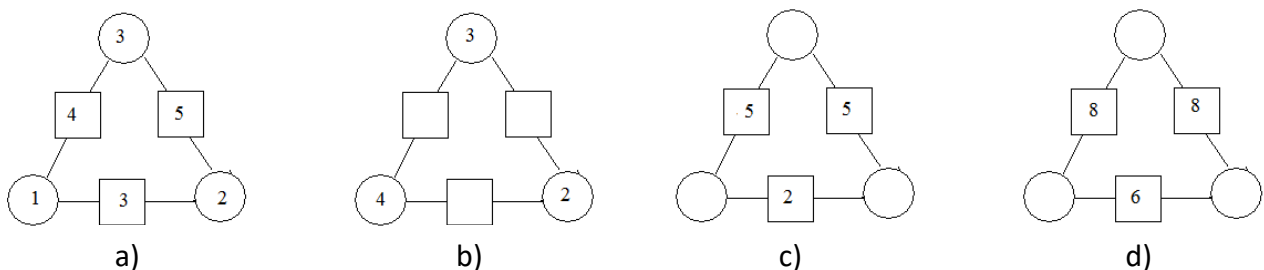


Figure 1. a) The structure of an arithmagon task, b) an introductory task in which the sums in the middle of the sides were to be solved, c) and d) the two tasks in which a simplified arithmagon with two same sums were to be solved.

This task is a non-standard task which requires new thinking. Pupils have to think backwards in order to figure out how the numbers should be located in the arithmagon. The main idea is to understand that if the numbers in the middle of the sides are equal also the numbers at the bottom corners have to be equal. In our analysis, the key aspect is the request to invent a method to find the missing numbers and to describe it. When pupils write down their solution method they have to think back what they have done and why this method works. By doing this they rehearse explaining their own thinking.

4.2 Data gathering and analyses

One of the authors (LN) observed and video recorded the teachers' actions. After the lesson (45 min) the pupils' solution papers were collected and given to the researcher. The pupils' solutions were checked and the written answers were classified. Four categories were found:

1. Two same numbers
2. Addition
3. Vague expression
4. No explanation.

Examples of pupils' explanations are given in [Table 1](#). The videos recorded during the lessons were watched and the transcribed text read through many times. In order to form as exact understanding of the lessons as possible different ways to classify the material were discussed together. Finally, the teachers' actions in the launch and explore phase were decided to be classified by paying special attention to how the teachers requested the pupils to explain their solution.

5 Findings

We first discuss pupils' performance in solving the arithmagon problem and their explanations. After this we describe the teachers' actions in launch and explore phase in order to understand the connection with teachers' actions to pupils' solutions. We pay attention especially to how the teachers emphasized that the pupils' assignment was to explain how they could always solve arithmagons with two same numbers.

5.1 Pupils' performance

The solutions of the two main arithmagons (Fig. 1c & d) were obtained from 94 pupils. About half of the pupils, 41, gave some kind of written explanation. These answers were carefully read many times before classifying them in four categories given in Table 1. In the best explanations, category X.1 'Two same numbers', the pupils had paid attention to the fact that these arithmagons contained two same numbers. The pupils had difficulties to express themselves clearly. For example, in subcategory X.1 we interpreted that in the answer like *I calculated first the numbers at the bottom line* the pupil had notified the fact that there are two same numbers. In category X.2 'Addition' we placed the answers in which the pupils noted that they had used addition in their calculations when they had tried to find numbers to the corners. The third category X.3 'A vague expression' contains the answers in which the pupils had written that they just calculated. These expressions are more like descriptions than strategies to find a solution. Most of the pupils did not write anything but some just wrote that they did not know, and these answers were included to category Y, 'No explanation'.

Table 1. The distribution of the pupils' explanations in the four categories with examples

Category	Examples	Number of pupils
X	Explanation	41
X.1	Two same numbers <i>There are always two same numbers in the triangles.</i> <i>I started by adding the topmost number, because this one number has to fit with two numbers.</i>	13
X.2	Addition <i>I just did + calculations.</i> <i>I added the corner numbers because so I got the numbers in the sides.</i>	16
X.3	A vague expression <i>I just calculated.</i> <i>Finally I just understood it.</i>	12
Y	No explanation <i>I don't know.</i>	53

There were big differences between the classes. Most of the pupils in Katie's, Sophie's and Lily's classes gave an explanation whereas in Ruby's and Julia's classes only two pupils wrote an explanation, and in Eva's and Sarah's classes no-one wrote an

explanation. In order to understand the differences we started to analyze the teachers' actions during the problem solving lesson.

5.2 Teachers' actions

Katie went through tasks 1a & b. After that, the pupils worked with tasks 1c & d and three additional tasks similar to those invented by Katie. While giving the task assignment she stressed the explanation as a part in it:

What kind of strategy did you use? How did you find the numbers? Now you should write it down. Did you find a strategy when there are those two numbers in the sides? Did you find that they can always be resolved in some kind of similar way? So think now what it was that in all these cases you were doing in the same way. What was going on in your thinking?

Katie guided some of her pupils especially to look for the solution strategy. By questioning she required more details like here from Emily, whose explanations were vague. She tries to get Emily to notice that the corner numbers are equal.

Katie: Where did you start? What did you do here?

Emily: I added.

Katie: Is that enough? How do you explain that?

Emily: It is a plus calculation.

Katie: What plus calculation? What numbers are those then? Next to each other, or what?

Emily: I don't know. $1+4 =5$, $1+4 =5$, and $1+1 =2$. All are plus calculations.

Katie: What do you notice about those corners?

During the explore phase Katie emphasized the point of two same numbers and finding an explanation to the solution. She also gave activating support to the pupils like here to Emily. Only after the pupils had written their strategies how they had solved the arithmagons, they were allowed to move to compose additional problems. Quite many pupils were able to point out two same numbers in their explanation. Emily wrote:

"I used addition. I noticed that there are always two same numbers in the triangles."

Katie had changed the task assignment by giving three extra arithmagons that helped the pupils to recognize the meaning of two similar numbers. Katie requested very strongly the pupils to write down their explanations. She also gave support for this by

asking questions, and encouraging pupils to write more precise explanations.

In the launch phase **Sophie** helped pupils to pay attention to central aspects of the task by questioning with the whole class. She pointed to the arithmagon with two same numbers (Fig. 1d) in the screen.

“Your task now is to think what numbers come to these corners. Which numbers do give the answers like here 6 and 8 and 8?”

She clearly instructed the pupils to find an explanation to their solution when the arithmagon contained two same numbers:

You should quite independently, by yourself, think what will be the solution. And when you have thought that, you should also think that if there is some rule how this kind of arithmagon can always be solved. When there are two same numbers and one different number, what is the rule how it can be solved?

Unfortunately, Sophie gave also the additional tasks in the same sheet, and the pupils were more interested in inventing arithmagons to their mates than concentrating in writing explanation.

During explore phase, Sophie advised and repeated many times with loud voice that everybody has to write down how one has solved the problems. She tried to activate the pupils' thinking but she gave e.g. the following rather slight instructions:

Sophie: Yes, you have to think such numbers, that this is correct in every direction. Write down how you solved it, where you started? Try to remember.

John: I added... I do not know.

Sophie: So write that then.

Sophie stressed that it is important to write down an explanation for their solution. She also supported pupils work by questioning but she did it in a more general level than Katie. She also gave right away the extra more interesting task for the pupils and did not check that they wrote down the explanations. Two pupils were able to produce an explanation with the idea of addition:

“I add the numbers and there comes the number that is between them.”

Lily's pupils just read the task sheet by themselves and started working. Thus Lily did not request about explaining their thinking. After about 30 min from the beginning of the lesson she noticed that the pupils had not written down their explanation. She

interrupted the pupils' working and stated to the whole group:

Now everybody has missed one very important point on the task sheet and it is just because you are not used to do it. That important point is here in the middle of the sheet. It can be solved in many ways. How did you solve it? Write it here. Try to write how you thought. This is the important thing. It is here the really important thing, how you thought.

Doing this, Lily emphasized to her pupils the importance of providing a written explanation.

After that she continued to advice the groups, emphasized to write the invention of the strategy with two same numbers but supported their thinking in a rather general level.

Lily: This is a good start. Here it says in many different ways. First you had different numbers. How did your thinking change with two same numbers? Because two same numbers may be solved in another way than the case when all numbers are different. This is a good starting sentence that you already have written. Continue this explanation further.

Pupil: Our explanation did not change.

Lily: Think how it could be changed.

Similarly to Katie, we see Lily pushing her pupils to provide more details but she did not pose questions that could have helped pupils to a higher level in their explanations. Therefore pupils' explanations were very short like: "I added."

In the launch phase **Ruby** went through the examples 1 and 2 on the blackboard with the pupils. She reminded the pupils about the solving method and emphasized that the pupils should write down how they solved these problems. However, she did not pay attention to the special case of two same numbers.

Who can calculate backwards? In an arithmagon the result would be known but not the numbers that give the result. In the next arithmagon task the results are already given. Now together with your mate you have to think aloud how this result has been obtained. With which numbers is the result possible? Is there some way how these arithmagon problems can be solved with backwards calculation. You can write there for example that 'I think that the arithmagon tasks are solved so and so'. You have now some time to think and solve this task together with your mate. When you know how these arithmagons can be solved, call out aloud 'I invented!'

Ruby was the only one to take up the method to find the addends when the sum is known. She also was quite explicit bringing forth the importance of verbalizing the

explanation. In the explore phase the pupils started in pairs to solve the main problems, and Ruby walked around giving advice to the pupils. She remarked many times that the pupils have to write down how they had solved the arithmagons. But she paid no attention to the demand of two same numbers.

Tom: I solved both.

Ruby: You did. Try to invent some way how you solved this. Why did just those results come to your mind? Just write it. Discuss with your mate how you found this result.

Tom: Now I know how I got those.

Ruby: How?

Tom: Using addition, it just came straight to my mind.

In this case we see that although Ruby requires an explanation, she, unlike Katie and Lily, did not push her students to think more details. Unfortunately she also gave the pupils a more complex problem, the general arithmagon, to be solved because she paid no attention to the existence of two same numbers. Nevertheless two pupils were able to produce an explanation containing the idea of two same numbers:

“I started by adding the topmost number, because this one number has to fit with the two numbers

In the launch phase **Julia** went through the examples questioning with the whole class. Then she gave the arithmagon tasks (Fig. 1c & d) without paying any attention to the case of two same numbers. She emphasized that the task is to find a rule according to which an arithmagon can be solved. Unfortunately the task she gave to the pupils was to find a rule to solve a more general arithmagon:

In the next task, however, the numbers in the corners are missing (Fig. 1c) and now your task is to find out in what different ways the numbers in the corners can be found. And you and your pair have to discuss together, if there is some general rule or a way how the numbers in the corners can always be found. Is there some way that it can be solved?

In the explore phase after delivering the task sheets Julia walked around the class and discussed with every pair. The conversation with a pair was as follows:

Julia: Have you boys found already some solutions?

Harry and Charlie: Yes.

Julia: Have you found reasons, justifications, that will always work? How did you Harry solve it? How did you reason those numbers?

Harry: Down there two threes because $3+3=6$.” (see Fig. 1c)

Julia: Did you find some common thing how these are easy to solve?

Charlie: Same number.

Julia: Is it possible to explain such a thing when those given numbers are not same ones. Try if you can make such arithmagons in which the given numbers are not same ones.

As can be seen from this extract by asking questions Julia pushed her pupils to provide more details in their explanation. However, she also posed the more challenging task and asked reasoning for a more general arithmagon. When the pupils justified the case with two same numbers she just passed it. Regardless, two pupils wrote an explanation stressing addition:

“I added the two numbers and made the sum.”

In the launch phase **Eva** gave the pupils an assignment to construct own arithmagons, both those with numbers in the corners and those with numbers in the sides. She read straight from the task sheet also the part about finding a method but she neither emphasized nor returned to it later.

Now we have to start thinking how we could construct these ourselves. And at the same time when you are thinking and constructing them you should also try 'to conjecture some method with which you can always solve the numbers in the corners of any arithmagon in which the numbers on the sides are given and there are two same numbers on the sides'. Now you have to invent your own tasks based on these tasks we just did; either such in which the numbers in the corners have to be solved or such in which the numbers in the sides have to be solved, i.e. either easy ones or a bit more difficult ones.

In the explore phase she delivered sheets with empty arithmagons, walked around and stressed that the numbers should not be equal:

Here you have used same numbers. Could you make it so that numbers are not equal?

You can use all the numbers in the world. Try.

Therefore, Eva like Julia asked solution for a more general arithmagon but she did not request any explanation. None of the pupils wrote an explanation.

Sarah showed the task sheet and used plenty of time in the first two examples (Fig. 1a&b). After that she went through the first main arithmagon (Fig. 1c) by asking numbers one at a time from the pupils. The pupils seemed to be quite frustrated.

Sarah: Now we have the sums in the squares here in the middle, and in these circles you should put the missing numbers. Raise your hand when you have figured it out.

Pupil: 1.

Sarah: What would then come there [points to the upper corner]?

Pupil: 4.

Sarah: Why 4?

Pupil: Because $1+4=5$.

Sarah: When we are working with a problem solving task it is important that we can justify why we come to a certain result. Here we found that of course $1+4=5$. How would you start thinking this other problem [Fig. 1d]?

Here we see that Sarah was explicit in requesting a justification only to a part of the problem. She paid no attention to explaining the whole problem. The pupils worked eagerly in solving the second main problem (Fig. 1d) but wrote no explanation. After that they started to construct arithmagons for their mates to solve.

5.3 Relation between teachers' request and guidance of explanation and pupils' performance

The teachers guided their pupils in different ways and emphasized different things in their instructions. The pupils' answers reflect on the one hand the task that the teacher presented in the lesson, but on the other hand they also respond to the teacher's conception of the task. In [Table 2](#) we have cross-tabulated the way how the teachers requested written explanations, how they supported the pupils to explain their solution and the pupils' performances.

Table 2. The cross-tabulation of the teachers' demand and support for written explanations, and the pupils' performance.

	Katie	Sophie	Lily	Ruby	Julia	Eva	Sarah
Explanation requested	Yes	Yes	Yes	Different task	Different task	No	No
Support for explanation	Deep questioning	Questioning	Questioning	Questioning	Deep questioning	No	No
X.1 Two same numbers	11	0	0	2	0	0	0
X.2 Addition	4	2	8	0	2	0	0
X.3 Vague expression	1	8	3	0	0	0	0
Y. No reasoning	1	2	4	14	11	8	13
Number of pupils	17	12	15	16	13	8	13

The differences between the teachers may explain at least a part of the differences in their pupils' achievements. As can be seen from [Table 2](#) most of the three teachers', Katie's, Sophie's, and Lily's pupils wrote down at least some kind of explanation. These teachers had specially emphasized the restriction of two same numbers. They also tried to activate the pupils' thinking by asking questions. Altogether 11 pupils in Katie's classroom had documented well their strategy (X.1). It is obvious that when a teacher guides the pupils, especially with the aim how they solved the problem, like Katie did, the pupils write better explanations. Another reason for this achievement could be that Katie gave more material, three more problems, to the pupils to find the connection. She also gave the task in sequences, and time for quiet pondering:

"Now work by yourself and give peace for others because these tasks are such that demand pondering."

After the pupils had solved the arithmagon problems Katie guided them to think and write the strategy on their task sheet. She paid attention to the pupils' solutions and asked questions based on them and by doing that helped pupils to produce more accurate explanations.

Sophie gave her pupils two assignments: to think about the strategy, and to construct arithmagon problems to their mate, simultaneously in the launch phase.

The majority of the pupils just gave a vague explanation ‘I just calculated.’, even though Sophie emphasized writing down the strategy. The second task tempted the pupils more so that they did not concentrate to think about their strategies. In Lily’s lesson the pupils had solved the two arithmagon problems before Lily took up the request to write down the strategy for solving. Most of the pupils wrote down their explanation but they had to try to recall what they had thought. Sophie’s and Lily’s questions were also more general than Katie’s questions as they were not based on pupils’ answers.

Ruby gave her pupils the method, calculating backwards, but unfortunately, she did not emphasize the restriction of two same numbers at all. Ruby and Julia requested explanations for a general arithmagon with three different sums. Sarah talked about explanation in general just in the beginning of the lesson and never returned to it later. Eva paid no attention to the request of written explanations.

6 Discussion and conclusions

The third-graders had no problems in solving the main arithmagons, and they were eager in constructing additional arithmagons with their classmates. However, they had difficulties to explain their thinking in the written form even though the importance of explaining their reasoning is included in the Finnish curriculum (NBE 2004) as a core task in mathematics teaching already in grades 1-2. Less than half (44%) of the pupils were classified to category X ‘Explanation given’, and most of them (68%) only stated that they used addition or that the solution just came into the mind. Quite few of the third graders (14%) had written an acceptable mathematical explanation for the solution of the main arithmagons. We also noticed that pupils’ performances varied between teaching groups. These findings are in line with earlier research (Evans & Houssart, 2004, Pehkonen, 2000).

It seems important (see Table 2) that the teacher should pay special attention to requiring explanation as part of pupils’ learning. In all classes where explanation was requested pupils wrote down their explanations. It is interesting that also in classes where the teacher (Ruby and Julia) requested explanation for different task the pupils were able to write down correct explanation. Secondly, teachers’ questions helped pupils to write down explanations. Especially deep questioning which activated pupils’ thinking helped pupils to provide more details in their explanations.

In our research project (see Laine et al., 2018) the participating teachers conducted an open problem once a month in their class. The implementation of the tasks was discussed in the researchers' and teachers' meetings before and after lessons. Teachers planned independently their own lessons and their ways to implement the task were clearly different. Teachers had understood the importance of activating in the project meetings. That is why most of them tried to activate pupils in their thinking by posing questions. Instead, it seems that not all the teachers had prepared well enough their lesson. Ruby and Julia had not noticed the special case of two same numbers and, therefore, they asked their pupils to find solutions and explanations when all the numbers were different. Sarah and Eva for their part let their pupils to construct additional arithmagons to their mates immediately after they had solved the main arithmagons. In the meeting after the arithmagon lesson Sarah told that she had totally missed the part of writing down the explanations. Whereas Eva told in the meeting that her pupils were so keen to construct arithmagons with three numbers that they had no time to write down their explanations. Preparation to teach a non-standard problem requires perhaps a different aptitude to instructional situations than a 'normal lesson'. The teachers should carefully familiarize themselves with the task by solving it in order to be prepared for pupils' comments and questions about the task. In that way they would be able to pose good questions that help pupils to explain their thinking.

It is important to notice that explaining own thinking should be a regular and natural part of mathematics lessons (FNBE, 2016). However, it seems that this was the first time when the pupils were asked to write down their thinking. Written explanations help pupils to insure that their idea is reasonable. It also helps them to remember and confirm new mathematical understanding (Bicknell, 1999). That is why it would be desirable that pupils would themselves feel a need of justifying their solutions in order to understand them better. Tasks and routines that promote discussion and sharing of ideas are useful in creating a culture of sense-making and reasoning (Parrish, 2011). For example problem-solving tasks and games are useful for creating a motivating context for comparing different strategies and for rehearsing justifying own thinking (Olson, 2007) However, based on our long experience as teacher educators, teachers are not used to this kind of teaching. It is possible that producing explanations is not easy for the teachers, either. It would be interesting to analyze how teachers' abilities to guide explaining developed during the research project.

References

- Bicknell, B. (1999). The writing of explanations and justifications in mathematics: differences and dilemmas. In J.M. Turan & K.M. Turan (Eds.). *Making the difference. Proceedings of the 22nd Annual Conference of Mathematics Education Research Group of Australasia* (pp. 75-83). Adelaide: MERGA.
- Evens, H. & Houssart, J. (2004). Categorizing pupils' written answers to a mathematics test question: "I know but I can't explain". *Educational Research*, 46(3), 269-282.
- FNBE (2004). *National Core Curriculum for Basic Education 2004*. Retrieved from http://www.oph.fi/english/curricula_and_qualifications/basic_education
- FNBE (2016). *National Core Curriculum for Basic Education 2014*. Finnish National Board of Education. Porvoo, Finland: Porvoo Kirjakeskus Oy.
- Grønmo, L.S., Lindquist, M., Arora, A. & Mullis, I.V.S. (2015). TIMSS Mathematics Framework. In I.V.S. Mullis & M.O. Martino (Eds.). *TIMSS 2015 assessment frameworks*. (pp. 11-28). US (Boston): TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College and International Association for the Evaluation of Educational Achievement (IEA)
- Hanna, G. & de Villiers, M. (2008). ICMI study 19: proof and proving in mathematics education. *ZDM – The International Journal on Mathematics Education*, 40(2), 329-336.
- Hemmi, K., Lepik, M. & Viholainen, A. (2013). Analysing proof-related competences in Estonian, Finnish and Swedish mathematics curricula – towards a framework of developmental proof. *Journal of Curriculum Studies*, 45(3), 354-378.
- Laine, A., Näveri, L., Pehkonen, E., Ahtee, M., & Hannula, M. S. (2018). Connections of Primary Teachers' Actions and Pupils' Solutions to an Open Problem. *International Journal of Science and Mathematics Education*, 16(5), 967–983. <https://doi.org/10.1007/s10763-017-9809-3>
- Lannin, J. Ellis, A. & Elliot, R. (2011). *Developing Essential Understanding of Mathematical Reasoning for Teaching Mathematics in Grades Pre K-8*. Reston, VA.: NCTM.
- Maher, C. & Martino, A. (1996). The development of the idea of a mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*, 27, 29–63.
- Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.). *Handbook of Research on the Psychology of Mathematics Education: Past, present and Future*, pp. 173-204. Rotterdam: Sense.
- Martin, G.W and Kasmer, L. (2009). Reasoning and Sense Making. *Teaching children mathematics* 16(5), 284–291.
- Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. London: Addison-Wesley.
- Olson, J.C. (2007). Developing Students' Mathematical Reasoning through Games. *Teaching Children Mathematics* 13(9), 464-471.
- Parrish, S. (2011). Number Talks Build Mathematical Reasoning. *Teaching children mathematics* 14(3), 198–206.
- Pehkonen, L. (2000). Written arguments in a conflicting mathematical situation. *Nordic Studies in Mathematics Education*, 2000(1), 23-33.
- Schneider, W. & Artelt, C. (2010). Metacognition and mathematics education. In *ZDM* 42(2), 149-161.
- Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D.A. Grows (Ed.) *Handbook of Research on Mathematics Teaching and Learning* (pp. 334-370). New York: Macmillan.
- Simon, M.A. & Blume, G. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior*, 15, 3-31.

- Stein, M. K., Engle, R. A., Smith, M. S. & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning*, 10, 313–340.
- Thomas, S. N. (1973). *Practical reasoning in natural language* (3rd Ed.). Englewood Cliffs, NJ: PrenticeHall.
- Yackel, E. (2001). Explanation, justification and argumentation in mathematics classrooms. In van den Heuvel-Panhuizen, M. (Ed.). *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education Vol 1*. (pp. 9-24). Utrecht, Holland: PME.
- Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 22, 390-408.