Relationship between affective-motivational constructs and heart rate

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The following survey study uses a quantitative research design to investigate motivational and affective aspects of students (aged 14–17) in a mathematical workshop on graph theory. Motivational and affective aspects are related to heart rate measurement (using the digital medium of a pulse watch) in mathematical knowledge development processes in an empirical-oriented mathematics class. Interestingly, a link between constructs on motivational and affective aspects and a heart rate measurement is describable. This gives further impulses for investigation and could be used in the future to determine the teaching phases or tasks in which students are particularly motivated.

Keywords: heart rate measurement, easiness, enjoyment, helpfulness, graph theory

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1 Introduction

Today, learning is understood as an active process (Scherer & Weigand, 2017). It is generally acknowledged that students constitute their mathematical knowledge in processes of action and negotiation (Krummheuer, 1984). In this context, students' mathematical knowledge development processes depend on individual domains of experience (Bauersfeld, 1988). With the approach of Burscheid and Struve (2020) these can be described as the construction of theories about certain phenomena of empiricism – empirical theories. With the cognitive psychology approach of "theory theory" by Alison Gopnik, it seems quite meaningful to describe the behavior and knowledge of students in theories. Burscheid and Struve (2020) state that with knowledge in this context not the knowledge formulated by the person is meant, but the knowledge that observers impute to the person in order to explain their behavior. The person – for instance students – behave as if they had the knowledge/theory (Burscheid & Struve, 2020). This can be related to Gopnik's (2003) studies when it states here that "children develop abstract, coherent, systems of entities and rules, particularly causal entities and rules, [...] they develop theories" (Gopnik, 2003, p. 5).

With reference to these theorizations and knowledge development processes, this article presents a pilot study that aims to show the extent to which connections exist between affective and motivational aspects and the physiological component of a heart rate measurement in mathematical knowledge development processes in an empirical-oriented mathematics class. The importance of motivational and affective aspects – and not only exclusively cognitive factors – in teaching-learning processes first emerged in the mid-twentieth century (Goldin, Hannula, Heyd-Metzuyanim, Jansen, Kaasila, Lutovac, Di Martino, Morselli, Middleton, Pantziara & Zhang, 2016). In this context, Di Martino also states: "intentional actions involve complex relationships between affective and cognitive aspects; therefore, it is crucial to develop methods able to grasp this complexity" (Goldin et al., 2016, p. 3).

In order to clarify the question of the connection, the article first goes into the theoretical background with regard to the descriptive concept of domains of subjective experience (DSE for short) according to Bauersfeld (1983; 1985), empirical-oriented mathematics classes and how we want to deal with the term motivation in this article. With reference to current accounts regarding motivational and affective components in mathematics education, the section concerning *Research approaches and hypotheses* follows with a statement of the research purpose and clarification of the study's hypotheses. Methodological decisions regarding data collection and the setting used are made clear in Section *Methodology*, and the results and subsequent analysis are presented in Section *Analysis and results*. The article concludes with a review of our hypotheses, a summary of the results, and an outlook on the investigation of further relationships between instructional phases, tasks, and student motivation (Section *Discussion and conclusion*).

We would like to point out here that the results of this paper are also partly published in German in Pielsticker and Reifenrath (2022).

2 Theoretical background

Studies by Bauersfeld (1983; 1985), Coles (2015), Steinbring (2015), or Voigt (1984) could show that there are differences between teachers' perspectives and students' perspectives on mathematics lessons. Thus, Voigt states, the ambiguity of objects is not just a feature of individual episodes or individual tasks. The ambiguity of objects can be a fundamental and long-lasting feature of classroom talk when teacher and students interpret the objects in systematically different ways (Voigt, 1994). In this paper, we want to focus on the students' perspective and assume, in terms of a constructivist approach to learning, that students develop and negotiate their knowledge in dealing with the reality that surrounds them.

Decisive for this is the descriptive concept of domains of subjective experience (DSE) according to Bauersfeld (1988). In the article results and problems of microanalyses of mathematical classes, Bauersfeld (1985) presents his concept of DSE by formulating theses; simplified, these are individual cognitive and affective knowledge structures that students build up domain-specific. The starting point of a constructivist concept of learning and the approach of DSE according to Bauersfeld offers an orientation framework for our contribution.

With the concept of DSE we can describe how students develop their knowledge in a constructivist and interactionist sense. The core idea is that learning is a domain-specific process and thus can be described as bound to a particular situation and context. DSE include meaning, language, objects, and actions, which include cognitive and motivational or emotional dimensions. According to Bauersfeld, it can be stated that "[...] learning is characterized by the subjective reconstruction of social means and models through the negotiation of meaning in social interaction and in the course of related personal activities. New knowledge, then, is constituted and arises in the social interaction of members of a social group (culture), whose accomplishments reproduce as well as transmute the culture" (Bauersfeld, 1988, p. 39). Language plays an important role in linking the individual DSE. According to Tiedemann (2016), a term has a meaning, especially in the context of other terms, but each term has a specific language use. In the sense of Bauersfeld's approach, affective knowledge structures play a decisive role in the development of mathematical knowledge in addition to the cognitive ones. Especially if we assume that a knowledge experience is "total" (Bauersfeld, 1985, p. 11). Rennie (1994) measures affective outcomes from visits to hands-on learning experiences in her study. Similar to Bauersfeld, she argues that in addition to cognitive outcomes, affective outcomes should also be considered for an overall picture of hands-on learning experiences. In her study, students visited a laboratory, which makes it a consideration of out-of-school learning opportunities. In this setting, Rennie sees motivation and willingness to engage as the most important affective outcomes. For example, the term motivation is used to describe the will-to-succeed across multiple contexts (Eccles, Wigfield, & Schiefele, 1998). A study by Renninger, which is addressing interest and motivation to learn in free-choice environments of informal science learning states: "Motivation to learn usually refers to the energy behind conscious decisions to achieve in school" (Renninger, 2007, p. 3). To describe affective outcomes like motivation, Rennie uses the components enjoyment, helpfulness, and easiness. We will follow the use of these components to describe the affective knowledge structures of the students in our study. In our context, this also involved hands-on learning experiences, which are based on the concept of empirical-oriented mathematics classes (Pielsticker, 2020). The workshop outlined in section *Methodology* was developed, designed and conducted in terms of empirical-oriented mathematics classes. The basis for empirical-oriented mathematics classes is the approach of empirical theories according to Burscheid and Struve (2020). Empirical-oriented mathematics classes is a teaching in which the teacher intendedly makes educational decisions to work with empirical objects as the mathematical objects of math classes. In mathematics classes, the empirical objects (e.g., 3D-printed tiles, graphs on a paper) do not intend to illustrate mathematical concepts that are abstract in nature, but they are rather the subject of the lesson (Pielsticker, 2020). The teaching-learning process from a mathematics class of this study takes this conception into account.

The purpose of this paper is to present, in a quantitative study, affective knowledge structures of students (through constructs measuring motivational and affective aspects), brought together with a heart rate measurement. Isoda and Nakagoshi (2000) had related students' emotional changes to the observation of heart rate in their study on problem posing in mathematics classrooms. In doing so, the authors note: Heart rate “is good quantitative indicator of the intensity of emotion” (Isoda & Nakagoshi, 2000, p. 89). Isoda and Nakagoshi looked at five types of changing emotional intensities in their study. For example, they observed a rapid intensity of changing heart rate when the students started to think in the given problem posing situation. Or a rapid and strong intensity of changing heart rate was observed when it came to social interaction (students exchange ideas and one gives an explanation). In our study we want to focus on motivational and affective aspects, combine them with a measurement of heart rates and describe affective knowledge structures.

2.1 Research approaches and hypotheses

Since affective and motivational aspects are important in view of our theoretical background from previous sections (and in doing so are especially related to the concept of DSE) it is useful to take a look at previous research and developed constructs in this area. A good overview of this is provided by Goldin et al. (2016), among others, who highlight that concepts and theories in the affective domain can be mapped along three dimensions. The first dimension is "identifying three broad categories of affect: motivation, emotions, and beliefs" (Goldin et al., 2016, p. 1). In this context, the last dimension comprises the theoretical level, which can be divided into three main levels: "physiological (embodied), psychological (individual), and social" (Goldin et al., 2016, p. 1). In this regard, Hannula also notes that affect in the mathematics context primarily addresses the psychological level, whereas the physiological level "is not very popular among mathematics educators" (Goldin et al., 2016, p. 2). In our study, we also address this level, among others, through heart rate measurement, thereby picking up on Hannula's observation that while some research exists in the area of affective and motivational components, there are also "insufficiently explored venues that call for additional research" (Goldin et al., 2016, p. 2). In this study, affective constructs are related to this same physiological component. In order to classify the affective constructs, we are investigating in this context, it is interesting to take a look at the widely used TMA model for the construct "attitude". According to this three-component model by Di Martino, attitude has three dimensions: "emotional dimension," "vision of mathematics," and "perceived competence" (Goldin et al., 2016, p. 6), which are interrelated. The constructs easiness, enjoyment, and helpfulness examined in our study, which will be explained in more detail below, can be located in the area of the emotional dimension and perceived competence. In addition, the constructs show a clear relation to motivation research. Goldin and others suggest, in addition to the general consideration of affective structures "that the focus of motivation research be shifted from the study of longer-term attitudes and beliefs toward that of in-the-moment-engagement" (Goldin et al., 2016, p. 18). Both with our collection of data on heart rate measurement and on easiness, enjoyment, and helpfulness, we take up the situational aspect (regarding "in-the-moment engagement") of motivation research. It is worth mentioning here that this distinction into "long-term" and "in-the-moment" is also found in many important individual motivational factors (interest, perceived instrumentality, etc.) (Goldin et al., 2016, p. 20). Thus, the concern of the study is to pick up motivational and affective aspects and to relate them to physiological components. Goldin and others also see the intersection of motivation and affect as "the future of the field" (Goldin et al., 2016, p. 23). In the present study, therefore, to measure motivational and affective aspects in knowledge development processes of students in empirical-oriented mathematics classes, we introduce the three constructs already mentioned, easiness, enjoyment, and helpfulness (Rennie, 1994; Woithe, 2020), and combine them with our measurement of heart rate. The choice of these constructs was largely based on a pilot study by Rennie (1994), which aimed to measure cognitive and affective outcomes in students attending a science education center and to develop a brief, easy-to-understand, and assessable measurement instrument for this purpose. The three constructs turned out to be central (Rennie, 1994). To capture the relevant constructs, we followed Woithe's items because, similar to our teaching-learning process, there was no explicit group work and therefore the items were adapted to the context (Woithe, 2020). In general, the survey context of Woithe, who studied the constructs in the context of workshops at the CERN S'Cool LAB, is also comparable to the teaching-learning process we developed in terms of the intervention duration of about 4.5 hours (Woithe, 2020) and via the structural design as a workshop. For the terms, at least for the constructs easiness and helpfulness, first a specification and for enjoyment a hint seems useful. According to Woithe (2020), the 2006 Pisa study rated "enjoyment of science" as “one of the measures of students' intrinsic motivation to learn science” (Woithe, 2020, p. 26). Here we can see our reference to the measurement of motivation. Helpfulness describes the perceived benefit of activities for students and is "closely related to value-oriented component of interest and serves as an indicator for a hold-component of interest development" (Woithe, 2020, p. 26). Regarding easiness, it can be said that it is related to "the cognitive load of the activities" (Woithe, 2020, p. 26) and Woithe states that “perceived easiness should not be too high to make sure activities cognitively activate students through challenging but doable tasks" (Woithe, 2020, p. 26). We therefore examined the following hypotheses in our study:

1. The heart rate (HR) deviates from the resting pulse of students in mathematical teaching-learning processes. (H1)
2. HR correlates with students' easiness in the given mathematical teaching-learning process. (H2)
3. Enjoyment correlates with easiness. (H3)
4. Helpfulness correlates with easiness. (H4)

In our first hypothesis H1, HR represents the abbreviation for heart rate or respectively the pulses in beats per minute (also used below). The hypothesis is based on a study by Isoda and Nakagoshi (2000), who demonstrated the importance of heart rate change in describing emotional change in students in a case study. Since we assume a change in emotional and affective components in the course of our mathematical teaching-learning process, this should also be shown by an increased heart rate. For our hypothesis H2, we look again at the study by Isoda and Nakagoshi. They state that “changing HR can be expressed in terms of arousal of the student’s mind” (Isoda &

Nakagoshi, 2000, p. 93) and that gradual increasing of HR represents concentrated thinking. The hypothesis picks up on the fact that this may also make the task perceived as easier. For our hypothesis H3, we can state: Woithe's study (2020) already analyzed a connection between helpfulness and enjoyment. As studies by Gläser-Zikuda and Mayring (2003) indicate, "if students enjoy learning [...] they are also more likely to perceive it as meaningful" (Woithe, 2020, p. 26). In hypothesis H3, we are interested in whether there is also a correlation between enjoyment and easiness. In H4, we wanted to know whether a correlation between helpfulness and easiness could also be established following the indications and results of the study by Woithe (2020) on helpfulness and enjoyment.

3 Methodology

3.1 Participants

Data were collected during several sessions of a workshop designed for this purpose in eight different learning groups. These learning groups were classes of secondary (German Sekundarschule) and high schools (German Gymnasium) in NRW Germany and represented grades 8 to 11, with the workshops taking place in their usual classroom environment. The workshop was designed to last 3,5 hours and was accompanied by two lecturers and the supervising teacher. Overall, the study thus has a sample size of N=73, with the analysis based on a sample size of N=46. The reason for the difference in the general sample size and the sample size used for the analysis is that the data (especially the questionnaires) were not complete (filled out) and usable for our analysis in every case.

3.2 Materials and measures

In order to be able to measure motivational and affective aspects in knowledge development processes of students in empirical-oriented mathematics classes, we methodically used a quantitative approach based on the measurement of the students' heart rate. A relationship between motivation and heart rate has already been demonstrated and researched in different studies. The data on this is initially not limited to the field of mathematics education but can be found primarily in sports science/sports medicine (Wang et al., 2015; Dadaczynski et al., 2017), as well as in computer science (Monkaresi et al., 2017), artificial intelligence (Patel et al., 2011), and psychology

(Scheibe & Fortenbacher, 2019). Specific to the field of mathematics education, studies on the relationship between heart rate and motivation can be found that take a case-study approach (Isoda & Nakagoshi, 2000), and on the relationship between perception of difficulty and motivation in arithmetic (Carroll et al., 1986). The latter is of particular interest to the present study because motivation is not defined as a concept that can be clearly grasped. Therefore, the study examined the three constructs easiness, enjoyment and helpfulness, which Rennie (1994) and Woithe (2020) have already used in similar workshops and research studies in advance. The heart rate was measured using the "Fitbit charge 4", a fitness watch which, as one of its functions, enables heart rate measurement over a definable interval and also outputs the data graphically.

3.3 Heart rate tracking devices – Fitbit

The pulse measurements and thus the activity and the demands on the students were carried out in this work with tracking devices. Fitbit Inc. is a US manufacturer of electronic vital function measuring devices, so-called "tracking devices", founded in May 2007 in San Francisco, California (https://www.fitbit.com/global/us/home). Tracking devices of the first generations still showed a huge variance in measurement accuracy whereas newer generations of tracking devices show a deviation from tests under laboratory conditions of only up to 4 % (University Aberystwyth, 2019). Numerous studies now prove that tracking devices have a positive effect on the likelihood of success in implementing a healthier lifestyle (Ridgers, McNarry, & Mackintosh, 2016). Each of our trackers had online access to the Fitbit website. In addition, all data was anonymized and worked with, for example, Tracker A, Tracker B, etc. The data such as heart rate, date and time could be retrieved via the website and the respective online access of the tracker at Fitbit. Accordingly, this data could be exported to Excel and used for the analysis. The representation in Figure 1 is based on the graph from Fitbit.

Figure 1. Data and representation of the values of the trackers (based on https://www.fitbit.com/global/us/home).

3.4 Resting pulse

Another important reference for our analysis is the resting pulse. The resting pulse is the pulse at which no activities or movements are performed. This means that it is the pulse measured in the absence of physical and mental stress. Ideally, the resting pulse is measured immediately after waking up. The resting pulse rate of adult's averages around 70 BPM, while that of adolescents averages around 80 BPM (Pape, Kurtz & Silbernagl, 2005). Since the analysis of this paper was conducted with adolescents in the eighth and ninth grade, the resting pulse rate of 60 to 100 BPM is decisive (Healthwise Staff, 2020). The present pulse values are therefore not a resting pulse in the true sense of the word. Nevertheless, a pulse can be read that is very close to a resting pulse. In the processing phases, the students reach phases in which they have a relaxation pulse (see Figure 3). This means that the pulse flattens out and becomes calmer.

3.5 Affective-motivational constructs

To measure motivational and affective aspects in knowledge development processes of students in empirical-oriented mathematics classes, we bring together the three constructs and our measurement of heart rate. The separate constructs easiness, enjoyment and helpfulness were recorded using a sequenced questionnaire integrated into a workbook. In total, this consisted of 19 items that could be assigned to the constructs in groups and related to different phases of the workshop. The constructs were always queried after the respective phase to obtain as fresh an impression as possible. In addition, each of the 19 items has an even number of characteristic values (1 "not at all true" - 6 "completely true") in order to avoid an "error of central tendency" with Likert scales. An example of the first four items can be found in Figure 2. (Sa1: It was easy to work on the above task, Sa2: It was easy to understand, what it was about, Sa3: I enjoyed doing the exercise, Sa4: The above exercise helped me to expand my mathematical knowledge).

	Please indicate how much the following statements apply to you.	not at all true	not true	rather not true	rather true	true	completely true
Sa1	It was **easy** to work on the above task.	☐	☐	☐	☐	☐	☐
Sa2	It was **easy** to understand what it was about.	☐	☐	☐	☐	☐	☐
Sa3	I **enjoyed** doing the exercise.	☐	☐	☐	☐	☐	☐
Sa4	The above exercise **helped** me to expand my mathematical knowledge.	☐	☐	☐	☐	☐	☐

Figure 2. First four items of the sequenced questionnaire (translated from German to English).

The reliability of the scales was satisfactory; the corresponding *Cronbach's alpha* is shown in Table 1. According to Field (2017) and Hair et al. (2018), the values for *Cronbach's alpha* are in the good range (> 0.7) and would not improve by removing an item. In this sense, the items on the constructs (also according to Rennie, 1994; Woithe, 2020) measure what they are supposed to measure and are consistent. The construct easiness had a higher number of items because of its importance for our hypotheses.

Table 1. Reliability of the scales.

Constructs	Number of items	Cronbach's alpha
Easiness	9	.782
Enjoyment	5	.789
Helpfulness	5	.728

Both data series were quantitatively recorded in different learning groups. For preparation and analysis of the data set we used SPSS. The assignment of the data series was done via an anonymized code.

3.6 Procedures

Data collection took place between August and December 2020. In terms of subject matter, the workshop is based on graph theory and addresses central questions about the functioning of a navigation system and the determination of optimal paths. The subject area was chosen, among other things, because it requires little prior experience on the part of the learning group and is easy to elementarize to a certain extent.

The workshop's content was divided into three sections, which can also be found in the workbook that the learning groups received. The first part deals with basic concepts of graph theory and clarifies them, whereas the second part focuses on the optimization of graphs and paths and leads to the third part, which deals with the application of algorithms. The workbook not only fulfilled content-related purposes but also served data collection purposes, since in addition to the content, which was prepared in appropriately sequenced boxes (information boxes, elaboration boxes, exercise boxes), the sequenced questionnaire items for recording the constructs easiness, enjoyment and helpfulness could also be found after the respective phase. The measurement of the pulse data within the workshop or the time of their collection followed a defined pattern, which fits into its organizational conception (see Figure 3). In the entire teaching-learning process, a short introduction for the learning group to the functioning of the heart rate monitor took place in the introductory phase. The start of the heart rate measurement was then after the creation of the workbook code before the start of the content work and the first part. During the following sequence, heart rates were constantly measured until the end of the workshop or the teaching-learning process. An exception was the breaks, in which the heart rate measurement could also be paused. A decisive factor was the resting pulse's determination as a comparative value for the heart rate and its changes. This took place after the end of the workshop and before the heart rate monitor was switched off, as the drop in heart rate was clearly visible in the data.

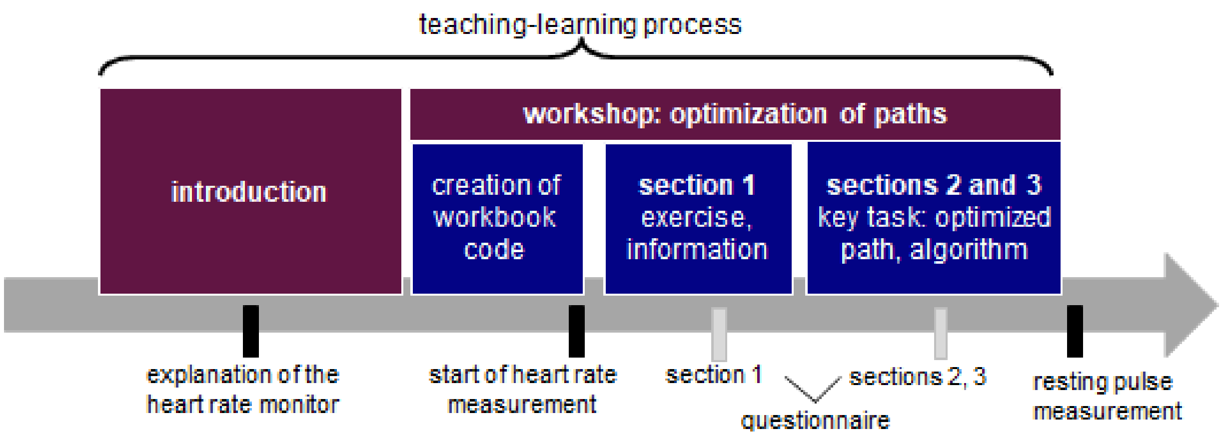

Figure 3. Organizational structure of the teaching-learning process.

4 Analysis and results

4.1 Descriptive results and correlations

In preparation for our *T-test*, we checked our data set on HR for metrics and normal distribution (Field, 2017; Hair et al., 2018). We can state that the heart rates are metric and normally distributed. The entire sample (N=46) was tested using the Shapiro-Wilk test (the value was > 0.1). Moreover, with Table 2, we can confirm the *first hypothesis* (*H1*), "The HR deviates from the resting pulse of students in mathematical teaching-learning processes" for our data set. For the students, the HR in our mathematical teaching-learning process deviates from the resting pulse of the latter. The significance is <0.01. For illustration purposes, a boxplot of the pulse data of a single participant with a resting pulse of 79 bpm is shown in Figure 4 for the entire teaching-learning process. The red line corresponds to the resting pulse and is well below the median for the specific case, for example.

Table 2. Data of the T-Test.

Variable	Average	N	p value
Resting pulse	70.33	46	
Pulse (entire work-shop)	87.89	46	
Difference	17.6		<.000

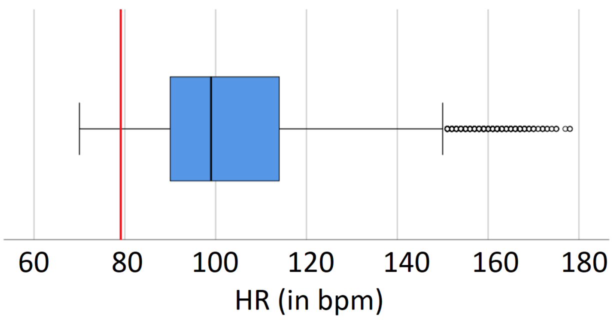

Figure 4. Boxplot of the pulse data of a single participant with a resting pulse of 79 bpm (indicated in red) for the entire teaching-learning process.

Furthermore, we examined five variables for our analysis (see Table 3). The value of the HR for the entire workshop (also known from the *T-test*), easiness, which is relatively high with a mean value of 5.07 on a straight scale from 1 ("not at all true") to 6 ("completely true"), enjoyment, and helpfulness (with a mean value slightly below easiness) and gender. Regarding the latter, we can state that more male test persons participated. Here, the gender "female" was coded by 1 and the gender "male" by 2. The gender "diverse" was not specified by the participants.

Table 3. Descriptive data for evaluation of the workshop.

Variable	Average	N
Pulse (entire workshop)	87.89	46
Easiness (entire workshop)	5.07	46
Enjoyment (entire workshop)	4.79	46
Helpfulness (entire workshop)	4.17	46
Gender	1.57	46

We then calculated a *Pearson correlation* to check whether there is a correlation between the variables. This was done mainly in preparation for a subsequent regression analysis. Correlations significant at $p \leq 0.05$ are indicated in bold and *Pearson correlation* coefficients are used for correlations between metric variables. According to the effect sizes according to Cohen (1988), we can then state in our analysis (correlation matrix, Table 4): Easiness and enjoyment correlate with a medium effect (above 0.5), as do helpfulness and enjoyment. Furthermore, helpfulness and easiness correlate with a small effect (up to 0.5).

Table 4. Correlation matrix.

	Variable	N	1	2	3	4	5
1	Pulse (entire workshop)	46	1				
2	Easiness (entire workshop)	46	.247	1			
3	Enjoyment (entire workshop)	46	.024	.560	1		
4	Helpfulness (entire workshop)	46	-.133	.352	.574	1	
5	Gender	46	-.084	-.100	-.075	-.247	1

Below in Figure 5 is a linear regression to our value 0.560 (see Table 4).

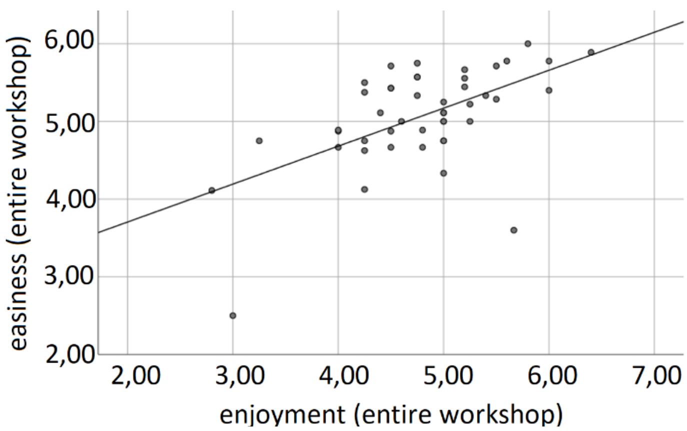

Figure 5. Simple scatter plot with linear regression of easiness and enjoyment (graphic belongs to value 0.560).

In our multiple regression analysis (see Table 5), we address *H3* “Enjoyment correlates with easiness” and *H4* “Helpfulness correlates with easiness” in addition to *H2* “HR correlates with students' easiness in the given mathematical teaching-learning process”.

Table 5. Multiple regression analysis on the dependent variable easiness (entire workshop). Here: * $p < 0.10$; ** $p < 0.05$ (see Hair et al., 2018 for significance level α).

Independent variable	Standardized coefficients β	p value
Pulse (entire workshop)	.246	.059*
Enjoyment (entire workshop)	.499	.002**
Helpfulness (entire workshop)	.093	.560
Gender	-.019	.884
R^2	.375	
Corrected R^2	.314	
N	46	

We can confirm our *second hypothesis* (see Table 5). The independent variable HR correlates with the dependent variable easiness. We see a positive correlation, with a beta value of 0.246 and a significance of $p<0.1$. We can therefore say that a higher HR correlates with a higher perception of easiness. According to the work of Khamis and Kepler (2010) and Hair et al. (2018), we can state with $N=5k+20$ that we can include four independent variables for the regression calculation (for a "statistical power"). Our four variables are HR, enjoyment, helpfulness and gender. This brings us to the confirmation of *H3*. With a positive beta value of 0.499 and a significance of $p<0.05$ (see Table 5), we can conclude that: A higher the enjoyment correlates with a higher perception of easiness. Unfortunately, we have to reject our *hypothesis H4* and cannot confirm it in our regression. There was no significance here with $p=0.560$. Furthermore, it should be mentioned that gender serves as a control variable to eliminate this effect from our regression and also to control for it.

5 Discussion and conclusion

For our four hypotheses regarding the measurement of motivational and affective aspects in knowledge development processes of students in empirical-oriented mathematics classes, we can conclude as follows: We were able to confirm our *hypotheses*

H1, H2 and *H3*. The confirmation of *H1* might be seen as an indication for cognitive engagement with regard to Isoda's and Nakagoshi's statement that "changing HR can be expressed in terms of arousal of the student's mind" (Isoda & Nakagoshi, 2000, p. 93) and that gradual increasing of HR represents concentrated thinking. The confirmed correlation in *H2* between heart rate and perceived easiness can be interpreted as follows. Presumably, hormones such as adrenaline, noradrenaline or cortisol, for example, lead to an increase in performance, but this was obviously perceived as positive. This is so-called eustress. This is positive stress because, for example, the tasks were performed well, a result was obtained, or one feels good and confident in the situation. We had to reject *H4* for our data set. We can confirm the results of Rennie (1994) and Woithe (2020) and extend them by measuring the constructs in a mathematical workshop. We have thus placed the three constructs in a different context and expanded them. We have also examined the results of Isoda's and Nakagoshi's (2000) case study in the area for N=46. A higher number of subjects would certainly be useful for more detailed statements. Naturally, our results are subject to some limitations. First, we used our three constructs and the heart rate to infer learner motivation. There were only a limited number of items for each construct in the survey study. For further research it would have to be considered that not only the constructs easiness, enjoyment and helpfulness but also many more aspects need to be considered when investigating motivational and affective aspects. Second, the heart rate can also be dependent on other factors. For instance, variables like time of measurement, after or before a meal, previous school lesson (e.g., physical education), breathing, age, body size or also grades of the students. The age of the students would be another interesting variable. Third, our analyses are based on a relatively small sample of 46 students. Despite these limitations, we believe that our results are valuable to discussion in mathematics education. It is one of the first quantitative studies to bring together constructs for measuring motivational and affective aspects (in an empirical-oriented mathematics classes) with a heart rate measurement (and thus digital tools); thus providing the linkage of affective constructs and physiological components addressed in Section *Research approaches and hypotheses*, addressing Hannula's "insufficiently explored venues that call for additional research" (Goldin et al., 2016, p. 2). In this regard, our results show that they provide an extension for already established constructs describing motivational and affective aspects by heart-rate measurement and put them on a broader basis for discussion.

With this and beyond, the study offers future links. For example, it is possible to add facial features similar to the case study by Isoda and Nakagoshi (2000) or to combine the results on affective knowledge structures with a cognitive dimension in the concept of DSE according to Bauersfeld (1988). In the long run, it would be interesting for (mathematics) teachers to know which phases of the lesson or which tasks (e.g., problem solving or drill training) particularly motivate learners. Thus, we hope that our results provide some valuable insights for further studies of motivational and affective aspects in mathematics education.

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