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Research article

Undergraduate physics students' understanding of abstract and applied mathematics: A case study on vectors

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Abstract: Mathematical competence is considered crucial for physics learning; besides knowing abstract mathematics, students need to apply mathematical skills to different physical concepts. It is hypothesised that the development of this applied physical competence requires some threshold of mathematical competence. However, the relation between these two competences remains unclear. Furthermore, research on undergraduate physics education is scarce within the Finnish national context. To bridge these research gaps, this study addresses the development of both abstract mathematical and applied physical competence in a physics intervention course in a Finnish university. The focus is on the key mathematical concept of vectors, which is covered in introductory physics courses and considered highly difficult, as students tend to acquire misconceptions of the topic and struggle to apply the mathematical construct in the physical context. The pre- and post-test results show that most of the learning outcomes during the intervention course were related to improved applied physical competence. Furthermore, common misconceptions regarding vectors are identified. Analysing the joint development of mathematical and physical competencies supports the assumption that the improvement of applied physical competence depends on some foundational level of mathematical understanding. Furthermore, the students' prevailing misconceptions and only modest overall improvement of their physics understanding highlights the need for more deliberate teaching practices.

Keywords: vectors, mathematics as a service subject, physics students, intervention study

1 Introduction

The transition to university-level mathematics is widely recognised to be challenging for students (e.g., Gueudet, 2008), particularly for those who study mathematics as a minor subject or use it primarily as a tool within a different discipline. Engineering students, for instance, often report motivational difficulties and limited competence in proof-based mathematics (Harris et al., 2015; Tossavainen et al., 2021). Similar challenges are observed among physics students as insufficient mathematical skills can hinder success in physics courses, while strong mathematical skills are associated with better performance and future success (Pietrocola, 2008; Sadler & Tai, 2007; Salehi et al., 2019). However,

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the histories of mathematics and physics have long been interconnected, as the disciplines have developed hand in hand as scientific fields. For example, Palmgren and Rasa (2024) conceptualise mathematics as a foundational structure in physics, comparable to the role of language. Therefore, it is important to understand how physics students can both learn abstract mathematics and apply it in the physical context.

Still, STEM (science, technology, engineering, mathematics) students outside of mathematics often see mathematics and physics as separate subjects. Bergsten and Jablonka (2013) found that engineering students tend to perceive mathematics and subject-specific courses as disconnected, viewing mathematical skills as beneficial for general problem-solving but not so much for their core disciplinary competencies. This perception aligns with what Barquero et al. (2013) describe as an *applicationist epistemology*, that is, a strict separation between mathematics and other disciplines. Such a separation is not optimal for students' learning, as it can create motivational and conceptual barriers (Barquero et al., 2013). In fact, research suggests that when students perceive mathematics as integrated and relevant to their own field of study, they are more likely to adopt meaningful approaches to learning and to develop a cohesive understanding of mathematical concepts (Macbean, 2004). In this vein, learning physics can be seen as inseparable from learning mathematics (Palmgren & Rasa, 2024). Therefore, to better facilitate students' learning, it is important to address the complexity of how students can jointly develop both mathematical and physical understanding.

1.1 The role of mathematics in learning physics

Mathematical competence is central to learning physics, as it provides the formal framework upon which physical theories are constructed and enables the representation of physical phenomena in mathematical terms (Palmgren & Rasa, 2024). In physics, multiple external representations, such as text, graphs, tables, formulae, and diagrams, are routinely used to present information. These representations not only support further reasoning and theoretical development but also facilitate the generation of new insights and explanations (Palmgren & Rasa, 2024). Therefore, the ability to interpret this information and translate it across representations is critical for learning physics.

This ability is known in the physics education literature as *representational competence* (Kozma & Russell, 2005). Crucially, representational competence does not concern the ability only to solve equations, for example, but to use the equations in a meaningful way. That is, students should be able to use representations to describe physical quantities and map (i.e., translate) the features of one representation to another (Kozma & Russell, 2005). Moreover, the understanding of physics concepts is partly tied to the particular representation used, as each representation may offer unique information or fulfil a specific function (Ainsworth, 2014; Lichtenberger et al., 2017).

Lacking sufficient representational competence has a significant impact on students' learning, achievement, and conceptual knowledge in physics (Burkholder et al., 2021; Meltzer, 2002; Edelsbrunner et al., 2024). Crucially, representational competence seems to be a necessary, albeit insufficient, prerequisite for learning and developing conceptual knowledge (Meltzer, 2002; Edelsbrunner et al., 2024; Edelsbrunner & Hofer, 2024). Meltzer (2002) found that among first-year university students, mathematics skill, but not pre-instruction physics knowledge, was associated with learning gains. Edelsbrunner and colleagues (2024) concluded that high representational competence is a prerequisite for high conceptual knowledge but not vice versa.

1.2 Physics students' difficulties in learning about vectors

Vectors are important mathematical objects in physics and are ubiquitous in first-year university courses. Vectors are used to represent quantities that have a direction and magnitude — such as velocity, force, or fields — making them one of the most important mathematical concepts to master in first-year physics. The vector concepts used in introductory physics courses are presented in Table 1. Table 1 also links each vector concept to a physics concept, in our case, in the context of mechanics.

Table 1. Vector concepts in introductory physics courses and examples of related mechanics concepts.

Vector concept	**Physics concept**
Sum	Network
Unit vector	Direction
Dot product	Work
Magnitude	Length, speed
Codirectionality	Positive and negative work
Scalar multiplication	Momentum
Difference, change	Displacement
Coordinate system, division into components	Circular motion, net force
Cross product	Angular momentum

Even though vectors are central to first-year physics, many students struggle to master basic vector concepts, such as two-dimensional vector addition, and apply them to physics concepts even after a full semester of study (Flores et al., 2004; Nguyen & Meltzer, 2002). For example, students confuse subtraction with addition when asked to graphically perform vector calculations (Barniol & Zavala, 2014a). A common error with the dot product is to interpret it as the magnitude of the bisector of the two vectors or as the bisector vector (Barniol & Zavala, 2014b). Students might also confuse the dot product with the cross product. In addition to struggling with specific vector representations (graphical, algebraic, etc.), students have difficulties expressing an algebraic solution graphically (Liu & Kottegoda, 2019), that is, translating across representations.

In a Finnish context, Virtanen et al. (2024) and Mosorin (2024) have explored students' knowledge of vectors in the mathematics matriculation examination. Although

Virtanen and others found that the students performed quite well (they scored on average 9.1/12 in the task), both studies found that students struggle with certain basic concepts regarding vectors, such as understanding the component form (e.g., treating unit vectors as if they were variables) or the notation of the vector norm. In both studies, errors with calculating the dot product were prominent, such as not understanding that the result of a dot product is a scalar and not a vector (Mosorin, 2024; Virtanen et al., 2024).

Students might have an intuitive grasp of vectors, possibly based on their understanding of some basic physics concepts. For example, velocity might provide an intuitive, concrete anchor for understanding vectors as objects with magnitude and direction. Yet, beyond this elementary understanding, students cannot apply vector calculations in the more precise manner required by physics courses. To foreshadow our results, even inferring the change in velocity (which is subsequently needed to infer the direction of acceleration and, further, the direction of force) causes difficulties for students. The famous example of gyroscope precession provides a case where the behaviour of the system is very unintuitive, and understanding it is rooted in understanding the situation mathematically, requiring the use of a cross product. This is often the case in physics, where intuition provides little guidance, and mathematics is indispensable for the understanding of the situation.

Consequently, difficulties in basic vector concepts and notation may hinder the learning of more advanced skills as well as the application and understanding of these concepts in physics. This is especially problematic if a certain level of representational competence must be acquired to achieve an adequate conceptual understanding in physics (Edelsbrunner & Hofer, 2024). As students' pre-instructional knowledge is one of the key determinants of learning (Simonsmeier et al., 2022), and first-year physics courses typically deal with vector notation from the start, it is essential that university instructors take students' pre-instructional knowledge into account in their teaching.

1.3 Research questions

To address the joint learning of mathematics and physics, this study follows a pedagogical intervention course comprising 12 weeks of workshops focusing on mathematics and its application in physics studies. The aim is to investigate how physics students can develop both abstract mathematical competence and the ability to apply mathematics in physical contexts. By combining quantitative and qualitative methods, the study seeks to respond to the following research questions:

1. How do the students' abstract mathematical and applied physical competence develop over the course of the intervention?
2. What common misconceptions can be identified from the students' responses related to vector concepts?

3 Methods

3.1 Context

The study was conducted at the Department of Physics and Astronomy at the University of Turku, where students are required to take two mathematics courses that are specifically tailored to support their physics studies. We shall call these the first and second courses. Typically, first-year physics majors enrol in the first course immediately at the beginning of their studies. The first course focuses on high-school-level algebra, calculus, and vectors, but it also introduces complex numbers as a new topic. The first course lasts eight weeks, and the coursework consists primarily of lectures, recitations, and an exam. The second course is implemented similarly to the first course, but it introduces new topics, such as series expansions, differential equations, and linear algebra. In recent years, concerns have arisen about students' mathematical competence. Grades have begun to decline, and students have seemed to struggle, especially with the second course, which has resulted in their having insufficient mathematical skills to undertake further studies.

To address these concerns, a new intervention course, that is, 12 weeks of mathematics workshops, was designed to better prepare students for their subsequent studies. The workshops consisted of two-hour small group sessions where the students worked on problem sets, aided by a teaching assistant. The teaching assistants were typically older students with little or no previous teaching experience. The problem sets were designed to strengthen both the students' mathematical competence and their skills to make use of that competence in physics contexts. The four topics followed the contents of the first course: 1) algebra, 2) calculus, 3) vectors, and 4) complex numbers. Each topic was covered in three workshops, during which the topics were applied in different physics contexts, such as mechanics and electromagnetism. The vector problems covered themes such as distinguishing between a vector and a scalar, vector sum and difference, vector components, unit vector, length of a vector, dot product, and cross product. The problems concerned both the graphical and the mathematical processing of vectors. Participation in the workshops was voluntary for students who received the highest or second-highest grade in the first course (grades 4 and 5 on a scale from 1 to 5). For others, it was mandatory to participate in at least six workshops and at least one session on each of the four topics. The workshop series was tied to the second course, such that all the students were awarded bonus points if they participated in more than six workshops. The workshop course was organised for the first time during the data collection for this study.

3.2 Data collection

The data were collected at the beginning and end of the 12-week workshop course. The students were given a pre-test a week before the first workshop and a post-test a week after the last workshop. In these tests, the students voluntarily answered an electronic questionnaire. The questionnaire included background questions followed by multiple-choice items measuring the students' competence in using vectors in both abstract mathematical and applied physical settings. The students were awarded one bonus point for the second course for responding to the questionnaire. The students gave informed active consent to participate in the research.

The students' abstract mathematical and applied physical competence was measured with 25 items, of which 12 were taken from the Test of Understanding of Vectors (TUV) questionnaire, a validated tool for measuring basic understanding of vector concepts (Barniol & Zavala, 2014a, 2014b). Additionally, an expert panel of three physicists and a mathematician who also had expertise in subject didactics developed 13 new questions to measure topics not covered in the TUV questionnaire, such as recognising parallel or perpendicular vectors and handling physical vectors with units. Of the final 25 items, 12 were given in a purely abstract mathematical context and 13 in an applied physical context. The topics of these items and their original TUV item numbers are listed in Table 2. See Appendix 1 for all the vector competence items.

Table 2. List of items in the vector competence questionnaire. For each item, we list the original item number in the TUV questionnaire (if applicable), context type (abstract mathematics or applied physical), and topic of the measured mathematical skill.

Item	TUV	Context	Topic	Item	TUV	Context	Topic
1	1	physics	vector addition	14	new	physics	scale, units
2	2	abstract	unit vector	15	15	abstract	cross product
3	6	physics	dot product	16	14	physics	components
4	new	physics	magnitude, units	17	new	abstract	equality
5	16	abstract	vector addition	18	18	abstract	cross product
6	8	abstract	dot product	19	new	physics	equality
7	new	abstract	parallel vectors	20	new	physics	unit vector, units
8	11	physics	scalar multiplication	21	19	physics	vector subtraction
9	new	abstract	dot product	22	new	abstract	vector addition
10	13	physics	vector subtraction	23	new	abstract	components
11	3	abstract	dot product	24	new	physics	vector addition
12	new	physics	coordinate systems	25	new	abstract	perpendicularity
13	new	physics	dot product				

3.3 Participants and data analysis

Only the students who answered the questionnaire at the beginning and end of the 12-week workshop course and gave consent to participate in the research at both time points were included in the study (n = 35), representing 63 percent of the students who passed the course. The participants were mostly first-year physics majors (30 students), with four students majoring in another STEM subject and four in their second year or higher. The participants had received grades from 1 to 4 in the first course. It is notable that four participants attended the course voluntarily, as they had received a grade of 4, and the workshop course was not compulsory for them. Out of the three vector workshops, the participants attended between zero and three workshops (0 workshops: 11%, one workshop: 20%, two workshops: 51%, and three workshops: 17% of the participants). On average, the participants attended 1.74 (58%) of the vector workshops.

The students' responses were analysed with RStudio 2024.09.1. The *coin* package (Hothorn et al., 2006) was used to compute the non-parametric Wilcoxon signed rank test to compare the pre- and post-test results of the total scores as well as the abstract mathematical and applied physical subscores. The quantitative results were accompanied by a qualitative investigation of common misconceptions among the students related to vector concepts. This analysis focused on the most frequently selected distractors in the test, which served as indicators of persistent conceptual difficulties. Drawing on the authors' subject matter and didactical expertise, the patterns in these incorrect responses were analysed to identify the underlying misconceptions driving these student errors.

4 Results

4.1 A quantitative investigation of the competence development

The mean values for the total score, as well as the subscores from the abstract mathematical and applied physical tasks in the pre- and post-tests, are presented in Table 3. The results indicate that the students responded correctly to 53 percent of the tasks in the pre-test and to 59 percent in the post-test. On average, the total score increased by approximately one and a half points during the workshop period, and this increase is statistically significant (MD = 1.43, $p < .01$). This overall increase in the total score was due to a comparable statistically significant increase in the applied physical subscore (MD = 1.46, $p < .001$). In contrast, the abstract mathematical subscores show, on average, no notable change (MD = -.029, $p = .81$).

Table 3. The means and standard deviations for the total competence score as well as for the abstract mathematical and abstract physical subscores. For comparison of the pre- and post-test results, the Z- and p-values from the Wilcoxon signed rank tests are provided.

	Total score (max. 25)			**Abstract mathematical (max. 12)**			**Applied physical (max. 13)**		
	Pre	**Post**	**MD**	**Pre**	**Post**	**MD**	**Pre**	**Post**	**MD**
Mean	13.34 (53%)	14.77 (59%)	1.43 (Z = -2.62, p < .01)	6.86 (57%)	6.83 (57%)	-.029 (Z = .24, p = .81)	6.49 (50%)	7.94 (61%)	1.46 (Z = -3.37, p < .001)
SD	3.83	4.03		2.16	2.28		2.33	2.26	

The students' pre-test scores were used to categorise them into three competence classes for both abstract mathematical and applied physical domains. The classification thresholds were defined as follows:

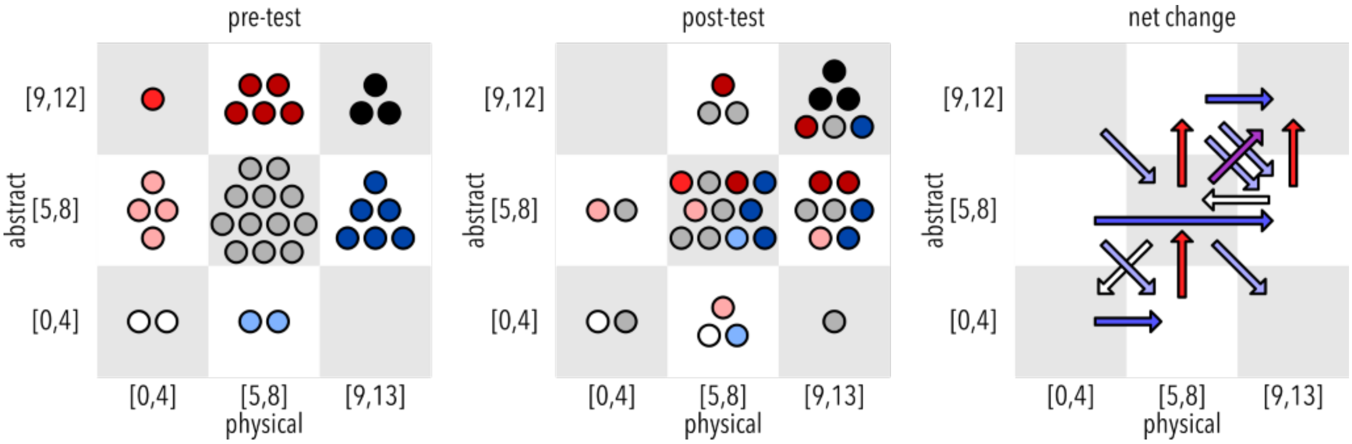
- low competence (0–4 points, corresponding to the mean minus one standard deviation)
- medium competence (5–8 points)
- high competence (9–12 points for abstract mathematical tasks, and 9–13 points for applied physical tasks, corresponding to the mean plus one standard deviation).

These thresholds were applied to both the pre-test and post-test data. The students were assigned to these competence classes, and their transitions between classes during the workshop period were subsequently analysed to evaluate their competence development (see Table 4). A diagram illustrating the net transitions between the competence classes was created to visualise student movement across different competence classes during the workshop period (see Figure 1). The diagram suggests that, on average, improvements in abstract mathematical competence (indicated by red arrows) tended to occur independently of gains in applied physical competence. In contrast, increases in applied physical competence (blue arrows) were typically associated with either stagnation or regression in abstract mathematical competence.

Table 4. The number of students who transitioned between classes during the workshop period.

	Total score	**Abstract mathematical**	**Applied physical**
Upward competence class transition	9	5	12
Downward class transition	2	7	5
Net transition	7	-2	7

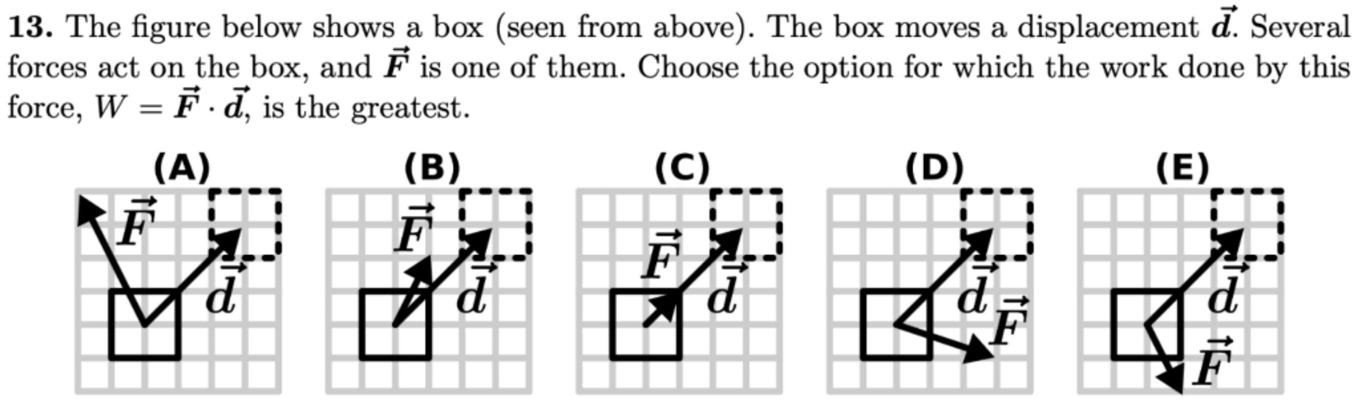
Figure 1. The students in their corresponding competence classes in the pre- and post-test data and their net transitions between the classes during the workshop period.



4.2 A qualitative investigation of common mistakes and misconceptions

The students' competence was measured in both abstract mathematical (12 tasks) and applied physical (13 tasks) settings. The task-level investigation shows that the average points per question ranged from 0.17 to 0.8 in the pre-test and from 0.17 to 0.95 in the post-test. As expected from previous studies, the dot product caused the students some difficulties. While they mostly understood the scalar nature of the dot product, they struggled to interpret it, as shown in tasks 11 (abstract mathematical context) and 13 (applied physical context). In task 11, the students were given a graphical representation of two vectors and asked to choose between statements representing different interpretations of the dot product. Many students (over 34% in the pre-test and over 24% in the post-test) interpreted the dot product to represent 'the magnitude of a vector between C and D pointing up to the right' instead of 'the projection of vector C onto vector D multiplied by the magnitude of vector D', which is the correct answer. In the physics context (task 13 presented in Figure 2), the students had to infer in which of the options the work done by a force along a displacement vector was the greatest, which entailed comparing the magnitudes of the dot products. A fourth of the students chose option A, where the magnitude of the force vector was the greatest. This may reflect a misunderstanding of the dot product, as they failed to consider the angle between the two vectors. Some (around 14% in the pre-test) also chose option C, where the vectors were parallel. This may reflect difficulties in understanding the projection of vectors or, for example, thinking that only forces parallel to the displacement do work. The students continued to struggle with this task in the post-test with very little improvement in the scores.

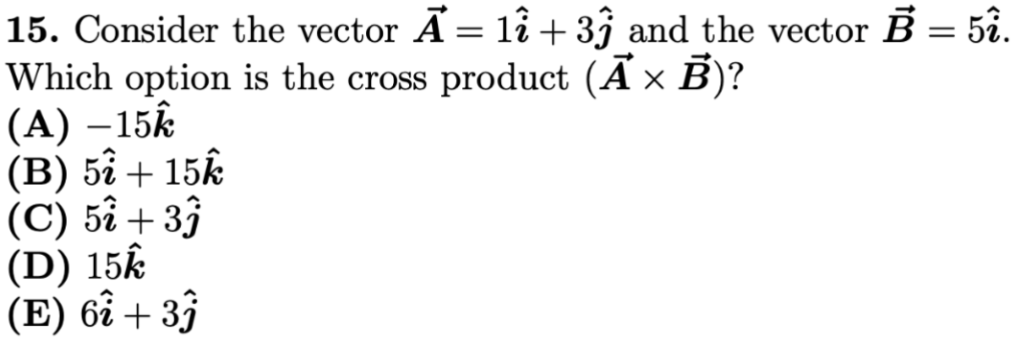
Figure 2. Task 13 in the test.



13. The figure below shows a box (seen from above). The box moves a displacement $\vec{d}$. Several forces act on the box, and $\vec{F}$ is one of them. Choose the option for which the work done by this force, $W = \vec{F} \cdot \vec{d}$, is the greatest.

Other persistent problems in the abstract mathematical context were related to calculating the cross product (tasks 15, 7, and 18). For example, task 15 (see Figure 3) asked the students to choose the correct vector depicting the cross product of two vectors. A common error was to choose the option that was opposite the cross-product vector. In addition, many students chose options that were not perpendicular to the input vectors, implying some fundamental conceptual-level difficulties with the cross product. In task 18, most of the students failed to pick the right formula for the cross product. This difficulty was mirrored in task 7, where almost half failed to recognise that the cross product is zero if the vectors are parallel.

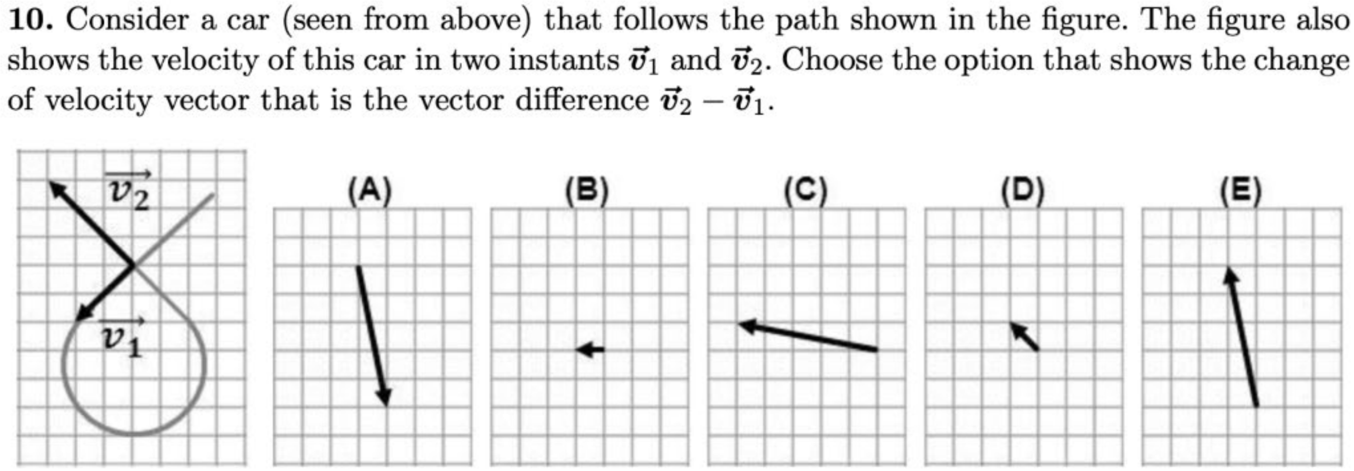
Figure 3. Task 15 in the test.



15. Consider the vector $\vec{A} = 1\hat{\imath} + 3\hat{\jmath}$ and the vector $\vec{B} = 5\hat{\imath}$.
Which option is the cross product $(\vec{A} \times \vec{B})$?
(A) $-15\hat{k}$
(B) $5\hat{\imath} + 15\hat{k}$
(C) $5\hat{\imath} + 3\hat{\jmath}$
(D) $15\hat{k}$
(E) $6\hat{\imath} + 3\hat{\jmath}$

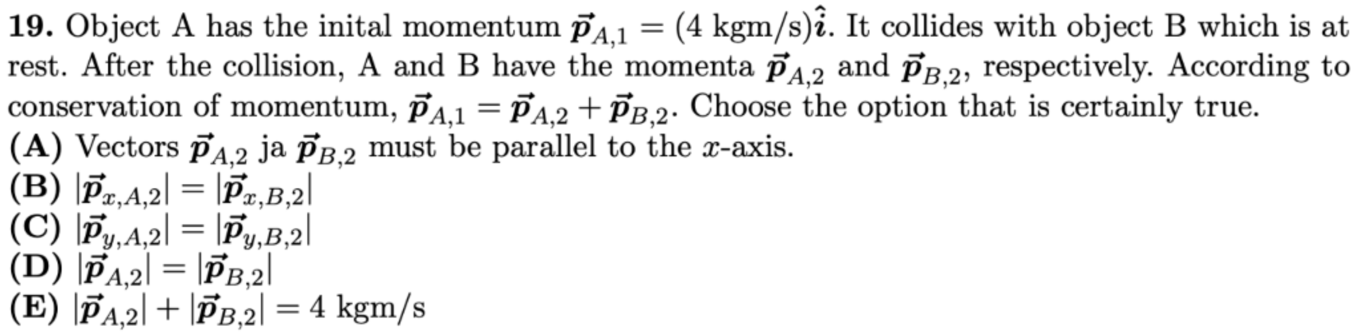
In the applied physical tasks, one difficulty was confusing the sum and difference of vectors in tasks requiring graphical interpretation (tasks 1 and 10). Tasks 1 and 10 asked the students to choose the option corresponding to the resultant force and change in velocity, respectively (Figure 4). In both tasks, many students confused the sum and difference vectors. In task 10, some students (over 14% in the pre-test) also calculated the difference in the wrong order (corresponding to option A, see Figure 4) – velocity in the beginning minus velocity in the end, rather than vice versa (option E). While the students improved in both questions, the velocity question was still one of the most difficult in the post-test. Task 19 (Figure 5) was the most difficult. Over 96% of the students failed to understand the vector nature of the conservation of momentum, that is, that in collisions, the sum of momenta vectors is conserved. Instead, most of the students (over 60% in the pre-test) chose an option indicating that the magnitude of momentum is conserved (option E, Figure 5). Some (20% in the pre-test) chose option A, failing to consider that objects can gain momentum in the y-direction as long as the vector sum in the y-direction is zero.

Figure 4. Task 10 in the test.



10. Consider a car (seen from above) that follows the path shown in the figure. The figure also shows the velocity of this car in two instants $\vec{v}_1$ and $\vec{v}_2$. Choose the option that shows the change of velocity vector that is the vector difference $\vec{v}_2 - \vec{v}_1$.

Figure 5. Task 19 in the test.



19. Object A has the inital momentum $\vec{p}_{A,1} = (4\ \text{kgm/s})\hat{\imath}$. It collides with object B which is at rest. After the collision, A and B have the momenta $\vec{p}_{A,2}$ and $\vec{p}_{B,2}$, respectively. According to conservation of momentum, $\vec{p}_{A,1} = \vec{p}_{A,2} + \vec{p}_{B,2}$. Choose the option that is certainly true.
(A) Vectors $\vec{p}_{A,2}$ ja $\vec{p}_{B,2}$ must be parallel to the x-axis.
(B) $|\vec{p}_{x,A,2}| = |\vec{p}_{x,B,2}|$
(C) $|\vec{p}_{y,A,2}| = |\vec{p}_{y,B,2}|$
(D) $|\vec{p}_{A,2}| = |\vec{p}_{B,2}|$
(E) $|\vec{p}_{A,2}| + |\vec{p}_{B,2}| = 4\ \text{kgm/s}$

5 Discussion

This study investigated the development of physics students' abstract mathematical and applied physical competence in the context of a pedagogical intervention focused on vector concepts. The results provide insights into how these two competences evolved over 12 weeks and the types of misconceptions students have about vectors.

The quantitative findings indicate a modest overall improvement in the students' physics competence, with the total score increasing by 1.43 points from pre- to post-test ($p < .01$). This gain was primarily driven by improvements in applied physical competence (MD = 1.46, p = <.001), while abstract mathematical competence remained largely unchanged (MD = -.029, p = .81). This finding suggests that the intervention was more effective in supporting the students' ability to apply mathematical reasoning within physical contexts than in enhancing their abstract mathematical understanding. The competence class analysis further supports this interpretation. The students who transitioned to a higher level of applied physical competence often did so without a corresponding increase in abstract mathematical competence. Conversely, improvements in abstract mathematical competence tended to occur independently of the applied physical competence. This asymmetry highlights a potential disconnection in students' ability to transfer abstract mathematical knowledge to physical applications, a phenomenon previously described in the literature as applicationist epistemology (Barquero et al., 2013) referring to a situation where mathematics and physics are perceived as separate domains.

The observed pattern, that is, where applied physical competence improves without parallel gains in abstract mathematics, align with earlier research suggesting that students often struggle to integrate mathematical knowledge into physics learning (Bergsten & Jablonka, 2013; Palmgren & Rasa, 2024). This may indicate that there is a certain threshold for mathematical competence before it can be successfully applied to other contexts, such as physics. Such results are mostly based on cross-sectional designs and should therefore be regarded as tentative (Edelsbrunner & Hofer, 2024). However, the present study adds to this with a longitudinal approach and is in line with the previous findings. However, students failing to integrate mathematics and physics might reflect broader educational or disciplinary cultures where these two subjects are treated as separate (Redish & Kuo, 2015). For example, in schools, teachers' readiness to integrate different subjects into the curriculum might vary, and mathematics may be seen as the most difficult subject to integrate with others (Niemelä, 2022). Also, students may rely on procedural and contextual cues rather than deep mathematical reasoning when solving physics problems. In any case, more research is needed to further understand the joint development of abstract mathematical and applied physical competences.

The qualitative analysis of the students' incorrect responses revealed persistent misconceptions related to vector concepts. As in previous studies of students' difficulties with vector concepts, the dot product, sum and difference of vectors, and cross product proved to be difficult in our study as well. The dot product has previously been found to be among the most difficult vector concepts (Barniol & Zavala, 2014a; Virtanen et al, 2024). Similar to the study of Barniol and Zavala (2014a), we found that the students interpreted the dot product to correspond to the magnitude of a vector between the vectors. This incorrect interpretation might be mirrored in physics tasks requiring the application of the dot product, such as task 13 in our test, which required the students to infer the work done by a force along a line. The difficulties the participants exhibited are similar to those found by Virtanen et al. (2024) in their study among Finnish high school students. It is noteworthy, however, that the participants had already completed their first mechanics course at the university. It is somewhat worrying that they still struggled with some very basic concepts, such as sum and difference, regarding vectors. Similar findings have been reported in the US context by Flores and colleagues (2004). The fact that students struggle with mathematics and its application to physics, despite having completed university-level courses in both subjects, is concerning and should be taken seriously when developing undergraduate physics education.

This study has limitations that should be considered when interpreting the findings. Aside from the small number of participants in this study, the implementation of the intervention workshop course should be considered. The students who participated in this research had differing attendance rates in the workshops. Furthermore, each small group had different participants and teaching assistants, most probably causing differing group dynamics and learning environments. Thus, the participants were not a homogeneous group, so further conclusions on the learning gains specifically caused by the intervention course should be made with caution. However, the results show that there were no major improvements to the students' mathematical skills during the intervention; thus, it is necessary to reassess the intervention design and determine whether the activities truly correspond with the aims of the intervention. A concrete action that has already been implemented was training the teaching assistants to ensure greater consistency of workshop instruction practices.

Taken together, the findings of the present study suggests that while applied physical competence can be developed through targeted instruction, its sustainability and depth may depend on some foundational level of abstract mathematical understanding. This supports the hypothesis that a threshold of mathematical competence is necessary for meaningful development in physics learning. As Palmgren and Rasa (2024) suggest, future pedagogical interventions should aim to bridge this gap between abstract and applied domains by fostering students' ability to reason across representations and contexts. The (only) modest overall improvement of the participants' physics competence observed in the present study further demonstrates this need and calls for more deliberate teaching practices; although the intervention provided students with time and space to engage with the mathematics and physics material, it proved insufficient to help them achieve substantial learning gains. Still, the present study illustrates the value of research-based pedagogical development: a problem was identified, an intervention was designed and tested, and the findings point to the need for more intensive intervention, along with preliminary guidelines for its implementation. This intervention can, in turn, be researched again. Understanding the iterative nature of pedagogical development is valuable for mathematics and physics lecturers as well as for anyone developing undergraduate physics education.

Research ethics

Author contributions

J.L.: conceptualisation, investigation, methodology, data curation, formal analysis, project administration, writing – original draft, writing – review and editing
H.K.: conceptualisation, investigation, methodology, writing – original draft, writing – review

and editing
T.K.: conceptualisation, investigation, methodology, data curation, formal analysis, writing – original draft, writing – review and editing
T.H.: conceptualisation, investigation, methodology, data curation, visualisation, writing – original draft, writing – review and editing

All the authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all the research participants.

Data availability statement

The data are unavailable due to privacy restrictions.

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Conflicts of Interest

The authors declare no conflicts of interest.

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