Static, dynamic and interactive elements in digital teaching materials in mathematics: How do they foster interaction, exploration and persistence?

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Contemporary comprehensive mathematics teaching material covering whole courses has developed substantially from the early versions that roughly were 'books as pdf' with some complementary online material. In teaching materials that are offered in online web portals (digital teaching platforms) a variety of dynamic and interactive elements can be utilised, offering new ways to engage with mathematics. Despite this recent development, the variety of affordances of the digital environment are utilised to a surprisingly small extent. The pros and cons with digital teaching materials in mathematics are debated, and publishers advertise with arguments about algorithms that lay out an ideal learning path and about joyful content. Critical for students' learning while working with teaching materials is however that they find it meaningful to use the materials, a persistence in the interaction with the materials, and furthermore that the willingness to explore mathematics remains. In this study students' interaction with digital teaching material with various kinds of dynamic and interactive elements supplementing the static parts in the presentation of new content is explored. Differences in students' attention to mathematical facts, essential in the problem solving, is captured using an eye-tracker. Analyses of differences in attentive behaviour depending on the kind of digital element that are used for presentation reveal that the type of digital element that students attend the least to is static elements. Differences in what is offered to and what is demanded from a reader when mathematical facts are presented using various digital elements is discussed and potential implications from the results are suggested.

Keywords: feedback, dynamic, attention, film, eye-tracking

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1 Introduction

The purpose of using teaching materials of different kinds is to support learning. From a student perspective however, this purpose is sometimes blurred by goals such as to get the right answer or to finish a section fast. Such goals can hinder learning, not least in a digital environment where active choices to 'open' or 'start' a particular part of the teaching material is needed to access all parts of the material. From this perspective teaching materials in print are beneficial because all content is displayed on the pages. A page-based layout, as in print, does however not surpass all affordances of the digital media. The digital media provides opportunities to visualise mathematics in new ways and to invite the reader to experience dynamic change (Dyrvold & Bergvall, 2023). Every learning opportunity does thus necessitate that students interact with the material, that they invest the required time, and make an effort to understand. Because of the particular features of digital teaching materials, knowledge about how students choose to use different dynamic features, and to navigate between different elements, is valuable to understand the potential these materials have to support learning.

The purpose of this study is to contribute to the understanding of what the presence of different dynamic elements in mathematics text means for students' interaction with the material. The current study focuses particularly on elements that are not static, that is *dynamic elements* that invite to interaction and/or present content that change over time (as e.g. films). The categorisation of different element types addressed in the research questions is presented in Table 1 in Method.

> RQ1. How does students' interaction with mathematics differ depending on the dynamic element type used for presentation?

Not all parts of digital teaching materials are dynamic. In fact, a large share of the materials offered in digital teaching materials utilize static elements like text in print, but on screen. The second research question do therefore explore reading of static text in a context with included dynamic elements.

> RQ2: Is there any difference in how students read static parts of an item due to presence of dynamic element types in other parts of the item, and if so, how?

1.1 Digital teaching material – in contrast to material in print

Teaching material in print has successively been complemented and partly also replaced with digital material. The use of digital teaching material is an intrinsic part of digitalization in schools (e.g., see European commission, 2021) and digital teaching material adds functions not possible to offer in print material. The potential of digital materials is particularly prominent in mathematics because of the possibilities to present relations between concepts and their representations dynamically. Besides the affordances of the digital media enabling interaction with and experience of mathematics in new ways, students' willingness to engage with dynamic material can also be beneficial in a learning situation. On the other hand, however, dynamic material may be distracting and can therefore potentially disrupt reading if displayed together with other material on screen.

The current study focuses on how students interact with digital materials that utilises different dynamic and interactive functions. Teaching materials, in print or digital, are supposed to offer rich learning opportunities to students and it is a reasonable argument that digital teaching material should in some manner offer better learning opportunities if the print material should be replaced. Previous studies who contrast print and digital teaching material highlight affordances of print materials such as the option to write in ink (Laughlin Davis et al., 2021), that text in print is easier to read (Abuloum et al., 2019), and the more natural sequencing of content in print (Gould, 2011).

The digital media, on the other hand, have many useful features that makes digital teaching material a very strong contestant to the traditional printed textbook. Such features, used in mathematics teaching material are for example, automated feedback (Van der Kleij et al., 2015; Stevenson, 2017), animations (Mamolo, 2019; McAlpin et al., 2019), dynamic visualisations of mathematical relations (Demir, 2018; Poon & Wong, 2017, Çeziktürk, 2020), drill-and-practice games (Beserra et al., 2019), and options to receive hints or access definitions (Stevenson, 2017; Arroyo et al., 2013). Analyses of contemporary digital teaching material do however reveal that much of the potential of the digital media is not taken advantage of (Glasnovic Gracin & Krišto, 2022; Mato-Vázquez et al., 2018; Dyrvold & Bergvall, 2023; Dyrvold, 2022).

1.2 Learning with dynamic and interactive material

Intervention studies contrasting learning with print material and digital material reveal different affordances of the digital media. For example, dynamic mathematics software (GeoGebra) has proven to be a powerful tool to help students think mathematically and to act like mathematicians and deduce hypotheses with the help of dynamic representations (Çeziktürk, 2020). Analyses of students' reasoning revealed that the connections that can be identified in multi representational mathematics software are crucial for the reasoning. Baccaglini-Frank (2021) also found that dynamic mathematics software contributes to students' development from a ritualistic discourse to a more explorative participation and thereby contribute to the construction of abstract mathematical objects. The usefulness of digital material is also revealed in a study evaluating evidence-based instruction, where results indicate that the instruction was helpful only for acquiring and maintaining mathematical knowledge if digital support was offered in the learning situation (Reinhold et al., 2020). Another affordance of digital materials is the possibility to offer virtual manipulatives, which for example can support understanding of patterns and concepts (Alagic, 2013). Furthermore, the option to digitally manipulate objects can increase students' motivation to persist. A meta-analysis of affordances of teaching materials offering dynamic representations of mathematical objects reveals that these materials can motivate students to persist at mathematical tasks. Other affordances revealed in the analysis are that these kinds of materials can encourage creativity, contribute to constraining students' attention to relevant content, and visualise relations between objects in relation to students' actions (Moyer-Packenham & Westenskow, 2013).

The digital media are useful for providing feedback of various kinds (e.g., Pinkernell et al., 2020; Ruthven, 2018) but rapid short feedback can also hinder learning if trial and error behaviour is used (e.g., Rezat et al., 2021b; Pinkernell et al., 2020; Nurmi & Jaakkola, 2006) whereas more elaborated feedback has proven useful for learning (Van der Kleij et al., 2015). The results of an in-depth analysis of individual learning with material offering automated feedback highlight that the design of the feedback is crucial to receive the desired effect (Rezat, 2021). Several studies reveal a positive relation between time spent reading digital teaching material and course grades (e.g., Junco & Clem, 2015), and furthermore, the time spent using certain parts of digital tools also predicted achievements in mathematics (Bokhove & Drijvers, 2012).

1.3 Interaction with dynamic and interactive teaching material

Digital material allows dynamic and interactive functions not possible to include in print, functions that also have proven to support learning. Students' interaction with these materials can however be shallow or without engagement and if so, the intended learning may be absent. But the opposite is also possible. It is argued based on the results of a meta-analysis that to foster motivation and persistence seem to be an under-emphasised affordance of virtual manipulatives (Moyer-Packenham & Westen–skow, 2013). An expected raised interest and willingness to engage is also a prevalent argument for use of digital games in learning. A study evaluating games in mathematics learning reveal that when a game mitigates anxiety, motivation and learning are enhanced (Huang et al., 2014) but there are also results revealing that a raised interest in a mathematical game-based activity can markedly decrease across sessions. Based on these results, Rodríguez-Aflecht et al. (2018) argues that game-based learning shall not be used to motivate students but rather used based on proven learning outcome.

If a chosen digital teaching material is engaging but does not scaffold learning, students' invested time is not fruitful, but the opposite is also true; if the teaching material has the best potential but students choose to not engage with it, the learning potential gets lost.

Design aspects of teaching materials are not new, but due to the diversity of dynamic functions and the modalities offered in digital media there is a huge increase in the kinds of learning opportunities that are possible to offer in a digital environment, in contrast to in print. This diversity gives rise to questions about which kind of digital features are useful in relation to which kind of mathematical activities and about the combination of dynamic and static elements in digital teaching platforms covering whole courses. Analyses of digital teaching platforms in mathematics reveal that dynamic and interactive functions are used only to a limited extent (Dyrvold, 2022; Mato-Vázquez et al., 2018), but also that when used such functions can extend the learning opportunities for example through activities where students can interact with the material or by adding a personal voice in a film (Dyrvold & Bergvall, 2023).

There is a growing research field focusing on digital teaching materials and many studies focus on some particular technology, investigating learning effects (e.g., Bray & Tangney, 2017;) or use in classrooms (e.g., Vahey et al., 2020). When it comes to teaching material that include a diversity of static and dynamic elements and students' use of such materials less is known, especially regarding students' interaction with the materials. Some recent studies focus on digital textbooks offering different kinds of elements (Pohl and Schacht, 2017; Brnic & Greefrath, 2022), others focus on students viewing time in different parts of online digital textbooks (O'Halloran et al., 2018; Kanwar & Mesa, 2022). The dynamic media used for such teaching material allows the reader to create their own reading path and to choose what to see and not. Reinhold et al. (2020) raises a question about ecological validity in studies about technology use in mathematics education, questioning whether results about potential benefits of technology use holds outside the experimental environment. For teaching material including a variety of static, dynamic, and interactive elements where students can choose whether to actively engage with all parts of the material, to what extent, and in what order, the use of digital features can be rather different than in an experiment. This dynamic and multimodal feature of digital teaching platforms in mathematics means that knowledge about their affordances, constraints and possible benefits must be built based on a variety of research studies. Knowledge built based on an accumulated bulk of research is nothing new, the issue here is that the variety of functions possible to include in digital teaching platforms means that separate studies can only comprise certain aspects of the material. For example, research reveals that rapid right/wrong feedback can foster trial-and-error behaviour whereas elaborated feedback and an option to choose to receive feedback step by step has been suggested as a better alternative (Rezat et al., 2021b; Heeren & Jeuring, 2019; Rezat et al., 2021a). If feedback options are included in a digital teaching platform with several other functions, however, the different types of feedback may be useful in relation to different parts of the material. For example, a few questions with rapid short feedback on the answer, in relation to theory presented as film, may foster active engagement.

Because of all options of reading paths and of what to visualise and interact with in digital teaching platforms, the opportunities to learn with the materials can differ substantially between users depending on their choices. Even materials with the best potential to support learning risk to miss their goal if the users do not invest time in the material and are not willing to explore and to be persistent. Designing digital teaching material that evidently aids all students in their learning is therefore a demanding task. Many studies do not reveal any convincing learning gains related to use of some digital teaching material despite thorough design of the materials. For example, materials developed with research based instructional design principles did not reveal a learning effect for all student groups (Reinhold et al., 2020). Another study synthesising results from 35 single-case studies on virtual manipulatives found that the use of commercially developed materials had a larger effect on students' mathematical accuracy after practice than researcher-developed materials (Shin et al., 2021). Together these results signal the complexity of both design of and learning with digital learning material.

In the current study our aim is to contribute knowledge about how students choose to interact with different kinds of dynamic and interactive elements. In digital teaching platforms students can choose what to display and the presence of one element is likely to have an impact on how other elements are read. Accordingly, the focus on interaction with different types of digital elements in the current study also includes analyses of students' distributed attention between various digital elements within a mathematics item.

2 Theory

2.1 Social semiotics

The current study is part of a research project with a social semiotic theoretical frame (see, Bergvall & Dyrvold, 2021). Taking this perspective on language means that various semiotic resources available in communication are considered as means for meaning making; these resources are signifiers whose meaning is dependent on the social and cultural context they are offered in (Halliday & Matthiessen, 2013; van Leeuwen, 2005).

The resources offered as means for meaning making do request different kinds of interaction from the reader. Halliday (1985) stresses that speech acts, also from text, are dialogic. Two commonly used speech acts are 'offering information' and 'demanding information'. These two speech acts are frequently used in mathematics teaching materials. For example, in theory sections, information is often offered as statements or agreed upon facts. In exercises, on the other hand, the speech act of demanding information is frequently used, in the form of questions or requests to engagement of some kind (see also Bergvall & Dyrvold, 2021). In a digital environment it is reasonable to also complement these categories with the speech acts related to 'offering goods and services' and of 'demanding goods and services' (cf. Halliday, 1985), because in a digital environment the dialog between the text and the reader includes other acts than offering or demanding information. For example, the reader may be demanded to use interactive elements to access information. All types of digital elements utilised in the section with facts (Figure 1, Method) in the items used in this study offer information. More dynamic and interactive digital elements do also, to a varying extent, demand 'digital acts', and in some cases also information in terms of responses, and the offered information is only provided after particular 'digital acts'. The theory constitutes an analytical framework in such a way that students' interaction with digital elements expressing different speech acts are compared.

2.2 Eye-movements and attention

Attention and gaze are strongly associated. The acuity in the centre of the fovea is far better than in the periphery, and accordingly, when some element is of particular interest it generally makes sense to fixate the gaze on it (e.g., see Pashler, 1999). Subjects can also attend away from where the gaze is fixated and shift the attention without moving the gaze. This divergence does not imply that shifts of attention and eye movements are unrelated, rather it is an expression of a person's ability to attend also to stimuli in the periphery. Two pairs of categories of attention and the relation between attention and gaze contribute to understanding the relation between the two. Firstly, visual attention can be either overt; that is a conscious act of physically directing the gaze to a stimulus, or covert; that is a mental shift of attention not related to a physical movement of the gaze. Typically, covert attention precedes spatial eye movements (e.g. Rai & Le Callet, 2018).

Secondly, attention can be either voluntary (endogenous attention) or involuntarily (exogenous attention) but despite different triggers, these types of attention often have similar perceptual consequences. Some exceptions indicate that endogenous attention is fairly flexible, in contrast to exogenous attention (Dugué et al., 2020). Endogenous attention emanates from the subject's mind whereas exogenous attention is caused by external stimuli. The current study is not designed to achieve data that discriminate between these categories, but these different kinds of attention are important to keep in mind when interpreting the results. For example, a student that interacts with a task can with an intention to grasp the offered content direct endogenous attention to a particular dynamic element. On the other hand, in interaction with dynamic elements particular visual displays may cause exogenous attention.

In summary, gaze fixations do not capture covert visual attention, because covert attention is the selective processing of information without change in gaze. Such covert visual attention does however precede a shift in visual attention to the particular location, and the coupling of attention and eye movements is mandatory. Interestingly, this relationship holds both for eye movements with exogenous control and with endogenous control (e.g., Hoffman, 1998). Attention and eye movements are not completely interdependent, but the relation is sufficiently prominent to be used as a foundation to learn about how texts are read. This widely accepted relation is often referred to as the eye-mind hypothesis.

3 Method

Data for this study was gathered using an eye-tracking analysis of 124 students in grade nine, drawn from four different schools in different parts of Sweden, while working on a set of five mathematics items. Data from 3 students were excluded due to poor calibration or missing eye-tracking data. One student rushed through all tasks in four minutes and data from this student was also excluded. This resulted in 120 students being included in the analysis. The group of participants do well represent the diversity in regular class in Sweden, with varying grades (for more information about the participants see also Dyrvold & Bergvall, 2023). The within-subjects design where all participants do all items and use all types of dynamic elements (Table 2) allows for inclusion of participants with different achievement levels without compromising the reliability of the study.

The students were informed that the overall purpose of the study concerned work with digital teaching platforms in mathematics and that the analysis would be carried out with eye tracking analysis to follow the participants' work with five mathematics items. The students were also informed that all participants are de-identified and that they can withdraw their consent at any time. All students who had reached the age of 15 gave consent to participate in the study (or their guardian in other cases).

3.1 Item design and included element types

The five mathematics items were designed to touch on areas of mathematics that are new to the students. The level of difficulty of the items was determined based on a review of Swedish textbooks in grade nine and thereafter adjusted in collaboration with two experienced teachers and textbook authors. The five items are about the inscribed angle theorem, maximum and minimum of quadratic functions, set theory, the relation between power and roots, and permutation and factorials. All five items had the same structure, consisting of four parts: introduction, task, facts essential for solving the task (hereafter called *Facts*) and answer options. Each of these four parts constitutes an area of interest (AOI). The eye-tracking analysis was built on how the student's gaze moved between these AOIs. These areas (that were not visualised on screen) were drawn closely around the constituents to minimise false positives (fixations that do not belong to the AOI) but not too small to avoid missed fixations that belong to the AOI. The shaded overlay around "Task" in Figure 1 visualises such an AOI.

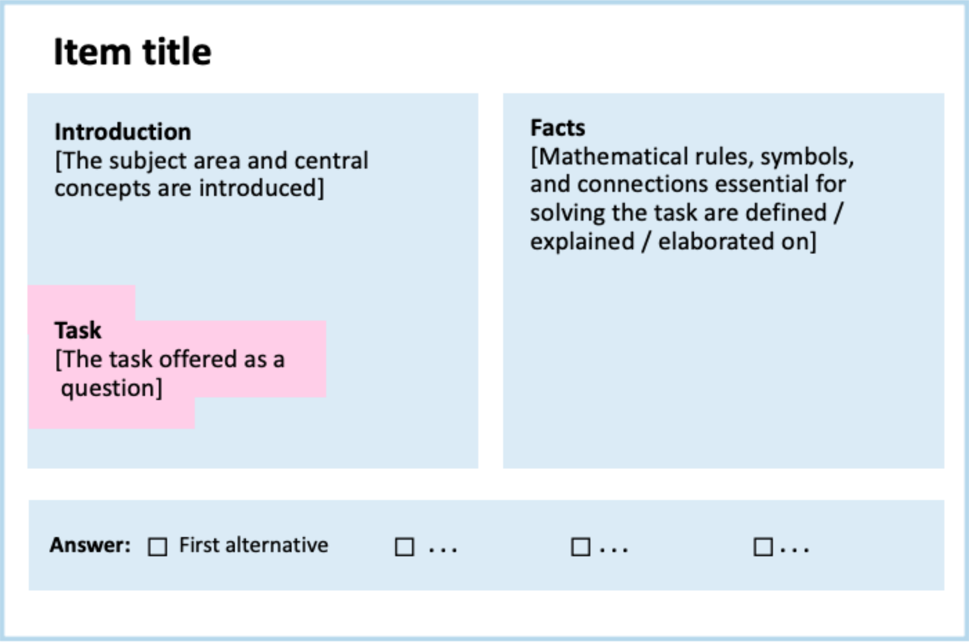

Figure 1. Basic design of the items and visualisation of an area of interest (AOI).

The presentation of the Facts using different element types (ETs) did not demand equal space, because different constituents were needed in the different versions. Because the number of fixations is likely to increase in larger AOIs (Holmqvist et al., 2011), these differences in occupied space were adjusted by somewhat enlarged constituents and row spacing. However, because different types of elements (Table 4) demand different kinds of constituents to present the Facts, equally sized AOIs would not have contained the same amount of information. Tightening constituents to achieve the same size if the AOI could instead diminish readability, and accordingly the best compromise between size and readability was sought in the design of the AOIs.

The Facts were designed in five versions, based on a typology of elements designed with an increasing interactivity and dynamism (Table 1) (Dyrvold, 2022). Elements are defined as a coherent part of a text that can contain both words, symbols, and images, where the components can be static and/or dynamic. Some of the elements are also interactive. Consequently, there are five versions of each item, and each version is dynamic and interactive to varying degrees depending on the element type (I–V) used in the Facts. All students were offered all five items in the same order, but the order of element type was varied (Table 2).

Table 1. Typology of elements used in Facts (Dyrvold, 2022).

Element type	Dynamic and interactive characteristics
I	static presence: elements are presented similar to a printed counterpart but on screen
II	opted presence: elements whose appearance is dependent on the readers' actions (e.g. click on a button)
III	dynamic presence: elements that change over time, typically content in a film or an animation
IV	dynamic feedback: elements that respond to and whose appearance is dependent on choices made by the reader (e.g. responses to students' answers)
V	continuous dynamic feedback: elements that change continuously over time depending on choices made by the reader (e.g. changing a slope in a coordinate system or moving geometric objects).

Because a central aspect of the method was to compare students' interaction with digital material depending on element types used for presentation, the different versions of Facts for a particular item should ideally offer the same information. Obviously, that is not possible because different semiotic resources contribute meaning in different ways, but as far as possible the offered meaning was kept similar. The typology (Table 1) allows for some variation within a particular ET. The same kind of dynamic and interactive features was however used within each ET in the study to increase consistency. For ET I, a static presentation of the Facts was used. For ET II the same static presentation was used but the Facts were hidden behind a button with the instruction "Click here to open". For ET III the same constituents (words, images, and symbols) were used as in ET I–ET II, but the content was presented in a film where the information appeared part by part simultaneously as a voice read the information out. Symbols were referred to by their names, but images were not referred to in the film. ET IV in all items consist of the same Facts as in the other ETs but the initial information was incomplete. In relation to some central aspect of the Facts two options of how to complete the facts were offered. The student was expected to 'tick' one of the alternatives and press "Check". If the response was "False" the correct alternative could be chosen to receive "Correct". In ET IV therefore, the students contributed to create correct Facts. In ET V a mathematical relation central in the Facts is visualised aided by a dynamic change of some kind. Typically, the students drag a slider that dynamically visualises for example values as 1, 2, 3. These dynamically changing values correspond to for example exponents in a mathematical expression and $2^2=4$; $2^3=8$ and so forth, is visualised corresponding to the values chosen by the slider.

The use of different ETs to present Facts means the reader is invited to different kinds of interaction depending on ET. Using ET I–ET III means the reader is 'offered information' and the demand of interaction is fairly small (i.e. to open a box [ET II] and to start a film [ET III]). In Facts where ET IV or ET V are used, parts of the information are instantly offered, but further information is offered only after some demands of interaction or information is met by the reader. For ET IV the reader must contribute to make the Facts complete, while receiving feedback regarding its accuracy, and for ET V the reader is demanded to move or drag constituents to be offered further information about the relation between the mathematical content in the moved spot and other central content (e.g. selecting a number of fruits [three] result in visualisation of the faculty [3!] for the number of ways the fruits can be ordered). A prominent difference between ET IV and ET V is that ET V offer information that dynamically change depending on the reader's action whereas ET IV changes instantly (not dynamically) in response to the reader's clicks.

To diminish potential carryover effects because of the within subjects' design, partial counterbalancing of the items was used. Five different items and five element types used to present the Facts gave 25 versions of the items (Table 2). These 25 versions were combined in ten different timelines where the order of the mathematics items was the same, but the element type used to present the Facts were altered.

Table 2. Items with different element types to present Facts combined in timelines.

Timeline	Item A	Item B	Item C	Item D	Item E
1:	Facts ET I	Facts ET II	Facts ET III	Facts ET IV	Facts ET V
2:	Facts ET II	Facts ET IV	Facts ET I	Facts ET V	Facts ET III
3:	Facts ET IV	Facts ET V	Facts ET II	Facts ET III	Facts ET I
…			…continued…		

Before working with the mathematics items, the students were provided individual information about the eye-tracking equipment, and an example item was shown on a screen. The students were informed that all items to be solved had the same type of layout and the main parts of the item (introduction, task, facts, and answers) were pointed out on the screen. The students were also informed about how the dynamic functions in the tasks work, either by being shown the dynamic functions by the experimenter or by having them try using the functions themselves. This instruction was provided in relation to a test item with a very easy task, an item that also was included before the five items during the eye-tracking. The inclusion of this test item before the five items contributed to diminish unwanted effects caused by distractions while the participants got acquainted with the digital environment.

3.2 Eye-tracking apparatus and measures

A portable eye-tracker was used because data collection was made at schools to make it equally accessible for all students to participate without a large effort. The eye-tracker uses binocular eye-tracking with a sampling frequency of 60 Hz with a precision of 0.10 degrees and accuracy of 0.3 degrees in optimal conditions. Tobii Pro Lab software (Tobii Pro AB, 2014) was used during data collection which provided good support to ensure the right distance between eyes and screen, by the provision of a virtual headbox to adjust to. The viewing distance was ≈ 68 cm and the monitor size was 15.6 inches.

Calibration procedure and AOIs. Students were allowed to wear glasses because the eye-tracker tolerated glasses well. If the calibration of a participant failed, it was repeated at maximum two times. Independent of the calibration results, all participants completed the assigned timeline but participants with bad accuracy values were excluded. The limit for exclusion was set at >1.0 degrees. This limit is reasonable because only four AOIs are used in every item and the distance between them is in most cases larger than 3 cm and never less than 2.5 cm (e.g. see Hessels et al., 2016).

Fixation filer. The threshold for fixations was set at 30 degrees/second. Adjacent fixations were merged with limits of a maximal angle between fixations of 0.5 degrees and maximum time between fixations of 0.5 degrees. Fixations shorter than 60 msec. were discarded.

3.3 Analysed variables and statistical analyses

The data from fixations used in analyses are accumulated fixation duration (AFD) within each of the AOIs and total number of fixations (NF) within each of the AOIs. Both AFD and NF are analysed because these variables represent similar but not the same kinds of attentive behaviour. Analysis of both variables is also valuable because similar results in relation to both variables is an indication of high reliability. Accordingly, the first two variables used in analyses are AFD and NF on the Facts. These variables are extracted in relation to the five types of element (ET) used to present the Facts. In addition to these variables, ratios of AFD on different AIOs in items with different types of elements (ET) used to present Facts are analysed. The ratio used in this variable is AFD on introduction (I) and task (T) in contrast to AFD on the Facts (F) ([AFD I+T]/[AFD F]). This means a larger ratio represents a minor share of AFD on the Facts in contrast to a smaller ratio. This variable is valuable in relation to potential high ADF on an ET because if the presence of a particular ET leads to increased attention and interaction, this increased attention is likely to be reflected also in a larger share of AFD on the ET in contrast to on the introduction and task (that is exactly alike in all versions of an item).

Because the focus in the current study is on various types of digital element types (Table 1), the analysed variables represent the five distinct different ET used to present the Facts. The design with five versions of every item and counterbalanced order of the items in timelines (Table 2) ensures that varying difficulty of the items depending on different content in Item A–E does not affect the reliability of the results.

To investigate whether there are differences in AFD, NF and ratios of AFD, between the five ETs a Kruskal-Wallis H test was used. A normality test revealed that the data do not meet the normality assumption which is why ANOVA was not used. Kruskal-Wallis H test is a non-parametric alternative to an ANOVA that can be used to determine if there are statistically significant differences between two or more groups of an independent variable on a continuous or ordinal dependent variable. In the current analysis, a null hypothesis suggesting no differences between the different ETs was investigated with a significance level of .050. A post hoc pairwise comparison was used to analyse differences between ETs (see Field, 2009). Effect sizes are calculated for pairwise comparisons, using Cohen's (1992) rule of thumb for effect sizes: r=0.10 small effect, r=0.30 moderate effect, and r=0.50 large effect.

4 Results

The first research question in this study concerns how students' interaction with mathematics differs with the element type used for presentation. Based on the eye-mind hypothesis, there is a connection between gaze and thought (Hoffman, 1998). By investigating how the accumulated fixation duration and total number of fixations on the different ETs differ, we gain information about students' interaction with the teaching content when it is presented using different ETs. A previous study has highlighted how different ETs have the potential to offer meaning in different ways (Dyrvold & Bergvall, 2023), which may have implications for the time and engagement students choose to spend on mathematics content when presented with different ETs.

Table 3. Descriptive statistics on accumulated fixation duration (AFD) and total number of fixations (NF) for Element type I – V.

Element type		**AFD**	**NF**
ET I	Mean	17092.46	50.15
	Median	14506.5	47
	SD	9794.373	26.65
ET II	Mean	17377.28	51.08
	Median	16288.5	47
	SD	10312.64	30.368
ET III	Mean	28424.48	80.29
	Median	25499	73.5
	SD	15775.58	41.136
ET IV	Mean	37419.12	106.6
	Median	33318.5	94.5
	SD	19807.28	61.098
ET V	Mean	31548.48	77.73
	Median	27639.5	71
	SD	19717.69	48.683

n = 120, Values for AFD presented are milliseconds

Table 3 shows that the mean and median values for both AFD and NF are lower in ET I and II compared to the other ETs. The Kruskal-Wallis H test showed that there was a statistically significant difference in AFD between the ETs, $H(4) = 147.6$, $p= <0.001$. There was also a statistically significant difference in NF between the ETs, $H(4) = 138.13$, $p= <0.001$, and thus that the null hypothesis should be rejected for both AFD and NF. The mean ranks for each ET used in the Kruskal-Wallis H test is displayed in Table 4.

Table 4. Mean rank of accumulated fixation duration (AFD) and total number of fixations (NF) on element type (ET).

	AFD		NF	
Element type	**Mean rank**	**Rank order**	**Mean rank**	**Rank order**
ET I	197.53	5	205.93	5
ET II	203.66	4	208.87	4
ET III	335.98	3	343.39	2
ET IV	415.90	1	422.85	1
ET V	349.44	2	321.47	3

The mean ranks for ET I and ET II are fairly similar and, a post hoc pairwise comparison (Table 5) reveals no differences between ET I and II, neither for AFD, nor for NF. These two ETs contain the same text, the only difference between the ETs is that the student must click to access the information in ET II, while it is statically presented in ET I. The largest effect sizes are found for the comparison between ET IV and ET I–ET II, and thus the contrast in number of fixations and fixation duration that stands out the most is on ET IV in compared to the more static ETs. All comparisons between ET I or ET II with the other ETs (ET III-V) reveal significant differences both in fixation duration and number of fixations. These differences signal that the more dynamic and interactive ETs (ET III-V) receive more attention both in form of accumulated fixation duration and number of fixations. The test also shows that there is no difference in fixations between ET III and ET V. One similarity between these ETs is that both, to a large extent, offer continuously dynamically changing content (see also Analysis and discussion).

Table 5. Pairwise comparison of accumulated fixation duration (AFD) total number of fixations (NF) on element types (ET).

	AFD					NF				
Comparison[a]	**Test statistic**	**Z**[b]	**Sig.**	**Adj. sig.**[c]	**r**[d]	**Test statistic**	**Z**[b]	**Sig.**	**Adj. sig.**[c]	**r**[d]
ET I- ET II	-6.137	-.274	.784	1.000	-.018	-2.942	-.131	.895	1.000	-.008
ET I- ET III	-138.450	-6.187	.000	.000	-.399	-137.458	-5.163	.000	.000	-.333
ET I- ET V	-151.917	-6.788	.000	.000	-.438	-115.538	-6.143	.000	.000	-.397
ET I- ET IV	-218.371	-9.758	.000	.000	-.630	-216.917	-9.693	.000	.000	-.626
ET II- ET III	-132.313	-5.912	.000	.000	-.382	-134.517	-6.011	.000	.000	-.325
ET II- ET V	-145.779	-6.514	.000	.000	-.420	-112.596	-5.032	.000	.000	-.388
ET II- ET IV	-212.233	-9.483	.000	.000	-.612	-213.975	-9.562	.000	.000	-.617
ET III - ET V	-13.467	-.602	.547	1.000	-.039	21.921	.980	.327	1.000	.063
ET III - ET IV	-79.921	-3.571	.000	.004	-.231	-79.458	-3.551	.000	.004	-.229
ET V- ET IV	-66.454	-2.969	.003	.030	-.192	-101.379	-4,530	.000	.000	-.292

[a] Each row tests the null hypothesis that the sample 1 and sample 2 distributions are the same.
[b] Standardised text statistic.
[c] The significance values was compared to significance values adjusted by the Bonferroni correction for multiple tests, with the same results.
[d] r is the effect size for pairwise comparisons (Field, 2018).

The second research question “Is there any difference in how students read static parts of an item due to presence of dynamic element types in other parts of the item, and if so, how?" was investigated using a ratio between accumulated fixation duration (AFD) on the static parts of the item, that is Introduction and Task, and the Facts. The assumption is that a low ratio means that the students spend more time on the Facts in relation to the rest of the task, which is interpreted as if the interaction with the Facts is informative enough to reduce the time spent on reading the introduction and solving the task. Table 6 shows descriptive statistics on the ratios, and the ratios for ET I and II are on average slightly higher than the ratios for ET III–V.

Table 6. Descriptive statistics on ratios of accumulated fixation duration (AFD) between areas of interest within items for different element types (ET).

Element type		AFD on Introduction and Task/ AFD on Fact
ET I	Mean	2.59
	Median	1.57
	SD	8.44
ET II	Mean	2.96
	Median	1.45
	SD	9.00
ET III	Mean	1.20
	Median	.85
	SD	2.39
ET IV	Mean	.77
	Median	.73
	SD	.43
ET V	Mean	1.49
	Median	.81
	SD	2.28

n = 120 (ETI. ET III. ET IV. ET V), n = 119 (ET II because AFD on Fact was 0 on ET II for one student)

The mean ranks of rations between accumulated fixation duration (AFD) on Introduction together with Task and Facts for each ET used in the Kruskal-Wallis H test is displayed in Table 7. The Kruskal-Wallis H test showed that there was a statistically significant difference in ratios between the ETs ($H(4) = 151.7$, $p < 0.001$). The ratio is between AFD on the three areas of interest: (Introduction + Task)/Facts.

Table 7. Mean rank of ratio for accumulated fixation duration (AFD) between areas of interest within items for different element types (ET).

Element type	Mean rank
ET I	411.51
ET II	392.77
ET III	235.77
ET IV	191.18
ET V	269.54

The result of a pairwise comparison of the mean ranks of ratios is presented in Table 8. A possible difference between ET I and the other ETs illustrates the difference between being offered only static text on screen and being offered also various types of dynamic elements in digital learning materials. The pairwise comparison indicates no difference between ET I and II, while ET III–V differ from the former in terms of ratios on ADF. All significant differences, except for between ET IV and ET V, have a moderate or high effect size. The ratios are lower for the more dynamic ETs, indicating that more time is spent on Facts compared to Introduction and Task. Table 3 shows that the students spend more time on the more dynamic ETs III–V and, moreover, Table 6 shows that the relative time of fixations on Facts in contrast to other text elements in an item is larger when ET III–V are used for presentation in contrast to ET I and II. A long AFD can, based on the eye-mind hypothesis, be interpreted as the student spending more time processing the mathematics content when ET is more dynamic.

Table 8. Pairwise comparison of ratio for accumulated fixation duration (AFD) between areas of interest within items for different element types (ET).

Comparison[a]	Test statistic	Z[b]	Sig.	Adj. sig.[c]	r[d]
ET IV - ET III	-44.583	-1.995	.046	.460	-.129
ET IV - ET V	-78.358	-3.507	.001	.005	-.226
ET IV - ET II	-201.590	-9.004	.000	.000	-.581
ET IV - ET I	-220.325	-9.861	.000	.000	-.637
ET III - ET V	-33.775	-1.512	.131	1.000	-.098
ET III - ET II	-157.006	-7.013	.001	.000	-.453
ET III - ET I	-175.742	-7.866	.001	.000	-.508
ET V - ET II	-123.231	-5.504	.001	.000	-.355
ET V - ET I	-141.967	-6.354	.001	.000	-.410
ET II - ET I	-18.736	-.837	.403	1.000	-.054

[a] Each row tests the null hypothesis that the Sample 1 and Sample 2 distributions are the same.
[b] Standardised text statistic.
[c] Significance values have been adjusted by the Bonferroni correction for multiple tests.
[d] r is the effect size for pairwise comparisons (Field, 2018).

5 Analysis

All analyses in the current study reveal significant differences in fixations between the more static ETs, ET I–ET II, and the more dynamic/interactive ETs, ET III–ET V. There are however no significant differences in fixations between ET I and ET II. The reader needs to click-open the Facts in ET II. When opened, the Facts are identical for ET I and ET II. If there were a significantly higher number of fixations (NF) and/or accumulated fixation duration (AFD) for ET II that could be a positive effect of a perceived agency or ‘investment’ in the item, because actively opening the Facts is an investment of effort and engagement in the item. No such differences between ET I and ET II were identified and accordingly an option to limit the amount of information by hiding the Facts seems not to increase attention to the mathematical content.

In contrast to ET I–II, more attention (NF) is paid to ET III–ET V and there is also a larger share of fixations (AFD) on the Facts in items where ET III–ET V are used for presentation. At the least, these results reveal that the participants chose to attend a great deal to the Facts presented using ET III–ET V. A goal with teaching materials is to get the readers persistently engaged to learn with the material, and in this respect ET III–V meet the expectations.

An analysis of the speech acts that come into play when ET III–ET V is used gives another lens on the results. All three ETs *offer* information whereas ET IV–ET V *demand* more from the reader, that is, not only information but also, in Halliday's terms, 'goods and services' (in this context some digital acts). ET III differ from ET IV–V because ET III (film) demand a minor 'digital act' namely, to start the film, whereas ET IV–V demand both information and digital acts. Facts presented using ET IV as well as ET V do not offer sufficient information to solve the task without the readers' digital acts. ET IV offer incomplete statements and the reader needs to choose how to make the Fact complete, and thereafter check if the choice is correct via a button. If the wrong option is chosen, several trials is demanded to receive the correct information. This means there is not only a demand of digital acts from the reader, but also information because the reader has to choose options based on their knowledge. ET V utilizes drag-and-drop or sliders that when used results in visualization of dynamically changing content. Digital elements of type ET V are possible to use in a digital act without responding to demands of information by using the element type explorative. It is likely, however, that the reader also responds to demand of information, by choosing digital acts based on knowledge about the mathematics. An example of such an act of voluntary offering of information is when students pause a movement to visualise a particular mathematical relation or a critical value.

In summary, the analysis of speech acts reveal that ET IV demands the most from the reader. The analysis of fixation duration and number of fixations (AFD and NF, Table 3) also reveal the highest values for ET IV. The significantly higher values for ET IV indicate that the students choose to meet the demands of both digital acts and information, and that they thereby engage with mathematics. It is possible the students use trial and error behaviour (e.g. see Pinkernell et al., 2020) and choose not to use their own understanding while reading, but at the least ET IV demand information from the readers and they can choose to interact consciously.

6 Discussion

This study concerns five element types, and what their presence in mathematics text means for students' interaction with the material. The five element types are used to present Facts and are designed to reflect increasing dynamism and an increasing invitation to interact, from ET I to ET V (definitions of ET I–V can be found in Table 1, Method). Accordingly, the ETs differ in how information is offered and demanded (i.e. speech acts) from the student. Recall that ET I define static elements like print material but presented on screen. All other ETs have some dynamic features, not possible to offer in print.

Before a more in-depth discussion of the results, a reminder of what eye-tracking can and cannot say, is in place. Firstly, fixations do reliably represent attention, but the attention can be due to for example either complex constituents or attractive, joyful constituents. Secondly, many fixations and/or high accumulated fixation time represent persistent attention but does not reveal the extent to which the offered content is understood. The reasons why a participant choose to focus on the Facts can however vary. Persistent interaction can be caused by hard struggle to understand or by amusement from using a highly interactive ET. While interpreting these results it is therefore important to keep in mind that we study interaction, exploration, and persistence, not the reasons why the participants interact with the element types in particular ways. However, persistence is a prerequisite to grasp new and demanding content, and this study highlights more and longer fixations on content when some element types are used (ET III–ET V). The study do not analyse learning, but according to the theory behind eye-tracking (e.g., Pashler, 1999; Hoffman, 1998) the number of fixations and fixation duration is tightly related to either endogenous or exogenous attention. Based on the high interdependence between eye movements and attention the results can be used to understand more about how texts are read.

The results of the current study show that students spend time and attention on dynamic resources to a greater extent compared to when the information is offered as static text. Previous analyses of digital teaching materials in mathematics have shown that different dynamic elements have different potential to offer a reader meaning (Dyrvold & Bergvall, 2023). Regardless of the extent to which some dynamic resource can support meaning making, students must spend time reading and processing the information to take advantage of its potential. The importance of spending time has been highlighted in other studies revealing positive relations between achievement in mathematics and time spent on digital material as well as on using particular digital tools (Junco & Clem, 2015; Bokhove & Drijvers, 2012). Time is a blunt measure, but at the same time, it is easily understood that learning will not happen if the required effort is not invested. Accordingly, the indications of preference for dynamic and interactive elements revealed in the current study is worth reflecting over in the design of digital teaching materials.

ET III–V can be assumed to have an inherent demand of time, which has a direct bearing on the conclusions possible to draw based on the results. There are however reasons to assume that the extended attention on dynamic ETs is not solely caused by inherent demand of time in these ETs. The extent to which, for example ET V contributes to make mathematics accessible varies between the different items, but despite these differences, the differences in attention according to ET are significant. For example, in the easiest item about the inscribed angle theorem, the Facts of ET I is somewhat straightforward to interpret, whereas the version of the same item with Facts of ET V demand the reader to make connections between two dynamically changing angles. In more demanding items however, there are many meaning relations between constituents in the Facts, and in the more static versions of the Facts (ET I and ET II) the reader must understand all these relations, reasonably by circulating in the text, something that requires a great deal of attention. In ET V for such demanding items, on the other hand, the meaning relations are partly made apparent through dynamically changing objects in Facts. For example, in the item about permutations and factorials, the information in Facts can be rather hard to interpret for a student who is unfamiliar with the content. In static Facts, it is likely that the reader needs to read back and forth to fully understand the relations between quantity, permutations, factorials, and the meaning of the sign “!”. In Facts using ET V on the other hand, the reader is supposed to move fruit in and out of a casket and the permutations and the related factorial are instantly visualised. Accordingly, the consistent higher attention to Facts presented with ET V, possibly reflect an extensive exploration of the mathematics content rather than barely meeting the demand for interaction. These arguments support the claim that students attend more to content presented using interactive and/or dynamic elements (i.e. ET III–V) based on their own urge to gain understanding needed to answer the task, rather than based only on demands of time spent on the ETs because of their inherent features. In contrast to game-based learning that is criticised for only raising temporary interest, not to scaffold learning (e.g., Rodríguez-Aflecht et al., 2018), ETs that invites to exploration and thereby makes relations in mathematics more apparent have more of a scaffolding function. The scaffolding function is however conditioned by persistence.

One example of a dynamic element that students pay a great deal of attention to is ET III in the current study. A reason to interpret the time spent on ET III and as actively opted is that students in a pilot study (Dyrvold & Bergvall, 2022) expressed an experience that the film provided *more* information compared to the static ETs, despite the text on screen being the same. A conclusion is that the auditory resource in the form of a text reading voice in the film was experienced as "more" information that provided support for the reader. The pilot also revealed that participants chose to see the film repeatedly or to pause the film.

In items where ET IV is used, the student is requested to complete statements by choosing an option that makes a statement complete and correct. If the wrong option is chosen, the student needs to try again. Apparently, this kind of ET demands prolonged attention, especially if the wrong option is chosen in the first trial, and this demand of attention likely explains much of the fixations (NF and AFD) on Facts presented using ET IV. Because the results show that students pay a lot of attention to ET IV, one conclusion about the use of ET IV is that students are persistent enough in the strive to access the information needed to complete a task and that they are willing to make an effort and contribute to make offered Facts complete. From a didactical perspective, such investment of attention can potentially support learning more deeply compared to if the Facts were only read. While completing the facts through the options students need to evaluate and reflect over central concepts and mathematical relations. Being aware of wrong conclusions while learning new content is valuable (Boaler, 2015).

The downside of ET IV is that it can be used with a trial-and-error behaviour without learning gains. Previous research revel that automated feedback can be beneficial for learning but also that feedback in the form of short rapid response may not constitute the support needed, or that it even can be detrimental for learning if trial and error behaviour is used (Rezat et al., 2021a; Pinkernell et al., 2020). More elaborated feedback has been suggested to avoid such behaviour (e.g. Rezat, 2021; Heeren & Jeuring, 2019; Rezat et al., 2021b) but those suggestions are adapted to feedback in tasks. In the design of the items in the current study response options in ET IV, are used not as feedback in tasks but as guidance to complete facts needed to solve a task. This is quite a different way to use digital response options. A possibility is that the demand of ‘digital acts’ to complete the needed information is sufficient when it comes to this type of feedback as guidance to how the mathematical facts is to be understood.

For ET V on the other hand the demand of ‘digital act’ is minor, and the offered information is provided instantly. Because of the dynamic manner of ET V there are connections between constituents in the offered mathematics that the students need to grasp part by part. The demand of ‘digital act’ while reading content presented using ET V does therefore also entail a demand to be part in making sense of the offered information. A reader does of course always need to make sense of the offered meaning, but in contrast to static ETs where all information is present simultaneously, the meaning making while reading ET V can be considered a more constructive process. This process of meaning making can be an explanation to students’ persistence while working with items with ET V as well as to their attention (high AFD and NF), because active learning engages. A previous meta-analysis about affordances of teaching materials with dynamic representations of mathematical objects highlight both that such materials can contribute to constrain students’ attention to relevant content and that these materials can motivate students to persist at mathematical tasks (Moyer-Packenham & Westenskow, 2013). Together with other studies highlighting the benefits of dynamic material while learning mathematics (Alagic, 2013; Baccaglini-Frank, 2021) the meta-analysis strengthens the conclusion that ET V is likely to not only increase attention but also contribute to learning. The current study contributes to these previous studies by emphasising that dynamic elements have a great potential also in presentation of theory, not only in tasks which seems to be the most common way to use them (e.g. Dyrvold, 2022).

In conclusion, the results of this study highlight that the inclusion of more dynamic and interactive elements in digital teaching material can be useful to increase students’ persistence and time spent on the mathematics content. The most prominent results from the statistical analyses are significantly higher attention (in the form of number of fixations and accumulated fixation duration) on Facts presented using dynamic and interactive element types (RQ1) compared to when static, or less dynamic, elements are used. Moreover, the increased attention on dynamic and interactive element types seem to reflect a persistence to sort out the information offered through these element types, either mathematical or linked to the dynamic function (the increase in accumulated fixation duration is allocated to the element types, not to the whole item) (RQ2).

In this way, our study elucidates that a more extensive inclusion of dynamic and interactive elements in digital teaching materials *can be* advantageous. This kind of resources are unique to digital teaching materials, and it is important to explore their function and advantages in teaching. The results can be useful as a guide when deciding in what manner these resources are beneficial, and when a static version reminiscent of a printed textbook is preferable. Thus, these results have implications for the development of digital teaching materials, but also for the teaching practice. When school leaders and teachers choose teaching resources, digital teaching materials offering dynamic and interactive elements must be considered based on an awareness that these elements can increase students' engagement in mathematics. Whether this engagement also leads to increased learning is not shown by this study and to investigate that further studies are needed.

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