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Research article

"Equal can be unequal sometimes": Grade 2 students' conceptions about working with sharing

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Abstract: This paper aims to analyse and discuss grade 2 students' conceptions about working with sharing. The students worked with cases in an education that differed from their regular mathematics classes, working silently with a textbook at their desks. When working with the cases, they worked in pairs, were encouraged to discuss, and then got to present their solutions in front of the whole class. The cases were problems that could have several valid solutions and answers. We interviewed 25 students, ages 8–9 years, and data was analysed using thematic analysis generating five themes. The first theme was about the importance of making sharing fair and that the cases were realistic. The second theme was about solutions being valid. The third theme covered conceptions about the cases as context or as concrete materials versus working in a textbook, which was connected to the fourth theme: discussing and thinking versus being quiet. The fifth theme was about working at your desk versus the whiteboard. The results signal that open problems can alter students' conceptions about mathematics and challenges when creating spaces for such teaching.

Keywords: collective mathematical reasoning, conceptions, division, primary students, sharing

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1 Introduction

The significance of factors such as beliefs, self-perceptions, motivation, and attitudes on students' mathematical achievement has been studied for decades (e.g., Guimond & Roussel, 2001; Hannula, 2006; Pajares, 2003; Ryan & Deci, 2000; Stage & Kloosterman, 1995; Valentine et al., 2004; Zimmerman et al., 1992). The key takeaway is that your thoughts about yourself and your abilities influence your ability to solve mathematical problems. This influence operates in both directions: a positive attitude can enhance performance, while negative beliefs can create obstacles and lead to lower performance (Capuno et al., 2019). There are, therefore, good reasons to study aspects of affect and mathematics education. In addition, the different ways students engage with mathematics depend on their different affective dispositions with the subject (Ingram, 2015). Looking only at Swedish students, there have been some previous attempts focusing on various aspects of affect to highlight the connection between affect and how mathematics is perceived and dealt with, such as the different motivations students express (Nyman & Sumpter, 2019), and how different expectations function as a mediator for various choices students make when solving mathematical tasks (Sumpter, 2013). However, although careful design when planning and conducting the studies, the conclusion is that different affective constructs do not operate as singular entities. Several researchers stress how motivation, emotions, and beliefs are intertwined with each other (e.g., Hannula, 2006) or internally, such as different types of motivation being combined in one statement (Nyman & Sumpter, 2019). It means that it can be difficult to separate different constructs from each other, which provides various methodological challenges.

Looking at empirical studies, one worrying issue is that students' affective disposition in many countries decline with age (e.g., Metsämuuronen, 2013; Sumpter & Sollerman, 2023; Wilkins & Ma, 2003). This is true for Sweden, too, where grade 5 students have significantly more negative views towards mathematics compared to grade 2 students (Blomqvist et al., 2012). The measured decline generates different questions, such as what causes and keeps the interest alive. In addition, recent reports stress that negative attitudes, emotions and so forth are established in early schooling (Larkin & Jorgensen, 2016). A study from Finland concluded that third-grade students express conceptions that mathematics is either nice/ easy or dull/ difficult but also that they, as students, can do mathematics (Pehkonen et al., 2011). It is, therefore, a more complex picture than just 'good vs bad'.

In addition, Hofmann and Ruthven (2018) conclude that if one wants to change teaching, one has to understand what socio-mathematical norms exist in the present teaching, norms that affect both teacher and students. When adding that few studies focus on young children's experiences with mathematics (Takeuchi et al., 2016), the conclusion is there are several gaps in the area of young children's affect in mathematics education, both with respect to how they experience different teaching and how they see themselves as actors in mathematics. Previous research has indicated that teaching shifting towards problem-solving often also results in more positive affect. (e.g., Asami-Johansson, 2015; Higgings, 1997; Ingram, 2015; Wong et al., 2010). It implies that changing teaching, using different types of tasks, and different mathematical content can transform students' conceptions. One example of a mathematical content that offers good opportunities to create open problems is sharing as there are several possible solutions to a sharing situation (Sumpter & Hedefalk, 2023).

The present paper aims to offer information about what young students think about working with open problems dealing with sharing compared to their traditional mathematics education, which here means working with the textbook, quietly at your desk. The research question is: what different conceptions do students express with respect to problem-solving and collective mathematical reasoning when working with sharing?

2 Background

Given that it can be difficult to separate between different affective constructs in data (e.g., Hannula, 2006; Nyman & Sumpter, 2019), the aim here is to give a more provide an overarching view of students' experiences more than trying to separate if they express, for instance, beliefs, attitudes etc. Therefore, we would like to use the theoretical concept conceptions defined as an umbrella concept (e.g., Philipp, 2007). In previous research in mathematics education, one commonly used definition of conceptions is Thompson (1992), who states that it is "conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences" (Thompson, 1992, p. 132). Some years later, Philipp (2007), building on Thompson's (1992) work, defines conceptions as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p.259). With the addition of 'a general notion', we can use conception as an umbrella term, and conceptions may have both affective and cognitive dimensions. Conceptions can then be compared to other constructs aiming to capture similar phenomena, such as mathematical views (Roesken et al., 2011). It means that we have no intention to try to separate between emotions, motivations, beliefs, or attitudes but instead talk about students' conceptions as an umbrella concept. It means that conceptions incorporate a range of more or less cognitively stable and emotionally loaded constructs, meaning that it is a construct with both affective and cognitive dimensions that have been inferred or derived from specific instances. Hence, students' conceptions consist of their belief systems, preferences, and so forth, reflecting their experiences.

This means that students' conceptions reflect their experiences of mathematics education (Pehkonen, 2001). What is promising, especially given the reported decline in motivation, interest, emotions, etc., with age (e.g., Metsämuuronen, 2013; Sumpter & Sollerman, 2023; Wilkins & Ma, 2003), is that several research studies report that students' affect can change depending on the mathematics teaching (e.g., Hannula, 2006; Stipek et al., 1998). One example is Higgings (1997), who describes how students who participated in a one-year-long problem-solving displayed more positive affect compared to the control group that had traditional mathematics instruction. Similar results have been reported with Swedish lower secondary students who got to experience a Japanese problem-solving-oriented lesson structure (Asami-Johansson, 2015). Hence, students' affect, including different socio-mathematical norms, depends on teaching, such as what type of tasks one works with, how to work with them, for instance, in random groups, and what type of support to expect from the teacher (e.g., Liljedahl, 2022).

Here, we are interested in collective mathematical reasoning (e.g., Sumpter, 2016). The core of collective mathematical reasoning is joint work, which can be both interactive whole-class teaching or collaborative small-group work, where the aim is to make use of the participants' argument for various decisions made during the work. Research indicates that this type of mathematics teaching and learning can be considered effective pedagogy both with respect to different measures of achievement and also more qualitative measures (Schroeder et al., 2007; Slavin et al., 2009. Previous studies show that when students face different types of tasks and work in a collective manner, it can cause some affective challenges, such as feeling frustrated when your peers want something different (Ayalon et al., 2022). It has been established that it is important for teachers, when working in this way, to strengthen the norm of value and to give space to different ideas (e.g. Hofman & Ruthven, 2018). This is connected to the nature of the task. Here, we are focusing on sharing, and empirical studies on collaborative work where students face sharing tasks show that there often is tension between students advocating for different interpretations of what could be considered fair (Eriksson et al., 2023; Hedefalk et al., 2022). The different interpretations come from that fair share as a concept is not univocal (Chernyak & Sobel, 2016; Sumpter & Hedefalk, 2023), and what can be thought of as a 'fair share' is influenced by cultural aspects (Carson & Banuazizi, 2008). Hence, it should not be surprising that they can be big variations on what is considered a 'fair share' (Chernyak & Sobel, 2016), thereby a need to discuss different solutions.

Other observed issues in collaborative work were negative emotions arising from peer critique but also learning opportunities when correcting one's reasoning (Ayalon et al., 2022). In addition, such teaching means formulating arguments and listening to others. This is related to how the opportunity for talk is distributed. Previous studies have indicated the importance of creating a norm where students get to share their reasoning and thereby take part in discussions (Hofmann & Ruthven, 2018; Mercer & Howe, 2012). Participation is connected to whether one feels that the classroom is a safe space, but other factors contribute as well, such as one's first language. Language plays a role especially with respect to communicating one's reasoning (Lee & Lee, 2019). Affective factors that may influence whether students with another first language will communicate are self-confidence, speaking anxiety, and motivation (Mulyono & Saskia, 2021). This is particularly important when taking into account that one of the norms and values in the Swedish school system is that the school should provide situations so that each student "can make and express conscious ethical judgements based on knowledge on human right, fundamental democratic values and personal experience" (Skolverket, 2022, p.11), means that teachers need to be aware of that affective aspects can function as an obstacle, for all students but especially for students that do not have Swedish as a first language.

3 Methods

To generate data (e.g., Mason, 2017) to answer the research question, we conducted interviews with 25 grade 2 students (aged around 8-9 years old) from two different classes from one school. The recording of one interview was not in a good condition and therefore discarded, leaving us with 24 interviews to transcribe and analyse. The research project, where this study is a part of, is situated in an area considered one of the most segregated areas of Sweden, meaning that none of the children have Swedish as their first language. The classes participated in a research design where three teachers have had the goal to, through didactic modelling, stimulate students' collective mathematical reasoning in three cycles. The teachers have worked with six cases describing six different sharing scenarios. One example is case 6, where the children are asked to share eight biscuits among two soft toys (here dogs). When the children have shared the biscuits, a third recipient with explicit need (hunger and sadness) is introduced. The children are now facing a more complex sharing scenario where there are different needs and where resources are scarce (see Sumpter & Hedefalk [2023] for a longer description of the cases). The first cycle of working with cases was in a class that was not filmed or interviewed. It served as a pilot for the teachers, allowing them to try different didactic choices without the camera in the classroom. Then, they moved to the two classes that were filmed and the children interviewed, one after another. After each teaching session, the teachers and one of the authors of the paper discussed the outcome and made appropriate changes to stimulate children's collective mathematical reasoning, so that the children could give arguments and listen to each other about the different choices they made during the problem-solving session. The children from the two classes that were filmed completed a questionnaire at the start of the research project and were then interviewed after the classes had worked through all the cases. It means that one class had their interviews at the end of the fall semester (December 2022), whereas the second class had their interviews in early May 2023. Here, we will focus on the interview data, as it provides information about how the children experienced working on this particular kind of problem-solving.

The interview guide was a revised questionnaire developed to study students' conceptions, including motivation and emotions, and has been tested in several studies (e.g., Blomqvist et al., 2012; Nyman & Sumpter, 2019). The questions in the survey aim to capture how students think about mathematics and mathematics lesson, especially in relation to Swedish lesson, but also general issues such as what you do in a mathematics lesson. In the second interview, we added the questions "Now you have worked with biscuits and dogs, if I say that is mathematics, what do you say?", "What was fun [when working with the cases]?", and "What was most difficult?". The interviews were transcribed using the steps suggested by Mergenthaler and Stinson (1992). One of the steps that they recommend, given that the study is about conceptions, is that language has in some cases been changed so that the meaning of the children's replies is in focus, not the exact wordings.

The data was then analysed using an inductive approach of thematic analysis (e.g., Braun & Clarke, 2022). Thematic analysis is free from theoretical underpinnings, thereby allow flexibility in the coding process with respect to the aim of the study (Nowell et al., 2017). It provides an opportunity to see patterns in complex, rich data. Previous research has found that around 75% of codes were found in six interviews, and over 90% in 12 interviews where remaining codes were found with less frequency after those 12 interviews (Guest et al., 2006). This means that 25 interviews should yield sufficient saturated data.

We, all four authors, started by creating codes. The codes represented the essence in what was said, in relation to conceptions about working with the sharing-problems. Through this process of peer debriefing, we increased the likelihood of achieving both dependability and credibility in the generation of codes (Nowell et al., 2017). One example of a code was 'difficult to make it fair'. In the next step, we discussed what codes that could be grouped into different themes. In that sense, the coding was done using the idea that codes are building blocks for themes (e.g., Braun & Clarke, 2022), that is that the patterns of different statements together generate a theme of conceptions, where conceptions are defined as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p.259). The patterns mean that similar codes, such as utterances about specific mathematics or teaching, are grouped together and then internally revised so that themes are internally consistent but externally disjoint (Braun & Clarke, 2022). For example, the code 'difficult to make it fair' was interpreted as a conception about the chosen mathematical topic, sharing, but the chosen mathematical topic also included other codes. Combined, these codes created the theme 'Fair share is important and real'.

As stated earlier, the idea of building blocks means that we have no aim to detangle whether the themes of conceptions are beliefs, mental images and so forth. When all four agreed on saturation with respect to codes and themes (e.g., Braun & Clarke, 2022), three of the four authors analysed the interviews using the established different codes and themes. Any unsure data points were discussed and agreed upon using the principles of peer debriefing and team consensus (Nowell et al., 2017). In this way, we could achieve research triangulation, ensuring stability in the analysis process.

4 Results

The analysis resulted in five themes: fair share is important and real, different solutions are valid, working with cases versus working in a textbook, to discuss or to be quiet, and working at your desk or working with whiteboards. The chosen quotes are selected to illustrate the themes. The replies are marked with a letter to indicate different classes (L and M, just to avoid A and B) and a number.

4.1 Fair share is important and real

The first theme concerns the core of the content, fair share, and working with the cases. Several students described it as difficult, important, and fun. It is the actual work that is at the heart of the theme – to share is difficult when you want it to be a fair share:

> Interviewer: What was it that was difficult?
> Student M4: That it isn't.... it is three dogs and it is eight biscuits and then it is difficult. One can't give it to them then.
> Student L14: When we should share eight biscuits. And [then] one more came. That was difficult. It never became fair.
> Student M1: No, [to make it] fair is difficult.

As seen in the quotes, several students find sharing, the chosen mathematical content, difficult. This is a conception about applied mathematics: to make a fair share. Many of them use case 6 as an illustration, a case where they had to share eight biscuits between three soft toys. The conclusion made by Student M1 illustrates the struggle they were facing when doing this important work: the preference of what a fair share is became the reason for their struggles. As another student M11 concludes, "equal can be unequal sometimes", illustrating that when using division, it might end up with not fair sharing.

However, making it fair was also fun and real, and thereby important concerning reality and learning:

> Student L6:Shared them and counted them and giving it [to] them. [To] make [it] fair.
> Student L9: We have shared all the biscuits to the teddies. We have shared them all, even if we didn't know if they wanted to have them.
> Student L7:Because I thought it was good and fun since we learnt a lot. And [to] think.
> Student L3:To… that we got to learn and it was teddies. It was real.

The quote from student L9 stresses the importance of their work: ensuring that the biscuits were handed out to all the teddies, even if they did not have all the information about the teddies' different needs. Again, the application of mathematics in this particular case, sharing, provides a struggle. The students felt it was real.

In sum, these quotes illustrate a conception that working with the cases was not straightforward. The work was difficult, especially when striving for a fair share and when cases were complex. At the same time, many of the students found the work rewarding and meaningful since it was real and fun, and they learnt a lot.

4.2 Different solutions are valid

The second theme was that the children expressed that all solutions were valid. This theme encompasses different aspects of what it means with 'all solutions are correct'. The first one is about the student's solutions were always correct independent of how they solved it:

> Student L3:The solutions were always right. You answered [using] the biscuits every time. How far [one] wish to go.

The quote from student L3 illustrates that the solution strategy was flexible, and the students could decide for themselves when their solution was finished. This is an illustration of a conception of open problem-solving and the notion that all solutions were considered correct. In addition, all students got to present their reasoning, which is the second aspect of this theme:

> Student M10:We used the white-board and [we] got to stand [next to] it all the time. We all [got to] tell.

The students noted that not only were all solutions correct, but there was also a power shift when all students were allowed to walk up to the whiteboard and present their solutions. This means that all solutions had validity. The quote from student M10 suggests that the mental image of standing in front of the whole class and presenting your solution carries some importance. For some students, this includes expressing a difference between the teacher knowing the right answer and the students providing a valid solution. One student compared how mathematics teaching normally looks like:

> Student L4:Yes, but we have only shared to the dogs. And, the teacher has written on the white board and said that all [solutions] were right. Otherwise, the teacher writes on the white board and has all [the] right [answers]. One figure is always twice the area of the other.

The student L4 stresses the difference in who has the agency to determine what is a correct solution, the students or the teacher. This conception includes an understanding of when the power shifts. The shift of power was interpreted as positive, which is the third aspect of this theme:

> Student M6:We all got to tell. It was fun.

Just as illustrated by the response from student M6, most replies mention the joy of presenting their solutions to the other students. Combined, the theme is about all solutions being correct, that they were all valid and thereby worthy of presenting in front of the whole class, which was interpreted by most of the students as a positive thing.

4.3 Working with cases or with textbook

This theme is about sharing how this type of collective work with open-ended problems differs from regular mathematics teaching. In this case, regular mathematics teaching consisted of working alone at your desk with the textbook. This theme focuses on the difference between problem-solving and working with the textbook. Student L11 describes how the two lesson designs differ concerning writing:

> Student L11: That... one gets to write not so much and in math one has to write a lot. You have to write almost 100 pages. [...] One gets tired quicker at math [lesson]. It is not that you get to think and stuff and get to share [things] out.

The student signals having experienced two different lessons: in one lesson, you do not have to write so much, but in regular teaching, you have to write a lot in your textbook. This is a conception about mathematics teaching and learning. It appears that the students prefer the problem-solving lessons since they do not get tired as quickly. In traditional mathematics lessons, they have to sit still and write, and the students describe pencils and paperwork. The work with the cases also meant that the concrete materials were different:

> Student M11:The biscuits. We got the biscuits.
> Student M8:What is that? When [it] is dogs and biscuits, we have dogs and biscuits. Other in math class, [the] only thing is to write.

The students pick out the concrete material as different to regular teaching since in regular teaching, "[the] only thing is to write" [M8]. But, the difference is not just having different materials; it is also what the materials offer:

> Student L14: When we have other math, we do not have the dogs and the biscuits.
> Interviewer: So, what do you have then, when you work with other maths?
> Student L14: Then we do not have the dogs. Then we have to write and think.
>
> Interviewer: If we compare work with the biscuits with normal math…
> Student L1: It is different to work with biscuits and books. You draw and write and count.
> [talk about thinking]
> Interviewer: Do you think when you work with the biscuits as well?
> Student L1: Yes, but more when doing maths. Math is math. You think, then draws and then teacher correct your answers. But [when working with] biscuits [you] draw. [When working] with a big biscuit, a lot.

It appears that the dogs (or teddies) and the biscuits create a different learning environment for the children. As such, they signal a conception of changes in the mathematical learning environment. In the students' quotes, it is unclear what it means 'to think'. It appears that 'thinking' represents different things for different students. Some students also brought forward what is the same:

> Student L3: The difference is that one work with teddies and you work with your hands. […] What is the same is addition, subtraction, multiplication and division.

Student L3 points out that the difference is not in the mathematical content but in the concrete materials – that you got to work with your hands. The replies that were coded in this theme were often connected to the next two: to discuss or to be quiet, and only to work at your desk or to be able to present your work using whiteboards.

4.4 To discuss or to be quiet

This theme contains the responses that point out the difference between being able to discuss your work and solutions, as when working with the cases where the students were encouraged to work together, or regular mathematics teaching when the students have to be quiet during the work. Here, the responses from students M5, L13, and M1 illustrate this difference:

> Student M5: Like, we get to work alone and stuff.
> Student L13: We have to think.
>
> Interviewer: Would it be possible to discuss when having regular mathematics?
> Student M1: No, only write.

The students interpret the two lesson designs as two different sets of rules: in one design, you are allowed to talk and work together, and in the other design, you work alone and you have to be quiet. As such, the quote signals a conception of mathematics teaching.

Many students describe a situation where they sit still at their desks and work in silence. The theme also covers replies about how the children got to present their solutions to their peers:

> Student M6: We all got to tell. It [was] fun.
> Student L5: Tedious/ long. And listen. All.

As indicated by the chosen quotes, there are mixed emotions about presenting your reasoning. Student M6 stress the fun of everyone getting to present their solution, whereas student L5 see the tediousness of listening to all the presentations. Although some students talk about how difficult it was to listen to all solutions, others, such as student M6, stress the importance and fun of everyone getting the opportunity to present. Many of the responses in this theme were connected to the next theme, which covers the difference between working at your desk or working with whiteboards.

4.5 Desk or whiteboard

In this fifth and final theme, the students point out the difference between regular teaching, here meaning that they sit at their desk, quietly working with the textbook, or working with the cases where they get to discuss with their partner and then present their work on the big whiteboard in from of their peers. Sometimes, they also worked with small whiteboards at their desk, which helped them create a joint solution:

> Student M10: We used the whiteboards [small ones that the students have at their desks] and we were at [the big] whiteboard all the time. We all [got to] tell.

It appears that the whiteboard, especially to present your solution at the big in front of the whole class, has a special meaning. Student M10 continued explaining that it was easier to get the other students to understand when moving the paper biscuits that were fastened with magnets on the whiteboard when sharing the biscuits between the recipients.

Although most students found presenting at the whiteboard was fun, one student recall one incident:

> Interviewer: You are marking the sad face. Why are you feeling in such a way?
> Student L10: Because... because... some [other students] laughed when we were there. When we were at the whiteboard and the teacher had to tell them that they should not [laugh], but then they already [have laughed].

The student continued that it did not matter that the teacher told the students off—the damage was already done. The quote illustrates a conception about mathematics education, in particular, how difficult it is to create a safe space when working with collective mathematical reasoning, especially if this work involves sharing one's solution with others.

5 Discussion

The research question posed was: What different conceptions do students express with respect to problem-solving and collective mathematical reasoning when working with sharing? The analysis yielded five themes: fair share is important and real, different solutions are valid, a difference between working with cases compared to working in a textbook, often connected to whether you are allowed to discuss or to be quiet, and working at your desk or working with whiteboards. The last three themes combined present traditional mathematics as quiet, with students working alone at their desks with the textbook, which aligns with the findings of Pehkonen and colleagues (2011). On the other hand, working with the sharing cases ('dogs and biscuits') means that you work together and discuss your solutions, often using concrete materials, and then present your solution to the other students in the class, including listening to their solutions. The conceptions reflect mathematics education as a problem-solving teaching (e.g., Asami-Johansson, 2015; Hofmann & Ruthven, 2018). Several students explicitly said that presenting and listening was good but also difficult. Here, 'difficult' appears to be a proxy for different things, similar to when the grade 2 students in Nyman and Sumpter (2019) used boredom as a stand in for a handful of negative emotions and motivations. For instance, difficult could be tedious since you have to listen to several presentations, but also frustrating when wanting different things. These are affective challenges similar to those that have been reported in earlier studies (e.g., Ayalon et al., 2022; Hofmann & Ruthven, 2018). Results like these are of interest in light of the norms and values of Swedish schools: to foster future members of society who can listen to others and are able to formulate their own reasoning (Skolverket, 2022). One conclusion is that working with collective mathematical reasoning (e.g., Sumpter, 2016) is not a straightforward and easy goal for teachers, but an important one if to fulfil the overarching goal of the Swedish school system. The implication is that if teachers want to work with this, they need to be aware of that creating such a norm where students are willing to listen to each other might take time.

Overall, most students thought it was fun to discuss and present different solutions learning ' a lot', though it could be tedious. As such, the teaching was considered, as Wong and colleagues (2010) put it, friendly. One student (L10) brought up one incident that was connected to negative emotions. Given that research has pointed out different affective aspects that may influence students with a different language background and their willingness to speak in a classroom (Lee & Lee, 2019; Mulyono & Saskia, 2021), the student's reply illustrates such findings with respect to mathematical problem solving and collective mathematical reasoning (e.g., Sumpter, 2016). Since creating a collective mathematical reasoning space is not easy, our results suggest that doing it in a non-first language classroom adds more challenges. One possible implication is that teachers, especially in multilingual classrooms, need to be aware of how to create a thinking classroom (e.g., Liljedahl, 2022) where everyone feels secure to present their solutions (Ayalon et al., 2022).

The view of mathematics being working in a textbook –quiet and alone at your desk– as different compared to working together, with concrete material, discussing and then presenting your solution, signals that students do form socio-mathematical norms or mathematical views rather quickly (e.g., Hofmann & Ruthven, 2018; Roesken et al., 2011) and at an early age (Blomqvist et al., 2012; Larkin & Jorgensen, 2016). It appears that they have different affective dispositions towards traditional mathematics compared to problem-solving (e.g., Ingram, 2015), which is similar to the results of previous studies (e.g., Asami-Johansson, 2015; Higgings, 1997). The students signal it is a positive shift, including a shift in the power of who is right. Here, the focus has not been on measuring a change. Instead, we have aimed to describe students' conceptions after working with six sharing scenarios. Hence, we cannot say if there has been a change based on the results of the present study. We therefore suggest a quantitative follow-up study to assess whether students maintain a positive disposition towards mathematics (e.g., Blomqvist et al., 2012). Such a study will provide information if a small intervention – to work with six mathematical problems – can influence how children experience mathematics as a subject. Also, given that few studies focus on young children's experience of different mathematical teaching (Takeuchi et al., 2016), such a study would increase our knowledge of how different teaching is received.

Research ethics

Author contributions

L.S.: conceptualisation, investigation, methodology, project administration, validation, visualisation, writing—original draft preparation, writing—review and editing

H.E.: data collection and curation, formal analysis, writing—review and editing

M.H: formal analysis, writing—review and editing

P.M.: initial analysis, writing—review and editing

All authors have read and agreed to the published version of the manuscript.

Artificial intelligence

Grammarly has checked the spelling, but no artificial intelligence has been used to analyse data or write the paper. DeepL has been used to translate from Finnish to English.

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Institutional review board statement

The project has been checked according to the standards of research ethics through the system provided by Stockholm University, Sweden.

Informed consent statement

Informed consent was obtained from all research participants and their parents.

Data availability statement

Transcribed data can be shared upon request. Other data is unavailable due to privacy or ethical restrictions.

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Conflicts of Interest

The authors declare no conflicts of interest.

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