

Learning mathematics by project work in secondary school

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In project-based learning, pupils have two central learning objectives: to understand the content of the subject and to develop their twenty-first-century skills. This article concerns the use of project work in mathematics learning, considered here in the context of the Finnish national core curriculum, mathematical proficiency, and pupils' previous level of attainment. The research consisted of two case studies in which a coordinate system project and a statistics project were tested with secondary school pupils (N=59+58). The main findings show it is possible to study the mathematics of the curriculum and to develop all types of mathematical proficiency using project work. Additionally, the pupils' grades on the project work correlate positively with their overall grades in mathematics.

Keywords: project work, project-based learning, mathematical proficiency, curriculum

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1 Introduction

As a learning method, project work is not a new idea. The roots of project work can be found in American pragmatism, a movement that began at the turn of the twentieth century (Markham, Lamer, & Ravitz, 2006). Across the world, the popularity of project work has increased in recent years because of learning theories transitioning from behaviourism to social constructivism and the requirements of the modern work environment (Markham et al., 2006).

The principles of project work have long been established in the Finnish education system (e.g., Pehkonen, 2001). Especially at the turn of the 1990s, learning projects from kindergarten to the university level have been widely reported (e.g., Vähätalo & Kanervisto, 1988; Sarras, 1994) and systematically studied (e.g., Leino, 1992; Heinonen, 1994).

In Finland, the national core curriculum for basic education 2014, which was published by the Finnish National Board of Education (FNBE, 2016), emphasises the elements of project work, especially multidisciplinary projects. It requires *the inclusion of at least one multidisciplinary learning module for every school year* (FNBE, 2016, p. 33). The aim of the modules is *to link knowledge of and skills in various fields and, in interaction with others, to structure them as meaningful*



entities (FNBE, 2016, p. 32). The pupils should perceive the significance of topics they learn at school for their own life and community (FNBE, 2016). Teachers have to permit pupils to take part in the planning of the modules so that the pupils can highlight their interests (FNBE, 2016). Indeed, interest is a significant predictor of mathematical achievement (Middleton, Jansen & Goldin, 2016).

Additionally, the conception of learning in the Finnish national core curriculum underlines an active role for the pupil and the development of learning-to-learn skills. Teachers are to encourage pupils to increase their self-assessment and to instruct them to work in groups. The practice of working and thinking skills plays a major role in every school subject (FNBE, 2016).

The latest Finnish core curriculum highlights the importance of project work in schools, which has increased the need both to support and research project work as one way of studying school subjects. For example, the StarT programme, which is organised by LUMA Centre Finland, supports teachers with multidisciplinary project work in science, mathematics and technology (StarT, 2019). In the context of the StarT programme, Aksela and Haatainen (2019) and Viro et al. (2020) studied the teachers' view of project-based learning in practice. Correspondingly, StarT projects have been examined from the viewpoint of mathematics and academic literacy (Viro & Joutsenlahti, 2018a). The problematics and the development proposals of the mathematical projects in Finland have been presented by Viro and Joutsenlahti (2018b). That article focuses on the achievement of the learning objectives in project work rather than the learning of mathematics.

2 Theoretical framework

2.1 Project work

Project-based learning can be defined as a systematic teaching method that engages students in learning knowledge and skills through an extended inquiry process structured around complex, authentic questions and carefully designed products and tasks (Markham et al., 2006, p. 4). Here, student-directed and teacher-facilitated emphasis is placed on teachers' and pupils' sharing of responsibility for pupils' learning (Erdogan & Bozeman, 2015). Pupils have two learning objectives: to understand the contents of the subject and develop their twenty-first-century skills (Larmer, Mergendoller, & Boss, 2015). In the present article, we describe the concept of project work as an organising method of teaching adapted from project-based

learning.

By twenty-first-century skills, we mean the skills that are necessary for success in everyday life, both at school and in the modern workplace. These skills are also called success skills (Larmer et al., 2015) or transversal competence (FNBE, 2016). The contents of twenty-first-century skills vary among sources. According to Gold Standard PBL (Larmer et al., 2015), twenty-first-century skills consist of critical thinking, collaboration and self-management. Correspondingly, the p21 network (Partnership for 21st Century Skills, 2007) defines twenty-first-century skills as learning and innovation skills; information; media and technology skills; and life and career skills.

Project work has several advantages (e.g., Yetkiner, Anderoglu, & Capraro, 2008). However, studies in this area are typically quite small, and it is consequently difficult to generalise any results. First, project-based learning has a positive effect on pupils' motivation (Larmer et al., 2015). Second, pupils in project-based learning classrooms seem to learn science content better than pupils in the more traditional classroom (Drake & Long, 2009; Rivet & Krajcik, 2004; Schneider, Krajcik, Marx, & Soloway, 2002). Correspondingly, in mathematics, the pupils experiencing project-based learning are better able to use mathematics in everyday life situations (Boaler, 1998) and apply it in new situations (Hung & Jonassen, 2007). Additionally, project-based learning as a working method also makes it easier to remember the learned content longer (Wirkala & Kuhn, 2011).

In general, the implementation of project work is difficult at first. Both teachers and pupils need support in their work (Larmer et al., 2015). Project work has sometimes been criticised from the viewpoint of learning, especially learning the contents of the subject (Hakkarainen, Bollström-Huttunen, Pyysalo, & Lonka, 2005; Lamer et al., 2015). A project might be a “dessert project,” including hands-on activities where pupils make a low-quality product. These poorly designed projects are usually a waste of time and do not support the achievement of the learning objectives (Lamer et al., 2015). The result of the project might often be an output that is visually excellent, but where the learning process itself is not (Hakkarainen et al., 2005).

Some teachers want to assess project work, especially the learning of the content, with an exam (Erdogan & Bozeman, 2015). Many twenty-first-century skills are not measurable through standardised tests (Bell, 2010). Student-directed and teacher-

facilitated project work needs authentic evaluation: formative evaluation, self-assessment and peer reviews (Bell, 2010; Erdogan & Bozeman, 2015).

2.2 Mathematical proficiency

Another learning objective of project work is to understand the content of the subject. In the current article, we concentrate only on mathematics. Kilpatrick, Swafford, and Findel (2001) created a model to describe a pupil's mathematical proficiency. The model consists of five intertwined strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Moschkovich (2015), for example, widened the model to academic literacy, which consists of the five strands of mathematical proficiency (Kilpatrick et al., 2001), mathematical practices and mathematical discourse.

Conceptual understanding includes the comprehension of mathematical concepts, operations and the relations between them (Kilpatrick et al., 2001), and it is about the meanings that a pupil gives for mathematical solutions. It is essential to understand what a particular result means: why a procedure works, and why the result is the right answer. The pupil should understand why a mathematical concept is important and what kind of situation it can be used in. Additionally, a pupil should be able to connect new ideas to them that he or she is already familiar with (Moschkovich, 2015). The essence of this branch is usually described as 'understand' (Joutsenlahti & Sahinkaya, 2006). In project work, conceptual understanding may involve becoming familiar with new concepts in groups. However, it is also important to be able to connect the new concept to those the group has learned earlier.

Procedural fluency can be seen as knowing how to compute (Moschkovich, 2015), the ability to use mathematical procedures flexibly, efficiently, accurately and appropriately. This mechanical counting and simplifying is often emphasised in schools but is only one component of mathematical proficiency. Conceptual understanding and procedural fluency are closely related because a pupil remembers the procedures better if he or she understands them and can connect new knowledge with prior knowledge (Kilpatrick et al., 2001). Procedural fluency is described as 'can-do' (Joutsenlahti & Sahinkaya, 2006). Most mathematical projects require routine counting. A pupil must repeat familiar procedures and acquire routine ones or learn the new ones.

Strategic competence refers to the ability to formulate, represent and solve mathematical problems that are not routine exercises. Especially outside of school, a

pupil can come across problems that he or she has to formulate before he or she can solve them using mathematics. This strand of mathematical proficiency is also called problem solving (Kilpatrick et al., 2001). Here, mathematics in the projects is not necessarily insight at the beginning, but the group has to formulate the problem in a mathematical format.

Adaptive reasoning can be seen as the capacity to think logically about the relationships between concepts and situations. It is not only reflection, explanation and justification, but also intuitive reasoning (Kilpatrick et al., 2001). Adaptive reasoning can be described as ‘apply’ (Joutsenlahti & Sahinkaya, 2006). In the project work, a pupil connects information from different sources and applies it to the situation of the project. Finally, the groups have to evaluate their results critically.

Productive disposition means a pupil’s perceptions of mathematics as sensible, useful and worthwhile; this also includes a pupil’s confidence in diligence and his or her own efficacy (Kilpatrick et al., 2001). In the present article, the productive disposition is replaced by the view of mathematics advocated, for example, by Joutsenlahti (2005). As a concept, this view of mathematics is wider and includes pupils’ beliefs about mathematics; beliefs about oneself as a learner and a user of mathematics, and beliefs about learning and teaching mathematics. Mathematical projects should be authentic, and their theme should be linked to pupils’ everyday lives (Larmer et al., 2015); they should also require diligent and persevering working.

All of these strands interact with each other. In problem solving, for example, pupils use their strategic competence to formulate and represent a problem, but they need adaptive reasoning when determining the legitimacy of a proposed strategy. On the other hand, adaptive reasoning includes conceptual and procedural knowledge (Kilpatrick et al., 2001).

2.3 Direct proportionality and statistics in Finnish national core curriculum

Mathematically, the current article focuses on the concepts of direct proportionality and statistics at the lower secondary school level. The Finnish core curriculum for basic education (FNBE, 2016) introduces direct proportionality for the first time in grades 7–9. However, pupils are first introduced to the first quarter of the system of coordinates (FNBE, 2016) in grades 3–6.

Table 1. Direct proportionality and statistics in the Finnish national core curriculum (FNBE, 2016). The text that links to conceptual understanding is highlighted with green, the text to procedural fluency with yellow and the text to adaptive reasoning with turquoise.

	Key content area	Objectives of instruction	Assessment targets in the subject	Knowledge and skills for grade 8
Direct proportionality	<i>C4 Functions: Correlations are depicted both graphically and algebraically. The pupils familiarise themselves with direct proportionality. They get acquainted with the concept of the function. The pupils draw straight lines in the coordinate system. They learn the concepts of the angular coefficient and the constant term. They interpret graphs, for example, by examining the increase and decrease of a function. They determine the null points of functions. (p. 404)</i>	<i>O15 to guide the pupil to understand the concept of the variable and to acquaint him or her with the concept of the function. To guide the pupil to practise interpreting and producing the graph of a function. (p. 403)</i>	<i>The concept of the variable and the function as well as the interpretation and production of graphs. (p. 407).</i>	<i>The pupil understands the concept of the variable and the function and is able to draw a graph for a first-degree and a second-degree function. The pupil is able to interpret graphs diversely. (p. 407)</i>
Statistics	<i>C6 Data processing, statistics, and probability: The pupils deepen their skills in collecting, structuring, and analysing data. It is ensured that the pupils understand the concepts of the average and mode. They practice defining frequency, relative frequency, and median. The pupils familiarise themselves with the concept of dispersion. They interpret and produce different diagrams. (p. 405)</i>	<i>O19 to guide the pupils in determining statistical key figures --. (p. 403).</i>	<i>Statistical key figures</i>	<i>The pupil masters central statistical key figures and is able to give examples of them. (p. 408)</i>
	<i>C2 Numbers and operations: It is ensured that the pupils understand the concept of percentages. The pupils practice calculating percentages and calculating the amount a percentage expresses of a whole. (p. 404)</i>	<i>O13 to guide the pupil in expanding his or her understanding of percentage calculation. (p. 403)</i>	<i>The concept of percentages and percentage calculation. (p. 407)</i>	<i>The pupil is able to describe the use of the concept of percentages. The pupil is able to calculate percentages, the amount a percentage expresses of a whole, and the percentage of change and comparison. The pupil is able to use his or her knowledge in different situations. (p. 407)</i>

The studying of statistics already starts in primary school. In the curriculum, one objective in grades 1–2 is *to familiarise the pupil with tables and diagrams* (FNBE, 2016, p. 137). In grades 3–6, pupils must prepare and interpret tables and diagrams and use key figures, such as the greatest and smallest value, mode and average. Additionally, systematic data collection is a part of the key content areas. Understanding percentages, which is essential in statistics, develops in grades 3–6. [Table 1](#) summarises what the Finnish core curriculum for grades 7–9 states about direct proportionality and statistics.

Both subject areas are studied in versatile ways in lower secondary school. The study of direct proportionality starts from the beginning, but a secondary school teacher can assume that pupils have prior knowledge of statistics. In particular, knowledge of the roles of strategic competency and conceptual understanding are emphasised instruction objectives and knowledge for grade 8 in the Finnish core curriculum.

2.3 Research questions

The role of the authors has been active in the reform movement of project-based learning in Finnish mathematical education. The authors have created the project instructions to the Finnish lower secondary school level and educated the pre- and in-service teachers in project-based learning. Research-based development is essential, and we must know more about mathematics learning in project work in order to provide good project practices for schools.

The focus of the current study is to examine mathematics learning and the possibilities to learn mathematics in project work at a lower secondary school level. The strands of mathematical proficiency and the Finnish national core curriculum provide tools for the evaluation of the learning processes. The following research questions are posed:

- How is the mathematical content of the projects based on (a) project instructions and (b) the project implementation in line with the demands of the Finnish core curriculum?
- What strands of mathematical proficiency does the project work develop per the project instructions?
- Were there any connections between the grades of the pupils' project work and their previous grades in mathematics in project implementation? What kind of pupils succeed, or who do not?

There are two parts of the research: (a) the research of project instructions and (b) the observation of project implementations. In the first research question, we cover the mathematical contents of the projects from the perspective of both project instructions and project implementations. The second question focuses only on the project instruction, while the third looks at the implementation in the schools.

3 Methods

The research covers two project instructions and realisations of project work in mathematics at the Finnish lower secondary school level: a coordinate system project and a statistics project. The research was carried out in two parts: the design and implementation of the projects.

3.1 Design of projects

The mathematics teacher who participated in the project implementation designed the project instructions in close collaboration with the researchers. The national core curriculum and available time set some limits for the project. The teacher also asked the wishes of his pupils who participated in the project implementation be accounted for. The final instructions for pupils were detailed in both projects, but the pupils still had a choice regarding the details.

The coordinate system project starts with an experimental part. Here, groups can choose their own research topics from the directly proportional quantities in accordance with their interests. The instructions include some examples of suitable topics. The pupils can weigh candies, coins, water and cooking oil or rice and rice crispies on the scales; gauge the speed of a walker and runner, or look for the prices of vacation trips. For example, pupils weighed 1 dl, 2 dl, ..., 9 dl and 10 dl of water and the same volumes of cooking oil. Then, they plotted the measuring points to the coordinate system by hand, where the x-axis is mass (g) and y-axis volume (dl).

The measuring points of every group had to form two lines (two situations) because the quantities were directly proportional. After that, the pupils plotted the same graph using Excel. Excel also gives the equation of a line. Using their graphs, the pupils had to find out where two lines intersected.

As a part of the project, the pupils defined the concepts of an angular coefficient, the intersection of two lines, the intersection of the x-axis and the intersection of the y-axis. They also had to consider what these concepts mean in practice and in their

project. The slope of a line may, for example, represent the density. At the end of the project, the pupils aggregated their results, placed them on a poster and presented them to their classmates.

In the statistics project, the pupils made an online statistical study using Office 365. To start with, they chose the subject of their own research – the topic could have been, for example, the quality of school food or the use of social media. The pupils created an online questionnaire that had to include at least one numerical question. The other classmates answered the questionnaire. The groups then analysed their data quantitatively using Excel, counting frequencies, relative frequencies, modes, medians, averages and dispersions. They also made diagrams. The pupils made a PowerPoint presentation and a poster about their research. At the end of the project, the groups presented their outputs to the class.

In both projects, it was important for the teacher that assessing the project work be many-sided. The teacher assessed the project work using numerical grades: the whole group was given the same grade for the project output, but project work was assessed individually. The assessment of the project work consisted of work done in the lessons, self-assessment (statistics project), peer review (coordinate system project), learning diary, doing homework and points in a final test, in addition to the project output. Using the final test, the teacher wanted to verify that the pupils learned mathematics. The significance of the tests was a small portion of the whole grade.

The teacher's impression of the assessment, the goals in mathematics and twenty-first-century skills, and the general framework are collected in [Table 2](#). These goals were set during the design process.

Table 2. The description of the project.

	Coordinate system project	Statistics project
Prior knowledge	A point in the coordinate system.	Percentages.
The mathematical goal defined by the teacher	- Familiarising with the direct proportionality and learning to draw straight lines in the coordinate system. - Practice using Excel.	- Conducting statistical research. - Calculating statistical key figures by hand and using Excel. - Preparing and interpreting the tables and diagrams.
Other goals	Collaboration and communication skills, self-management, IT skills, problem solving and scheduling.	
Duration	About nine lessons. (45 min)	About 12 lessons. (45 min)
Working	In three-person groups.	In three-person groups.
Output	A poster and a presentation.	A PowerPoint presentation and a poster.
Assessment	The output of project work (two-thirds of the grade), a small final test (one-third of the grade), working in the lessons (± 0.5 grade), the peer review of their own group (± 0.5), homework (± 0.5) and learning diary. (max -0.25)	The output of the project work (65% of the grade), a small final test (8% of the grade), working in the lessons (± 1 grade), self-assessment (7% of the grade), learning diary (20% of the grade) and homework. (max. -0.5)

In the current research, both project instructions were analysed qualitatively from the perspective of mathematics found in the Finnish national core curriculum and from the viewpoint of mathematical proficiency. The analysis method was a theory-based content analysis (Tuomi & Sarajärvi, 2018). First, we examined the project instructions to look for the connection to the content area and the objective of instruction defined by the curriculum. We then separated the concepts from the skills. Second, we classified the stages of the projects according to the strands of mathematical proficiency. This division was based on two researchers' opinions of which strands the stage may especially develop.

3.2 Implementation of the projects

These projects were tested in two different schools in Western Finland, both of which had more than 500 pupils. The pupils worked in approximately three-person groups chosen by their teacher. The group members had heterogeneous skills in mathematics and in using Excel. The teacher talked about the goals with pupils at the beginning of the project. In both projects, the pupils were motivated to work with the possibility of choosing their research topics as a way to affect the difficulty level of their work and design an individual final output. The instructions only drew up guidelines and

boundary conditions. The teacher helped the groups find a suitable difficulty level and tried to keep the groups' distribution of work balanced.

The coordinate system project had a test run of 59 seventh-grade pupils and one teacher in the spring of 2018. Correspondingly, 58 eighth-grade pupils and one teacher participated in a statistics project in the spring of 2017. In both projects, the pupils worked in three classrooms supervised by their own teacher. Overall, there were 64 girls and 53 boys. Before the project began, the pupils were attuned to more traditional teaching in mathematics, and their previous grades in mathematics were known. The average was 8.3 in the coordinate system project and 7.6 in the statistics project. No diagnostic pre-test was used.

These test runs were analysed as two case studies using a mixed methods design. [Table 3](#) presents the research data, which consist of pupils' learning diaries, previous grades in mathematics, the grades of the project work, self-assessments and peer reviews. Additionally, we utilised the teachers' interviews and the groups' project outputs. In both projects, there were a total of 18 groups.

Table 3. Description of the data.

The data	<i>N</i> (coordinate system project)	<i>N</i> (Statistics project)
a. Previous grades in mathematics	59	56
b. Assessment of project work	57	58
c. Project outputs	18	18
d. Learning diaries	46	53
e. Self-assessments	48	57
f. Peer review	48	57
g. Final test	56	55
h. Interview with the teacher	1	1
i. Project instruction	1	1

Based on the interview with the teacher and the project outputs of the groups, both implementations were examined using the national core curriculum framework. We observed the final outputs to find a connection with the contents of the curriculum. Now, we got information on the achievement of the goals at the group level.

The pupils' previous mathematics grades and the grades for the project work and small final tests were analysed quantitatively using two-by-two frequency tables. The pupils, whose grades differed significantly (non-parametric statistical tests) from previous grades in mathematics, were qualitatively examined in addition to using their self-assessments, peer reviews, and their own and their group mates' learning diaries. Using a data-driven content analysis, the aim of the analysis was to find the

main reason for their high or low grades in project work. The pupils were classified into groups based on the researchers' impressions of these reasons.

Permission to use the research data was obtained from both the teachers and the pupils' parents. Personal data were processed according to the EU General Data Protection Regulation (GDPR).

4 Results

4.1 Mathematical base of the projects

The coordinate system project combined direct proportionality with drawing straight lines. The statistics project combined all the mathematical statistics found in Finnish basic education. [Table 4](#) summarises what kind of concepts and skills pupils practised during the project (based on data i) and how they are similar to the Finnish national core curriculum (FNBE, [2016](#)). The content area 6 for example, it is:

“C6 Data processing, statistics, and probability: The pupils deepen their skills in collecting, structuring, and analysing data. It is ensured that the pupils understand the concepts of the average and mode. They practice defining frequency, relative frequency, and median. The pupils familiarise themselves with the concept of dispersion. They interpret and produce different diagrams. They calculate probability.” (FNBE, [2016](#))

The area includes both understanding concepts and learning skills.

Table 4. The mathematical content of the projects from the viewpoint of the key content areas in the Finnish national core curriculum for basic education 2014 (FNBE, [2016](#)). C indicates the content area, and O is the objective of instruction in the curriculum (see [Table 1](#)).

	Curriculum	Concepts	Skills
Coordinate system project	C4, O15	Angular coefficient. Constant term.	Depicting correlations graphically. Drawing straight lines in the coordinate system. Depicting correlations algebraically. Interpreting graphs.
Statistics project	C6, O19	Average. Mode. Frequency. Relative frequency. Median. Dispersion.	Deepening pupils' skills in collecting, structuring and analysing the data. Interpreting and producing different diagrams.
	C2, O13		Calculating percentages.

Before the implementation of the coordinate system project, the pupils had little previous experience of a coordinate system, direct proportionality or drawing a graph in a coordinate system. Correspondingly, the pupils were familiar with percentages before the statistics project but had probably never made a statistical study or calculated statistical key figures.

Every project group made their own project output, and the processes were always of a different kind. The depth and richness of the mathematics used varied. Few groups totally missed the practising of some skills, and on the other hand, some groups handled them very deeply. Based on project outputs and learning diaries (data c and d), [Table 5](#) and [Table 6](#) present the skills practised and the concepts per group. This material does not give information on individual pupils.

Table 5. Skills practised during project work (based on data c and d). N (groups) = 18 per project. ‘Well-done’ projects met the requirements of the Finnish core curriculum for skills excellently (FNBE, 2016). If a skill is ‘missing,’ the group ignored the topic.

		<i>f</i> (missing)	<i>f</i> (done)	<i>f</i> (well done)
Coordinate system project	Depicting correlations graphically	0	11	7
	Depicting correlations algebraically	0	16	2
	Interpreting graphs	2	11	5
	Angular coefficient as a concept	2	15	2
Statistics project	Depicting correlations algebraically	0	16	2
	Interpreting graphs	2	11	5
	Angular coefficient as a concept	2	15	2
	Calculating percentages	0	19	0

In the statistics project, there were two groups that did not use diagrams. On the other hand, two groups in the coordinate system project lacked the ability to interpret graphs and define an angular coefficient. In principle, almost all the groups achieved the required standard and became familiar with the mathematical concepts, but only a few groups treated the issues thoroughly.

Table 6. Seeing the concepts in the groups' project outputs. N (groups) = 18.

		f (missing)	f (explained)	f (calculated)	f (explained and calculated)
CSP	Angular coefficient	0	0	1	17
	Constant term	0	0	18	0
Statistics project	Average	0	5	4	9
	Mode	1	1	3	13
	Frequency	0	0	18	0
	Relative frequency	0	0	18	0
	Median	0	4	3	11
	Dispersion	2	7	2	7

In the statistics project, all the groups calculated the angular coefficient and constant term, but any group did not explain what the constant term is. 17 groups explained what an angular coefficient means.

The concepts of average, frequency, relative frequency and median were found in every group's statistics project; a couple of groups lacked the concepts of mode and dispersion. Most groups calculated statistical key figures, although not every group explained what these meant. In several of the project outputs, procedural fluency was emphasised at the expense of conceptual understanding. There were also some groups that only explained what a key figure means, but they did not calculate an example.

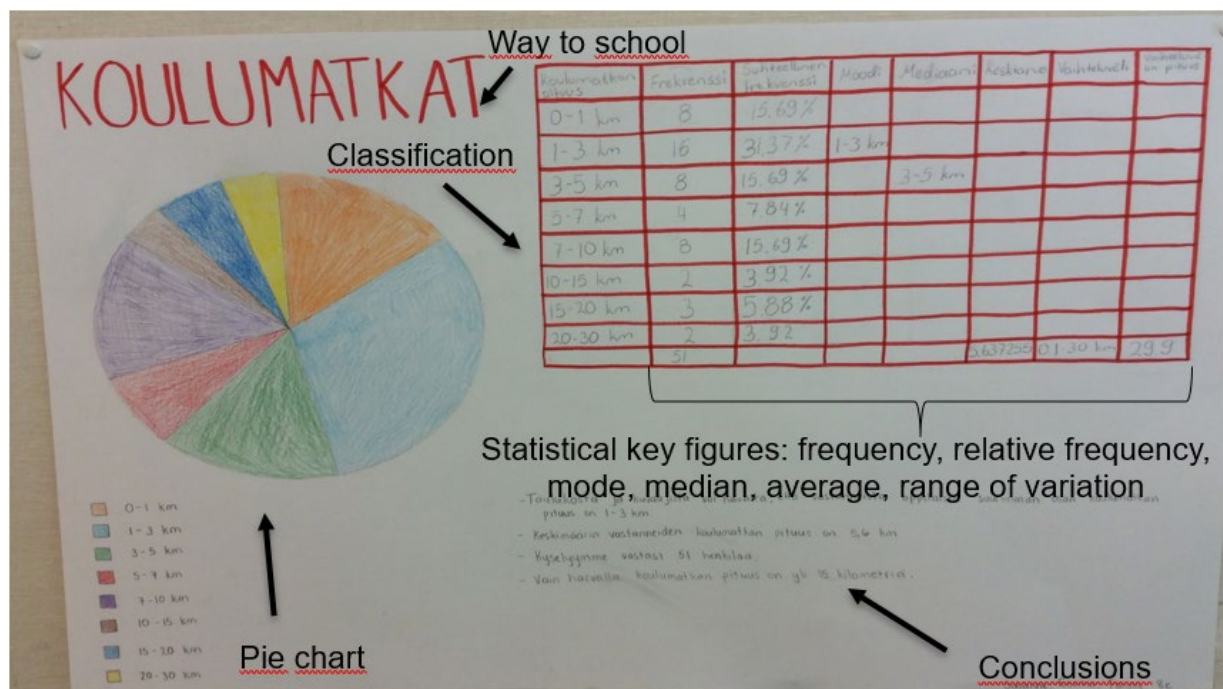


Figure 1. Poster displaying an output of the pupils' statistics project.

Figure 1 presents a typical output of the statistics project; this group had studied the length of the journey to school. In the poster, the pupils calculated statistical key figures, illustrating the frequencies with a pie chart. The groups also drew some conclusions from their study. In addition to the poster, the pupils made a more comprehensive PowerPoint presentation in which they explained what their calculated key figures meant.

4.2 Development goals of mathematical proficiency

According to written project instructions (data *i*), the coordinate system project started by organising the measurement and the statistics project by planning the questionnaire. The groups had to consider how they could study the problem mathematically; in other words, they needed strategic competency.

After that, the groups created their own view of the mathematics required by using the Internet, textbooks and the help of their teacher. They also had to apply mathematics in their textbook to the project problem. This stage involved a combination of conceptual understanding, adaptive reasoning and procedural fluency. In the statistics project, the groups familiarised themselves with the conception of statistical key figures and worked out how to apply these concepts to their data. They then calculated, for example, the frequencies, relative frequencies and modes of the data.

Correspondingly, in the coordinate system project, the groups studied how to draw a line in the coordinate system using their textbook and then adapted it to their data with pen and paper and by using Excel. The groups also had to read values in the line and think about what they mean. They clarified their understanding of concepts like the angular coefficient, the intersection of two lines, the intersection of the x-axis and the intersection of the y-axis while also comparing their results in Excel with their lines on paper.

Finally, in both projects, the groups evaluated their results and considered what the results meant in practice. **Table 7** details the required mathematical proficiency in both projects.

Table 7. The projects from the viewpoint of mathematical proficiency.

Proficiency	Coordinate system project	Statistics project
Conceptual understanding	<ul style="list-style-type: none"> Understanding direct proportionality Explaining the concepts of the angular coefficient, the intersection of two lines, the intersection of the x-axis and the intersection of the y-axis 	<ul style="list-style-type: none"> Explaining and understanding the concepts related to statistical key figures. E.g. <i>Explain your classmates what mode, median and average mean. What do they mean in your study? (data i)</i>
Procedural fluency	<ul style="list-style-type: none"> Plotting a point to the coordinate system. E.g. <i>Plot your points to the coordinate system (data i)</i> Drawing straight lines in the coordinate system. Reading values in the coordinate system 	<ul style="list-style-type: none"> Calculating statistical key figures by hand and using Excel Drawing diagrams by hand and using Excel
Strategic competence	<ul style="list-style-type: none"> Organisation of measurement Linking the results and physics 	<ul style="list-style-type: none"> Making the questionnaire. E.g. <i>Create a questionnaire. You must think how you formulate your questions. You need numerical data. (data i)</i>
Adaptive reasoning	<ul style="list-style-type: none"> Applying the examples of the textbook to the situation in the project Comparison between the line made by hand or Excel. E.g. <i>Compare the line made by hand and Excel. Are they different? How? (data i)</i> Estimation of the results 	<ul style="list-style-type: none"> Applying learned concepts to the situation in the project Estimation of the results
View of mathematics	<ul style="list-style-type: none"> Usability of mathematics as a tool 	<ul style="list-style-type: none"> Usability of mathematics as a tool

Figure 2 summarises the big picture of these two projects from a mathematical proficiency viewpoint. At the beginning of the projects, the pupils had to formulate the problem as a mathematical expression using strategic competence. Over the course of the project, they effectively studied mathematics in many different ways, developing their conceptual understanding and procedural fluency, and their competence with applying what they had learned to their project. The projects concluded with a discussion of the results. The pupils' view of mathematics was an

integral part of working throughout the entire project, highlighting the significance and importance of mathematics in everyday life.

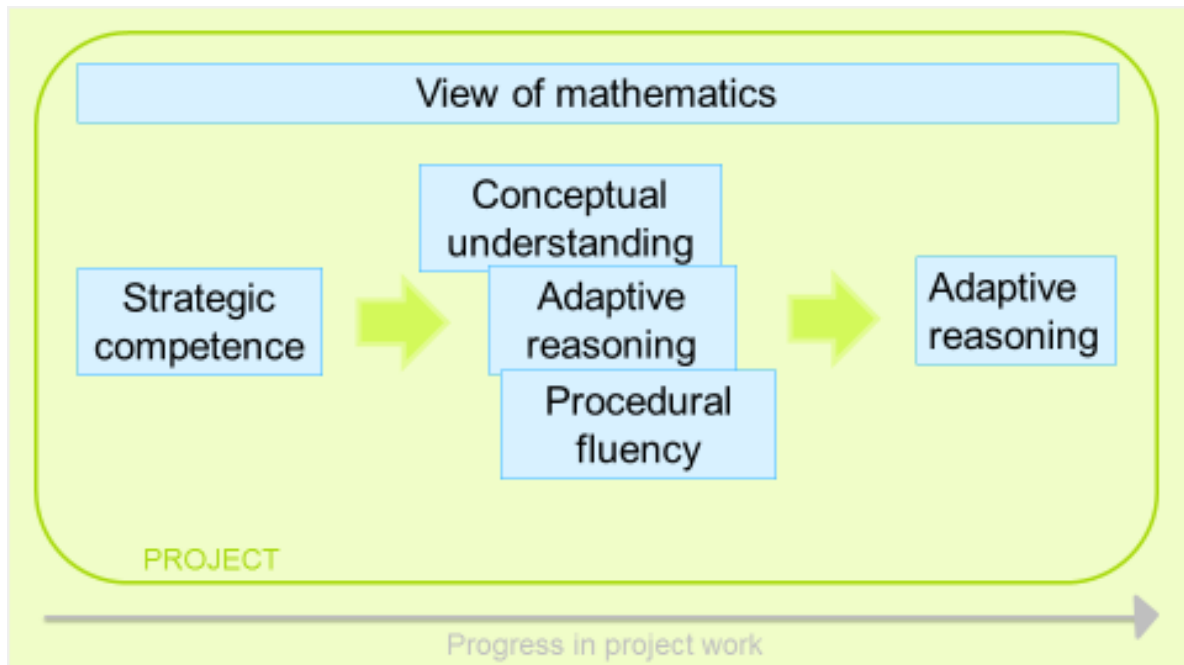


Figure 2. Mathematical proficiency needed during project work.

These data do not afford an opportunity to observe the attainment of assumed goals precisely. In the project implementation, the strands of mathematical proficiency were visible (based on data c and d). The boundary conditions set by the teacher ensured that every strand was needed in the project. In the coordinate system project, the meaning of strategic competence only held a minor role because the groups needed lots of guidance in the organisation of measurement, and some groups' topics were not connected with physics.

4.3 Connection between the project work grades and mathematics grades

The data from the studied projects were combined with increasing the size of the random sample. In both projects, the pupils worked in groups, with each group making a project output. The teacher assessed these outputs with number grades; hence, the made groups were allotted the same grade.

Figure 3 presents the data on the pupils' grades in their project output in relation to their previous mathematics grades (based on data a and b). The first quarter (I)

shows the pupils whose previous grades in mathematics were 8 or better and the project output grade 7.5 or better. On the other hand, the pupils with the mathematics grade being under 8 and the project output grade under 7.5 are shown in the third quarter (III). The pupils whose previous grade in mathematics was at least 8 but less than 7.5 for the project output are shown in the fourth quarter (IV), and in the second quarter (II), there are the unexpected achievers, whose previous mathematics grades were under 8 but had a project output grade of at least 7.5.

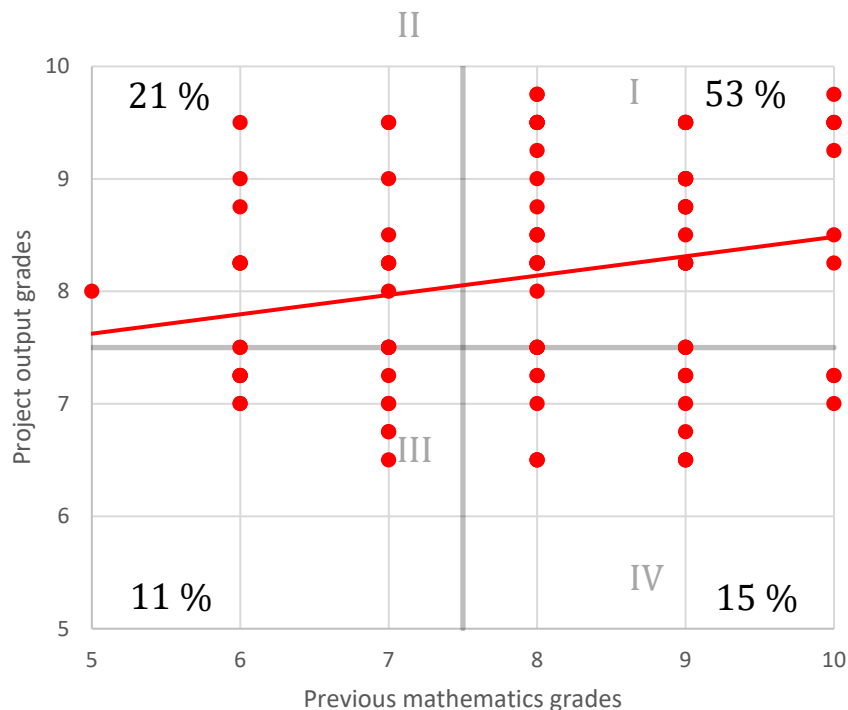


Figure 3. The project output grades in relation to the previous mathematics grades (N = 112).

The pupils' grades for the project output seem to be, on average, slightly better than their previous mathematics grades – about 74% of the pupils achieved at least 7.5. The difference is not statistically significant ($p = 0.392$, Wilcoxon). There is a weak positive correlation between these grades ($r_s = 0.223$, $p = 0.018$). Between the projects, there was no statistically significant difference ($p = 0.269$, Mann-Whitney U test) in the grades for the project outputs.

Each pupil's grade in the project work (Figure 4) was assessed using a combination of work in the lessons, peer review of their own group, self-assessment and learning diary, the small final test and homework, in addition to the project output. Figure 4 shows that there is a strong positive correlation between the pupils' total grades in the

project work and their previous mathematics grade ($r_s = 0.711$, $p < 0.001$). There was no statistically significant difference ($p = 0.079$, Mann-Whitney U test) for the whole project work between the projects (based on data a and b).

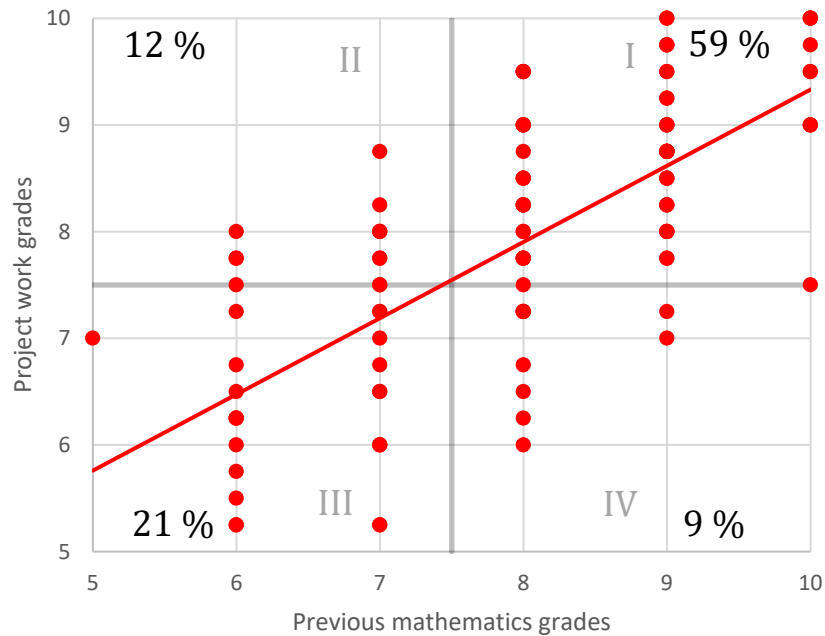


Figure 4. The project work grades in relation to the previous mathematics grades (N = 112).

Figure 5 indicates there is a moderate positive correlation between pupils' previous mathematics grades and the grade on the test after project work ($r_s = 0.624$, $p < 0.001$; based on data a and g). No significant difference was found between the projects in the final test ($p = 0.781$, Mann-Whitney U test).

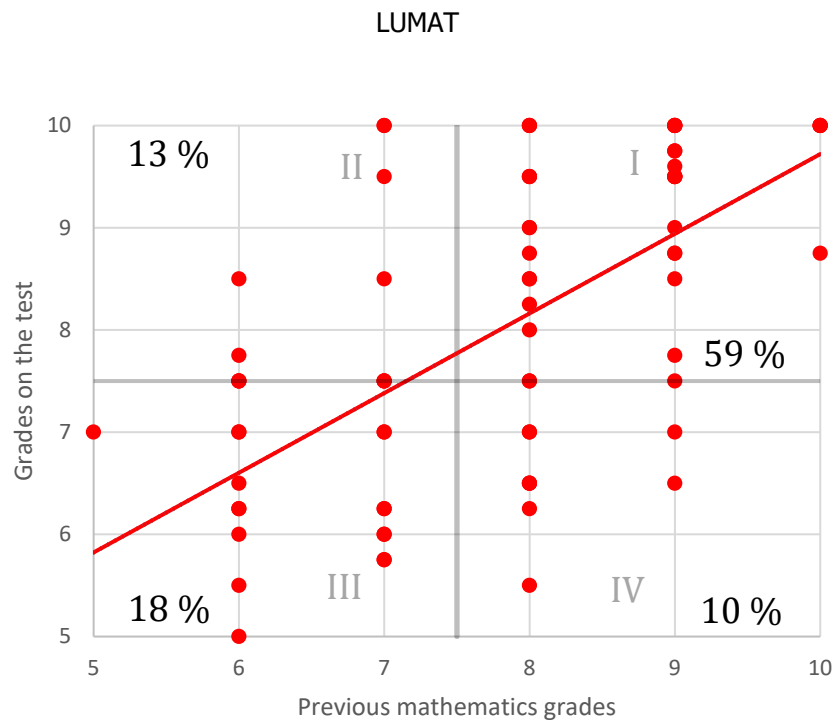


Figure 5. The grade on the test in relation to the previous grade in mathematics (N = 110).

From a research perspective, the significant groups are II and IV; it is interesting to see why some pupils succeeded and others did not. These reasons were examined using a combination of data a, d, e and f. The reasons are listed in Table 8. The first column is based on the project work grade, and the second is the test grade with relation to the previous mathematics grade.

Table 8. Feedback on belonging to groups II and IV. The first column is based on the project work grade, and the second is the test grade.

The project work grade (Figure 4)		The test grade (Figure 5)	
Group II (N = 13)	f	Group II (N = 14)	f
Better learning as a result of the support of the group	8	Support for his/her group	13
Taking advantage of others without learning	5	The working method is good	1
Group IV (N = 10)	f	Group IV (N = 11)	f
Taking advantage of others without learning	4	Taking advantage of others without learning	4
Other group members did not work	2	Other group members were a burden	2
Pupil did not do his/her best	2	There was too little time to learn	2
Absences	2	The group did not do their best	1
		Pupil did not study for a test	1
		Not to be classified	1

The make-up of the group had a major influence on the total grade. Most unexpected achievers in the project work experienced improved learning with the support of their group.

“It was easier to concentrate in groups.” (Pupil 1, data e)

On the other hand, a few pupils only took advantage of their group mates and got a better grade for the project output, though they did not participate in the group work. Some of them also got a better grade in the project work because the project output grade was good enough. We called them ‘passengers’ (Schuck, 2002).

The general reason for the expected weaker scores was also taking advantage of others without learning. These pupils could get a good grade on the project output, but the test results, working in the lessons and the peer review decreased their grade. They were also ‘passengers.’ These data do not tell reliably why some pupils did not participate as much in the group work.

“One pupil of our group worked less. I and other groupmates did nearly all.”
(Pupil 2, data f)

Based on the learning diaries (*d*) and self-assessment (*e*), the other reasons for weak scores were absences and a lack of effort.

“One pupil was at the project lesson only once. Another pupil wanted only to write. I had to do everything alone.” (Pupil 3, data d)

Some pupils showed a noticeable difference in the grades between the project output and the entire project work. Altogether, 12 pupils increased their grades and moved from group IV to group I. Most of these (9 of 12) belonged to groups where there were either disagreement and ‘passengers.’ The teacher was able to assess pupils’ mathematical content knowledge in the post-project test. A few pupils (3 of 12) did not produce a good project output but learned mathematics normally. The grades of 11 pupils decreased from group II, dropping them into III; in these cases, it was the other group members who were responsible for the good project output, but these pupils had not participated in the work and they were ‘passengers.’

In total, there were 11 pupils in group IV based on the post-test. The reasons given for doing poorly on the test were linked to teamwork problems (7 of 11). Other reasons were too fast of a working pace or the lack of revision for the test. It is important to note here that the teacher did not give any advance warning of the test.

The pupils in group II obtained better test results than expected – nearly all of these pupils (13 of 14) said (data *d*, *e* and *f*) that their good group encouraged working hard. One pupil learned better by working in groups than by listening to a teacher.

Pupils' own opinions on their learning were asked in the self-assessment. As a whole, 99 pupils (85%) answered the question. The views were distributed evenly among the students: 37 pupils (37% of the respondents) believed that they learned better through project work than through traditional teaching, 34 pupils (34% of respondents) thought they learned worse, and 28 pupils (28% of respondents) thought that it was the same.

“If I believe I don't need that kind of mathematics in my everyday life, then I don't bother to study. Now, I know that I need these mathematics, so I want to learn, and I am really learning.” (Pupil 4, data *e*)

“I learn more quickly in traditional teaching.” (Pupil 5, data *e*)

There is no statistically significant difference ($p = 0.121$) between the projects (based on data *e*).

After the coordinate system project, a control group ($N = 18$) took the same test as the intervention group ($N = 59$). This control group learned the same content traditionally before the test, and their average mathematics grade (avg. = 8.3) was the same as that of the intervention groups. In the groups' scores for the final test, there was no statistically significant difference when using the Mann-Whitney U test ($p = 0.950$). Based on the results, it is possible to assume that project work does not seem to dilute the quality of learning.

5 Discussion

In summary, it is possible to study mathematics using project work as a working method. The coordinate system and the statistics projects are good examples of projects in which the mathematical objectives are clear. In a well-designed project, the objectives of project-based learning can also be achieved (Larmer et al., 2015). However, the depth of mathematics handling was shown to have many differences between the studied groups. Some groups only calculated, and the others explained more what and how they calculated.

Kilpatrick et al. (2001) defined the mathematical proficiency of an individual. In the current article, we assume that the mathematical proficiency of a group is at least the sum of the group members' skills. A pupil said the following in his learning diary:

“We work as a team, and everybody helps according to their skills. With the help of everyone's little skills, we made very good project output.” (Pupil 6, data d)

In both projects, every branch of mathematical proficiency was needed at the group level. The groups could share pieces of work, in which case a pupil may not participate in some of the tasks. Now, we examine only the instructions, not implementation. Here, the instructions offer the possibility to develop every branch. In the future, further data collection is required to determine exactly how well the expectations can be realised in practice.

The examined project instructions were quite closed. It would be important to examine pupils' mathematics learning during more student-centered and open projects. Is it possible to achieve the learning objectives in mathematics if pupils can influence more their own projects? How can teachers instruct their pupils on the handling of essential mathematics without well-designed project instructions?

Taken together the implementation in the schools, the current study indicates that it is good to assess project work in various ways. The project output does not show the whole truth at the individual level, so we also need other assessment methods, such as the self-assessment and peer review. Compared with previous success in mathematics, an unexpected good or weak grade in project work is generally because of the groups. A hard-working group can support and inspire a pupil to work and learn more, but on the other hand, a strong group may encourage a pupil to be a 'passenger.' If the other pupils in the group are a lot weaker, a pupil can experience these members as a burden.

At all, the 'passenger' phenomenon is interesting, and this article does not tell reliably why some pupils are 'passengers.' The 'passengers' could be both weak and gifted in mathematics. Some 'passengers' got a good grade in the project work, but most of them only got a better project output grade due to their group. There were also 'passengers' who had good previous grades in mathematics, but they got a lower grade in the project work. These gifted pupils might have suffered from inefficient group mates, where the result would be a low grade for the project output. The difficulty level of the project might have been too easy for them. In the future, it would be important to know why somebody is a passenger.

On the other hand, assessing project work is a useful addition to teachers' evaluation methods. The Finnish core curriculum (2016) emphasises using a versatile assessment method. The success of group work was seen in the project work and project output grades. Group work – or collaboration – is a twenty-first-century skill, or a part of the transversal competence, in the curriculum.

As a whole, project work supports mathematics learning while diversifying traditional teaching methods and the evaluation of mathematics. It can also encourage inquiry- and phenomenon-based learning when in accordance with the curriculum. According to the research, the teachers must focus on the tutoring of group work and helping pupils to go towards their learning objectives.

The most important limitation of the current study was the small sample size, and it should not be assumed that its findings could be applied in another context. Also, when we compare the grade in the project work with a pupil's previous mathematics grade, there may have been in connection with a different aspect of mathematics, so the grades are not fully comparable.

Future work should focus on learning in project work on a larger scale. Nowadays, multidisciplinary learning modules and projects are being made annually in Finland (FNBE 2016), so this type of research material would be available.

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