# Languaging and conceptual understanding in engineering mathematics

Kirsi-Maria Rinneheimo and Sami Suhonen

Tampere University of Applied Sciences, Finland

The ability to apply mathematical concepts and procedures in relevant contexts in engineering subjects sets the fundamental basis for the mathematics competencies in engineering education. Among the plethora of digital techniques and tools arises a question: Do the students gain a deep and conceptual enough understanding of mathematics that they are able to apply mathematical concepts in engineering studies? This paper introduces the use of languaging exercises in the engineering mathematics course 'Differential Calculus' during the spring semester 2020, at Tampere University of Applied Sciences, TAMK. In this study, the students' conceptual understanding and learning of differential calculus is researched. In the learning process, the languaging method is used to deepen the conceptual understanding of the concepts of differential calculus. Pre-test/post-test setup was used to see the possible gain in conceptual understanding. During the course, students did online assignments, which included languaging exercises. Students described the concepts of differential calculus using natural language, pictures, or a combination of them. The students were also asked to fill in a self-evaluation form to collect their perception of their own knowledge of mathematical skills. Mid-term and final exams summarized the acquired knowledge. The study aimed to enhance the learning outcomes and to gain a deeper understanding of mathematical concepts by exploiting the languaging method.

Keywords: languaging, mathematics, engineering, conceptual understanding

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Correspondence: kirsi-maria.rinneheimo@tuni.fi

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## 1 Introduction

The way we teach and learn mathematics has changed in the past few decades. Technological tools have enriched the resources available for teaching and learning through 'computer aided' devices, through appropriate software, and through learning platforms. Today's students are more accustomed to learning with the help or the aid of state-of-the-art technologies. Using tools and calculators to solve exercises speeds up the calculations and provides usually more accurate results. Among this plethora of digital techniques and tools arises several questions: Do the students gain a deep and conceptual enough understanding of mathematics that they are able to apply mathematical concepts in engineering studies? Do the students just master the tools without understanding what they are doing and what does the result mean, e.g. I solved a derivative – but what does it actually mean?

According to the literature (Woods et al., 1997; Bok, 2006) and the authors' own experience, students seem to be able to mechanically repeat the known procedures to solve problems, to carry out assignments quite well – but they do not necessarily learn to think. In engineering mathematics, the foundation of learning mainly evolves from thorough understanding of mathematical concepts and the ability of exploiting abstractions to solve engineering problems. The fundamental aim of mathematics in engineering education is mathematics competencies, which means the ability to apply mathematical concepts and procedures in relevant contexts (Alpers et al., 2013).

This paper presents how the method of languaging is implemented to clarify mathematical concepts and to promote deeper learning. By making concepts of the subject more concrete to students, the aim is to clarify mathematical expressions and lead to the students' better understanding of the subject.

In a previous study it was shown that languaging exercises do have an effect on knowledge of the theory (Rinneheimo et al., 2020). In that study, an independent-samples t-test was conducted to compare if the students gain a better knowledge of the theory with the help of the languaging exercises. As a result, there was a significant difference in the scores for using the languaging exercises during the course and not using the languaging exercises during the course.

In this paper, a deeper view on the understanding of the concepts with the help of languaging method and self-evaluation has been taken. This paper focuses on promoting higher understanding of concepts by utilizing languaging exercises.

## 2 Theoretical background

### 2.1 Mathematical thinking and languaging exercises

Mathematical thinking is usually expressed with symbols, expressions, calculations etc. (by symbolic language). Languaging in mathematics refers to expressing a student's mathematical thinking through different ways, such as writing/orally using natural language, by pictures, or by a combination of these (by natural language, mathematical symbolic language, or pictorial language) (Joutsenlahti, 2010; Joutsenlahti et al., 2013). O'Halloran (2015) has presented that language assists in reasoning the mathematical process and its results. Symbols describe mathematical relations and visuals present images to concretize mathematical relations (O'Halloran, 2015). In this study, the languaging of mathematics forms an approach to making meanings of mathematical concepts and procedures. This meaning-making process enables the students' mathematical thinking and knowledge construction (Morgan, 2001; Schleppegrell, 2010; Joutsenlahti et al., 2017). Solving a mathematical exercise or presenting the solution to a mathematical exercise by using different languages assists a student to organize their own mathematical thinking and eventually gaining a better understanding of that mathematical concept or procedure (Joutsenlahti et al., 2015; Joutsenlahti et al., 2017).

There are different types of languaging exercises and the exercises used in this study are presented in Table 1.

**Table 1.** Languaging exercises (Joutsenlahti, 2010; Joutsenlahti et al., 2013; Joutsenlahti et al., 2014)

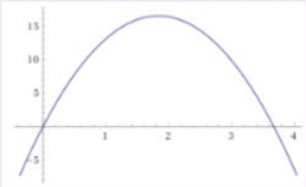
| Type of the languaging exercises | Description of the exercises |
| --- | --- |
| Argumentation of the solution. | Student writes or selects a natural language explanation for the solution in place of using symbolic language (or vice versa). Pictorial language could also be used. |
| Explaining in your own words. | Student provides an explanation by using natural language. |
| Adding missing parts of the solutions | The problem solution is uncompleted, and the student adds the missing parts. |
| Seeking errors. | Student has to find errors or missing items in the given solution and to correct the errors. |

Some examples of the languaging exercises used in the Differential Calculus course are presented in Figures 1 – 3. In Figure 1 is presented two languaging exercises where student interpreted the graph. From the graph of the function $h(t)$ (height $h$ (m) is a function of time $t$ (s)) the students were asked to explain in their own words: 1.) what is the difference between the markings $h(1)$ and $h'(1)$, 2.) how would they define the derivate for the function at the point $t = 3$ graphically, numerically and symbolically. They also needed to think about the unit for each reply. From the graph of the function $f(x)$ the students were asked a) what is the average rate of change of the function $f(x)$ from $x = 0$ to $x = 3$ and b) what is the rate of change of the function $f(x)$ at the instant that $x = -1$ and $x = 1$.

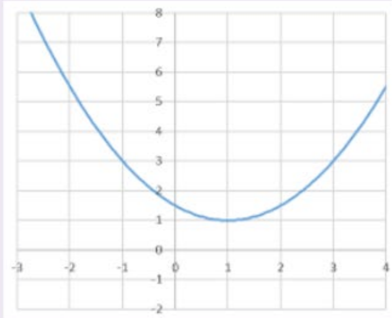
Kun pieni kappale ammutaan maanpinnalta pystysuoraan ylöspäin alkunopeudella 18 m/s, sen korkeuden *h* riippuvuutta ajasta *t* kuvaa likimain funktio

$h(t) = 18\frac{\mathrm{m}}{\mathrm{s}} \cdot t - 4{,}9\frac{\mathrm{m}}{\mathrm{s}^2} \cdot t^2.$

Funktioyhtälössä muuttuja on aika *t* ja funktion arvo on korkeus *h*. Ohessa on myös funktion kuvaaja.



Ohessa on funktion *y* = *f*(*x*) kuvaaja. Määritä kuvaajan avulla



(käytä desimaaliluvuissa desimaalipistettä)

a) funktion *f* keskimääräinen muutosnopeus välillä $x \in [0,3]$

b) funktion *f* muutosnopeus eli derivaatta kohdissa *x* = -1 ja *x* = 1.

*f* '(-1) =

*f* '(1) =

Figure 1. Examples of the languaging exercise interpreting the graph.

In Figure 2 is an example of the languaging exercise “Seeking errors”. Students have to find errors or missing items in the given solution and to correct the errors. The exercise has been modified from task 10 of the longer mathematics course matriculation exam from spring 2017.

Tiedetään, että $h(x) = g(f(x))$, $f(x) = e^x$ ja $g(x) = 2x^2 + 1$. Kaksi opiskelijaa laskevat derivaatan $h'(x)$ seuraavalla tavalla:

| Opiskelijan 1 ratkaisu: | Opiskelijan 2 ratkaisu: |
|---|---|
| $f(x) = e^x$<br>$g'(x) = 4x$<br>joten $h'(x) = g'(f(x)) = 4e^x$ | $h(x) = g(f(x)) = 2(e^x)^2 + 1 = 2e^{x^2} + 1$<br>$h'(x) = 2e^{x^2} \cdot (2x)$<br>joten $h'(x) = 4xe^{x^2}$ |

Kolmas opiskelija saa laskimella vastaukseksi $4e^{2x}$.

a) Kenen vastaus on oikein?

b) Etsi väärien ratkaisujen virheet ja selitä omin sanoin korjatut ratkaisut.

Figure 2. An example of the languaging exercise seeking errors (Ylioppilastutkintolautakunta, 2017).

In Figure 3 is a part of the languaging exercise “Adding missing parts of the solutions”. In this kind of exercise, the problem solution is uncompleted and the students add the missing parts.

Erään kondensaattorin varaus ajan funktiona on $q(t) = 5 \cdot 10^{-6}\mathrm{C} \cdot \sin\left(\frac{100\pi}{\mathrm{s}} t\right)$.
Laske virranvoimakkuus $i(t) = \frac{\mathrm{d}q}{\mathrm{d}t}$ ja virran muutosnopeus $\frac{\mathrm{d}i}{\mathrm{d}t}$ ajanhetkellä $t = 0{,}9$ s.

*Ratkaisu:*

Lasketaan virranvoimakkuus ajanhetkellä $t = 0{,}9$ s:
Derivoi funktio $q(t) = 5 \cdot 10^{-6}\mathrm{C} \cdot \sin\left(\frac{100\pi}{\mathrm{s}} t\right)$

$$i(t) = \frac{\mathrm{d}q}{\mathrm{d}t} = q'(t) = \boxed{\phantom{xxxxxxxxxx}}$$

Figure 3. An example of languaging exercise adding missing parts.

The use of languaging has given good results in mathematics education (Joutsenlahti et al., 2013; Joutsenlahti et al., 2014; Sarikka, 2014; Joutsenlahti et al., 2016). Languaging exercises make the student think about what they are doing, not only mechanically calculate the exercise (Rinneheimo et al., 2019). One challenge of mathematics teaching is how to describe mathematical thinking and how to make it visible. The languaging exercises enable making the students’ mathematical thinking processes visible and also support the development of these processes (Joutsenlahti et al., 2017).

Hiebert and Lefevre (1986) divided the mathematical knowledge to conceptual knowledge and procedural knowledge. Conceptual knowledge has been defined as understanding of the principles and relationships that underlie a domain, and procedural knowledge consists of the symbol representation system of mathematics and the algorithms and rules for completing mathematical tasks (Hiebert et al., 1986). Kilpatrick, Swafford and Findell (2001) described students’ mathematical proficiency with five components as follows:

- conceptual understanding – comprehension of mathematical concepts
- procedural fluency – the ability for flexible, efficient, accurate and appropriate calculation
- strategic competence – problem solving

- adaptive reasoning – ability for logical thinking, reflection, explanation and
- justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These five components of the mathematical proficiency can be seen as one way of describing the features of the mathematics. This study focuses on the skills' conceptual understanding and procedural fluency as follows: the student has the ability to use mathematical concepts in the right context and manages the procedures behind the concepts. In this study, these skills are discussed as conceptual understanding and the capability has been studied with the languaging exercises, as illustrated in Figure 4.

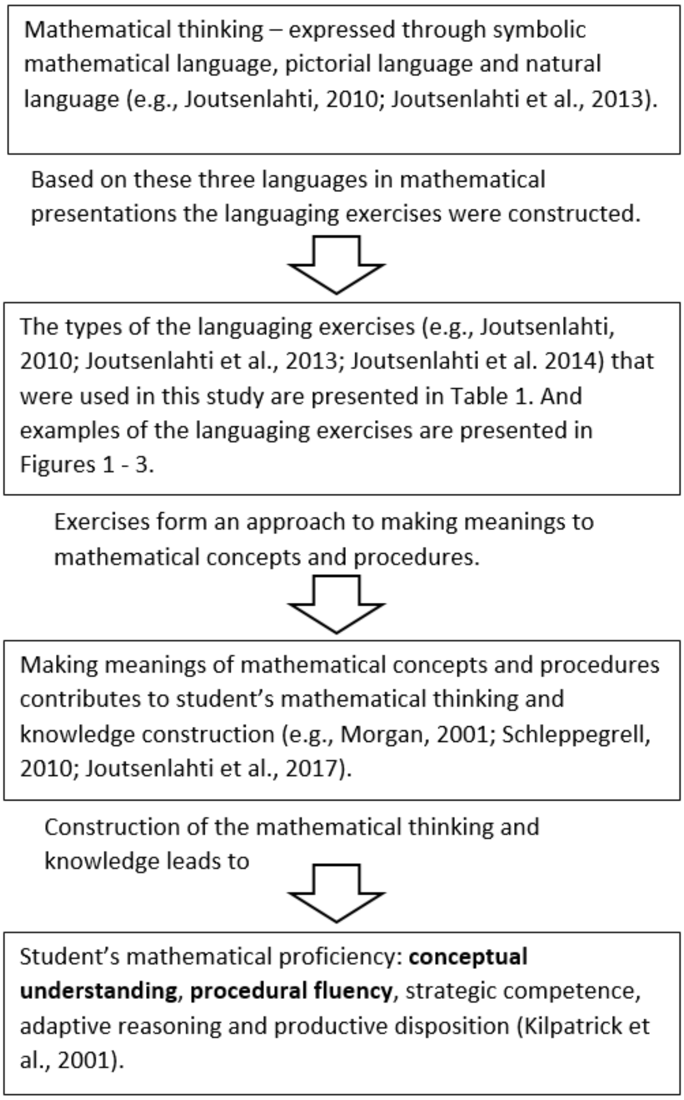



Figure 4. Building the conceptual understanding.

The languaging exercises have been formed using three languages (Joutsenlahti, 2010; Joutsenlahti et al., 2013). The exercises form an approach to making meanings of mathematical concepts and procedures (Morgan, 2001; Schleppegrell, 2010; Joutsenlahti et al., 2017), which contributes to conceptual understanding (Kilpatrick et al., 2001).

### 2.2 Meaning making and conceptual understanding

The purpose of using languaging exercises that express a student's mathematical thinking through three languages (natural language, mathematical symbolic language, and/or pictorial language), is to develop the student's own meaning making process and lead to the conceptual understanding. Boudon (2016) pointed out in his study that writing mathematics does not only strengthen the student's conceptual understanding, but can also develop their ability to communicate the meaning of such concepts. According to Morgan (2001), writing and the use of natural language in the solutions of mathematical exercises develop conceptual understanding, the attitudes of the learners towards mathematics improved, and they also facilitate the assessment work of teacher.

Also, according to Moschkovich (2015), explaining meanings, constructing arguments and justifying procedures leads to conceptual understanding. Research has shown that the use of natural language and drawings helps most students in solving mathematical exercises (Joutsenlahti et al., 2016). Languaging exercises and presenting mathematics in writing enables a student to structure and clarify their mathematical thinking (Joutsenlahti, 2010; Kangas et al., 2011).

## 3 Research process

### 3.1 Research questions

In this study, the students' conceptual understanding of differential calculus concepts is researched, and the capability has been studied with the languaging exercises. In this article, we concentrate on the following research questions:

1. How does the students' languaging ability develop throughout a course?
2. How did the mathematical languaging clarify mathematical expressions?
3. How did develop the conceptual understanding?

In the following chapter, we present the data collection process and the analysis of the data. The key idea in the teaching process and data collection was collect data from several sources during the whole course.

### 3.2 Data collection and analysis

This paper introduces the use of languaging exercises in the engineering mathematics course 'Differential Calculus' taught at Bachelor's level during the spring semester 2020 at TAMK. In this study, there were two engineering student groups and the number of active students was altogether 64. Course materials were a book, a formula book, a symbolic calculator and as additional material, online exercises and timetable in Moodle learning platform.

The data was gathered from the several sources:

At first pre-test/post-test setup was used to see the possible gain in conceptual understanding. In the tests, students described, by natural language or by interpreting a graph, the concepts of differential calculus.

Secondly during the course, the students had six compulsory online assignments to be completed as homework. These assignments were prepared by using different question types in Moodle and most of the exercises in these online assignments were languaging exercises. This study compiles 14 languaging exercises from these online assignments. The topics of the assignments used in this study were graphical, numerical, and symbolic differentiation, and applied exercises. In the exercises, the students were asked to explain course concepts in their own words, or to seek errors and explain in their own words the correction to the error. Also, students interpreted graphs and in some exercises the solution to the problem was explained with natural language and the student was asked to complete or select from the list the missing calculations or symbolic presentations. Examples of the used languaging exercises are presented in Figures 1–3.

Students were also asked to fill in a detailed self-evaluation form weekly to collect their perception of mastery of that week's topics. In the form, each week's learning objects were described using natural language. The then students typed a letter a - d to the cell according to their perception of the mastery of the topics (a = green: I have learnt this so well that I could teach it to my peers. b = blue: I feel I understand this topic. c = orange: I think I have understood this partially, but it is partially unclear. d = red: I need more practice to understand this.). Part of the form is presented in Figure 5.

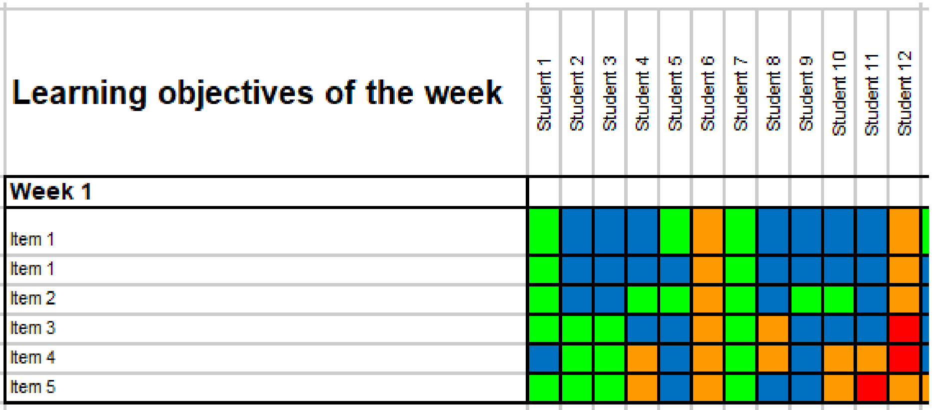



Figure 5. Self-evaluation form (Peura, 2018).

During the course there were two exams (mid-term and final), which summarized the acquired knowledge. The first exam contained mechanical calculations, such as differentiate the given function, and a languaging exercise, which asked the students to interpret a graph. The second exam also contained a languaging exercise, where students explained, with their own words, mathematical concepts of the course. The second exam mainly consisted of applied exercises where the students first needed to invent the mathematical model of the assignment and then to solve it. The data collection is summarized in Table 2.

**Table 2.** Collection of the data.

| Data sources | Languaging exercises | N |
|---|---|---|
| **1) Pre-test/post-test** | 6 (Figure 6, in chapter 4.1) | 53 |
| **2) Online assignments** | 14 | 64 |
| **3) Self-evaluation form** | In the form each week's learning objects were described using natural language (in Figure 5 is part of the form). | 59 |
| **4) Exams** | mid-term: included languaging exercise, which asked the student to interpret a graph<br>final: included languaging exercise, where students explained with their own words' mathematical concepts of the course | 64 |

The data were analyzed by mixed methods. The MS Excel program was used for typical statistical analysis (e.g., in comparing distributions, arithmetic mean, variation, median, correlation, frequencies). The qualitative analysis was made by theory guided content analysis (e.g., categorizations). Classification into the four categories was used while analyzing the students' answers to pre-test/post-test, online assignments, and exam replies as follows: wrong/do not know (0 points), just a little right/only some idea of the task (1 point), partly correct (2 points) and correct (3 points). Self-evaluation form's replies were categorized as follows: 0 = I need more practice to understand this, 1 = I think I have understood this partially, but it is partially unclear, 2 = I feel I understand this topic and 3 = I have learnt this so well that I could teach it to my peers.

Based on the data it was possible to interpret what kind of meanings the students constructed for the given mathematical expressions, and to evaluate how had they understood the mathematical concepts. The students were also asked to fill in a self-evaluation form to summarize their perception of their own knowledge of mathematical skills.

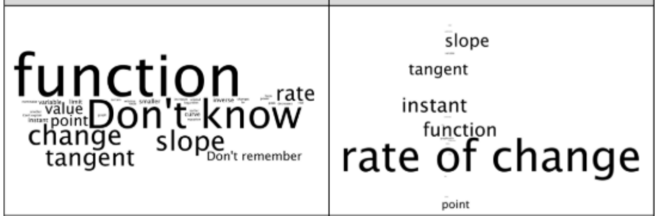
The students were aware of this study while data was collected during the course. They were able to choose whether their answers could be used in the study. All students gave permission to use their answers in the study. The students were informed that at all stages the processing of data is completely confidential and from the results of the study, the information provided by an individual student could not be identified. While students filled in a detailed self-evaluation form they used nicknames as the table was visible to all students. Students informed the teacher of their nickname. This research data will be used (in an anonymous manner) in this publications and in correspondence author's dissertation research/ when all the necessary data-based research has been done and then the data will be destroyed.

## 4 Results

This chapter presents the results of using languaging exercises on the course. First, the pre-test and post-test results are investigated for finding out how the students perceive their learning of the course topics. Second, the correlation between languaging skills and learning outcomes is investigated. Third, students' skills in different types of exercises (symbolic calculus, languaging and applied mathematics) are presented in relation to final grade. And finally, the self-evaluation form is used to analyze the students' perception of their own mathematical skills.

### 4.1 How does the students' languaging ability develop throughout the course?

On the course, the pre-test/post-test setup was used to see the possible gain in conceptual understanding, but these tests did not affect the final grade. The test was exactly the same in the beginning and the end, and it consisted of six languaging exercises, where students explained in their own words the concept of derivative, interpreted a graph, and explained how the derivative of the given function is defined graphically, numerically, and symbolically. The exercises are shown in Figure 7. Figure 6 presents a word cloud of students' answers to open-ended question about derivative. For this figure the number of correct keywords in students' answers were analyzed.




| Pre-test results | | Post-test results | |
|---|---|---|---|
| Rate of change | 7 | Rate of change | 35 |
| Function | 19 | Function | 21 |
| Instant | 0 | Instant | 16 |
| Slope | 9 | Slope | 14 |
| Tangent | 8 | Tangent | 13 |
| Point | 0 | Point | 9 |
| Don't know/remember | 16 | Don't know/remember | 1 |
| *(empty)* | 16 | *(empty)* | 3 |

Figure 6. Word cloud of how the students understood the concept of derivative (N = 53).

From Figure 6 we can perceive that languaging through writing has improved during the course. The students are able to formulate the concept of derivative at the end of the course in a more versatile and correct way. Pre-test shows that at the beginning of the course the most common reply was that derivative is related to a function. There were many blank and Don't know/remember answers in the pre-test and only a few in the post-test.

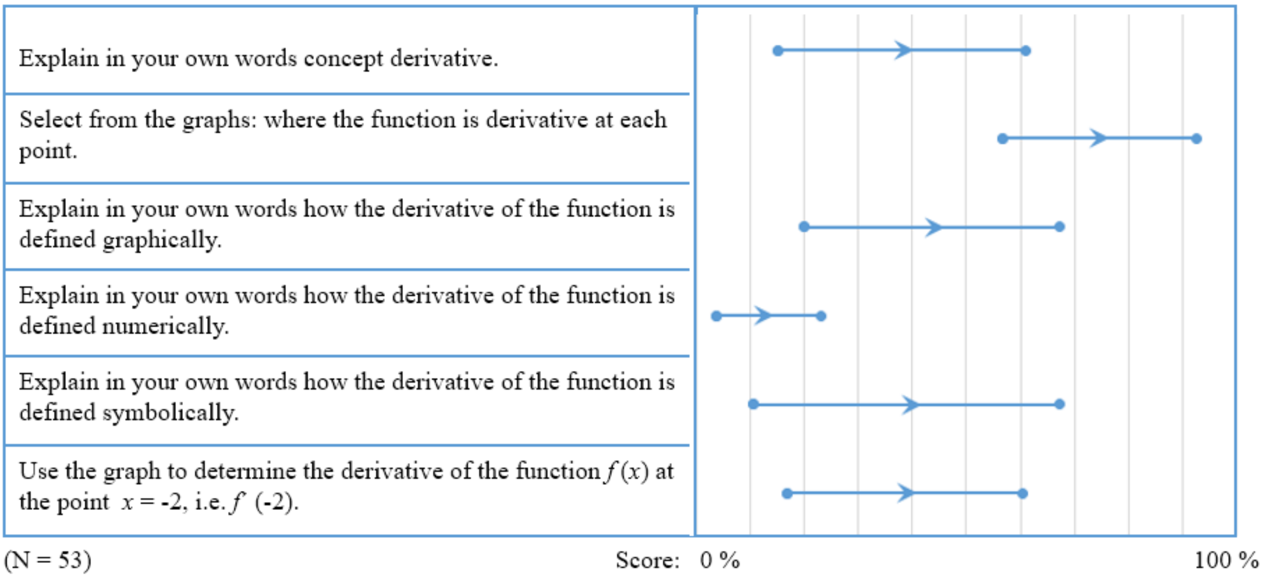



Figure 7. Gain chart.

In Figure 7 are the averages of responses of 53 students to pre-test (left end of the line) and post-test (right end of the line). The left end of the line is the result of the responses to the exercise averaged over all respondents and the right end the responses to the final test, accordingly. Thus, the length of the line represents the average “amount of learning” during the course. It can be seen that explaining derivative symbolically, graphically, and the concept of derivative improved the most during the course. The exercise explaining in their own words the derivative graphically relates to the last exercise, where the students were actually asked to interpret the derivative of the function at the given point from a graph. In both exercises, the students improved very well during the course. Pre-test reveals that explaining in one’s own words what numerical derivative means was the least known matter and the learning outcome was also low here. The likely reason for this was that numerical solving was not practiced more than in a couple of exercises during the course. The second question had been answered well in pre-test and also in post-test. This question had three answer options, so this was a different type of question from the others, where the students had to explain in their own words or interpret the graph.

These results (Figure 6 and 7) indicates that the students’ ability to correctly express mathematical concepts by writing has improved. When students express their thoughts out loud and by writing, they remember things better and they are able to apply them later (Lee, 2006).

## 4.2 How did the mathematical languaging clarify mathematical expression and how did develop the conceptual understanding?

Next, the correlation between the online exercises that were languaging exercises and the exam points (Figure 8) was calculated. Figure 8 shows the exam points ($y$) as a function of points of languaging exercises ($x$). The Pearson's correlation coefficient $r$ = 0,68 (N = 64) tells a moderate positive linear correlation between the final assessment and languaging exercises. Exactly the same online languaging exercises were used on the Differential Calculus course during spring semesters 2018 and 2019. Also, the final exam on those years was delivered in a similar way with similar kinds of exercises and the correlation (Pearson's correlation coefficient) between the grading and languaging exercises was as follows: 2018 $r$ = 0,62 (N = 58) and 2019 $r$ = 0,62 (N = 73).

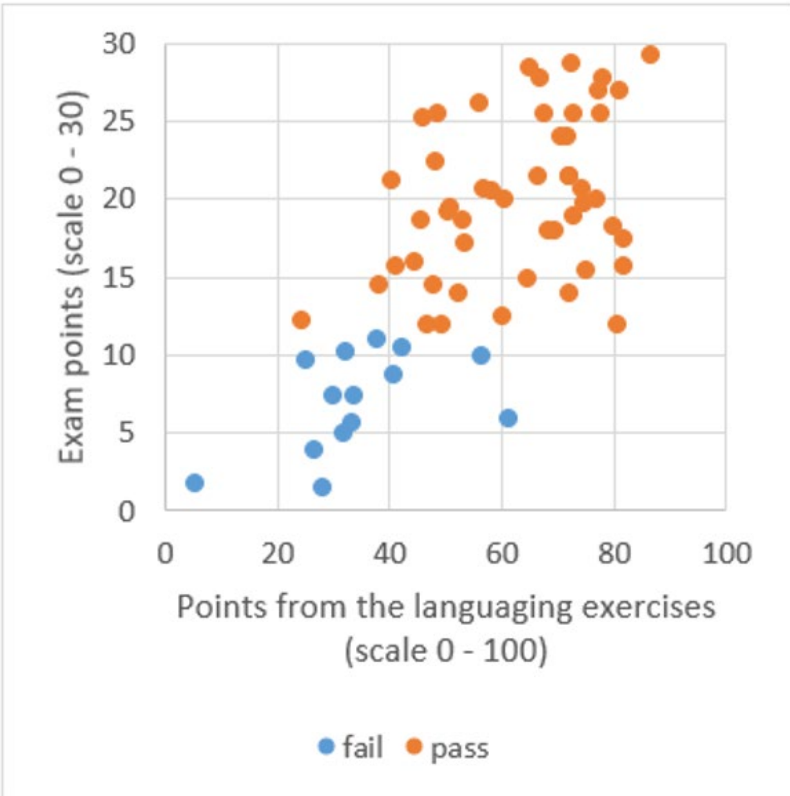



Figure 8. Correlation between the exam points and languaging exercises (N = 64).

For a more in-depth study of how languaging exercises effects learning of concepts, the exercises in the final exam were investigated further (Figure 9). Figure 9 presents the average points of the exam exercises in different final grade categories from 0 (fail) to 5 (best). The blue line describes the average points in symbolic calculations, orange line in languaging exercises, and grey line in applications. The orange line shows a clear step between grade 0 and 1 (increased 35 %). After this step, the curve shows only a minor increase in grade categories 1 - 5. Based on this shape of the curve, the competence of the languaging exercises has a clear effect on passing the course. This raises the question whether acquiring a certain level in languaging skills forms a threshold for understanding mathematics. To investigate more this very interesting finding, the current data was supplemented with data from two previous years. The languaging exercises were exactly the same every year and the exercises in the exams were similar. Even with this three times larger data set the result is the same: there is a clear step in the languaging category between the grade 0 and 1 (36 %, N = 195).

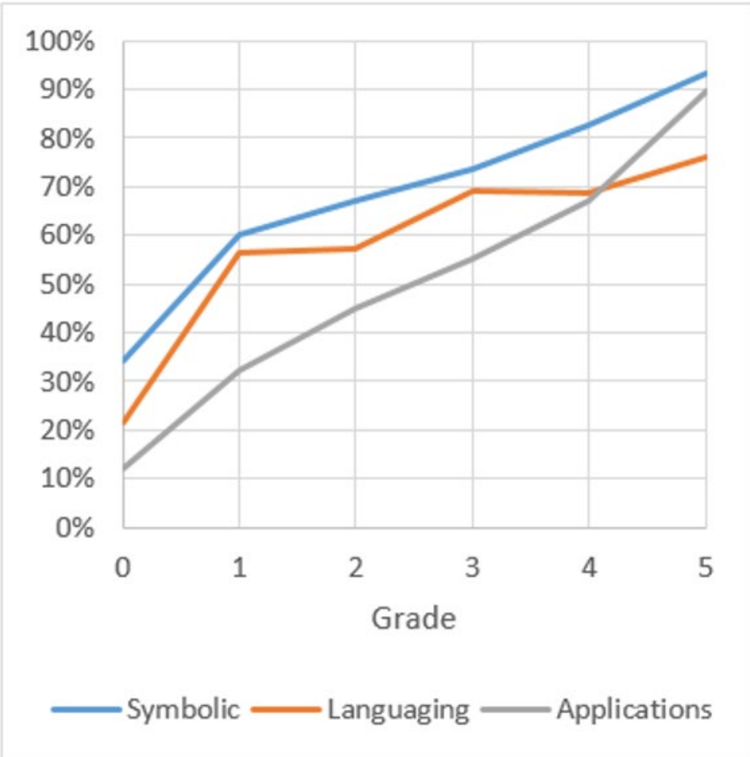



Figure 9. Exam exercises in three category (N = 64).

Research questions 2 and 3 (see page 8) dealt with the questions if mathematical languaging clarifies mathematical expressions and does that lead to the students' better conceptual understanding of the subject, which would help them to apply mathematics. Students, who did not pass the course, were able to do some mechanical calculations (symbolic) but did not get many points from the languaging exercises and even fewer from the applications (Figure 9). From Figure 9 we can also perceive that students with a grade of five (5) stand out from other students in terms of competence in application exercises. They also got the best points in all categories. It seems that exercises where the students needed to apply their knowledge (grey line) has the highest discrimination power.

The students were also asked to fill in a detailed self-evaluation form weekly to collect their perception of their own knowledge of mathematical skills. The table was visible to all students. Therefore, only nicknames were used on the table. This table served many pedagogical purposes: it made the students evaluate their own knowledge about the key issues of the week, it made them think through languaging of the covered concepts, as the subjects of the week were explained by using mainly natural language, it showed them that others are perhaps struggling with the same topics as well and for the teacher, it showed which topics students had found the most difficult. The teacher then had the possibility to give extra guidance for the subjects that were found difficult.

Table 3 presents the averages of self-evaluations regarding the specific topic. The average of self-evaluations is calculated by first substituting the phrases (designated with letters a, b, c and d) with numbers 0 - 3 and then calculating the averages. Table 3 shows the same thing as Figure 9: the students estimated that they are good at doing mechanical calculations (symbolic calculations) and application tasks are more demanding.

**Table 3.** Averages of self-evaluations (scale from 0 (I need more practice to understand this) to 3 (I have learnt this so well that I could teach it to my peers.), N = 59

| Topic | Average (and standard deviation) of self-evaluations |
|---|---|
| Regression and limit | 2,15 (0,79) |
| Introduction to derivative (what is derivative - graphically, numerically and symbolically) | 2,40 (0,64) |
| Symbolic calculations (derivative rules) | 2,35 (0,77) |
| Applications (partial derivatives and error estimation) | 2,11 (0,76) |
| Applications (finding maxima and minima using derivatives, Max-Min problems) | 2,10 (0,83) |
| Applications (the derivative as a rate of change, tangent line, rates of change per unit time) | 1,72 (0,69) |

The Pearson's correlation coefficient between the final exam points and self-evaluation was a moderate positive linear correlation ($r$ = 0,61, N = 59). The students with higher grades (4 – 5) showed only slightly better self-evaluations compared with the group average. Students, who did not pass the course, seem to be overconfident of themselves, whereas the best ones seem to be somehow unsure of their knowledge and skills. Thus, the students seem to evaluate their knowhow “average”. Similar results have been found in another study of engineering students’ self-evaluations (Suhonen, 2019). This explains the rather small differences between averages in Table 3. Nevertheless, the Table 3 shows which topics are the most difficult ones to the students.

## 5 Discussion

In this study was presented the use of languaging method, in which the ways to express mathematical thinking are expanded beyond mathematic symbolic language. The objective was to observe how engineering students understand the concepts of differential calculus based on this method.

The use of languaging exercises on the mathematics course enables the teacher to interpret in more detail the students’ thinking and provides also a way for the teacher to evaluate the students’ understanding of the concepts. Also, the self-evaluation form provided the teacher valuable information of the difficulties the students encountered at the time while the subject was being covered, and thus the teacher was able to react and try to help the students proactively.

The analysis from the pre-test/post-test setup indicates that the students had learned expressing the meanings of the mathematical concepts by natural language. The findings also indicate that the students, who passed the course, were able to express mathematical concepts by natural language and to explain the meaning of the concepts.

Languaging exercises enable various types of ways to enhance the students’ mathematical thinking. Consequently, using different ways to express the mathematical concepts gives students a much clearer overall understanding of the mathematical concept in question. It seems that this helps especially those students, who struggle with mathematics (threshold of passing the course). Joutsenlahti and Kulju (2017) suggested that broadened ways of expressing mathematical thinking may help especially those students who have difficulties with mathematics and for whom mathematical symbolic language is difficult to comprehend.

The course Differential Calculus used various methods for learning mathematics alongside the languaging exercises, such as videos, visualizations, and learning analytics. According to Moschkovich (2013), exercises that provide opportunities to participate in mathematical activities, which use multiple resources to do and learn mathematics support, among others, the mathematical reasoning and conceptual understanding. Kilpatrick, Swafford and Findell (2001) have pointed out that conceptual understanding is the ability to present mathematical solutions in different ways and the ability to evaluate how to utilize different presentations for different purposes. The understanding of mathematical concepts and the relationships between concepts will create sustainable development from the point of view of learning, which leads to the students being able to apply the mathematics later on in their engineering studies.

In the EDUCAUSE Horizon Report (2021), Horizon panelists were asked to describe key technologies and practices they believe will have a significant impact on the future of postsecondary teaching and learning. Six items rose to the top of a list as follows: Artificial Intelligence (AI), Blended and Hybrid Course Models, Learning Analytics, Microcredentialing, Open Educational Resources (OER) and Quality Online Learning. Three of these six technologies and practices identified in the report (learning analytics, OER, and AI) are returning entries from previous years' reports.

Learning analytics is a growing trend in all education. Many higher education institutions use digital learning management systems to deliver their courses, as was the case also in this study with the Differential Calculus course. These systems collect large sets of data about learners and their actions on the platform. Learning analytics, the data, offer a view, for example, to studying and learning activities, but learning management system cannot record such activities as reading the course book or carrying out calculations on paper. The data is a very valuable source of information to teachers, instructional designers, and the students themselves. However, it mostly tells about studying and does not reveal what has been actually learned and what is the student's perception of their own learning. In this study, the learning analytics view was combined with student self-evaluation, analysis data of online languaging exercises, and the actual learning outcomes. This forms a more comprehensive manner to look at the studying and learning and offers a way to try to find patterns and correlations. Further research is needed, but languaging exercises could be seen usable while creating education materials for learning and teaching mathematics, with the help of learning analytics and online learning to reveal mathematical thinking and conceptual understanding.

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