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Research article

Positioning in mathematics: 6-year-old students collaborating on combinatorics

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Abstract: In mathematics education, student collaboration is often emphasised as desirable. The focus of this paper is on students' positioning during collaborative problem solving and problem posing on the mathematical content combinatorics. The study utilises the Learning Design Sequence model for lesson design and positioning theory as an analytical framework. In a sequence of lessons, students worked with both problem solving and problem posing, using different ways of representing their solutions, such as pictures, physical objects, and digital animations. Two cases of collaboration between 6-year-old students are used to illustrate their positioning during this sequence of lessons. One conclusion is that, when collaborating, students' experiences of each other as learners in general and mathematics learners in particular have influence on their positioning which in turn may influence their possibilities of meaning making in mathematics.

Keywords: positioning, mathematics, collaboration, problem solving, digital technology

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1 Introduction

The empirical material in this study derives from an intervention in which 6-year-old Swedish students were introduced to problem solving and problem posing in mathematics. In mathematics education research in general, and specifically in studies on problem solving, collaboration between students is frequently emphasised as something desirable (e.g., Liljedahl, 2021). However, there are also studies indicating that collaboration when working on problem solving can be challenging for young students (see, for example, Palmér & van Bommel, 2015).

In the sequence of lessons used as an example in this paper, the students are supposed to solve a combinatorial task where they are to consider in how many ways three toy bears in different colours can be arranged in a row on a sofa. To make the task meaningful for young students, it is presented as a conflict between the toy bears, who cannot agree on where each toy bear should sit on the sofa. One toy bear then suggests that they could change places every day. The task for the students is to determine how many consecutive days the bears could sit on the sofa in different arrangements. Thus, it is an enumerative

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combinatorial task where the students are to count the permutations for n = 3. In the sequence of lessons, the students first work on the task together in pairs using paper and pen. Then, they record their solutions in a digital animation, and finally, they are to pose a similar task to a peer. Thus, they work with both problem solving and problem posing, using different ways of representing their solutions, such as pictures, physical objects, and digital animations. Students' solutions to this task have previously been studied by Palmér and van Bommel (e.g., 2018, 2020) as well as Ebbelind et al. (2023). However, the focus of this paper is not on the students' solutions per se but on their positioning when collaborating on the sequence of problem-solving and problem-posing tasks presented above. Two research questions are elaborated on: 1) In what ways do the students position themselves and others in the sequence of problem-solving and problem-posing lessons? 2) How (if at all) does students' positioning appear to influence their possibilities of meaning making in mathematics?

2 Literature review

As mentioned, in a sequence of lessons, the students in this study first use paper and pen when working on problem solving. After that they record their solutions in a digital animation. Finally, they work on problem posing by posing a similar task to a peer.

The importance of young students becoming familiar and comfortable with problem solving and problem posing has been pointed out by several researchers (see, for example, English, 2004; English & Sriraman, 2010). However, few studies focus on problem solving in early mathematics, and even fewer focus on problem posing in early mathematics (e.g., Cai et al., 2015; English & Sriraman, 2010). Yet, incorporating problem posing as part of problem solving has been shown to develop students' problem-posing as well as their problem-solving skills (e.g., Ellerton et al., 2015; Palmér & van Bommel, 2020). Also, some studies have shown the successful use of problem posing in early mathematics education (e.g., Palmér & van Bommel 2018, 2020).

In Sweden, the use of digital technology is included in the curricula for all grades. Studies of digital technology in mathematics education show varied results – positive, neutral, and negative which can be linked to the digital tool used, the way it was used, and the purpose for which it was used (Baki et al., 2011). For example, simply adding an interactive whiteboard does not turn a classroom into an interactive teaching environment. On the contrary, an interactive whiteboard can lead to less interaction (Bourbour, 2015; Higgins et al., 2007). According to Clements et al. (2024), digital technology, if used in an interactive, supportive and research-validated context, can make substantial contributions to early childhood education. Craft (2012) emphasizes the importance of young children becoming producers of technology rather than consumers. This point is also emphasized in a literature-based study of issues and challenges in the relationship between technology and pedagogy in early education (Murcia et al., 2018). Being producers can help children become critical and reflective about how they use digital tools in the future. Further, digital technology can support children's learning of different content, as it can combine visual presentations, animations, and sounds, offering a multimodal environment in which children can simultaneously see, hear and react to what they are experiencing. Such multimodal environments enable young children to interconnect representations (Trigueros et al., 2014). Also, open, challenging, and exploratory questions have proven to be successful in connection with the use of digital technology in early mathematics education (Sarama & Clements, 2009). Further, studies indicate that when both analogue and digital resources are used, these often add value to each other (Maschietto & Soury-Lavergne, 2013).

As will be further elaborated on in the next section, in this study, we use positioning theory to analyse collaboration when students work on a problem solving and problem posing. Using positioning theory implies that we focus on how individuals are situating themselves or being situated by others in diverse situations. Campbell and Hodges (2020) have used positioning theory in a study on students' interactions during group work in mathematics with a focus on how students' positions and storylines influence their participation and collaboration. The results show five different types of collaborations among students when solving a challenging mathematical task in groups of three. These include confirming the initiatives of one group member, competition between group members, all members contributing with ideas with little coordination or consideration of each other's suggestions, two members working together effectively while the third either disengages or is excluded, and all members collaborating, sharing and refining ideas. Of these, the last was the most equitable and productive collaboration while collaboration characterized by all members contributing with ideas with little coordination or consideration of each other's suggestions was least productive. These findings emphasize the importance of fostering effective collaboration in mathematics education. Further, Campbell and Sears (2024) have used positioning theory to examine how social factors influence students' participation in collaborative mathematical tasks. They identified five factors of importance for successful collaboration: (1) shared language, (2) access to the chalkboard and resources, (3) tone of voice, (4) teacher intervention, and (5) peer contestation. Wood (2013) studied how students' positioning within a classroom setting can shift on a moment-to-moment basis due to peer and teacher positioning. Such shifts influenced the learning experiences of the students. Positive positioning fostered engagement and learning opportunities while negative positioning limited active participation and reduced learning opportunities. DeJarnette (2018) has used positioning theory to compare collaboration in traditional group work and group work when programming. The study shows that when working with programming, one student often handles the digital tool and lower-level tasks while the other provides higher-level guidance. In the study, students' positioning was influenced by the context, the design of the task, and the norms for collaboration. Even when the final results were the desired ones, the tasks did not necessarily foster deep collaboration or shared problem solving. The study highlights the importance of teachers encouraging discussions on the intended mathematical ideas. Also, sharing mathematical insights collectively were of importance. The role of the teacher in mathematics collaboration has been investigated in studies using positioning theory showing that how teachers position their students, for example promoting shared responsibility, exploration of advanced strategies, and peer explanations, influence students' opportunities to engage in and learn mathematics (Tait-McCutcheon & Loveridge, 2016).

3 Learning Design Sequence and positioning theory

In this study, the Learning Design Sequence (LDS) model is used as a basis for the design of the lessons, and positioning theory is used as an analytical tool.

3.1 Learning Design Sequence

In the study, the LDS model (e.g. Björklund Boistrup & Selander, 2022; Selander, 2021) was used as a tool for planning the lessons where the students would work with problem solving and problem posing. One important premise of this theory is the multimodal perspective, which assumes that multiple modes are used in interaction. From this perspective, teachers' choices in teaching are seen as design *for* learning, while students' choices are described in terms of design *in* learning. The teacher's design for learning includes choices concerning activities, learning tools, and other resources provided for the students. The students, as designers in learning, make choices on how to take part in the activities as well as what resources to use and how.

The LDS model comprises three main parts: i) framing and setting, ii) primary transformation unit, and iii) secondary transformation unit. A teacher's overall design for learning relates to, for example, curriculum documents and various institutional norms and regulations. The teacher also has a purpose for the planned activities, in our case to have the young students explore combinatorics through problem solving and problem posing. During the primary transformation unit, students participate in various activities, use available resources, and transform the content by reading, listening, acting, speaking, drawing, writing, etc. The primary transformation unit may lead to students creating some form of representation to communicate their ideas about the content. During the secondary transformation unit, the students' representation and activities during the primary transformation unit may be the focus of meta reflective conversations between the teacher and students.

Furthermore, in this model, the teacher's choices (e.g., of activities) position students in various ways (e.g., capable of taking responsibility for their learning). Similarly, students position themselves, the teacher, and each other in different ways through their choices.

3.2 Positioning theory

According to positioning theory, the roles people adopt in social relationships, such as classroom discourses, are not fixed but emerge in interaction. Further, systems of meaning "regulate rather than cause human behavior, with the implication that individuals have some choice as to which rules and norms they follow in relation to the projects they have in hand and what they take the social and physical environment to be" (Harré & Moghaddam, 2014, p.129). Positioning involves either individuals situating themselves or being situated by others in diverse situations. As such, positioning is a dynamic process influenced by factors such as familiarity with a situation, experience level, or perceived status. According to Meschitti (2019) there are three ways to position oneself in interactions:

willingness (the extent to which someone is prepared to position others and themselves), capability (the extent to which someone can take the position offered), and power (the extent to which someone is permitted, allowed, or encouraged to take the positions). In essence, how a person positions themselves or is positioned evolves and varies based on specific situations or contexts, sometimes occurring almost simultaneously (Davies & Harré, 1999).

Positioning theory includes a three-part framework of *storylines*, *speech acts*, and *positions*. Storylines are narrative conventions that emerge as people interact and often adhere to established recurring narrative structures that are either part of the shared cultural repertoire or are created through the participants' interactions. In a mathematics classroom, the storyline is about the positions being created, as well as the learning and the mathematics content itself. Speech acts are significant actions, movements, and forms of communication that comprise interaction. An example of a storyline (presented in detail in the result) is a student promoting an idea on how to continue a collaboration on a mathematical task. This idea aligns well with the shared cultural repertoire of the classroom, which emphasises persistence. The teacher aligns with the storyline of continuing to work and as such, thus, her action becomes significant in the established narrative of the boy. Therefore, speech acts build and reproduce storylines that invoke rights and duties, that is, positions (Harré et al., 2009).

4 The study

This qualitative case study was conducted with 45 students from two Swedish preschool classes (6-year-olds). In Sweden, 6-year-old students are enrolled in preschool class, which is an obligatory year of schooling in the transition between preschool and compulsory school. The teachers in these classes have, for several years, been collaborating with two of the researchers in this study. Due to this collaboration, they are well acquainted with the combinatorial task, since they have conducted it previously with other groups of students. However, the part of the task where students create digital animations was new to the teachers.

Following Swedish ethical research guidelines, parents gave written consent for their children's participation in the study (Swedish Research Council, 2017). To ensure the students' own consent for participation, their verbal and non-verbal expressions were carefully considered during the sequence of lessons in the classroom.

4.1 The sequence of lessons

The learning design sequence of the combinatorial task was divided into three lessons. The first lesson started with the teacher showing the students three toy bears in three different colours and explaining the dilemma of choosing seats on the sofa (setting). Then the students were divided into pairs to solve the task together (the primary transformation unit). The students were given paper and coloured pens. With these, they were expected to create representations based on the toy bears, thus transforming from one mode of representation (manipulatives) to another (visual representation on paper). The students could choose how they wanted to document their solutions (e.g., drawing pictures of bears or representing the bears with lines or dots). The first lesson ended with the teacher gathering all the students and inviting some of the pairs to present their documentation (the secondary transformation unit). During the presentation, the teachers posed questions focused on similarities and differences in the students' documentations, how many different solutions (permutations) the students had found, how they had structured their solutions, and whether and how they knew that they had found all permutations (referred to as "combinations" during instruction).

The second lesson was conducted the following day, with students working in the same pairs as in the first lesson. This lesson was led by one of the researchers, who asked the students to re-design the solution they had come up with the day before (setting) in a digital animation using a tablet. When creating the animation, the students used the documentation they had produced the previous day (the primary transformation unit). The toy bears were photographed because the students wanted to attach the heads of the bears to the bodies in the application used to create the digital animations. After the animations were created, the students and the researcher watched them together (the secondary transformation unit).

The third and final lesson was conducted on the same day as the second lesson and with the same pairs. The students were reminded of the task they had worked on in the two previous lessons, and they were asked to pose a similar task to a peer (setting). The students were free to document their task using any material they wished (the primary transformation unit). In some cases, the students asked the teacher to complement the documentations of their tasks by writing the posed question.

4. 2 Analysis

The empirical materials analysed included the video documentations from the three lessons, along with the students' paper-and-pen documentations from the first lesson, the animations from the second lesson, and the posed tasks from the third lesson.

To analyse these empirical materials, we used questions developed by Wagner and Herbel-Eisenmann (2009). First, we asked: Who does mathematics, and when in the sequence do they do it? By asking this, we were able to identify when the participants engaged in the problem-solving and problem-posing tasks. Second, we asked: What mathematical processes are the students engaged in? By asking this, we were able to identify the different actions the students performed. Finally, we asked: Who are these students doing these things for, and why? By asking this, we could identify what was valued by the students and the different positions that were available and taken by the students in the collaboration.

5 Results

In the following, two cases of two 6-year-old students are used to illustrate in what ways students position themselves and others in the sequence of problem-solving and problem-posing lessons and how this may influence their possibilities of meaning making in mathematics.

5.1 Lesson one

In both cases, two students sit next to each other at a table. On the table there is one piece of paper, lead pencils, and three coloured pens: one blue, one red, and one yellow – the same colours as the plastic bears the teachers showed when introducing the task (figure 1a).

Group 1 Felicia and Anthony
Anthony says to Felicia that she needs to draw a big sofa, which she does. Felicia then finalises the drawing of the first permutation.

> Felicia: Now we are done.
> Anthony: No, we cannot have only one [solution]. We cannot have only one. It is not only one.

At the same time the teacher comes by. Felicia shows her their paper saying, “We are done.” Anthony says “No.” The teacher asks, “What was the task? In how many different ways [can the bears sit on the sofa]?”

> Anthony: It was only Felicia who said it.
> Teacher: You have to agree.
> Anthony: This is one way [they can sit].
> Teacher: Exactly, this is one way. You are not finished. The question is in how many ways. You have to agree.

The teacher walks away. Anthony starts to draw a new sofa. Felicia takes the paper from him and complains about the way he has drawn the sofa. The teacher walks by and takes the paper from Felicia and gives it to Anthony. Anthony draws a new permutation while talking to Felicia, saying that he is drawing dots instead of bears. Felicia is watching what he is doing. Then he gives the paper to Felicia, saying, “Now you can draw the next one.” While she is drawing, he does not look at what she is doing; instead, he turns his back towards her, and he is looking around in the classroom (figure 1b). When Felicia has finished, Anthony looks at the paper and says, “No, now you drew in the same way [...] Now we did the same thing [solution] again.” Then he shows Felicia how to draw a new permutation and says, “You must put the yellow [bear] in the middle.” When they have three permutations, he turns to the teacher and asks, “Are there three ways or are there more?”

Teacher: How many [permutations] have you found?
Anthony: The yellow [bear] has sat in all the places [on the sofa], the blue has sat in all the places, and the red has sat in all the places.
Teacher: Okay!
Felicia: Is there one more way?

The teacher walks away without answering the question. Anthony then concludes that "There are more ways because otherwise she would not do that." After giving it some thought, he draws three more solutions on the paper while Felicia looks at what he is doing. Thus, their final documentation shows six permutations (figure 1c).

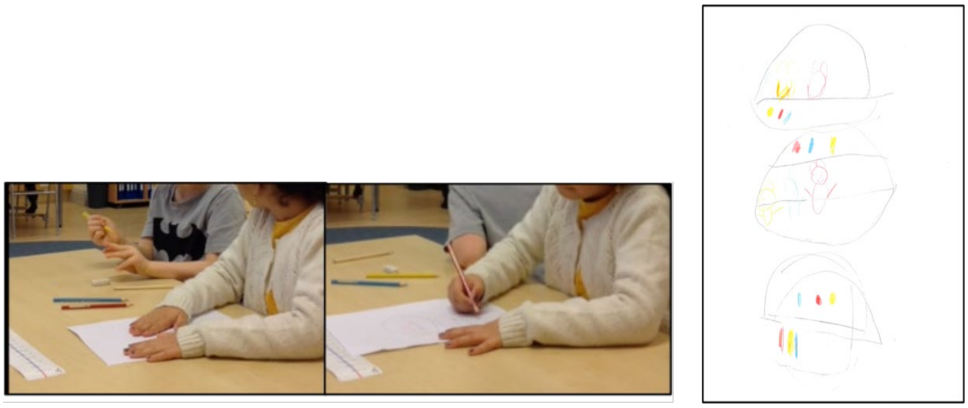

Figure 1. a) The students working on the task. On the table there is one piece of paper, lead pencils, and three coloured pens. b) While Felicia is drawing, Anthony does not look at what she is doing; instead, he turns his back towards her, and he is looking around in the classroom c) The students' documentation of the six permutations.

During lesson one, Anthony and Felicia follow two different storylines. Felicia initially follows a storyline where she appears to believe that they have completed the task with one permutation. Anthony follows a storyline that focuses on drawing new permutations, appearing to be sure that there must be more than one. Then, through the way the teacher is acting, significant action, the teacher, through a speech act, both reproduces and builds up a storyline in line with Anthony's by promoting the need for multiple permutations. Also, she takes the paper from Felicia and gives it to Anthony. The teacher's significant action offers Anthony a position as someone who knows how to solve the task. Anthony positions himself by initiating the suggestion to draw a big sofa and later to draw new permutations. These actions position him as someone who knows how to solve the task. He aligns himself with the teacher's expectation, recreating the storyline of finding multiple solutions by drawing new permutations and insisting on the need for more than one solution. Felicia initially aligns herself with Anthony's position but later resists his ideas, complaining about the way he draws the sofa, suggesting a different approach. As such, Felicia, through her actions, tries to move away to another storyline. However, she fails to get a new position and draw new permutations. To summarise this part of the sequence of lessons, both actors are engaged in the problem solving, but only Anthony has grasped the mathematical content of combinatorics. Further, the students value different things, where Anthony (and the teacher) value and jointly build a storyline, finding new permutations, while Felicia values drawn pictures. These two positions can be understood as the two available based on the significant actions, speech acts, of the teacher and Felicia not understanding the mathematical content.

Group 2 Dina and Paul

Dina puts the three coloured pens on the paper and starts to move them into new positions (figure 2a). She looks alternately at the example the teacher showed during the introduction, at the pencils, and at Paul.

> Dina: You could do like this? I think we have to do like this.
> Paul: You could draw one blue here? And then one green here. *He points at the paper where the pens lie.*

Paul starts to draw a green square where the green pen lies. Then, Dina draws a red square where the red pen lies. Finally, Paul draws a blue square where the blue pen lies. Then the teacher comes by.

> Teacher: Yes. Tell me how did you think?
> Dina: That the blue one sits here. The red one sits here. The green one sits here. *Points at the drawn boxes.*
> Teacher: Yes. Can you think of any other way they can sit? And how could you show that? *No answer from the students.* Now you have one way. But they wanted to sit in several different ways. To make it fair.
> Dina: You could do like this. *Shows with the green pen how she can draw a green box around the red box.*
> Teacher: Yes, then you show that it can sit there too. That is one way. *Walks away*
> Paul: I can be blue.

The two students take turns drawing squares in different colors. They talk while drawing, discussing that there are three different ways. They draw one new square around the previous at a time until they have three squares in different colors in each place (figure 2b). Each layer symbolizes a permutation, as further indicated by the numeral three. Then the teacher comes by.

> Teacher: Tell me, how were you thinking?
> Dina: That everyone would get a seat. So, we drew the colors on each other.
> T: Exactly. So, everyone can sit in all the seats. How many ways will that be?
> D: Three. *Tells the teacher the three ways while pointing at the documentation.*
> Teacher: Could there be any more ways than these three? Think about it a bit. I think it is a great solution you have come up with.

Again, Dina puts the three coloured pens on the paper and starts to move them into new positions. She looks alternately at the example the teacher showed during the introduction, at the pencils, and at Paul. However, they do not find any new permutation and Paul instead suggests that they can colour their documentation. For the next 15 minutes, they collaborate on colouring the squares on the documentation.

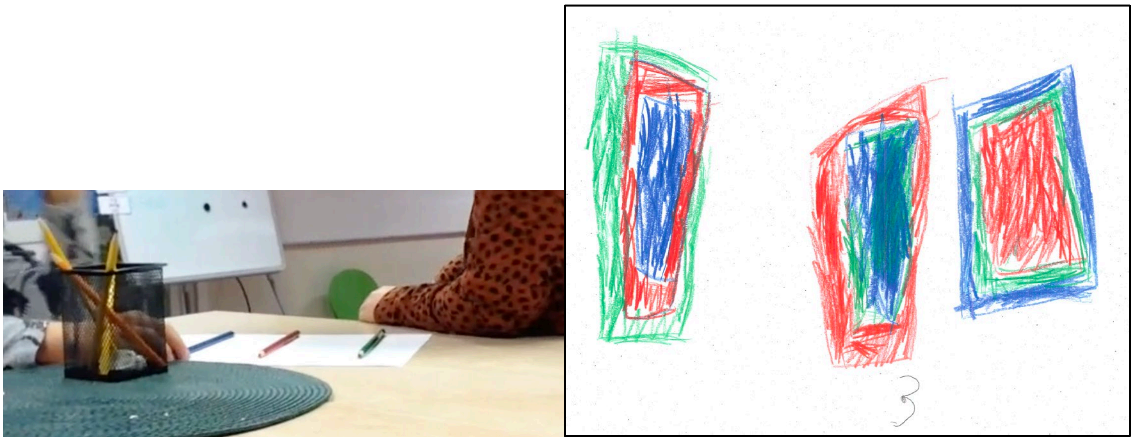

Figure 2. a) The students working on the task with the pens. b) The students' documentation of the solutions. The inner squares show the first permutation (blue, green, red), the next layer the second permutation (red, blue, green) the outer layer the third permutation (green, red, blue). Each layer symbolises a permutation, as further indicated by the numeral three.

During lesson one, Dina and Paul follow the same storyline where they both believe that they have completed the task with three permutations. Dina is the one who takes the initiative to use the pens, and she is the one who moves them. When doing this, she is constantly talking to and making eye contact with Paul. It is Paul who suggests how they can draw their documentation, and they take turns drawing. The teacher tries to introduce another storyline through a speech act, promoting the possibility of more permutations. The teacher's action offers both Dina and Paul positions as someone who knows how to solve the task. However, Dina and Paul instead turn the combinatorial task into a drawing task, not following the teacher's attempt to build up another storyline. To summarise this part of the sequence of lessons, both actors are initially engaged in and collaborate on the problem solving where they both to some degrees have grasped the mathematical content of combinatorics. Further, when neither of them takes the problem solving further after finding three permutations, in tacit agreement they transform the task into a drawing task. As such, they build up another storyline through their actions.

5.2 Lesson two

Group 1 Felicia and Anthony
The researcher asks Felicia and Anthony if they remember what the task was about. Both

students explain that it was about how many different ways three bears could sit on the sofa. The researcher confirms this and says they will record a story about how the three bears can be seated on a sofa. Anthony says, “Yes, like, once upon a time there were three bears. Then you say, first the blue sat, then the red sat, then the yellow sat.” The researcher confirms what Anthony says. Then Anthony points at the tablet on the table and says, “I am not good at that” (they have been shown the application before). At first, Anthony has both the paper with the documentation and the tablet, and he asks Felicia, “Am I to do everything?” Felicia nods. But when Anthony starts recording, she takes the tablet, wanting to help. But then something happens with the application/recording, and they have to start all over. The researcher helps them start over by saying, “Is Felicia to do [the animation] and you tell [the story]?” Anthony confirms this by nodding. Anthony holds the documentation and tells the story, while Felicia moves the figures in the animation based on his story (Figure 2a). On one occasion he corrects Felicia by repeating what he said before and looking at her at the same time: “The yellow [bear] sat last [to the right], I said.” When they have finished, Felicia looks at the researcher and asks, “Was it good?” The researcher asks them what they think, and Felicia answers, “Yes, no one disappeared.” She is referring to the creation of the animation where the figures must be kept within a frame as they otherwise disappear. After this, they look at their animation together (Figure 2b and 2c).

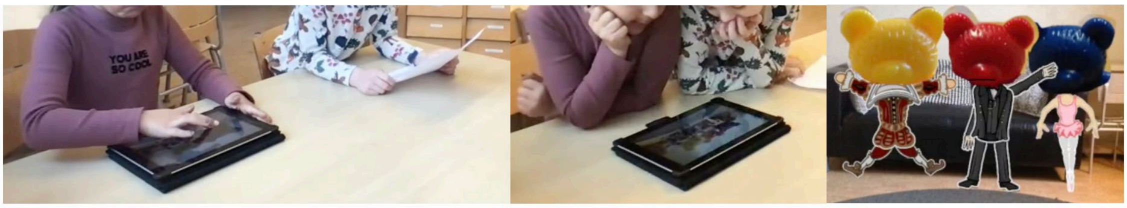

Figure 3. a) The students create their digital animation. b) The students look at their digital animation. c) example from the digital animation

Above, the storyline revolves around recording a story about the three bears. The researcher sets this storyline through his communicative actions. Anthony immediately suggests a traditional storytelling format, which the researcher aligns with, while Felicia engages in animating the story. Anthony's storyline involves hesitation with technology and his subsequent role as the storyteller. As the storyteller, he guides Felicia through his actions, helping her create the animation. Anthony positions himself as someone who contributes to the task by suggesting how the story should be told and, at the same time, expressing his limitations with technology. He aligns himself with the task by offering suggestions for how the story should be told. In doing so, he is building the storyline through his actions and ultimately assuming the role of storyteller. Felicia's storyline focuses on her role in creating the animation. She positions herself as someone who actively participates in the task by moving the figures in the animation based on Anthony's storytelling. Felicia aligns herself with the task and seeks feedback on the final product. The collaborative nature of the task highlights the interdependence between Anthony and Felicia as they work together to complete the animation under the guidance of the researcher. To summarise this part of the sequence of lessons, both actors are engaged in the recording. Still, as Anthony is the one who grasped the mathematical content of combinatorics, he positions himself as the storyteller. At the same time, through his significant actions, he positions Felicia as the one who knows how to work with the application. Both students in this sequence value the same thing, that is, making a good animation, and there are two different roles available and needed in the situation.

Group 2 Dina and Paul

Both the researcher and the teacher are present. The teacher introduces the task to Dina and Paul. She points at their documentation (figure 3a) and reminds them of the task.

> Teacher: When you did the task, you found three different ways. The students nod. How many did we come up with when we did it together?
> Paul: Six.
> Teacher: Six. Yes, there were more than three. What made it more than three? How could you think? Do you remember?
> Dina: That everyone can sit in one place.
> Teacher: Then there were three ways. But what if there were even more ways?
> Dina: I don't know

The teacher says that one bear can stay in the same place while the other two shift places. Then they open the application on the tablet. The teacher points at the documentation and tells how the bears should be placed. Dina moves the figures on the screen according to the teacher's instructions. After recording the three permutations in the documentation, the teacher asks if they can find more ways. Dina changes places on two bears. The teacher confirms that this is a new way and says, "you can do the same with two others, then it will be a new way". Now the researcher suggests the students to make a new recording where only their voices will be heard. The students are mostly silent during this recording but after each permutation Dina says, "that was one way". Dina is the one handling the application (figure 3b). After recording the three permutations from their documentation she says "that is what we thought then. Those were the three ways we thought of". Both Dina and Paul look at the teacher and the researcher, the researcher asks, "can you think of more ways now?" Dina and Paul take turns and record four more permutations where one is a duplicate of one of the first three. Then Dina says, "we are done".

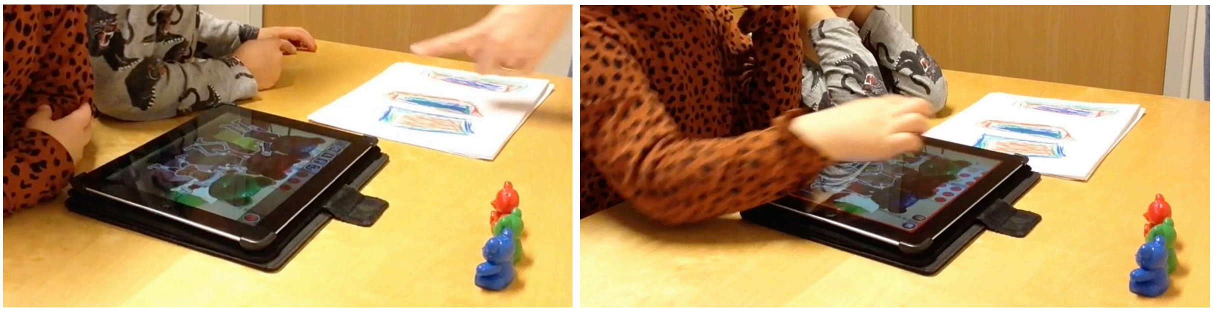

Figure 4. a) The teacher pointing at the documentation telling how the bears should be placed. b) The student making the first recording.

Above, the teacher, the researcher, and the students have three different storylines. The teacher, through her actions, is instructing, wanting the students to understand the mathematical content of how to find all six permutations and thus to solve the task. The researcher wants the students to demonstrate their understanding of the task. The focus of the students is on their solution of the task in the first lesson. Based on the entire sequence, it seems like the students understand that there are more than three permutations and how to obtain them. Still, their focus is on the three permutations they found initially, the ones on their documentation. First, Dina positions herself as someone who, based on the teacher's instructions, moves the characters in the animation. She aligns herself through her actions with the teacher' storyline. Later, when recording without instructions from the teacher, both students position themself as someone who moves the figures in the animation. The students collaborate to complete the animation and seek feedback on their final product. To summarise this part of the sequence of lessons, both students are engaged in the recording, and they seem to have grasped the mathematical content of combinatorics.

5.3 Lesson three

The student pairs are seated next to each other at different tables in the classroom. On the tables, there is one white paper and several coloured pens. The students have been told to pose a task similar to the one they have been working on during the two previous lessons.

Group 1 Felicia and Anthony

> Anthony: Should we do ice-cream cones and see how many kinds of sprinkles you can have on them?
> Felicia: Didn't the teacher say it was to be about bears?
> Anthony: No, you do not have to [have bears in the task]. We are supposed to do a similar task.

Felicia then reaches out to the teacher and asks, "Was it [the task] to be about bears?" The teacher answers, "What did you work on before?" Anthony looks at the teacher and says, "Then we should not do one with bears, Felicia." They decide to make a task with ice creams, and Felicia starts to draw an ice cream. Anthony wants to show her what a cone should look like, but she refuses to hand the paper and pen to him. When she has drawn one ice cream, though, she does hand the paper to Anthony, who continues to draw. Felicia, however, does not accept his drawing. Instead, she takes the paper and draws the rest of the ice creams. Anthony suggests a posed question on combinations of ice cream flavours and toppings. However, Felicia refuses this solution. Then the teacher comes by, and Anthony tells her about another task about three ice creams and three ice cream trucks and tells the teacher what she should write on their paper: Draw a line from the correct ice cream to the correct ice-cream car. (Figure 5a and b).


Figure 5. a) Anthony tells the teacher how she should pose the problem in writing on their paper. b) In the image, the students have drawn three ice creams and three ice cream trucks. The posed task: *Draw a line from the correct ice cream to the correct ice-cream car*.

In this final lesson, the storyline initially revolves around negotiating the task, with Anthony suggesting a posed task showing that he understands the task and the mathematical content. Nonetheless, he decides to go along with Felicia's idea when she seeks clarification from the teacher. Anthony's storyline involves suggesting a task centred around ice-cream cones, while Felicia's storyline centres on her role as the illustrator, initially refusing Anthony's guidance but eventually accepting it. Anthony initially positions himself as someone who suggests a task and later as a guide, offering instructions to Felicia as she draws the ice creams. Felicia initially positions herself as someone who seeks clarification from the teacher, indicating a desire to adhere to the instructions given. Felicia's significant actions in this episode are speech acts that build a storyline that sets the positions available, and Anthony aligns with. To summarise, both actors are engaged in the problem posing, but it is still apparent that only Anthony has grasped the mathematical content of combinatorics. Further, the students still value different things, where Anthony values posing a task with the correct content, while Felicia values nicely drawn pictures. As before, these two roles can be understood as the two available ones, based on the actions of the teacher and Felicia not understanding the mathematical content. The result is a posed task without either the same content or the same context as the original task.

Group 2 Dina and Paul

> Teacher: What task would you like the classmates to get?
> Dina: We would like to do something with animals.
> Teacher: With animals. Great. Then you have a start. Then you have to think about what animals and what the classmates are to do when they get your paper.
> Paul: Should we have a dog? I can draw dogs.

For about 20 minutes, Dina and Paul take turns drawing animals on the paper (figure 5). The teacher comes by, asking them what the classmates are to do when they get their paper. Dina and Paul answer that they are thinking about that. They continue to take turns in drawing on the paper. Later, when the teacher comes again asking the same questions the students answer that it is about where the animals live.

> Pauly: It could be a kind of similar as the toy bears.
> Dina: If you move this. On paper. And then you are to find where they should be.
> Paul: Yes. We draw like this. A bird, or I do not know.

The students continue to draw on the paper. They draw three animals, two birds and one dog, and two dwellings, one tree and one house. At the end of the lesson, the teacher helps them to write down the task on paper. Paul says, “to draw a line”. Dina says, “some should be right and some wrong”. Paul formulates the task that the teacher then writes down, “draw lines so that the things end up in the right place” (figure 6).

> Teacher: Was that how you thought, too, Dina?
> Dina: Yes.
> Teacher: That is good, so you agree.
> Teacher: What do you find similar between your task and the bear task we did before?
> Paul: There is a bear here too.
> Teacher: Yes. Anything else you think is similar?
> Dina: I think it is similar when you have to draw lines and move things.
> The teacher confirms that this is similar. The students cannot think of anything else that is similar and the lesson ends.

Figure 6. The students draw three animals, two birds and one dog, and two dwellings, one three and one house. The posed task: *Draw lines so that the things end up in the right place.*

In this final lesson, the students' storyline initially revolves around drawing animals. The teacher reminds them of the task, to pose a question for the classmates to solve. Both students position themselves as someone who knows how to solve this task but when collaborating, their focus is on what animals and dwellings to draw. They recreate the storyline of drawing animals throughout the lesson even though the teacher tries to intervene. To summarise this part of the sequence of lessons, both actors are engaged in the problem posing but the mathematical content of combinatorics is not that visible. Both students value the same thing, nicely drawn animals and dwellings. The result is a posed task without a clear connection to the mathematical content of the original task.

6 Discussion

In this paper, two cases when 6-year-old students work on problem solving and problem posing are used to elaborated on the ways the students position themselves and others and how (if at all) students' positioning appear to influence their possibilities of meaning making in mathematics. The results of the study should be understood in the context of the Swedish preschool class, that is, a school year in the transition between preschool and compulsory school for six-year-old children. Of importance is also the experience of the teachers who have long experience of teaching both problem solving and problem posing. Although the focus of the article is not on the teachers but on the students, the teachers are part of the context, and despite their experience, it is not straightforward how to notice and influence what happens between students when they collaborate. And, as visible in the results and discussed below, the speech acts of the teachers clearly influence the positioning between the students. The students both align with and do not align with the teacher's speech acts. This could be further elaborated on in future studies, for example in interventions in which teachers focus on positioning explicitly with the intention of promoting mathematical inclusion of both students.

Overall, using positioning theory, the analysis shows how the students position themselves and each other during the three lessons. Of the three ways to position oneself in interactions highlighted by Meschitti (2019) all are visible (willingness, capability and power).

In the first group, Felicia follows and builds her usual storyline, portraying herself in actions as the well-behaved girl who can draw and write nicely and listens diligently to the teacher's instructions. However, in the sequence of lessons, it becomes evident that she has not grasped the task or the mathematical content, unlike Anthony. He, on the other hand, understands the task and works mathematically to find the permutations, which Felicia does not appreciate because it challenges her position. Maybe this can be explained by the fact that Anthony, according to the teacher, usually does not follow a storyline where he is the one who understands the tasks and listens to what the teacher tells him to do. Thus, the "usual" storylines of these students may influence how they negotiate their roles and relationships, and how they align with different positions. When they are to pose a task, Anthony again shows that he understands the mathematical content. However, Felicia does not show any indications of understanding the task, and since the teacher has not heard what Anthony suggested regarding the ice creams, their posed task is not in line with the original task. It is evident that Anthony builds a storyline in lesson one with the teacher's support. Thus, their previous experiences of themselves and each other as learners of mathematics may be influenced by and may influence their positioning. Even though Anthony has the capability in the final lesson, Felicia and the teacher have the power. In this case, their positioning during collaboration appears to hinder the focus on mathematics and thus also both students' meaning making in mathematics.

In the second group, both students, according to the teacher, follow their usual storyline, portraying themselves as students who listen diligently to the teacher's instructions, attempt to solve the tasks, and are able to draw neatly. In the sequence of lessons, they collaborate, both on the mathematical content and on the drawings. In the second lesson, they show that they can illustrate their initial solution as well as understand the mathematical content of the task. Even though the students do work on the task, they spend most of the time focusing on drawing, 15 of 22 minutes in the first lesson and 35 of 40 minutes in the third lesson. As the students in collaboration are constantly working on their documentation and answer on the teacher's questions when she comes by, it is likely that the teacher thinks they are working more on the mathematics content than they actually are. Regardless, their actions during the sequence of lessons recreate their positioning during collaboration and seem to contribute positively to both students' meaning making in mathematics as they go from a solution with three permutations to showing an understanding of how to produce six permutations. Their collaboration is in line with the most equitable and productive collaboration from Campbell and Hodges (2020), that is all members collaborating, sharing and refining ideas. Maybe this is due to the two students having similar capability and willingness to position themselves and each other with duties and rights, thus creating equal power.

The examples selected show both when collaboration may impose and may not impose meaning making in mathematics. In the first group, the non-contributory cooperation could have been counteracted if the teacher had heard that Anthony understood the mathematics content. Then the teacher could have helped also Felicia to understand the task and the collaboration could have developed differently. In the second lesson, there were two possible roles that were both needed. This contributed to both students participating actively in solving the task, but still without Felicia showing an understanding of the mathematical content. Their collaboration is in line with results from DeJarnette (2018) showing how when working with programming, one student often handles the digital tool and the lower-level tasks while the other student provides higher-level guidance. The fact that Felicia still does not seem to understand the mathematical content appears also to influence possible meaning making in mathematics when the students work on problem posing. In the second group, the teacher is keen that the students understand the content. This is visible when she in the second lesson initially takes over the animation, but perhaps that is what contributes to the shown meaning making of mathematics of both Dina and Paul.

The examples in this paper show that the often-encouraged collaboration between students during problem solving and problem posing (Liljedahl, 2021) may not always lead to meaning making in mathematics. This does not mean we believe that collaboration should be avoided. But, while young students need to become familiar and comfortable with problem solving and problem posing (English, 2004; English & Sriraman, 2010), when and how to collaborate needs to be further investigated. The results from this paper indicate the importance of the teacher guiding the students in their collaboration and ensuring that both students have sufficient understanding of the mathematics content. Thus, one conclusion drawn from the empirical examples in this paper is that, even if students are not told how to solve a task (not valued in problem solving), the teacher needs to ensure that all students in a collaborating group understand what the task is about. Also, the study indicates the importance of carefully selecting who collaborates, on the basis of experiences in both mathematics and collaboration. Otherwise, the meaning making in mathematics of all collaborators may be negatively influenced. This could constitute a direction for future research, in intervention studies focusing on teachers' actions and in longitudinal studies examining the development of students' positionings connected to meaning making in mathematics.

Author contributions

The researchers have been collaborating in a project on digital animations in early education. For this paper, the work has been distributed as follows:

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All authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all research participants.

Conflicts of interest

The authors declare no conflicts of interest.

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