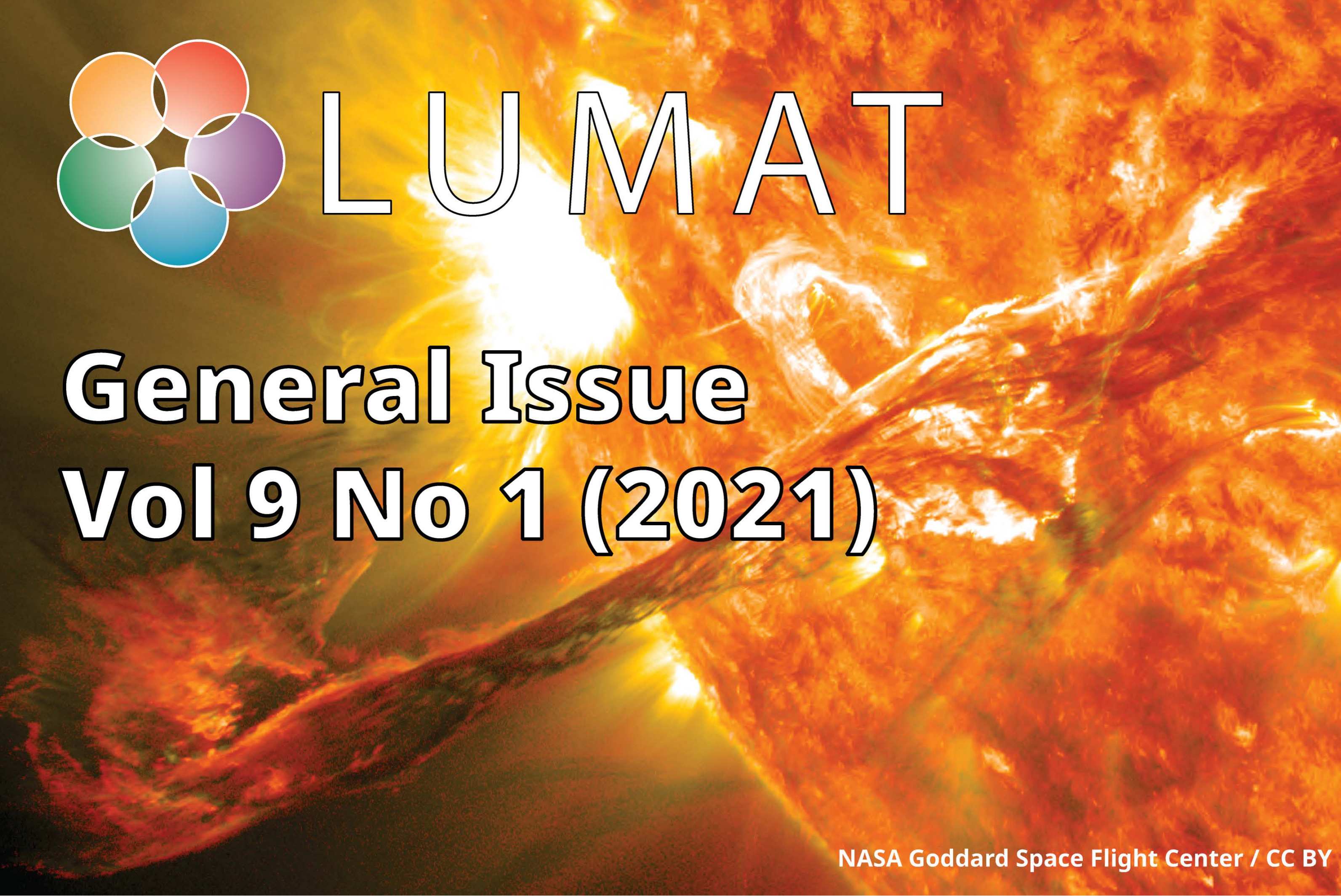

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Yngre elevers uppfattningar av det matematiska i algebraiska uttryck

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Det övergripande syftet med denna artikel är att analysera och beskriva yngre elevers uppfattningar av det matematiska i ett algebraiskt uttryck och utifrån det diskutera vad som kan utgöra kritiska aspekter för utvecklandet av mera kvalificerade uppfattningar. Artikeln bygger på data från ett forskningsprojekt där elever i förskoleklass, årskurs 1 och årskurs 4 intervjuades med syfte att analysera de aktuella elevernas kvalitativt skilda sätt att uppfatta det matematiska i algebraiska uttryck. Intervjuerna analyserades fenomenografiskt. Studiens resultat visar tre kvalitativt skilda kategorier av yngre elevers uppfattningar av det matematiska i algebraiska uttryck. Det matematiska i ett algebraiskt uttryck erfars som ”något som kan och bör räknas ut”, ”något som beskriver en relation mellan komponenter” och ”något som representerar en situation”. Vidare identifierades tre kritiska aspekter i relation till kategorierna. De kritiska aspekter som ger eleverna möjlighet att kvalificera sina uppfattningar för att utveckla ett mer komplext kunnande av algebraiska uttryck är att kunna urskilja att 1) ett uttryck består av olika komponenter som har olika funktioner, 2) en och samma variabel i ett uttryck har samma värde och 3) värdet på en variabel i ett uttryck bestäms relationellt. Att urskilja sådana kritiska aspekter kan hjälpa eleverna att kvalificera sitt kunnande. Således måste de kritiska aspekterna beaktas vid utformningen av undervisningen.

Nyckelord: algebraiska uttryck, algebraiskt tänkande, fenomenografiska uppfattningar, kritiska aspekter, tidiga skolår

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Younger students’ conceptions of the mathematics in algebraic expressions

The overall purpose of this article is to analyze and describe younger students’ conceptions of or ways of experiencing the mathematics in an algebraic expression and to discuss what can be critical aspects for the development of more qualified conceptions. The article is based on data from a research project where students in preschool class, Grade 1 and Grade 4 were interviewed with the aim of analyzing the students' qualitatively different ways of experiencing the mathematics in algebraic expressions. The interviews were analyzed with phenomenography. The results show three qualitatively different categories of younger students’ conceptions of the mathematics in algebraic expressions. The mathematics in an algebraic expression is experienced as ”something that can and should be calculated", ”something that describes a relationship between components”, and

”something that represents a situation”. Furthermore, three so-called critical aspects the students need to discern were identified in relation to the categories 1) an expression consists of different components that have different functions, 2) one and the same variable in an expression has the same value and 3) the value of a variable in an expression is determined relationally. Discerning such critical aspects may help the students to qualify their ways of knowing. Thus, the critical aspects need to be considered in the design of teaching.

Keywords: algebraic expressions, algebraic thinking, critical aspects, phenomenographic conceptions, primary education

1 Introduktion

En förutsättning för att utveckla kunskaper inom olika områden i matematik är en förståelse av algebra (Bråting et al., 2018; Hemmi et al., 2020). Algebra ses av tradition som ett av de mer krävande matematiska innehållen i grundskolans matematikundervisning och har i västvärlden därför tidigare introducerats rätt sent, ofta först i högstadiet (se t.ex. Bråting et al., 2019; Hemmi et al., 2020; Kilhamn & Röj-Lindberg, 2019; Stacey & Chick, 2004). Såväl svensk som internationell forskning har de senaste decennierna fört fram argument som pekar på vikten av att introducera algebra tidigt i skolan, främst i syfte att utveckla elevers algebraiska tänkande (se t.ex. Blanton et al., 2015; Bråting et al., 2019; Cai & Knuth, 2011; Davydov, 2008; Eriksson & Jansson, 2017; Kieran, 2018; Mason, 2008, 2018; Schmittau, 2004, 2005). Intresset för att introducera algebra tidigt i skolan kommer bland annat till uttryck i hur läro- och kursplaner i matematik[1] formulerats. I Sverige och de övriga nordiska länderna framträder likheter gällande kursplanernas centrala innehåll för algebra och när detta innehåll introduceras. I den finländska kursplanen från 2014 lyfts exempelvis att eleverna i årskurs 1–2 ska erbjudas möjligheter att utveckla sin förmåga att upptäcka likheter, skillnader och mönster (Utbildningsstyrelsen, 2020). Vidare ska de i årskurs 3–6 introduceras till begreppet *obekant* samt ges möjlighet att undersöka ekvationer. Liknande innehåll lyfts i den svenska (Skolverket, 2019), den danska från 2019 (Undervisningsministeriet, 2020) och den norska kursplanen (Utdanningsdirektoratet, 2020). Gemensamt för kursplanerna är att eleverna inledningsvis introduceras till ämnesinnehållet tal och mönster, i form av mönster i talföljder och talföljder utifrån en regel. I årskurserna 4–6 introduceras variabler och

[1] De nordiska läro- och kursplanerna använder olika begreppsapparater för likartade områden. Exempelvis används läro- och kursplaner samt kompetenser, mål och förmågor synonymt. I artikeln används de svenska benämningarna för motsvarande områden.

ekvationer där även likhetstecknets betydelse behandlas.[2] I Sverige betonas likhetstecknets betydelse tydligare jämfört med de andra länderna. Detta framträder exempelvis i en studie om svenska läromedel för de yngsta eleverna i form av uppgifter där värdet på ett okänt tal efterfrågas 4 + __ = 8 (Hemmi et al., 2019). Denna typ av uppgifter återkommer även i läromedel för årskurs 7–9.

Utifrån forskning om algebraundervisning (utvecklas nedan) vet vi att elever vanligen introduceras till algebra på en aritmetisk grund i form av exempelvis uppgifter riktat mot likheter där värdet på ett okänt tal efterfrågas. Samtidigt argumenterar allt fler forskare för att elever snarare behöver möta uppgifter och situationer som utvecklar ett algebraiskt tänkande. Exempelvis behöver elever få en möjlighet att utforska algebraiska strukturer (Blanton et al., 2015; Bråting et al., 2018; Kieran et al., 2016). Hur elever uppfattar algebraiska fenomen i form av det generella (symboler, strukturer och relationer) i algebraiska uttryck eller ekvationer fokuseras dock inte i särskilt många studier (se t.ex. Bråting et al., 2019).

1.1 Syfte och frågeställningar

Syftet med föreliggande artikel är att med hjälp av en fenomenografisk analys beskriva och diskutera yngre elevers erfarande av det matematiska i algebraiska uttryck. Med ett algebraiskt uttryck avses i denna artikel en meningsfull sammansättning av matematiska symboler (Kiselman & Mouwitz, 2008). Det innebär exempelvis att $a + b - c$ och $ax + b$ är uttryck, men även olikheten $a < b$ och likheten (eller ekvationen) $a = b + c$ (James & James, 1976). I studien som ligger till grund för denna artikel har vi använt algebraiska uttryck i form av likheter av typen $5x = y$. Vidare är syftet att identifiera potentiella kritiska aspekter i relation till mer kvalificerade uppfattningar av algebraiska uttryck. Syftet preciseras i följande två frågeställningar:

- Vilka kvalitativt skilda sätt att erfara algebraiska uttryck kan urskiljas?
- Vilka aspekter behöver eleverna erfara för att de ska ges möjlighet att kvalificera sina uppfattningar om algebraiska uttryck?

[2] I den reviderade svenska kursplanen i matematik som träder i kraft 1 juli 2021 introduceras obekanta tal och hur de kan symboliseras redan i årskurs 1–3 och variabler och användningen av variabler i exempelvis enkla algebraiska uttryck och ekvationer samt algebraiska metoder för att lösa enkla ekvationer introduceras redan i årskurs 4–6 (Skolverket, 2020).

2 Tidigare forskning om algebraundervisning

Även om algebra idag introduceras redan i grundskolans lägsta årskurser är det vanligast att det sker i vad van Oers (2001) benämner som en aritmetisk undervisningstradition, i form av det som kan beskrivas som pre-algebra (se t.ex. Gravemeijer, 2002; Kaput et al., 2008; Kieran, 2006; Lins & Kaput, 2004; Riesbeck, 2008; Schmittau, 2004, 2005). Med ett aritmetiskt tänkande som grund kan elever få svårt att föra ett mer generellt analytiskt resonemang om till exempel vad en variabel representerar eller hur man kan förstå ett okänt värde (Schmittau, 2004, 2005). Stacey och MacGregor (1999) betonar att när elever i senare årskurser introduceras till algebra, exempelvis i form av ekvationer, tenderar de att försöka lösa dessa med en aritmetisk problemlösningsmetod genom att lägga in bestämda värden för att på detta sätt kunna argumentera för en lösning.

Krutetskii (1976) beskriver de problem många elever har när de möter algebra på följande sätt:

> It was always very hard for our students to abstract themselves from concrete numerical expressions. Our students had difficulty (some more, others less, but all had difficulty!) understanding the very essence of algebra, which is an operation with numerical abstractions. It was hard for them to understand that letters in algebra are numbers deprived of their concrete expression. (Krutetskii, 1976, s. 253–254)

Krutetskii menar alltså att svårigheten för eleverna är förståelsen för att själva kärnan i algebra är operationer med numeriska abstraktioner, det vill säga att bokstäver i algebra är okända kvantiteter eller värden.

2.1 Tidig introduktion av algebra

För att motverka en del av de svårigheter som brukar uppstå senare i utbildningen förespråkas inom forskningsfältet *early algebra* att elever med fördel kan introduceras till algebraisk problemlösning och algebraiskt tänkande tidigt (Blanton et al., 2018; Blanton et al., 2015; Bråting et al., 2019; Cai & Knuth, 2011; Eriksson & Jansson, 2017; Kieran, 2018; Kieran et al., 2016; Mason, 2008; Radford, 2015). Vad som menas med termen ”tidigt” skiljer sig dock åt. Länge har ”tidigt” refererats till elever som är 10–12 år gamla medan det idag är vanligare att använda termen ”tidigt” i relation till algebra för elever i förskoleklass eller årskurs 1 (Blanton et al., 2015; Kieran, 2011; Morris & Sloutsky, 1995).

Enligt Blanton et al. (2015) anger Kaput (2008) två grundläggande handlingar för att beskriva ett algebraiskt tänkande: (i) att uttrycka generaliseringar i allt högre grad med formella och konventionella symbolsystem; och (ii) att föra och följa resonemang med symboler. Utveckling av algebraiskt tänkande inbegriper bland annat utforskandet av generella, grundläggande och teoretiska samband och strukturer (Blanton et al., 2015; Davydov, 2008; Kaput, 2008; Venenciano & Dougherty, 2014). Ett flertal forskare hänvisar idag till det matematiska program som utvecklats av El'konin och Davydov som en framgångsrik modell för tidig introduktion av algebra (se t.ex. Cai & Knuth, 2011; Kieran et al., 2016; Schmittau, 2004, 2005; Venenciano & Dougherty, 2014). Davydov (1990) beskriver detta som "ascending from the abstract to the concrete" (s. 173) och menar att elever, genom lärandemodeller och kollektiva reflektioner, först behöver lära sig att arbeta med generella strukturer och relationer i exempelvis algebraiska uttryck, för att senare använda dem i konkreta numeriska operationer. Kieran (2004) föreslår också att elever tidigt på olika sätt behöver arbeta teoretiskt med algebra.

> Algebraic thinking in the early grades involves the development of ways of thinking within activities for which the letter-symbolic algebra could be used as a tool, or alternatively within activities that could be engaged in without using the letter-symbolic algebra at all, for example, analyzing relationships among quantities, noticing structure /…/ generalizing, problem solving. (Kieran, 2004, s. 149)

Algebraiskt tänkande kan enligt Kieran alltså handla om användning av bokstavssymboler vid analys av exempelvis relationer mellan kvantiteter, strukturer och generalisering. Schmittau och Morris (2004) hävdar att en tidig algebraundervisning lägger grunden för ett pre-numeriskt tänkande (att lära sig att förstå olika tal som tal), i motsats till ett pre-algebraiskt tänkande som dominerar i aritmetiska undervisningstraditioner. Schmittau och Morris jämför den undervisning som Davydov och hans kollegor i Ryssland utvecklat med motsvarande undervisning i USA: ”Thus, while children in the US have pre-algebraic experiences that are numerical, Russian children studying Davydov's curriculum have *pre-numerical experiences that are algebraic*” (s. 61, emfas i original). Davydov (1975) benämner en sådan undervisning som ”algebraization of elementary mathematics” (s. 202). Idén med en pre-numerisk förståelse är att eleverna behöver utveckla en teoretisk grund för att förstå olika tal som tal och att inte se tal i exempelvis uttryck enbart som diskreta storheter, som heltal (Eriksson, 2015). Mot denna bakgrund behöver en undervisning som skapar förutsättningar för elever att utveckla förmågor som att resonera algebraiskt, att göra algebraiska generaliseringar samt att använda algebraiska representationer erbjudas elever (Greer, 2008; Kaput, 1999; Usiskin, 1988). För att ha en grund för undervisningens utformning behöver vi även kunskaper om hur elever uppfattar olika matematiska fenomen, som exempelvis algebraiska uttryck.

2.2 Uppfattningar av algebraiska fenomen

I en systematisk litteratursökning[3] framkommer det att forskningen gällande uppfattningar av fenomen kopplat till algebra är begränsad. En större del av studierna undersöker äldre elevers, lärarstudenters eller lärares föreställningar, uppfattningar eller missuppfattningar gällande algebra. Ett fåtal studier fokuserar yngre elevers uppfattningar och ännu färre studier har använt en fenomenografisk ansats. I det följande har vi begränsat oss till studier som berör algebraiska fenomen som uttryck, ekvationer och likhetstecken. Ett flertal studier som till exempel berör mönster, funktioner och substitution har sorterats bort.

Även om vår studie direkt intresserar sig för de yngsta elevernas uppfattningar av algebraiska uttryck har studier som riktar sig mot äldre elevers och lärares uppfattningar relevans för föreliggande artikel. Av de studier vi granskat finns uppfattningar som är av olika karaktär. Exempelvis framkommer det i en del av studierna att elever har uppfattningar som inte är av matematisk karaktär (tolkar bokstäver alfabetiskt eller som en förkortning) men även uppfattningar av att algebraiska uttryck implicerar numeriska operationer (Knuth et al., 2005; MacGregor & Stacey, 1997; Sfard & Linchevski, 1994; Stacey & MacGregor, 1997; Wagner, 1983). Stacey och MacGregor exemplifierar hur elever tar med sig vardagserfarenheter in i algebraundervisningen. Det kan till exempel inkludera elevers användningar av bokstäver i andra sammanhang, operationer med sammansatta symboler (t.ex. 7 1/2, 34, XII) och att läsa likhetstecknet som ett processtecken, det vill säga att något ”blir”. I studier som har undersökt elevers uppfattningar av likhetstecknet erfars uttrycket till vänster om likhetstecknet som en process och uttrycket till höger som resultatet (Frieman & Lee, 2004). Detta innebär att likhetstecknet signalerar att något ska ”ska göras”, det vill säga som ett processtecken (Kieran, 1981). Knuth et al. (2005) visar även på en förståelse av likhetstecknet som betecknande en relation, det vill säga att

[3] Huvudsökning gjordes i databasen Ebsco där sökorden *conception*, *perception*, *misconception*, *beliefs*, *phenomenography*, *phenomenographic* tillsammans med *algebraic* användes i olika kombinationer. Sökningar har också gjorts i SWEpub med motsvarande söktermer på svenska. Även kedjesökning utifrån från tidigare kända studier och de studier som identifierades i den systematiska sökningen har använts.

man skapar en likhet mellan de två sidorna av en given aritmetisk ekvation (Carpenter et al., 2003; Knuth et al., 2005). Ronda (2009) menar att det är centralt att elever har en förståelse av den relationella aspekten i en ekvation då detta är nästa steg för att kunna lösa uppgifter med två variabler. Att notera är att Attorps (2006) studie visar att även en del lärare erfar ekvationer som en procedur, som ett svar eller som ett uttryck.

En tidig studie som behandlar elevers uppfattningar av algebraiska bokstavssymboler, genomförd av Küchemann (1981, jfr även Wagner, 1983), beskriver sex skilda uppfattningar: 1) bokstaven tilldelas ett värde; 2) bokstaven används inte; 3) bokstaven används som objekt eller förkortning; 4) bokstaven används som ett specifikt, okänt tal; 5) bokstaven används som ett godtyckligt tal och 6) bokstaven används som variabel (för svensk översättning av Küchemanns kategorier se Olteanu, 2014). Även i Küchemanns kategorisering kan de tre första kategorierna kopplas till sådana erfarenheter som Stacey och MacGregor (1997) diskuterar. Det är således endast kategorierna 4–6 ovan som kan ses som mer matematiskt relevanta.

Ett flertal studier exemplifierar och diskuterar sådana uppfattningar som ses som outvecklade, felaktiga eller som bygger på missförstånd. Få studier tar sikte på vad elever som ser likhetstecknet som en processymbol behöver urskilja för att utveckla en mer algebraisk relevant förståelse. Forskning som använder variationsteori talar om vilka aspekter av ett kunnande som eleverna behöver lära sig – så kallade kritiska aspekter (se t.ex. Kullberg et al., 2017; Olteanu, 2007; Wernberg, 2009). Det som ännu inte urskilts är därmed det som i variationsteorin beskrivs som kritiska aspekter (Pang & Ki, 2016).

Mot bakgrund av den tidigare forskningen som refererats ovan finns det således skäl att studera de yngsta elevernas uppfattningar av fenomen som algebraiska uttryck, det vill säga innebörden i det kunnande eleverna förväntas utveckla.

3 Metod och analys

Data som ligger till grund för artikeln kommer från ett treårigt forskningsprojekt, 2017–2019, finansierat av Skolforskningsinstitutet (2020), där målet var att främja elevers algebraiska resonemangsförmåga (Eriksson et al., 2019; Stockholms universitet, 2020). För att få en grund för detta forskningsprojekt genomförde vi en mindre fenomenografisk kartläggningsstudie med elever från förskoleklass, årskurs 1 och årskurs 4. Det initiala syftet med kartläggningen var att utforska och beskriva de aktuella elevernas kvalitativt skilda sätt att uppfatta algebraiska uttryck.

Sammanlagt 20 elever intervjuades, tio elever från förskoleklass och årskurs 1 samt tio elever från årskurs 4 vid två olika skolor.[4] Eleverna intervjuades parvis där utgångspunkt för sammansättningen av elevparen var att de skulle bestå av en elev som bedömdes vara mer kunnig i matematik och en elev som bedömdes vara lite mindre kunnig. Urvalet gjordes i samråd med undervisande lärare med avsikten att skapa förutsättningar för kvalitativt skilda uppfattningar gällande algebraiska uttryck att framträda. Eleverna i förskoleklassen, årskurs 1 och de i årskurs 4 hade inte undervisats om algebra eller algebraiska uttryck innevarande läsår. Däremot hade alla elever tidigare mött uppgifter där värdet på ett okänt tal efterfrågas.

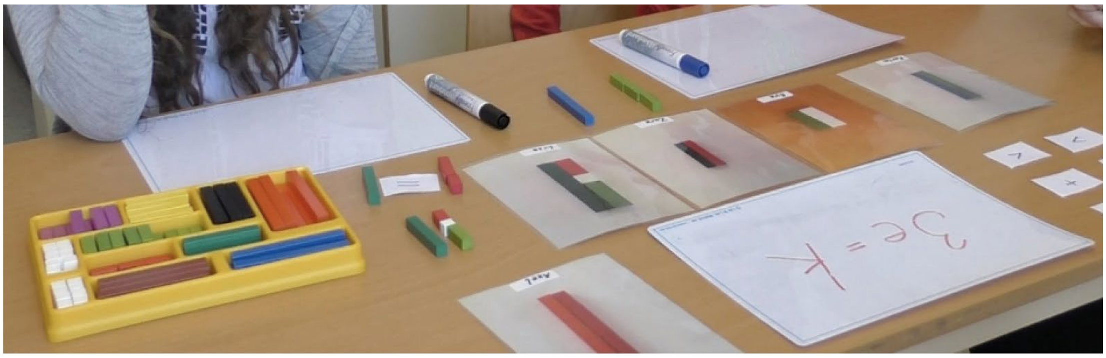
3.1 Elevintervjuer

Frågorna i intervjuerna konstruerades utifrån erfarenheter från en pilotstudie där eleverna arbetat med Cuisenairestavar[5] för att utforska relationella aspekter i algebraiska uttryck. Utgångspunkten var att intervjufrågorna skulle öppna upp för elevers erfarande av olika aspekter av strukturer och relationer. Vi utgick även från variationsteoretiska principer i utformningen av de konkreta exemplen som utgjorde underlag för frågorna. Intervjuerna kan således beskrivas som materialbaserade (se t.ex. Lindberg & Löfgren, 2010). En idé med materialbaserade intervjuer, som här är genomförda med hjälp av olika visuella representationer, är att i någon mån kunna säkerställa vilket fenomen som adresseras (Adawi et al., 2001; Ingerman et al., 2009; Jägerskog, 2020). Samtidigt innebär detta givetvis också en begränsning i vad intervjupersonerna kommer att tala om. Med små barn är det speciellt svårt att genomföra intervjuer så att det valda fenomenet kan hållas i fokus (Jaidin, 2018; Österlind, 1998). Som nämnts hade eleverna inte mött denna typ av algebraiska uttryck tidigare varför vi har utgått ifrån att den riggade intervjusituationen hjälpt till att hålla fenomenet ”algebraiska uttryck” i fokus. Frågorna utformades utifrån en idé om att eleverna skulle resonera om hur andra elever kunde ha tänkt när de löste uppgifterna med intentionen att detta också skulle öka förutsättningarna med att hålla fokus på fenomenet.

[4] Eleverna som intervjuades i förskoleklass och årskurs 4 skulle delta i forskningslektionerna följande termin.
[5] Cuisenairestavar är ett slags relationellt laborativt material som består av stavar i olika längder och färger, där varje längd har en viss färg (Küchemann, 2019) (se Figur 1).

3.2 Intervjufrågorna

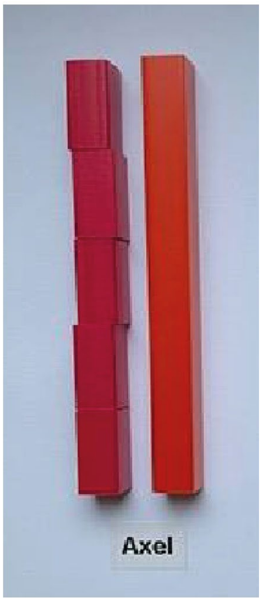
Intervjufrågorna kombinerades, som nämnts ovan, med material i form av bilder på skrivna algebraiska uttryck såsom $5x = y$, $5c = z$ och $k = 3e$ samt bilder där de algebraiska uttrycken gestaltades med hjälp av Cuisenairestavar (se Figur 1).



Figur 1. Materiell iscensättning av intervjusituationen under elevintervjuerna.

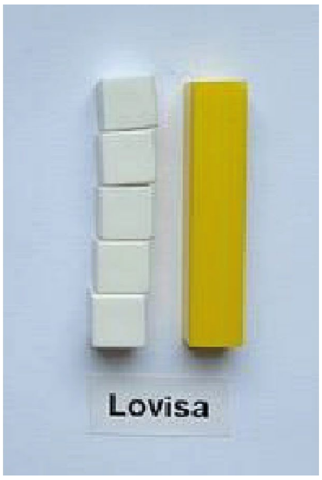
Även Cuisenairestavar i fysisk form fanns tillgängliga för eleverna. Vidare hade eleverna tillgång till små whiteboards med tillhörande whiteboardpennor. Samma intervjufrågor användes vid intervjuerna med samtliga elever, det vill säga i förskoleklass, årskurs 1 och årskurs 4. Vi ställde inga direkta frågor riktade till fenomenet. Vi frågade således inte eleverna ”Vad är ett uttryck?”, utan genom att eleverna fick arbeta med de algebraiska uttryck som var avbildade framträdde *vad* och *vilka* aspekter eleverna innehållsligt fokuserade.

Ingångsfrågan i intervjuerna var att vi sa att elever ”i en annan klass” hade fått se ett algebraiskt uttryck, $5x = y$, och att eleverna ”i den andra klassen” fick i uppgift att visualisera uttrycket genom att använda Cuisenairestavar: ”... i den klassen fick de [eleverna] en uppgift /.../ de skulle visa eller lägga det här uttrycket: fem x är lika med y [$5x = y$]”. Efter ingångsfrågan visade vi en bild på en stavkonstruktion med en orange stav med fem röda stavar bredvid (Figur 2) och la med fysiska Cuisenairestavar en likadan stavkonstruktion som bilden på Axels konstruktion. Sedan frågade vi hur den eleven som hade lagt stavarna så kunde ha tänkt: ”Axel la uttrycket så här. Hur kan Axel ha tänkt och resonerat när han gjorde det?”.



Figur 2. Axels stavkonstruktion i relation till uttrycket 5x = y.

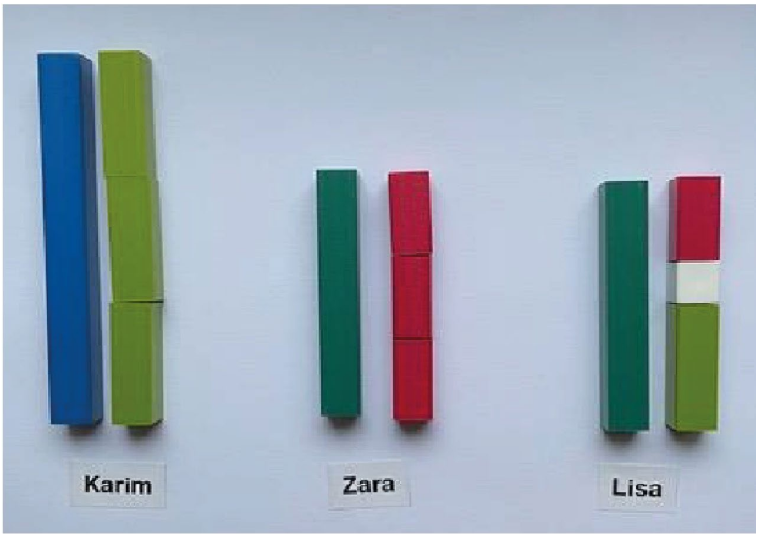
I den andra intervjufrågan använde vi en bild på en annan stavkonstruktion som också illustrerade uttrycket $5x = y$ (Figur 3). Uttrycket hölls således konstant medan stavkonstruktionen varierades. Vi sa: ”Lovisa la uttrycket så här” och frågade därefter ”Hur kan hon ha tänkt? Kan man göra så?”



Figur 3. Lovisas stavkonstruktion i relation till uttrycket 5x = y.

I fråga tre valde vi att hålla en stavkonstruktion konstant och variera uttrycket. Vi visade Axels stavkonstruktion (Figur 2 ovan) och sa: ”Pelle, en elev i en annan klass, fick se Axels stavkonstruktion och Pelle fick i uppgift att skriva ett uttryck till den. Han skrev uttrycket fem c är lika med z [intervjuaren skrev samtidigt $5c = z$ på en liten whiteboard]. Hur kan han ha tänkt när han skrev det?”

Den fjärde och sista frågan inleddes med att vi visade tre bilder på olika stavkonstruktioner som förslag till samma uttryck, $k = 3e$, och frågade hur de tre fiktiva eleverna, Karim, Zara och Lisa, kunde ha tänkt när de la sina olika stavkonstruktioner med Cuisenairestavar (Figur 4). De tre stavkonstruktionerna stämde relationellt med varandra men endast två av konstruktionerna stämde med uttrycket $k = 3e$.



Figur 4. Karims, Zaras och Lisas stavkonstruktioner i relation till uttrycket k = 3e.

Samtliga intervjuer videofilmades och transkriberades i sin helhet utifrån Linells (1994) beskrivning: ordagrant, talspråkligt neutralt samt organiserat i replikform. Även gester och referenser till materialet som eleverna hade tillgång till har beskrivits i hakparenteser (Nordin & Boistrup, 2018). Vid transkriberingen har elevernas namn fingerats.

3.3 Fenomenografisk analys

Elevintervjuerna analyserades fenomenografiskt (Marton, 1981). En fenomenografisk analys syftar till att förstå och beskriva kvalitativt skilda sätt att uppfatta eller erfara ett fenomen. Ett grundantagande är att människor, på basis av vad de har erfarit i livet, vilka situationer och problem de har mött, uppfattar ett fenomen på ett specifikt sätt, det vill säga att vi urskiljer olika aspekter ofta mer eller mindre komplexa sådana (Marton, 1981; jfr Eriksson, 1999). Inom fenomenografi skiljer man på första och andra ordningens perspektiv, där första ordningens perspektiv handlar om vad en person framför som sant eller värderar. Andra ordningens perspektiv, vilket är fenomenografins intresse, handlar istället om vad personerna erfarit och som därmed ligger till grund för hur de uttalar sig. Enkelt uttryckt handlar första ordningens perspektiv om det som står på raderna i en transkriberad intervju medan andra ordningens perspektiv handlar om det underförstådda, det som ligger bakom eller bortom raderna (Marton, 1981; Pang, 2003). Bland personer verksamma inom en och samma praktik, exempelvis matematiklärare, förskoleklasselever eller årskurs fyra-elever är det ovanligt att det förekommer en mycket stor variation av uppfattningar. Detta innebär att fenomenografin inte intresserar sig för en enskild persons uppfattningar, utan istället antas en och samma person kunna ge uttryck för en eller flera uppfattningar. En fenomenografisk analys resulterar således vanligen i ett begränsat antal, men kvalitativt skilda, uppfattningar (Marton, 1981, 2015, jfr även Eriksson, 1999) Analysarbetet med att identifiera de olika uppfattningarna förutsätter att både kunna fokusera vad de intervjuade talar om (fenomenet) och hur de talar om det. Resultatet från en fenomenografisk analys utgörs av ett antal kvalitativt skilda uppfattningar presenterade som sinsemellan relaterade kategorier. Sammantaget utgör kategorierna med deras inbördes relationer ett utfallsrum. Det innebär att det som kvalitativt skiljer en uppfattning från en annan är begripligt först när de relateras till varandra (Marton, 1981). Kategorierna i utfallsrummet följer vanligen en hierarkisk logik.

Variationsteorin kan ses som en vidareutveckling av fenomenografin. Den använder resultatet från en fenomenografisk analys för att urskilja så kallade kritiska aspekter. Detta innebär att en hierarkisk ordning på kategorierna implicerar ett mer differentierat urskiljande av olika aspekter. För att arbeta fram kritiska aspekter analyseras kategorierna i relation till varandra för att identifiera aspekter som är urskilda i respektive kategori och i förlängningen vilka aspekter som behöver urskiljas för utvecklingen av en allt mer differentierad uppfattning.

3.4 Vägen fram till kategorier

Inledningsvis närlästes de transkriberade elevintervjuerna flera gånger med fokus på att identifiera vad eleverna talade om. Intervjumaterialet visade indikationer på utsagor som antydde att uttrycken eleverna presenterades för bland annat uppfattades som en gåta, en kluring, alfabetisk logik, att versaler och gemener hade betydelse eller att bokstäverna kunde vara en förkortning. Eftersom det matematiska innehållet algebra, här i form av algebraiska uttryck, var nytt för eleverna var det också vanligt att eleverna inledningsvis gissade och kopplade till vardagserfarenheter (jfr MacGregor & Stacey, 1997; Stacey & MacGregor, 1997). I en fenomenografisk mening utgör dessa icke-matematiskt relaterade uppfattningar också uppfattningar av fenomenet i fråga, men för forskningsprojektets behov var denna typ av uppfattningar inte relevanta. De delar som inte innefattade matematiska aspekter lades därför åt sidan. Detta innebar att fenomenet preciserades från att inledningsvis ha beskrivits som *elevers uppfattningar av algebraiska uttryck* till *elevers uppfattningar av det matematiska i algebraiska uttryck*. Efter att denna avgränsning av data hade genomförts analyserades den kvarvarande sammantagna texten med elevutsagor för att genom en komparativ och simultan läsning få fram uppfattningar. Utifrån denna läsning utarbetades kvalitativt skilda kategorier som prövades mot den totala mängden elevutsagorna i materialet. När alla utsagor, eller delar av en utsaga, kunde relateras till någon av de identifierade kategorierna gjordes en beskrivning av respektive kategori.

De lärare som deltog forskningsprojektet men inte inledningsvis i analysarbetet fick i validerande syfte i uppdrag att sortera elevutsagorna i de olika kategorierna.[6] De utsagor som det inte rådde samstämmighet kring granskades åter av forskargruppen[7] och kategorierna justerades ytterligare. Kategorierna bearbetades och kalibrerades således i flera steg. Analysarbetet kan med andra ord beskrivas med det Wahlström et al. (1997) kallar för förhandlad samstämmighet (*negotiated consensus*).

Efter valideringsarbetet utformades de slutliga kategorierna med kategoribeskrivningarna och tillhörande exempel på utsagor. Sammantaget består utfallsrummet av följande tre kategorier: ”något som kan och bör räknas ut”, ”något som beskriver en relation mellan komponenter” och ”något som representerar en situation”.

3.5 Från kategorier av uppfattningar till kritiska aspekter

Utfallsrummet med sina tre kategorier analyserades därefter i ett nästa steg i syfte att fånga de aspekter som skilde de olika sätten att uppfatta det matematiska i algebraiska uttryck från varandra. Dessa aspekter utgör således de aspekter som är kritiska för att eleverna ska kunna utveckla ett mer kvalificerat sätt att erfara fenomenet på. Tre

[6] I ett första skede prövade och justerade lärarna i årskurs 1 och 5, och i ett andra skede lärarna i årskurs 6–9 och gymnasieskolan.
[7] I forskningsgruppen ingår Inger Eriksson, Roger Fermsjö, Jenny Fred, Verner Gerholm, Anna-Karin Nordin, Martin Nyman, Torbjörn Tambour och Sanna Wettergren.

kritiska aspekter identifierades som eleverna behöver kunna urskilja (Pang, 2003; Pang & Ki, 2016; Runesson, 2011, 2017). I nästa avsnitt presenteras det fenomenografiska utfallsrummet med sina tre kategorier samt de tre kritiska aspekter som identifierades.

4 Resultat

I det följande avsnittet presenteras resultatet i två delar. I den första delen presenteras de fenomenografiska uppfattningarna i form av ett utfallsrum. I den andra delen presenteras de kritiska aspekter som vi identifierat i relation till de olika kategorierna och som elever behöver ges möjlighet att urskilja för att utveckla ett mera kvalificerat kunnande av vad som kan vara *det matematiska i algebraiska uttryck*.

4.1 Elevers erfarande av det matematiska i algebraiska uttryck

Utifrån det preciserade fenomenet resulterade analysen av elevintervjuerna i tre kvalitativt skilda kategorier av yngre elevers uppfattningar av det matematiska i algebraiska uttryck. De identifierade kategorierna kan organiseras hierarkiskt och bildar tillsammans det illustrerade utfallsrummet i Figur 5 nedan. Kategorin ”något som kan och bör räknas ut” kan förstås som enklare i relation till de andra två kategorierna som representerar mera komplexa uppfattningar. I de två mera komplexa uppfattningarna av fenomenet urskiljer eleverna fler aspekter.

Figur 5. Schematisk bild över utfallsrummet med sina tre kategorier.

I det följande presenteras kategorierna och exemplifieras med excerpt från förskoleklass, årskurs 1 och årskurs 4.

4.1.1 Kategori 1: *... något som kan och bör räknas ut*

I den här kategorin uppfattar eleverna uttrycket som att det innehåller en uppmaning till dem att göra en beräkning av något slag. Eleverna uppfattar att det finns ett bestämt tal eller en bestämd siffra bakom bokstaven. Om man kan lista ut talet eller siffran genom att exempelvis gissa eller uppskatta så kan man lösa uppgiften som eleverna uppfattar anges i uttrycket. Vidare uppfattar eleverna att x är ett okänt tal. Kategorin exemplifieras nedan med tre excerpter. I excerpt 1 ges exempel på att eleverna ser stavarna som representationer av ett visst antal som ska adderas. Detta synliggörs när eleverna pekar och räknar de enskilda små Cuisenairestavarna, en efter en, i en konkret beräkning. Den ena stapeln består av fem röda stavar och bredvid ligger en orange stav som motsvarar samma längd som de röda stavarna tillsammans. I excerpt 2 ger eleverna uttryck för att variabeln x är en faktor i en multiplikation och menar att x inte kan vara ett multiplikationstecken. I excerpt 3 exemplifieras att eleverna antar att en siffra gömmer sig bakom x. Således skiljer de mellan ett tal man ska räkna ut och en dold siffra.

Excerpt 1 (intervjufråga 1)

Amir	”Kanske 5 + 5 är lika med 10?”
Intervjuare	”Hur tänker du när du säger 5 + 5 är lika med 10?”
Amir	”Att det här [pekar på de fem fysiska stavarna som är lika som på bilden (se Figur 2)] är 5 som är uppdelade. Den här [pekar på den långa staven] är inte uppdelad i 5, men den är ändå lika lång som de här [pekar på de fem stavarna], så då skulle man dela upp den här.” [drar som små streck med fingret på den långa staven]
Intervjuare	”Så då är de tillsammans lika långa [pekar på de fem stavarna] som den?” [pekar på den ensamma staven]
Amir	”Ja.”

(åk 1)

Excerpt 2 (intervjufråga 1)

Kim	”Jag tror att det där [pekar på *x* i uttrycket 5*x* = *y*] är en siffra, för jag kan inte tänka mig ett tal där det står fem gånger är lika med y [5• = *y*] det måste ju va typ fem gånger tre är lika med *y* [5•3 = *y*] då jag tror det där [pekar på *x* i 5*x* = *y*] är en siffra.”
Intervjuare	”Vad skulle då *y* vara?”
Kim	”Jaa asså, jaa du, det är en bra fråga …”
Lana	”Jag tänker mer *y* som bara det dolda talet, som man ska räkna ut, jag tänker inte att det kan vara en siffra nu, inte än.”

(åk 4)

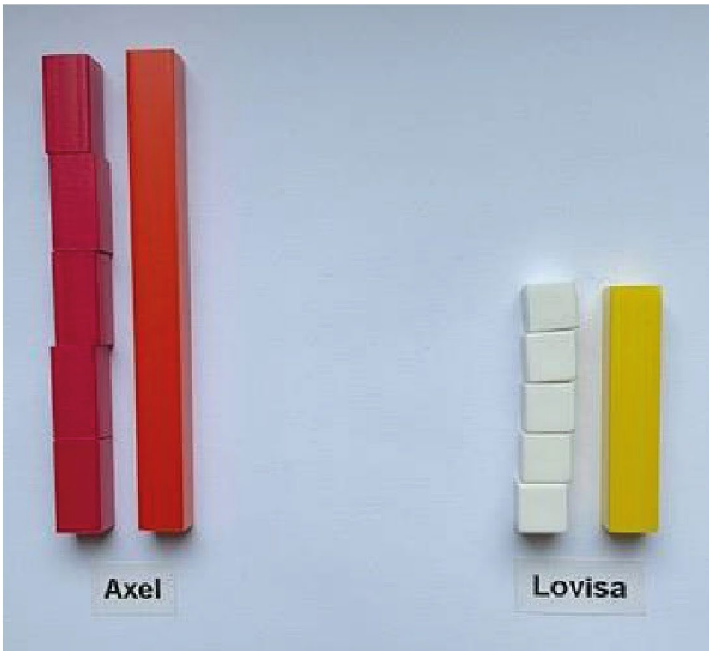
Excerpt 3 (intervjufråga 1)

Kim: ”Då så tänkte jag att det kanske, om det är nånting där bakom [pekar på *x* i uttrycket 5*x* = *y*] … så tänkte jag om det var typ femtiotvå (52), då skulle det vara en mer här [pekar på *y* i uttrycket 5*x* = *y*] som så att det blev femtiotre (53) här [pekar på *y* i uttrycket 5*x* = *y*], jag är jättedålig på att förklara /…/ jag tänkte det kan vara en dold siffra där bakom.”

(åk 4)

4.1.2 Kategori 2: *… något som beskriver en relation mellan komponenter*

Uppfattningar som utgör denna kategori indikerar ett erfarande av likhetstecknets betydelse eller funktion sett till det som finns på vardera sida om likhetstecknet. Eleverna uppfattar och tar med andra ord fasta på den relation som symboliseras av likhetstecknet, som till exempel i uttrycket $5x = y$. De erfar alltså själva likhetstecknets betydelse i uttrycket $5x = y$ och att x och y är variabler men inte att variablerna är representationer för något. Den siffra som finns i uttrycket anger bara hur många det är av något och om variabeln inte har någon siffra så är det bara en. En förståelse för relationen mellan delar och helheter inom uttrycket finns inbyggt i resonemanget. I excerpt 4 och 5 ger eleverna uttryck för att de är medvetna om att vilka symboler som väljs inte har någon betydelse och att till exempel $5x = y$ uttrycker samma relation som $5c = z$. Detta kan då också förstås som att det är egalt vilken symbol som används för att beteckna en variabel i ett givet uttryck.

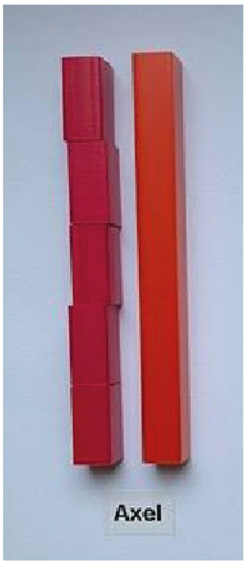


Figur 6. Axels och Lovisas stavkonstruktioner i relation till uttrycket 5x = y.

Excerpt 4 (intervjufråga 2)

Intervjuare	”5*x* är lika med *y* [relaterar till det skrivna uttrycket 5*x* = *y* på en bild framför dem]. Axel han la det så här och Lovisa la det så här [visar bilder på Axels och Lovisas stavkonstruktioner och lägger ut stavarna]. Hur kan de ha tänkt?” /.../
Theo	”Det står ju fem *x* då blir det fem sådana.” [pekar på bilden med Axels stavkonstruktion]
Rebecca	”Ja, 1, 2, 3, 4, 5.” [räknar de små röda stavarna]
Theo	”Och så en *y.”* [pekar på den långa orange staven]
Rebecca	”Ja.”
Intervjuare	”Så de tänkte att 5*x* är lika med *y*?” [pekar på bilden på stavarna för att visa]
Theo	”Ja.”
Intervjuare	”Sen var det en annan kille. Han fick det här [visar bilden på Axels stavkonstruktion], men han skrev inte 5*x* = *y* utan han skrev 5*c* = *z*. Hur kan han ha tänkt?”
Rebecca	”1, 2, 3, 4, 5.” [pekar och skrattar]
Intervjuare	”Vad tänkte du Rebecca?”
Rebecca	”Att det är bara samma sak [som 5*x* = *y*].”

(förskoleklass)



Figur 7. Axels stavkonstruktion i relation till uttrycket 5x = y som Pelle uttryckte som 5c = z.

Excerpt 5 (intervjufråga 3)

Intervjuare	”Så här skrev Pelle i den klassen [skriver uttrycket 5*c* = *z* på en whiteboard]. Fem *c* är lika med *z.”* [Petra läser med i delar av uttrycket]
Olga	”Va?!”
Intervjuare	”Hur kan Pelle ha tänkte när han gjorde det?”
Olga	”Femman kanske är de här fortfarande.” [lägger handen på de fem röda stavarna]
Petra	”Det tror jag, ja. För det är fem klossar. Då har vi det.”
Olga	”Men *c* och *z*, är ju lite som *x* och *y* [pekar på *x* och *y* i uttrycket], *c* och *z*.” [pekar mot *c* och *z* i uttrycket]
Petra	”Ja.” [samtidigt som Olga] /.../
Olga	”Fast, det står ju fem på båda.” [pekar på båda femmorna i uttrycken]
Petra	"Amen, det är ju det.” [håller på båda femmorna]
Olga	[ohörbart] ”Tagit olika bokstäver typ.”
Petra	”De har använt olika bokstäver.”
Olga	”Fast det kan ju.”/
Intervjuare	/”Kan man göra det?”

Olga	”Jaa.”
Petra	”Men de kanske bara vill använda olika uttryck.”

(åk 4)

4.1.3 Kategori 3: *... något som representerar en situation*

I denna kategori erfar eleverna att olika komponenter i ett algebraiskt uttryck bär på information som kan kopplas till en situation bortom själva uttrycket. En situation kan vara ett samband men också ett problem som kan modelleras med ett uttryck exempelvis $5x = y$.[8] Eleverna ser således att en konkret situation kan representeras av ett eller flera abstrakta algebraiska uttryck. Exempelvis uppfattar eleverna att en av de fiktiva eleverna har lagt sin stavkonstruktion kan exemplifieras som en situation som kan beskrivas med uttrycket $5x = y$ och att $5c = z$ kan representera en annan men även samma situation, men att en viss variabel måste representeras av samma symbol varje gång den förekommer.

Excerpt 6 (intervjufråga 3 och 4)

Intervjuare	”Är det bara samma sak? Finns det någon skillnad?”
Theo	”Ja att de där är olika.” [pekar på ”*x* och *c*” samtidigt som ”*y* och *z*”]
Intervjuare	”Men annars är det samma sak?”
Rebecca	”Ja, att det är fem och att det är lika med. Bara att det är fem *x* lika med *y*, ’*x* och *y*’ och ’*c* och *z*’...”
Intervjuare	”Sedan fick de flera uppgifter: de fick det här uttrycket $k = 3e$ och då var det så att Karim la så här, Zara la så här och Lisa så här.” [lägger ut bilder på Karims, Lisas och Zaras stavkonstruktioner]
Rebecca	”Jag vet varför: samma sak [som $5x = y$]. För det står ingenting där [pekar på *k*] då ska man ta en och när det står en siffra innan då ska man ta så många.” [visar tre små stavar samtidigt som han pekar på tavlan]
Theo	”Där är det ju tre.”

(förskoleklass)

Excerpt 7 (intervjufråga 3)

Fahime	”e?, men de här tre måste vara samma då.” [pekar på Lisas olika långa stavar]
Intervjuare	”Så du tänker att det inte stämmer?”
Fahime	”Nej, [skakar på huvudet], för att tre e är lika med k, men det måste vara tre av samma.”
Intervjuare	”Så den [Karims stavkonstruktion] stämmer, vad säger du, håller du med?” [vänder sig till Ester]
Ester	”Ja, de här tre ska vara lika.” [pekar på Lisas olika långa stavar]

(åk 1)

[8] I svenska läromedel i matematik för yngre åldrar beskrivs ofta konkreta vardagshändelser i form av till exempel räknesagor till vilka ett uttryck kan kopplas (se t.ex. läromedelsserierna Favorit matematik 2A (Ristola et al., 2012) och Matte Direkt Safari 2A (Falck et al., 2009)). I elevintervjuerna har möjliga situationer representerats med olika konstruktioner med Cuisenairestavar.

Excerpt 8 (intervjufråga 4)

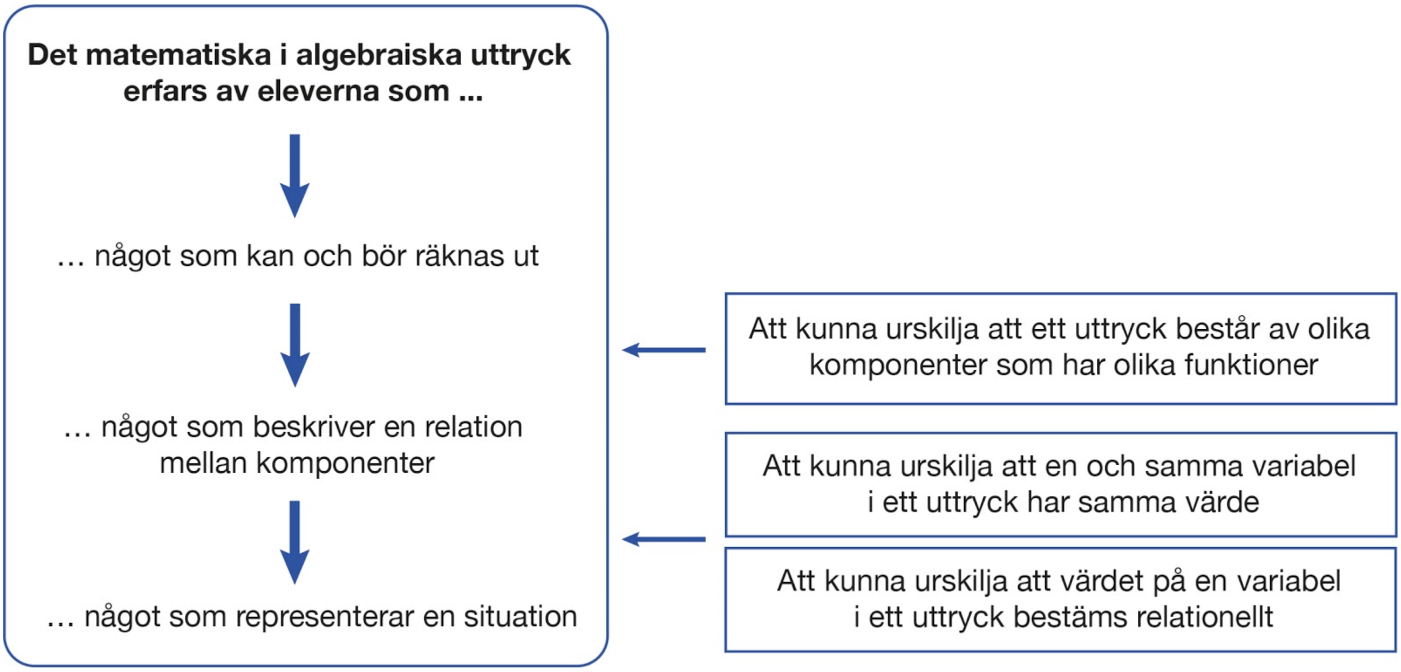
Olga	”Det är väl typ samma sak fast de har använt olika klossar.”
Petra	”Ja, det är ju.”
Intervjuare	”På vilket sätt är det samma sak?”
Olga	”För att, asså. Båda har så här två långa [håller upp den gula och lägger den mot den orange] och sen har båda så här fem klossar [pekar först på de röda och sedan de vita]. De här motsvarar ju en [håller upp en röd och lägger den mot en vit och håller sedan upp den igen] / eller två [håller upp en röd] eller två sådana här [pekar mot de vita] motsvarar en sån.” [håller upp den röda]

(åk 4)

Sammanfattningsvis kan sägas att i kategorin ”något som kan och bör räknas ut” uppfattar eleverna att ett algebraiskt uttryck innebär en uppmaning om att ta reda på vilka möjliga värden de obekanta talen representerar. Kategorin ”något som beskriver en relation mellan komponenter” handlar om att eleverna erfar likheter och likhetstecknet inom ett uttryck. I den tredje kategorin, ”något som representerar en situation”, erfar eleverna att samma uttryck kan representera olika situationer eller att samma situation kan representeras av olika uttryck men även att eleverna erfar strukturella likheter mellan olika uttryck.

4.2 Vad elever behöver urskilja

Genom att analysera uppfattningarna i relation till varandra har tre kritiska aspekter som skiljer de olika uppfattningarna åt identifierats. Tentativt indikerar dessa vad elever behöver urskilja för att utveckla ett mera kvalificerat kunnande av vad som kan vara *det matematiska i algebraiska uttryck*. De identifierade kritiska aspekterna i relation till kategorierna sammanfattas i Figur 8. De två kritiska aspekterna som skiljer kategori två från kategori tre är icke-hierarkiska men behöver båda urskiljas för att utveckla uppfattningar motsvarande kategori tre. Dessa urskiljs oberoende av varandra och inte nödvändigtvis i en specifik ordning.



Figur 8. Schematisk bild över identifierade kategorier (till vänster) och kritiska aspekter (till höger).

I den första kategorin ”något som kan och bör räknas ut” visar utsagorna att eleverna ännu inte har urskilt att exempelvis bokstäverna a, b och c i uttrycket $a + b = c$ är variabler, att + är en operator och att = uttrycker en relation.

Av utsagorna som bildar den andra kategorin, ”något som beskriver en relation mellan komponenter”, kan man se att eleverna urskiljer att uttrycken innefattar de matematiska komponenterna även om de inte kan benämna dem. Eleverna är således inte bekanta med terminologin som till exempel variabler och komponenter. Utsagorna visar också att eleverna urskiljer strukturen inom ett uttryck och att symbolerna x och y i $5x = y$ är variabler, det vill säga att deras värden kan variera. De erfar också att det finns ett samband mellan x och y och således att variablerna inte är oberoende av varandra, samt att det inte spelar någon roll vilka beteckningar för variablerna man väljer. Istället för x och y kan man välja exempelvis c och z. Relationen uttrycks då $5c = z$. Uppfattningen innefattar, som tidigare nämnts, å den ena sidan ett samband mellan två variabler, och å den andra att en och samma relation kan uttryckas på flera olika sätt, till exempel genom att variabelbeteckningarna varieras mellan uttryck.

Utsagorna i den tredje kategorin ”något som representerar en situation” visar att eleverna har urskilt att samma uttryck kan representera olika situationer eller att samma situation kan representeras av olika uttryck. Exempelvis kan en situation som beskrivs av ett uttryck $a = b + c$ även beskrivas $x = y + z$ om man byter beteckningar på variablerna. Den kan också beskrivas $c = a - b$ eller $b = a - c$. Utsagorna som bildar kategori tre visar även att eleverna urskiljer den strukturella likheten mellan $5x = y$ och $k = 3e$, det vill säga att ett visst antal av något (t.ex. x eller e) är lika med något annat (y respektive k). Urskiljandet innefattar att antalet och/eller symboler kan vara olika utan att strukturen ändras. För att utveckla en uppfattning som motsvaras av kategori tre (se speciellt excerpt 6 och 7) behöver eleverna urskilja både *att en och samma variabel har samma värde* och *att värdet på en variabel bestäms relationellt*.

5 Diskussion

I det följande diskuterar vi resultatet i relation till syftet och forskningsfrågorna: Vilka kvalitativt skilda sätt att erfara algebraiska uttryck kan urskiljas? Vilka aspekter behöver eleverna erfara för att de ska ges möjlighet att kvalificera sina uppfattningar om algebraiska uttryck?

5.1 Kvalitativt skilda erfaranden av det matematiska i algebraiska uttryck

Som redovisats i resultatet ovan framträdde tre hierarkiskt ordnade kategorier: 1) ”något som kan och bör räknas ut”, 2) ”något som beskriver en relation mellan komponenter” och 3) ”något som representerar en situation”. Liksom i ett flertal av de studier som kan kopplas samman med ett utforskande av elevers uppfattningar av algebraiska uttryck identifierade vi, innan preciseringen av fenomenet, två uppfattningar som inte kan ses som matematiskt relevanta (jfr t.ex. MacGregor & Stacey, 1997; Stacey & MacGregor, 1997). Då vårt fokus i projektet var att designa en undervisning som skulle öppna upp för en matematisk relevant förståelse av algebraiska uttryck preciserades fenomenet till elevers uppfattningar av *det matematiska* i algebraiska uttryck (Eriksson et al., 2019; Tambour, 2019).

I den hierarkiska ordningen som presenterats i utfallsrummet beskrivs den enklaste uppfattningen en uppmaning till att göra någon slags beräkning. Denna typ av uppfattning återfinns i ett flertal av de studier som ingår i forskningsöversikten ovan (Kieran, 1981; Küchemann, 1981). De två andra kategorierna i utfallsrummet är det inte lika lätt att finna en motsvarighet till i den tidigare forskningen. Exempelvis kan de sex kategorier som Küchemann presenterat inte enkelt relateras till vårt utfallsrum. Även om kategori 1 (något som kan och bör räknas ut) mycket väl kan jämföras med den kategori som Küchemann benämner som *bokstaven används som ett specifikt okänt tal* kan hans två följande kategorier *bokstaven används som ett* *godtyckligt tal* och *bokstaven används som en variabel* inte lika enkelt jämföras med de två mer kvalificerade uppfattningarna i vårt utfallsrum. Snarare går det att argumentera för att Küchemanns två sista kategorier utgör en förutsättning för de två mer kvalificerade av våra kategorier. En skillnad kan noteras är typen av uppgifter eleverna i dessa två studier fått möta jämfört med de uppgifter de intervjuade eleverna fick möta. Det framstår som att flera av de uppgifter Küchemann använder indikerar möjliga beräkningar i form av ekvationer, medan våra elever fått möta vad som kan beskrivas som ”rena” algebraiska ekvationer eller uttryck.

Med referens till Piaget är det inte ovanligt att lärare har en förväntan på att de yngsta eleverna inte ska kunna visa tecken på en mera utvecklad algebraisk förståelse (se t.ex. Wagner, 1983). Men i vår fenomenografiska analys kan vi inte se en sådan skillnad relaterat till deltagarnas ålder, utan exempel på de tre uppfattningarna som slutligen urskildes fanns representerade hos elever som intervjuades i såväl förskoleklass som i årskurs 1 och i årskurs 4. Av analysen framträder att redan i förskoleklass kan elever föra ett resonemang om algebraiska uttryck som kan kategoriseras som mera utvecklat. Med hänvisning till exempelvis Attorps (2006) studie kan även lärare ha en uppfattning som enligt Wagners resonemang endast borde återfinnas hos de yngsta. Det går således inte att ha en förväntan om att de yngsta elevernas utsagor endast skulle återfinnas i den minst kvalificerade kategorin

Med det preciserade fenomenet fick vi således fram tre kategorier av uppfattningar vilka gav tre kritiska aspekter som också kunde ge oss vägledning till designen av de kommande forskningslektionerna i forskningsprojektet.

5.2 Kritiska aspekter för ett mer differentierat urskiljande av innebörden av algebraiska uttryck

De kritiska aspekter som ger eleverna möjlighet att kvalificera sina uppfattningar för att utveckla ett mer komplext kunnande av algebraiska uttryck är 1) att kunna urskilja att ett uttryck består av olika komponenter som har olika funktioner, 2) att kunna urskilja att en och samma variabel i ett uttryck har samma värde och 3) att kunna urskilja att värdet på en variabel i ett uttryck bestäms relationellt. Det finns, som nämnts, en idé med kritiska aspekter att det är sådana aspekter som den specifika elevgruppen behöver urskilja för att utveckla mer kvalificerade uppfattningar (Pang & Ki, 2016; Runesson, 2011, 2017). Det innebär således att vid planering av en kommande lektion eller lektionsserie behöver undervisningen designas så att de kritiska aspekterna blir möjliga för eleverna att urskilja (Marton, 2015; Runesson, 2017). Det kan med dessa antaganden framstå som möjligen oväntat att elever i förskoleklass, årskurs 1 och elever i årskurs fyra skulle behöva lära sig samma sak. Dock har ett antal studier på senare år visat att när elever med stor åldersskillnad och därmed olika erfarenheter av matematik möter ett innehåll som är helt nytt för dem så uppvisar eleverna, på gruppnivå, behov av likartat undervisningsinnehåll (Kullberg, 2012; Runesson, 2017; Tuominen et al., 2018).

Enligt kursplanen lyfts algebra som innehåll redan från de första årskurserna, men hur det ska behandlas sägs inget om (Hemmi et al., 2020). Matematikundervisningen i Sverige karakteriseras vanligen som läromedelsstyrd och studier visar att elever sällan möter komplexa uppgifter som exempelvis inbegriper generella, grundläggande och teoretiska samband (Bråting et al., 2019; Hemmi et al., 2019). I det internationella forskningsfältet lyfts vikten av en tidig introduktion till algebra (Blanton et al., 2015; Davydov, 2008; Kaput, 2008; Kieran et al., 2016; Venenciano & Dougherty, 2014). Forskningsprojektet som den här artikeln har hämtat data ifrån utgör ett exempel på ett utforskande av hur algebra kan introduceras redan för de yngsta eleverna (Eriksson et al., 2019). Med en sådan ambition behöver undervisningen designas medvetet för att skapa förutsättningar för utveckling av elevers algebraiska tänkande och att detta kan utifrån våra data ske tidigt (Blanton et al., 2015; Eriksson & Jansson, 2017; Eriksson et al., 2019; Kieran, 2004, 2011). Att ha kunskaper om vad olika elevgrupper uppfattar algebraiska fenomen som och att på basis av det kunna identifiera möjliga kritiska aspekter ger läraren förutsättningar för att designa en meningsfull undervisning.

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Student participation in peer interaction – Use of material resources as a key consideration in an open-ended problem-solving mathematics task

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This study explores how students deal with material resources in their peer interaction when working in pairs in an open-ended problem-solving task. The productive use of material resources can be expected to support successful peer work. However, research into social phenomena in peer interaction is needed in order to identify and describe productive and less productive forms of dealing with material resources as students participate in open-ended problem-solving tasks. Consequently, this explorative study responds to this research need. Based on multimodal data, including video recordings, transcribed talk and the written contributions from four pairs of Year 7 students aged 12-13 years, the analysis focuses on different ways in which students deal with material resources while negotiating their participation as they respond to the task. The findings indicate that aspects of participation are a key factor for describing productive and less productive ways of dealing with material resources by the student pairs. Foregrounding aspects of participation for an increased awareness of potential obstacles to student-centred work is among this study's contributions for classroom practice and theory development.

Keywords: material resources, video study, participation, student pairs, open-ended problem-solving task

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1 Introduction

This study focuses on how students use material resources in their interaction when working in pairs in an open-ended problem-solving task. In open-ended problem-solving tasks, a mathematical struggle is built into the design of the task (Livy, Muir & Sullivan, 2018) and material resources are expected to be a key support for students' peer group work and mathematical thinking. Materials, such as pens, (blank) papers, calculators, and textbooks, can help students process multiple pieces of information, choose strategies and record their thinking (Ingram, et al., 2019). As students with different abilities can respond in different ways, open-ended problem-solving tasks should be educative for students (Sullivan & Clarke, 1992) and provide researchers with insights into student participation (Rogoff, 2008). From a theoretical perspective, the use of material resources should support participation in an open-ended problem-solving task as it provides a space for working with the information, strategizing and recording thinking (e.g. Kuntze, et al., in prep.). This study uses multimodal data, including video recordings, transcribed talk and the written contributions from four pairs of Year 11 students, to investigate how student pairs can use material resources to respond to an open-ended problem-solving task and how the use of material resources mediates productive and less productive forms of student participation.

2 Theoretical background

2.1 Mathematical objects and their representations

In mathematics education, students participate by engaging in mathematical practices and tasks, using objects to represent the abstract nature of mathematics (cf. Goldin & Shteingold, 2001). Mathematical objects can be represented in multiple ways and different representations of a mathematical object may show or emphasise different properties of the mathematical object in question (Duval, 2006). As a consequence, the use of multiple representations can enable students to build up knowledge about the (abstract) mathematical object behind its different representations, and enable them to solve problems flexibly (Lesh, Post & Behr, 1987): changing between representations can make properties of mathematical objects immediately visible which are rather hidden in another representation. For example, transforming a word problem into an equation which represents the same mathematical relationship may make its structure more transparent, and subsequent changes of representation by algebraic manipulations can lead to a representation in which the solution is immediately evident. Connecting different representations as well as changing between representations provides crucial learning opportunities (e.g. Duval, 2006; Lesh, Post & Behr, 1987; Ainsworth, 2006). Different ways of representing a mathematical object can be described by the notion of representation registers characterised by specific rules of how mathematical objects have to be represented (Duval, 2006).

In any mathematics-related social interaction, students have to somehow refer to mathematical objects (e.g. Duval [2006, 2017] Ainsworth, 2006). Changing between different representation registers, however, is demanding for learners (Ainsworth, 2006; Dreher & Kuntze, 2015a) and challenges successful student peer interaction. In a pair task, for example, students have to negotiate a shared understanding of the representation register(s) used in the task, and if a student creates a new representation (e.g. in a new register), the students have to renegotiate their shared understanding of this new representation. The different ways students use specific individual knowledge related to representation registers can enable or hinder the pair as they seek to solve the task (e.g. Kuntze, et al., in prep.). In addition to negotiating the representation of mathematical objects in a pair, however, students have to negotiate how to work together.

Chan and Clarke (2017) differentiate between three negotiative foci a pair task requires from student participants as they complete open-ended problem-solving tasks. These foci are mathematical, socio-mathematical and social. The mathematical focus refers to participation in the practices of mathematics and use of representations. The socio-mathematical focus recognises the situated construction of meaning and the influence of socio-mathematical norms which inform how students participate in a mathematics classroom (Yackel & Cobb, 1996). The social focus points to the way peer relationships have significant implications for student participation and negotiation of understanding (Clarke, 2011). Inspired by the socio-didactical tetrahedron of Johnson, Coles, and Clarke (2017), Figure 1 illustrates the different 'partners' involved in an open-ended problem-solving pair task with the edges pointing to the different kinds of negotiative foci.

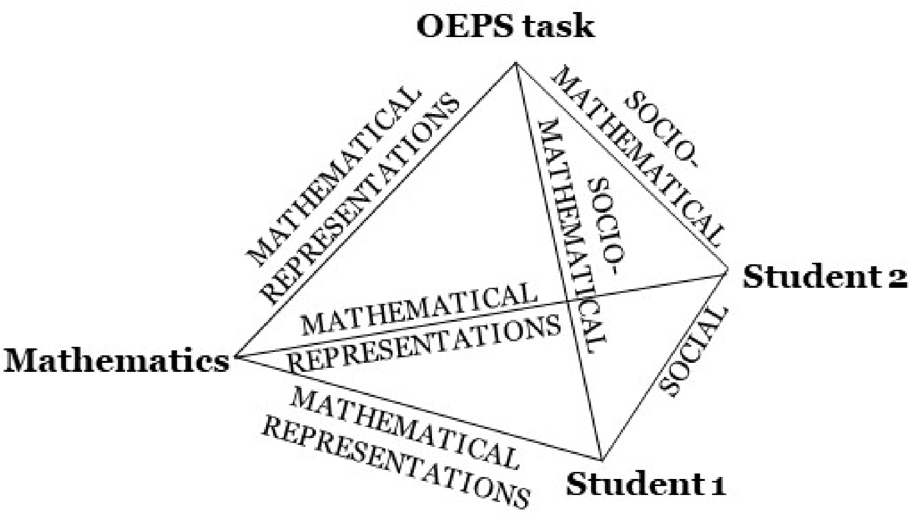


Figure 1. Partners involved in an open-ended problem-solving task

In Figure 1, the negotiation between the student pair is social as they decide how to work together. Negotiations drawing on the mathematics corner employ representations to concretise abstract mathematical objects. Between the open-ended problem-solving task and the students, the negotiation is socio-mathematical as the students determine the mathematical content of the task as well as the requirements of the task in terms of student participation.

2.2 Material resources and representations of mathematical objects

As students participate in open-ended problem-solving tasks, however, the options for representing mathematical objects are co-determined by the materials they are offered or allowed to use in the situation, and the way they actually use these materials. By the term 'material resources' or 'materials' (used as a synonymous expression) we refer to physical objects and media which allow or support representing mathematical objects, including such representations on/in these objects or media. For example, working materials and tools such as pens, sheets of paper, calculators, compasses etc. are such material resources, as they can be used as a help for representing mathematical objects. Conversely, not all representations of mathematical objects require such material resources: For example, representing a number by an action, a gesture or in spoken language is possible without specific materials – in this case, learners can represent mathematical objects using their hands or their voice, for instance (Radford, 2014; Díez-Palomar & Olivé, 2015).

Material resources can contain representations of mathematical objects: Printed textbook material or worksheets, for example, mostly contain representations of mathematical objects. If students represent a mathematical object on an initially blank sheet by taking a note of a symbolic expression or by producing a drawing, the sheet, as a material resource, is transformed and may from then on be more useful for the students' further work as it carries information the students can build on in subsequent steps, in their individual thinking and as a peer group. Of course, the usefulness of material resources for the work of a student pair can be expected to depend on how the students deal with it and on which representations they use and produce. In this way, the usefulness of material resources is not an objective category but depends on how the students actually use the materials.

Material resources should be considered in the framework of intentions related to their (possible) use and in the context of socio-mathematical norms (Clarke & Mesiti, 2013) related to the learning situation and to the classroom. For example, such socio-mathematical norms function like conventions, whether the use of a material is allowed or not, and they reflect a socially established understanding what can be done with a pen and a blank sheet of paper if it is handed out to the students in a pair work setting. Focusing on the use of material resources within an open-ended problem-solving task provides a novel approach to the concrete realisation of student participation.

2.3 Material resources and representations in student peer interaction

During an interaction process of peer students, a 'pool' of materials is available to them. For example, at the beginning of the working process on a task, the task assignment in a textbook may be available, together with blank sheets and a calculator. If the textbook is available, then of course other sections of the textbook with the corresponding content are at hand as well. Students can then produce additional material or alter the available material by, e.g., adding notes containing representations of mathematical objects on the sheet(s) of paper, typing on their calculators etc., which potentially enriches the pool of available materials (cf. Fig. 2) and the opportunity to think through (the use of) different materials (Rojas-Drummond, et al. 2008).

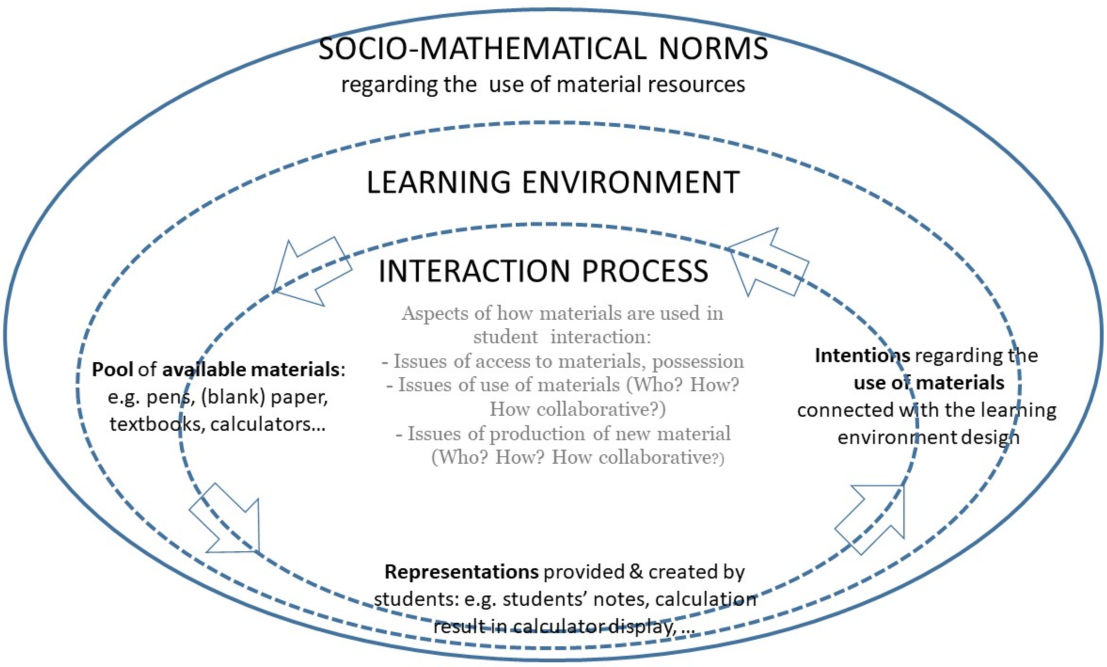


Figure 2. Aspects of how materials are situated and used in task-based interaction

Studies indicate however that how students participate is indicative of the socio-mathematical norms of the learning environment (Laine, et al. 2018; Rezat & Sträßer, 2012) as well as providing insights into the past, present and future development of students as individuals (Rogoff, 2008). The aim of this study is to investigate how students use materials in order to gain insights into the ways students build mathematical understanding through the use of representations, use socio-mathematical norms to guide their participation and engage in peer interaction.

Research Questions

The focus of this study is on the different ways pairs of students used materials to answer an open-ended problem-solving task. The research questions are:

1. How do the students use materials in response to an open-ended problem-solving mathematics task?
2. How does the situated use of materials in an open-ended problem-solving mathematics task differ between the pairs?

3 Methodology

The data for this study belong to a larger research project, 'Social Unit of Learning' (Chan & Clarke, 2017) to examine the social basis of problem-solving in mathematics as students work as individuals, in pairs and small group. This study is based on four pairs of students' participation in a 15-minute pair task as part of a 60-minute researcher-designed and teacher-facilitated session. The task the students are given is: *The average age of five people living in a house is 25. One of the five people is a Year 7 student. What are the ages of the other four people and how are the five people in the house related? Write a paragraph explaining your answer.*

The pairs had around 15 minutes to work on the task. The session was video recorded, all student work collected and speech transcribed for analysis. Prior to the pair task, the students had completed a 10-minute individual task and the small group task was still to come. The student pairs were assigned by the teacher before the session, but in the laboratory classroom once the teacher had read out the task, his role was to keep students on time and on task with minimal guidance. The pairs had an answer sheet which included a copy of the task as written above and a designated place for student names. Each pair also had access to blank sheets of paper for working out their answers. All four pairs each used one working out sheet for the task. The four pairs are included in the study as contrasting examples.

3.1 Verbal and visual data

Using both verbal and visual data as the dataset in this study recognises that oral language is contextualised by complex actions and omitting nonverbal actions can distort interpretation (Norris 2004; Goodwin & Goodwin, 2004). The rich dataset from the project provides a detailed record of what was said and done, by whom and in what manner for each moment of the allotted time. The camera angles and student seating were optimised to capture the faces of the students as they spoke and interacted. By carefully watching the videos, listening to the student comments and focusing on the material resources, it is possible to discern when and often what notations are added by the students to the working out sheet and answer sheet. The video data includes how the pairs place and use the material resources. Figure 3 illustrates snapshots typifying the pairs' positions in relation to the working out sheet: Pair 1 (leftmost) sharing the sheet and alternating between talking and writing; Pair 2 failing to share; Pair 3 both with a hand on the shared working out sheet; Pair 4 (rightmost) maintaining a distance from the working out sheet. All of the videos were shot in high definition and compressed as videos in 480 × 270 pixels with 25 frames per second. A filter has been applied to the illustrative screenshots to protect the privacy of the participants.

Figure 3. Screenshots from the video recording for Pairs 1 and 2 (left image) and Pairs 3 and 4 (right image).

3.2 Analytical steps

The dataset for this study includes videos of the four student pairs, copies of the working out sheet and answer sheet, and the transcriptions of the students' talk. Following familiarisation with the dataset, the initial analysis involved writing minute-by-minute accounts of the student activity throughout the 15 minutes designated for the task. Writing accounts for each minute for each pair requires the researcher to carefully consider what meaning is given to actions and interactions of the students (Sfard & Kieran, 2009; Goodwin & Goodwin, 2004). During this stage, body language and gaze, the use of material, the coherence of the actions and interactions as well as talk were included in the account (Sfard & Kieran, 2009). If the students pursue a line of thought together or talk at cross-purposes, this is included in the account. If one student points to the board on the lab classroom wall or physically removes the partner's hand from the paper, this is in the written account. If a debate between a pair stops as the teacher walks passed, this is included; or if both students add information to the working out sheet, or one student speaks aloud whilst the partner adds information to the paper, this is included. Once these accounts were completed, they were compared with the transcriptions and videos to ensure the accuracy of interpretation in terms of what happened when, as well as what was said and written.

The final step focused on the use of materials during the task. As Table 1 indicates in the Findings, the pairs used the materials for multiple purposes as they participated in the task. The short accounts (Livy, Muir & Sullivan, 2018) included in the findings illustrate the situated nature of the students' actions and interactions as well as draw attention to the significantly different ways in which the students participate in the task.

4 Findings

For each pair, the materials provided an important focal point as they participated in the open-ended problem-solving task. The paper materials offered a concrete starting point to begin approaching the task and for negotiating the peer interaction. Pairs 1, 2 and 3 use the task instructions to establish a shared focus and to take ownership of the assigned task and Pair 4 also read the task aloud as they attempt to move forward with the task. Each pair added notes to the working out sheet and at least attempted to form an answer to write on the answer sheet. The members of a student pair did not necessarily use the materials in the same way or for the same purpose. The findings first provide an overview of the different ways the student pairs use materials for a variety of purposes before addressing the situated way in which the pairs used materials as they participated in the task. The discussion then addresses individual ways the students use the materials.

4.1 Using materials for different purposes

The findings from this study indicate that the students used the materials in three ways: to represent mathematical objects, to form their answer to the household task and to manage their interactions. These three ways correspond with the negotiative foci outlined above and are written into the leftmost column of Table 1. Particular examples of how the students participated in the mathematical, socio-mathematical and social interactions of the task are outlined in the second column. The columns with Xs indicate how students from each pair participated. The situated participation of the student pairs is explained in section 4.2.

Table 1. Using materials for different purposes in pairs (P1, P2, P3, P4) and as individuals (represented by the initial letters of their pseudonyms)

Material resources used for...		P1: Ka	P1: Au	P2: Pe	P2: Po	P3: An	P3: Pa	P4: Jo	P4: Ar
Representing mathematical objects	Visualising thinking by, e.g. noting down key info, steps & symbols	x	x			x	x	x	
	Adding calculations and decisions	x	x	x		x	x		
	Revising ideas: adding changes, crossing out suggestions	x	x			x	x	x	
Forming an answer to the Household Task	Reading task aloud	x				x		x	x
	Revisiting task instructions to negotiate meaning of info or key terms	x				x	x	x	x
	Revising decisions & mediate formal answer	x	x	x		x	x		
	Parallel working space rather than synchronised					x	x		
	Rearranging answers /Order notions in a logical manner	x	x			x	x		x
	Co-authoring the paragraph	x	x						
Managing the interaction	Trying to establish authority of suggestion, e.g. ‘it says’				x	x			x
	Seeking agreement on use of papers				x	x	x	x	x
	Adding both names	x	x			x	x	x	x

	Leaning back so the partner can see /Paper within shared space	x	x			x	x	x	x
	Dominating use of paper/ Denying access of partner			x					
	Removing paper from partner/Crossing out partner's contribution			x		x			
	Attempting to participate: to have name, to see paper				x		x		
	Attempting to guide partner's participation				x		x	x	
	Observing how partner approaches task	x	x		x	x	x		
	Focusing attention	x	x	x	x	x	x	x	x
	Synchronising activities - notations are added as they speak, agreeing on what symbols represent	x	x			x	x	x	x
	Decorating the working out sheet together	x	x						
	Withdrawing, e.g. doodling						x		

Through the representation of mathematical objects the students participate in mathematics by visualising their thinking, adding calculations and revising their suggestions through the use of different representational registers. By using the materials to form their answer by checking the instructions, noting down their decisions and using the notes to write up the final paragraph to explain their answer the students participate in the socio-mathematical dimension. This dimension further develops as the students write down and revise their decisions, and the role of the materials changes from being an instructional guide to a record of the students' thinking process.

It is striking, however, that the most varied use of materials is with regard to managing the social interactions of the pairs. As the different examples included in Table 1 indicate, managing the social interactions of the pair could be a positive or negative form of participation. Placing the materials between the partners affording access to both students, for example, is a positive use of the materials whereas refusing to allow a partner to see the working out sheet undermines the potential of a productive social relationship.

4.2 Observed patterns of participation

The following accounts elaborate on the significance of the materials within the pairs to provide situated insight into the students' participation, as well as the way in which the interactions within the pair help or hinder their response to the task. The findings therefore provide a range of insights into student participation through their use of materials.

4.2.1 Pair 1: Shared and sharing

From the outset, Pair 1 Katie and Audrey (pseudonyms) used the material resources systematically. The working out sheet and answer sheet are immediately shared between the students and both their names are added on the sheets suggesting a positive foundation for their partnership has been established. The girls take turns to add key information and the procedural steps for working out the answer are added to the working out sheet as they talk together. As their ideas develop, Pair 1 cross out different suggestions and translate between different representational registers. Katie reads the task aloud and points again to the task on the board during the task (17:02) as a reminder for what they are doing adhering to the socio-mathematical requirements of the task. Although Katie adds more text overall, both girls constructively use the sheet and as one speaks, the other writes the words down. Pair 1 verify their answer by recalculating the different ages of the family members using their notes. The final solution is presented as a list (see Figure 4) and the coherent paragraph is written in sentences with minimal mistakes (only the incorrect spelling of "writtin" [sic]). In the final moments, the girls decorate the working out sheet which has played an important role throughout the process, almost as a celebration of their success and affirmation of their positive partnership.

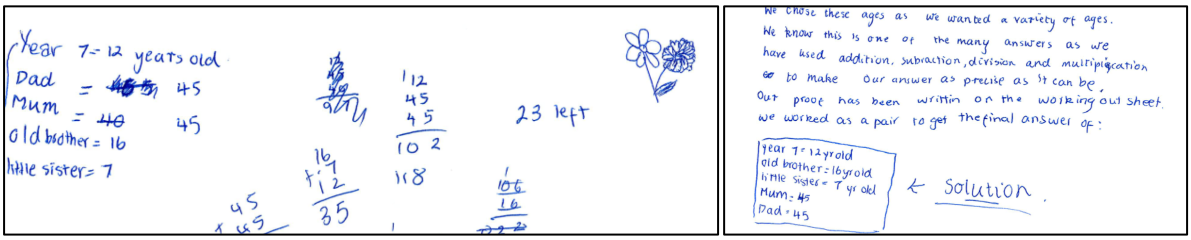


Figure 4. Extracts from the working out and answer sheets of Pair 1 (Katie and Audrey).

For Pair 1, the material resources provide a focal point, a record of their thinking and points of agreement along the way. By making their previous thinking visible, Pair 1 have a record to return to when they write up their answer. The generation of these resources means that Pair 1 can strategically use their notes to craft their co-authored response to the task, as illustrated in the extract in Table 2:

Table 2. Minutes 22:31-24:57 from the discussion between Pair 1, extract 1

Transcription		**Actions regarding material resources**
Katie:	"Whoo. Let's write explanation now. Why did we choose these ages?"	Katie begins to write, then pauses, looks at Audrey and the sheet again.
Audrey:	"We chose these ages as we wanted a variety (laughs)."	Audrey is looking at the task sheet as she begins to speak and Katie continues writing what is said.
Katie:	"We wanted …"	
Audrey:	"Because they were all just - ah, I just stabbed myself with a pen. No. Does this have to be [inaudible]."	
Katie & Audrey: (Laughter)		
Katie:	"Forty-four, forty-five doesn't make a variety."	
Audrey:	"Just say because we wanted a variety of ages. We know this is correct as […] as we have used addition to add them all."	
Katie:	"We …"	
Audrey:	"We used addition to …"	
Katie:	"No. We can't say it's correct because there could be many answers."	
Audrey:	"Oh, we know this is one of the many answers."	Katie points back to the working out sheet to justify what she writes
Katie:	"We know... (Laughs)."	
Katie:	"… the answers. As…"	Audrey continues to read as Katie writes and seems to continue the sentence when Katie is coming to the end of the written answer.
Audrey:	"As we have used addition to add these five numbers up."	
Katie:	"No. We used all of them, divide everything, times."	
Audrey:	"Subtraction. Division."	
Katie:	"Multiplication."	

Audrey:	“And multiplication to make sure our answer is precise.”	
Katie:	“Pretty sure I spelled that wrong (laughs).”	
Audrey:	“Pretty sure that's an ‘I’.”	
Katie:	“You spell it.”	
Audrey:	“To make our answer as precise as it can be. … Mrs XXX will be so proud.”	Audrey moves closer to check the spelling of ‘multiplication’.
		Momentarily both girls are writing on the working out sheet.
		As Katie writes this on the working out sheet, she glances at answer sheet and as soon as she finishes the sentence she grabs the answer sheet, draws boxes around their names, and ticks either side of their names as though it is positive affirmation from a teacher.

Katie and Audrey’s use of the material resources is interwoven with their talk; both talk and text-based activities are part of their response and by the end of the task the girls know they have worked well – ‘Mrs. XXX will be so proud’. This comment is significant as the project design involves bringing intact classes, students and teacher, into the lab-classroom. The mathematics teacher with this class, however, is male yet the girls refer to a female person and the tick they add to their work suggests Mrs. XXX would affirm their efforts, an absent, although influential presence in the coordination of Pair 1’s strategic participation. Using a range of strategies, Pair 1 generate resources that productively prepare the way for subsequent actions. In addition to their willingness to collaborate, the girls seamlessly share the roles to complete the task suggesting a harmonious social partnership and agreement regarding the socio-mathematical demands of the task.

4.2.2 Pair 2: Access denied

For Pair 2, Pedram and Poya, the material resources are sites of contention. Their social interaction is contested throughout the task. Pedram begins by taking both sheets and refusing to share with Poya despite many attempts by Poya to view and contribute to the written solution. Pedram places his arm around the material resources, a physical action that simultaneously acts as a barrier to their interaction as his partner cannot see the working out sheet or answer sheet. Occasionally, Poya manages to add something to the working out sheet, but his contributions are immediately crossed out by Pedram. Poya's efforts to access the working out sheet and complaint that he cannot see the sheet suggest he recognized the value of the material resources for participating in the task, but Pedram's dominance of the resources suggests he has little regard for the potential contributions of his partner. The answer sheet is no less contentious and only after the time for completing the task has finished does Pedram momentarily permit Poya to add something, although the sentence that Poya begins to write ('The father of the house is…'; see Figure 5) is crossed out by Pedram. Poya has to insist on having his name added to the sheet and spelt correctly by Pedram.

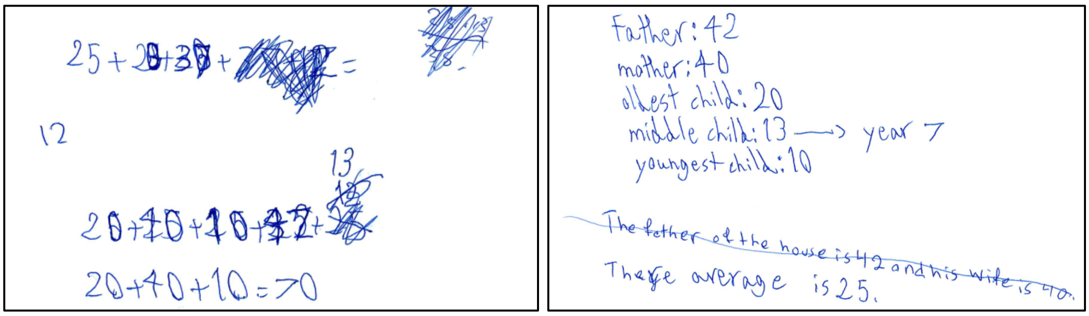


Figure 5. Extracts from the working out sheet and answer sheet, Pair 2 (Pedram and Poya).

The material resources of Pair 2 neither systematically visualise nor record their thinking process in response to the task. As can be seen in Figure 5, different ages are mentioned in their discussion, but as these ages are successively written on top of earlier suggestions, it is increasingly difficult to read and make sense of the notes. When Poya succeeds in glimpsing the list Pedram is writing on the answer sheet, Poya notices the absence of a written explanation and reminds Pedram of the socio-mathematical requirements of the task, but Pedram appears more invested in denying his partner access to the materials and the representational register is based on numerical addition with the listed ages of the household members added in the final moments. The written answer merely restates the calculations on the working out sheet with the addition of a single sentence 'There average is 25' written just before materials are collected. Poya's protest that Pedram is 'hogging the sheet' (23:50) depicts the limitations of the partnership and overall participation, as Table 3 illustrates:

Table 3. Minutes 24:45- 25:05 from the discussion between Pair 2, extract 2

Transcript		**Actions in relation to material resources**
Poya:	"Let me use the paper. Let me just see it. I can't - I - I don't understand what you're doing. Okay. Do this."	Poya leans forward to get the working out sheet and prepares to write
Pedram:	"Yeah. Don't write anything on there."	
Poya:	"Wait. No."	
Pedram:	"Don't write anything."	
Poya:	"No. That's the paper we're going to care about."	
Pedram:	"Okay. Yeah. I'm just going to write it."	Before Poya can add anything, Pedram takes the paper back and begins to write.
Poya:	"Write father."	
Pedram:	"Father."	Poya points to the page and tries to guide the writing.
Poya:	"Write mother."	
Pedram:	"No. What's father's age?"	Pedram moves Poya's hand away
Poya:	"Just do it later."	
Pedram:	"Forty-two - 42."	

Throughout the task, Poya attempts to use the working out sheet as the place for notations and ideas. Poya draws on a range of actions that would facilitate the use of the materials such as asking questions to see what was already written, making suggestions for what could be added and reminding his partner of the requirements regarding how to present the answer. These socio-mathematical actions have the potential to support the productive use of materials and to coordinate the social relationship of the pair as a resource to meet the demands of the task. The pair's interaction, however, is punctured with disagreements and Pedram uses the materials to exclude his partner from the task. By denying his partner the space to see or write, crossing through any permitted additions as well as writing over his own working out, Pedram's notes leaves little trace of the thinking process and limits the possibility of coordinating the different contributions. Whereas Pair 1 use the written record they have produced, created in concert with their thinking aloud, to produce a coherent paragraph rewriting their mathematical process as a short narrative, Pair 2 produce a list and repeat information given in the task description (see Fig. 5).

Throughout this task, Pedram and Poya use materials in contrasting ways. Whereas Pedram excludes his partner through his use of the materials, Poya consistently seeks access, responds to what he sees on the paper and attempts to make useful suggestions following the requirements of the task and writing a paragraph.

The final answer would have been improved if Pedram had listened to his partner. However, their contested use of materials makes their different approaches to and understandings of the task visible.

4.2.3 Pair 3: Shared arc

For Pair 3, Anna and Pandit, the material resources are a focal point for their participation in the task from the outset, although Pandit has to persuade Anna to let her share the working out sheet. Anna wants the written notations to be logically presented, yet as Pair 3 add their calculations and depictions of their thinking to the working out sheet, the material record of their different suggestions becomes increasingly chaotic (see Figure 6). When Pair 3 have decided on the ages of the household members, Pandit reviews the decisions they have made along the way by saying them aloud. At this point, Anna again takes the answer sheet and starts to write each step down. Although Pandit offers a commentary on what could be written, Anna does not write what Pandit says. Pandit watches as Anna goes through the calculations until Anna's flow is abruptly interrupted: 'Thirteen, that's 18, 17, 35. What?' (Anna, 27:10). As though Anna's question is an invitation to participate in the task again, Pandit takes take the working out sheet and redoes the calculation on paper, reassuring Anna that their answer is correct and Anna completes the written answer (see Figure 6) in a manner that is satisfactory to both partners.

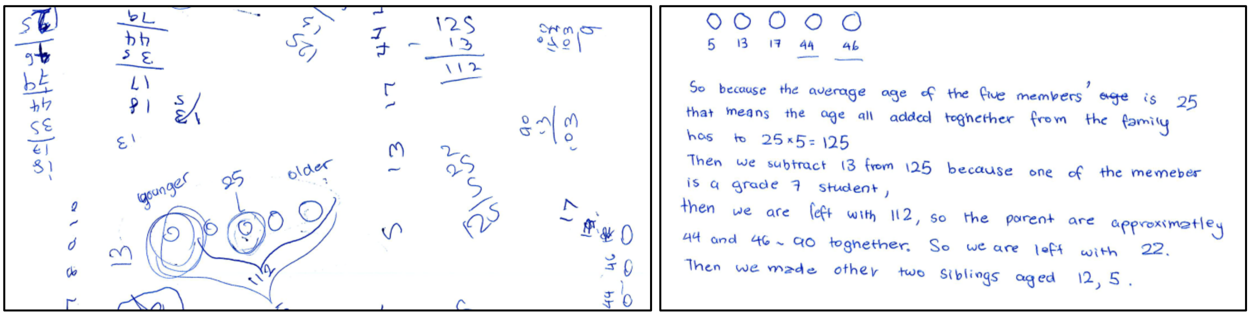
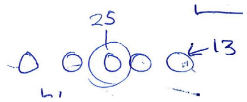
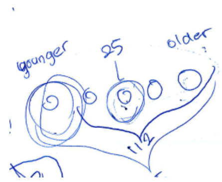


Figure 6. Extracts from the working out and answer sheets of Pair 3 (Anna and Pandit)

Pair 3 use the material to negotiate their participation in the task by clarifying the socio-mathematical requirements written into the task description. Anna then re-presents the mathematical information given in the task as five dots on the working out sheet representing the household members. Pandit observes what Anna adds to the sheet and as they both agree that the Year 7 member of the household is 13 years old, Pandit writes 13 on the working out sheet with an arrow pointing to the dot on the right initiating further discussion between the pair on the mathematical representation, as illustrated in Table 4:

Table 4. Minutes 18:04 -18:17 from the discussion between Pair 3, extract 3

Transcribed speech	Actions with regard to material resources
Anna: "Why do you put it on this side? That's like the older side." Pandit: "Oh my God, it doesn't really matter (laughs)." Anna: " I know. I mind it. Older." Pandit: (Laughs) Anna: "Um... younger." Pandit: (laughs). "You're a good girl..." Anna: "Doesn't matter."	 Anna points to where Pandit added 13 and starts to write a new row 

As Anna objects to Pandit's additions, Anna adds another row of dots to the sheet to clearly represent the younger and older members of the household. Using the written notes, Pandit begins to see how Anna is thinking. Both students focus on the socio-mathematical demands of the task, and as they negotiate the mathematical representations visually representing the information they have, Anna accepts that one dot represents the Year 7 student and insists that the 'year 7 dot' is placed on the 'younger' end of the row. Anna then adds 25 to the middle dot suggesting this is the age of another household member. Pandit then questions Anna's thinking in response to the written, not spoken, information added to the working out sheet. In Table 5, Pair 3 negotiate whether 25 represents the average or actual age of a household member:

Table 5. Minutes 18:19-19:11 from the discussion between Pair 3, extract 4 (Note. “//” indicates overlapping speech.)

Transcribed speech	Actions with regard to material resources
Anna: “Twenty five.”	Anna again circles the middle dot and writes 25.
Pandit: “Why are you saying that dude's 25? They don't have to be 25.”	
Anna: “ It - it - this one is 25 because that's the average.”	
Pandit: “Average doesn't have to - doesn't mean that one guy has to be 25.”	Pandit places her pen on the page.
Anna: “Oh okay, okay. That makes sense then.”	
Pandit: “Altogether it's 125 because like ... “	Pandit draws a bracket to include all of the dots and writes 125.
Anna: “Yeah, yeah, yeah.”	
Pandit: “And ...”	
Anna: “Now, I get it. I thought that was //just 25.”	
Pandit: “//Yeah, yeah. So, one dude's [inaudible]. That means the other four is 112.”	Pandit now draws a bracket including all but one of the dots (the 13 year old) Anna appears to keep Pandit’s hand away from the paper momentarily
Anna: “What do you mean? No. It can't - they can't all be like so equal.”	Pandit takes the paper in one hand and starts to point with her pen in the other hand to explain to Anna
Pandit: “They're not. Oh my God. Look, so 25's one guy, right. No. It's like for, you know, average means like ...”	Anna gently pulls the paper back towards herself
Anna: “I know, I know.”	
Pandit: “Yeah. So 25 times five is the total, right?”	Pandit gestures a circle around the row and then points to one and Anna starts writing
Anna: “Yeah. I know.”	
Pandit: “So, everyone's 125. And one guy is 13.”	The girls leap back from the page before leaning forward to continue with their calculations
Anna: “I know, one guy. So ...”	
Pandit: “How did you put minus 13? It's 112, oh my God.”	
Anna and Pandit: *laugh*	
00:19:06,08	
Anna: “Okay. One hundred and twelve so they are ...”	

In this extract, the talk between Pair 3 suggests they have reached a conclusion as Anna, somewhat defensively, repeats, 'I know, I know'. Adding different mathematical representations to their materials as they participate in and negotiate their understanding of the task, their thinking becomes visible and resources their further interaction. Anna's talk insists she understands average as part of the task, however, by labelling one dot as 25 Pandit sees that Anna has understood 25 as the age of a household member. In contrast, Pandit recognizes that the notion of average as an abstract representation of the shared ages of the different household members. Pandit then refers to the materials they have generated through their negotiations to help Anna change her mind. Pandit points to the circles as though she is trying to guide Anna's thinking through the representations on the page and draws Anna's attention to the total age of the household using 25 in her verbal explanation whilst circling the row of dots on the material resources: 'So everyone's 125'. Anna's new understanding is then visualized through her use of material resources as she deducts 13 from the total age of the household on the working out sheet. Anna places the four remaining household members within a bracket and adds 112 as the total age of the four remaining members. As Pair 3 share and map their understanding on the working out sheet (see Fig 5) they generate a record of their thinking and tools for focusing their further actions.

Towards the end of the task, Pair 3 simultaneously add ideas to the working out sheet. Unlike their initial interaction when they pay attention to one another and Pandit in particular seeks to establish a social partnership, now Anna and Pandit focus on their individual notations. Although both girls are focusing on the mathematics of the task, their social interaction is no longer in sync leading to a somewhat comical exchange. Pandit, focusing on the age of the youngest household member, asks Anna for her favourite number between one and five. Anna is paying little attention to what Pandit is doing or asking and Anna answers, 'Seventy-eight'. Pandit asks again for Anna's favourite number between one and ten. Anna says she does not know, before saying, 'So that's 9. Wait, what?' (21:23-21:25), as though she has only just become aware of Pandit's question.

Of the four pairs, Pair 3 most intensively use the working out sheet as a pair and as individuals. The initial social negotiation over how to use the working out sheet helps them to agree on basic ground rules regarding who can see, who can write. With this social agreement, a shared point of focus is established and the actions and interactions of Pair 3 transition from the social to the mathematical. As Pair 3 add notations to the working out sheet, a record of their thinking and decisions begins to take shape and their working out sheet includes the greatest variety of representational registers with illustrations, words, highlights, crossings out and a variety of calculations. Although the materials help the students visualise and share their understanding in different ways, for Pair 3 the materials also seem to distract them from their partnership and their tentative social agreement breaks down undermining the socio-mathematical requirement to collaborate. In Pair 3, when the social agreement breaks, Anna insists on writing the answer alone. Pandit withdraws by doodling on the working out sheet but when Anna comes to a sudden stop in the writing process, Pandit turns to the written record as a way to re-enter and to complete the task with her partner.

As with Pair 2, the partners within Pair 3 use materials in different ways to participate in the task. Pandit carefully negotiates entry into the task and asks Anna what she is doing, Pandit reads Anna's notes on the working out sheet and adds her own notations in a similar manner. Pandit demonstrates that she is able to use social skills, as well as mathematical and socio-mathematical understanding, to participate in the pair task. In contrast, Anna appears less willing to share initially, and although she responds positively to Pandit's questions, Anna prefers to work alone when writing the final answer. The mistakes that appear in the written text, including misspellings and incorrectly transferring the calculations, contrast with the systematic way Anna worked in the earlier stage. This perhaps indicates that Anna is working on the very edge of what she is able to do and that for her meeting the requirements of the task and social interaction are separate activities.

4.2.4 Pair 4: Hovering

The difficulties Pair 4 (John and Arman) have to find a concrete entry point into the task seemed to foreshadow their difficulties throughout the task. John and Arman begin by reading the task aloud and trying to understand the meaning of the task. John is able to identify keywords in the task description and asks what average means (19:04) and Arman incorrectly answers by saying that 'It is like the maximum' (19:09) and 'average is like the most likely so... most of five people living in a house is 25'. Using Arman's response, John calculates the total age of the household as 125 (25 x 5; see Fig. 6). As one person is a Year 7 student, assumed to be 12 years old, another person should be the maximum age (25) minus the Year 7 student's age, miscalculated as 25 – 12 = 17 (see Fig. 6). Although minimal notes are added to the working out sheet, John tries to use the material resources to participate in the task and to guide the participation of Arman by asking Arman to write and suggesting what could be written. Arman looks over the working out sheet but does not add any written notes. In the written answer, Arman adds the ages as a list which begins with 'Year 7 student' but this is then crossed out and appears to be at the top of a list. Although Arman rearranges the order of ages from the youngest to oldest, he seems disappointed and disinclined to participate in writing and minimal information is included in the final answer: 'The peoples in the house related' (see Fig. 7).

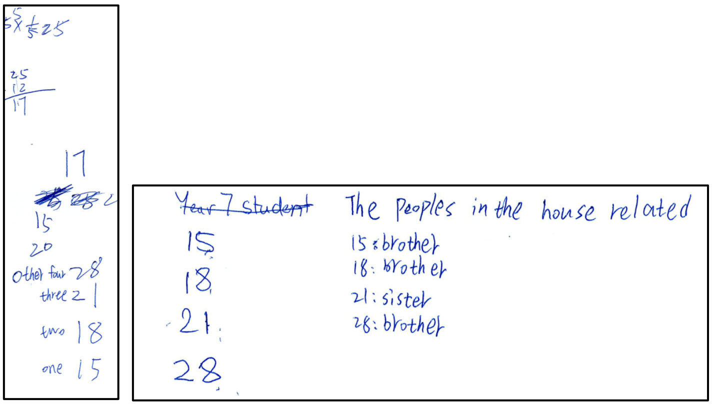


Figure 7. The working out and answer sheets of Pair 4 (John and Arman)

Although Pair 4 repeatedly turn to the task description appearing to seek the socio-mathematical guidance needed to enter into the task, they do not seem to find a way in. Towards the end of the allotted time, Pair 4 agree to 'just write' although they have little to add to the answer sheet with minimal notes added to the working out sheet. Their misunderstanding of the mathematical information in the task undermines their mathematical participation and the laboratory conditions mean that the teacher can only direct them to each other when they ask for help from the teacher. Pair 4 appear to have very limited resources to participate in the task and are unable to use the given materials or to generate further materials. Nevertheless, careful observation of the actions and interactions of Pair 4 suggests that John and Arman are trying to participate in the task (Fig. 8).

Figure 8. John writing in the air on the left and Arman counting with his fingers on the right

When the task is first introduced, John appears to translate the task into Chinese characters in the air (min. 16:24-16:29) before quizzically turning to the teacher who redirects John to Arman. During the task, John re-uses this strategy of asking Arman, for example, to write because he does not 'know how' (min. 22:51), at least not 'know how' in English. Although John struggles with the mathematical representations due to the terminology, John recognises the socio-mathematical requirement to write an answer and tries to use his social relationship with Arman to persuade his reluctant partner to participate in the task. For his part, Arman does not write anything more than John has already written and uses his fingers to calculate the age of university students in relation to a Year 7 student (Fig. 7). Arman's limited participation suggests he also struggles to make sense of the task and his resistance to use the material resources limits the generation of further resources and arguably indicates their need for the support of a more expert, not just social, other (Mariotti, 2009). John's questions, suggestions and hesitations indicate the skills and limitations of his participation. Arman's reticence to enter into the task hides the extent of his ability to participate, but arguably his limited responses to John's questions, his refusal to write and counting on his fingers indicate that his range of strategies for participating is limited.

5 Discussion

The focus of this study is on the use of materials within an open-ended problem-solving pair task and the situated use of materials within the pairs. Materials refers to the blank working out sheet and answer sheet with the task description, as well as the notations students added to their papers. The findings indicate that students use materials to include different representational registers (Duval, 2017), to support their initial engagement with the task and to generate further materials (Wertsch, 2007), and to manage the social relationships implied by the task design. In other words, the use of materials brings the mathematical, socio-mathematical and social foci (Chan and Clarke, 2017) of an open-ended problem-solving task together as interwoven and interconnected considerations. Moreover, the findings illustrate how the use of materials can offer a shared space for thinking and generating further resources, and potentially a space for 'shared thinking' (Rojas-Drummond, et al. 2008) although this requires students to synchronise their use of materials and establish a working partnership in order to manage the demands of the task together.

The particular contribution of this study, however, addresses how the use of materials provides insights into student participation both within the task and across a broader timeframe (Rogoff, 2008). Within the task, the students' notations concretise the focus of their attention and indicate how they make connections between different aspects of the task (e.g. Kazak, et al. 2015). As the task progresses, the notations become a material record of the thinking process and decisions made along the way (Ingram, et al. 2019) and the approach adopted by the students (Livy, et al. 2018). The use of materials, however, can be indicative of what is familiar to the students, their assumptions regarding the materials, the task and partnership as well as the limitations of their participation. For example, in this study the practical step of sharing the materials within the pair could not be assumed, even though this was written into the task design and the students were instructed to work in pairs. If one student refused to heed this socio-mathematical requirement, his/her pair had to draw on his/her social skills to 'win' his/her way into the task. Developing a shared understanding of the representational registers also required a degree of social accord, for example, Pandit was prepared to change which dot represented the Year 7 student to accommodate how her partner viewed the line of dots. The complex negotiations the students entered into as they responded to the task required ways of participating not written into the task design per se. These different ways of participating appear dependent on previous skills and experiences of students and become visible through their orientation to the task and responsiveness to different perspectives.

The descriptions of the student pairs highlight significant differences with regard to the approaches of the pairs, as well as individuals. John, for example, tries to make sense of the task by translating it in the air, by identifying unfamiliar key words, seeking help from the teacher and his partner. These different ways of participating indicate John's willingness to participate and to use all the resources that are available to him, but he cannot go beyond listing the ages of different family members without more support. Anna participates by quickly taking charge of the paper and in effect the task, and although she initially appears reluctant to share her thinking, as she answers Pandit's questions and comments on Pandit's contributions together they generate more resources. Anna's approach differs from John and Pandit highlighting the different repertoires of participation that can be present within an intact class. 'Repertoires of participation' as a notion acknowledges that the approaches of individual students can significantly vary in effect providing insight into how individuals have participated in the past and actively participate in the present (Frankel, 2012). Moreover, the findings indicate how materials can be used to undermine a partner's participation in a task as well as to withdraw from the demands of participating (e.g. Kuntze, et al., in prep.). Whether students should continue to participate this way in the future draws attention to the responsibilities of educators and educational researchers.

In this study, the open-ended nature of the task and limited presence of the teacher or artefacts (Wertsch, 2007) allows for a careful exploration of the different approaches of students and their individual repertoires. As part of the design of the study, the students had to decide for themselves how to participate in the task. For educators, observing the use of materials and the ability to read the record of student thinking expands teachers' views into student participation. If teachers pay attention to the use of materials they can gain insights into the established and developing repertoires of students, even if they are not physically beside them (Dreher & Kuntze, 2015a & 2015b) or able to attend to the concurrent thinking processes of multiple students. For educational researchers, investigating the use of materials provides a record of student development that is situated within the socio-mathematical norms of the learning environment and task, and provides insights into the history and future of student participation (Laine, et al. 2018). As such, the way in which materials mediate student participation and development enables educational research to acknowledge students as active participants and contributors to their own development as they engage with mathematics and the socio-mathematical demands of mathematics education. Moreover, acknowledging the ways in which students' repertoires of participation vary within the same community, can hopefully contribute to education to be an expansive endeavour, sensitive to the individual repertoires of students, and in turn contributing to the overall potential of educational communities.

Although a small-scale study, the findings are indicative of future areas of research. Mapping the repertoires of a whole class would provide clearer insight into the established approaches of the community (Radford, 2014; Laine, et al. 2018) and better guidelines for instructional practice. If many students, for example, resist sharing papers and ideas then teachers can invest in social participation; if students are unwilling to share unfinished answers or to enter into productive struggles (Livy, et al. 2018), teachers can support the socio-mathematical participation of students; and if students struggle to identify or translate key mathematical concepts into different registers (Lesh, et al. 1987), then educators can support participation through the use of mathematical representations. Material resources are only one aspect of participation in learning environments, yet students use of materials translates the participation of students into material records that can yield significant insights for teachers and educational researchers.

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Research Article **LUMAT General Issue 2021**

The effects of School location on students' academic achievement in senior secondary physics based on the 5E learning cycle in Delta State, Nigeria

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This study examined the effects of school location on secondary school students' academic achievement in Physics based on the 5E learning cycle. The design of the study was a non – randomized prêt-test, post-test control group quasi-experimental design. The population of the study was 66,345. Two hundred and forty-three students were sampled from six schools. Four hypotheses were tested at 0.05 level of significance. The hypotheses state that there is no significant difference in mean achievement scores in Physics between urban and rural students taught using 5E leaning cycle among others. The statistical tools used were mean, standard deviation and Analysis of Covariance (ANCOVA) were used in testing the hypotheses formulated. The result amongst others showed there is no significant difference between rural and urban students' achievement taught using 5E learning circle (Fcal. (113) = F crit (0.005), p>0.05). Based on the findings, it was recommended among others, that 5E learning cycle be adopted in Nigeria secondary schools as a teaching method and that faculties of education in various schools of higher learning should ensure that 5E learning cycle is included as a method of teaching Physics

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1 Introduction

The International Union of Pure and Applied Physics (IUPAP) defines Physics as the scientific study of matter and energy and their interactions with each other, which plays a key role in the future process of mankind. Advancement in physics often translates to the technological sector and, sometimes to the other sciences, Mathematics and Philosophy (IUPAP, 1999). For instance, advancement in Physics, relative to electromagnetism, has led to the spread of electrically driven devices; advancement in thermodynamics has led to the development of motorized transport; just as advancement in mechanics has also led to the development of calculus, quantum chemistry, and the use of instruments such as electron microscope in microbiology. Okoronta (2004) asserts that Physics is a vehicle for achieving long-term goals of science because it is instrumental to technological and socio economic growths across the globe. Physics, as a subject, is the foundation upon which the scientific and technological advancement of a nation rests (Ogunleye &Babajide, 2011). It is the link between all the science subjects at the secondary school level and technological courses at the tertiary levels of education.

In spite of the importance of Physics, its teaching and learning have been in decline in Nigeria as shown by its low enrollment. The reason is not far-fetched. Ogunleye and Babajide (2011) state that there are observable problems plaguing the learning of Physics in Nigeria. They include poor infrastructure, lack of qualified manpower, non – availability of, or poorly equipped laboratories, wrong teaching methods, among others. Such problems often lead to students' poor performance in external examinations, such as the West African Examination Council (WAEC) or Joint Admission and Matriculation Board (JAMB), (Abamba, 2012)

Research by Ogunleye and Babajide (2011) affirm that Physics students at the secondary school level continue to exhibit poor performances in the subject. The methods of teaching adopted by teachers go a long way in determining how students learn, and this ultimately affects their academic achievement. Eze (2003) blames the persistent low achievement on persistent use of traditional teaching methods, especially the lecture method which has been very ineffective in teaching pedagogy. Hence, Eze (2003) advocate a shift from the traditional methods to more effective ones that will engage the students' domains (affective, cognitive and psychomotor) of learning. Teaching is not just a process of passing information by the teacher or showing how much a teacher can express himself but one that affords students the opportunity to interact with both humans and available material resources. Agbowaro (2008) states that meaningful learning is active, constructive, intentional, authentic and cooperative. Therefore, it is imperative for teachers to employ methods that are student-centered. According to Bybee, Tylor, Gardner, Scaffer, Powell, Westbrook and Landes (2006), science teachers' globally strive to improve their instructional practices to enhance students' learning. According to them, science teachers, curriculum experts have identifying research findings to incorporate into materials in order to facilitate connections between the teachers, the curriculum, and the students. Bybee et al. (2006) state that the use of coherent and coordinated sequencing of lessons and instrument models have become popular in science education at present.

1.1 Origin and development of 5E

The 5E learning cycle is an instructional model that describes a teaching sequence that can be used for an entire program, specific units and individual lessons. It was developed by the Biological Science Curriculum Study (BSCS) through a team led by Roger Bybee in the late 1980's from the work of Atkin and Karplus who explored children thinking and their explanation of natural phenomena. By 1961, Karplus began connecting the developmental psychology of Jean Piaget to the design of instrumental material and science teaching. In 1901, J.M. Atkin shared Karplus' ideas about the teaching of science to young children. Their collaboration led to the development of a model of guided discovery that focused on exploration, invention, and discovery (Bybee et al., 2006; Kolis, Krusack, Stombaugh, Stow and Brenner, 2010; Ajaja and Eravwoke, 2012). In the 1980's, Lawson and others slightly modified the terms of the Atkin and Karplus model, though, in spite of the changes, the conceptual foundation of the learning cycle remained the same. According to Bybee, the new phases added to the SCS model are engagement and evaluation. The 5E learning cycle has become successful in improving students' achievement in science and in helping to improve the way students learn. According to Bybee (2011), the 5E learning cycle has been more successful than was imagined when it was originally developed and it is recognized internationally and applied in other disciplines other than science; adapted by curriculum developers outside the BSCS, and used by science teachers at all levels.

The 5E learning cycle consists of 5 stages which are engagement, exploration, explanation, extension and evaluation in that order.

1. Engagement: the teacher assesses the learners' prior knowledge, helps in engaging them in a new concept, using short activities in promoting curiosity and eliciting prior knowledge. According to them, such activities should connect past and present learning experiences, reveal prior conceptions, and organize students' thinking in achieving the learning outcomes of current activities (Bybee, 2011).
2. Exploration: this stage provides students with a common experience within which current concepts, processes and skills are identified and a conceptual change is facilitated. Learners may complete laboratory excise that allows them to use prior knowledge in generating new ideas, exploring possibilities and questions, designing and conducting preliminary investigations (Bybee, 2011).

3. Explanation: this stage focuses on students' attention on their engagement and exploration experiences, and enables them to demonstrate an understanding of concepts, process skills or behaviour (Bybee, 2011). It enables teachers to directly introduce concepts, processes or skills. Learners are allowed to explain their understanding of the concepts. The teacher's explanation or the curriculum may guide students towards a deeper understanding of the concept.
4. Extension (elaboration): teachers challenge and extend students conceptual understanding and skills. Through new experiences, students develop deeper and broader understanding, acquire more information, adequate skills, and apply their understanding of the concept by conducting additional activities (Bybee, 2011). Students conduct additional activities based on their new experiences.
5. Evaluation: according to Bybee et al. (2006), evaluation stage encourages students to assess their understanding and ability and provides opportunities for teachers to evaluate students' progress towards achieving the educational objectives. It is a diagnostic process which enables the teacher to determine whether the learner has attained understanding of concepts knowledge.

Since the development of 5E learning cycle, a lot of researches have been carried out to examine the instructional effectiveness of learning across different subjects in the sciences. Adams, Bevevino & Dangel (1992) explored the 5E learning cycle model approach and found that it encourages students to develop their own frame of thought. Caprio (1994) compared a class taught with traditional method in 1985 with one taught with 5E instructional model and found that the students taught by using 5E instructional model achieved higher.

Some studies conducted by using 5E instructional model revealed that the model increases the success of students, improves conceptual understanding and their attitudes (Kor, 2006 & Saglan, 2006 in Cardak et al., 2008). In another study, Seyhan & Morgil (2007) compared two classes taught with traditional methods with two classes taught with the 5E instructional method. They found that the experimental group had a much greater understanding of information, especially on questions that required interpretation. Keser & Akdeniz (2010) stated that the 5E learning cycle aids the teacher to structure and sequence potential learning experiences in a systematic and synergistic way that is consistent with a constructivist view of teaching and learning. They further said that the 5E learning cycle is not an essential part of students' learning but a scaffold or framework for the teacher. Hence, students must be provided with learning environments that encourage them to explain their ideas and understanding and give opportunity for them to extend their knowledge of concepts to other contexts (Boddy et al., 2003).

Cepni, Sahin & Ipek (2010) showed that instructional materials embedded with different techniques in the 5E learning cycle could be effective in removing alternative conceptions and providing conceptual changes more than existing material. Turki & Calik (2008) also found that students were highly motivated and their achievement were increased when 5E learning cycle model was employed in the teaching of exothermic and endothermic reactions. Tuna & Kacar (2013) observed students' scores in experimental group on academic achievement and permanence on trigonometry knowledge are higher than those of the control group statistically when 5E learning cycle was employed. The difference between the groups is statistically significant and in favour of the experimental group. Abdul, Muhammad, Khalid & Shahid (2010) worked on the teaching of Physics with the 5E learning cycle model. The study was aimed at measuring the effectiveness of the 5E learning cycle based on the constructivist approach in the teaching of Physics in public secondary schools. Results showed that the achievement level of students had a significant difference from the performance of students taught with traditional methods. They concluded that the instruction based on 5E learning cycle model yielded better student' performance than that of students taught by the lecture method. Ajaja & Eravwoke (2012) showed that students taught by using the learning cycle had a better achievement in Biology and Chemistry compared to their counterparts that were taught by the lecture method. Similarly, Balci, Cakiroglu & Tekkaya (2006) compared the effectiveness of 5E learning cycle with expository instructions and found that the activities of students in 5E learning cycle activated their prior knowledge and to overcome struggling with their misconceptions.

Ajaja (2013) showed that students in the 5E learning cycle and cooperative learning group significantly outscored those in the concept mapping and lecture group on both achievement and retention tests. Furthermore, students in 5E learning cycle and cooperative learning groups did not significantly differ on achievement and retention. Qarareh (2012) also observed that students taught by the use of the 5E learning cycle achieved better results than students in the group that was taught with the traditional method.

1.2 Limitations

However, since the application of 5E learning cycle in the teaching/learning process, a rage limitation has been observed. One such limitation is that teachers have been finding it difficult to use the model effectively such that major characteristics of the model are overlooked (Keith & Shelly, 2012). In addition to this, not extending the elaboration into novel areas beyond the specific concept has been identified educator using only verbal explanation during the 3rd stage (explain) has also been criticized (Keith et al., 2012; Fletcher in Somayeh & Shahram, 2015). Hence, researchers have called for proper training of both instructors and students before the commencement of instruction (Ajaja, 2013).

The model has also been criticized for being time-consuming both in implementation and in planning, with calls for increased time on the task before and during instruction (Ajaja, 2013; Claire, 2013). Kirschner et al. (2006) and Dodge, (2017) also observed the risk of developing new misconceptions by students with little background or experience in the concept. Furthermore, Ajaja (2012) observed that low ability students may find it very difficult to cope with the model and called for strong cooperation among members of a group under such program. Finally, the bulk of the limitations highlighted stern from the fact that most educators find 5E learning cycle novel and thus lack the skill required for the effective use of the model. There must be adequate tutelage for both instructors and students on their level of participation in all 5 stages of instruction

1.3 Urban and rural schools

The location of a school has a big role to play in the academic achievement of students at school. Akinyele (2011) stated that the immediate environment of a child plays a major role in his socialization. According to him, the area in which a school is located can affect the academic achievement of a student. In the same vein, Akpan (2001) has stated that school location is one of the major factors that affect students' academic achievements. A school located in a rural area is usually faced with problems like shortage of teachers, lack of laboratories, poorly equipped laboratories, among others in Nigeria. These shortcomings negatively affect both students' motivation and achievement. Evidence abound that the educational aspirations of students who study in rural are weaker than those of their urban counterparts (Hum, 2003; Arnold et al., 2005). Macmillan (2012) found that students in rural areas place less value on studies such that their achievements are affected.

Adesoji and Olatunbosun (2008) have pointed out that the relationship between the location of a school and students' academic achievements has been reported. Urban students perform better than their counterparts in semi-urban and rural schools (Adepoju, 2001; Ogunleye, 2002; Ndukwu, 2002). Corroborating this, Hu (2003) said that, compared with urban students, rural students tend to have lower educational aspiration, place less values on academics, and have lower academic motivation. Owoeye (2002) found a significant difference between the academic performance of students in rural areas and that of their urban counterparts.students in urban areas are better.

On the other hand, Ajayi (1999) studied the relationship between academic achievement and school location and found that the there is no significant difference between academic achievement of students in urban students and that of students in rural students. Yusuf and Adigun (2010) also observed that whether a student attends a rural or urban secondary school does not make any difference in his academic achievement. Owoeye and Yara (2011) posit that in Nigeria, education in rural areas is usually full of difficulties.

1. Teachers who are qualified don't like being posted to villages
2. Villagers prevent their children from going to school regularly because of the children's involvement in farming activities
3. Parents are reluctant to entrust their female wards to male teachers.
4. Lack of roads and communication facilities making it difficult to get books and teaching materials to the schools.

There is, therefore, disparity between the quality of teachers in urban schools and that of those in rural areas, and, this is reflected in students' achievement. The review has shown that more researchers hold the view that urban students do better than rural ones. This research is, therefore, designed to investigate whether the application of 5E learning cycle will significantly improve the achievement of students both in rural schools and urban schools irrespective of the location of the school.

Having established that students' academic achievement in the sciences, particularly in Physics is, in decline, it has become imperative to find methods and strategies that can curb this downward trend in students' achievement in that subject. Having examined the effectiveness of the 5E learning cycle in improving students' achievement in the learning of science subjects, and having and seeing the poor achievement in the rural areas, this work therefore seeks to examine the effects of school location on students' achievement in Physics based on the 5E learning cycle. The general purpose of this study is to examine the effects of school location on students' academic achievement based on the 5E learning cycle. Specifically, the study will find out whether

1. There is a difference in the mean achievement scores in Physics between urban and rural students taught with the 5E learning cycle model.
2. There is a difference between the mean achievement scores of urban and rural secondary school students taught with the lecture method.

2 Research hypotheses

The following null (H_o) hypotheses were put forward to answer the problems stated and tested at 0.05 level of significance

1. $Ho_{1:}$ there is no significant difference in the mean achievement scores of students in Physics between groups taught by the 5E learning cycle and those taught with lecture method.
2. $Ho_{2:}$ there is no significant difference in the mean achievement scores in Physics between urban and rural students taught with 5E learning cycle model.
3. $Ho_{3:}$ there is no significant difference in the mean achievement scores in Physics between urban and rural students taught with the lecture method.
4. $Ho_{4:}$ there is no significant interaction effect between method and school location on students' academic achievements in Physics.

3 Materials and methods

3.1 Design of the study

A research design is the plan or logical structure of a study (Okorodudu, 2013). According to him, the nature of the problem and the hypotheses to be tested as well as the type of sample and the subjects determine to a large extent the design to be adopted. The study is a non-randomized pre-test, post-test control group quasi-experimental design. The population of the study is sixty-six thousand, three hundred and forty-five (66,345). Two hundred and forty-six students were sampled using simple random sampling technique from six secondary schools in Delta State (two from each senatorial district out of the three senatorial districts) using stratified sampling.

3.2 Instrumentation

The research instruments designed by the researcher and used for this study include

1. Physics Achievement Test (PAT). The test was constructed by the researcher on topics in the concept of light waves. The topics treated include reflection of light waves, refraction of light waves and applications of light waves. The test consisted of 50 multiple-choice questions with options A-D or E from past West African Examination Council (WAEC) questions. A table of specification prepared showed that 48% of the question tested their knowledge of the concepts, 36% tested comprehension, and 16% tested application of the concepts.
2. Instructional packages for the instructors (lesson play). They include (a) comprehensive lesson plan on 5E learning cycle on the concept of light waves (b) comprehensive lesson plan based on the traditional method (lecture method).

3.3 Validity and reliability

Factor analysis is a statistical method used to describe variability among observed correlated variables in terms of a potentially lower number of unobserved variables called factors. The Physics Achievement Test (PAT) was administered to 31 participants who were involved in the pilot test. The instrument was found valid in content, construct and face. In establishing the reliability of the instrument, Kuder-Richardson formula 21 ($KR\ 2_1$) was used to estimate the internal consistency reliability of PAT. KR21 coefficient calculated was 0.71. Based on this value, the Physics achievement test was found reliable. The factor analysis of items in Physics Achievement Test (PAT) was processed so that the test could be estimated for content and construct validity. The factor analysis of the PAT began with the processing of Descriptive Statistics and Initial Communalities. The Descriptive Statistics of mean and standard deviation of total items retained for the 31 (intact class) participants who were involved in the pilot test of the PAT instrument at Delta State University Secondary School were reported. A total of 50 items were selected out of the initial total items of 78 that were factor analysed. These 78 items selected were computed for content and construct validity using Factor Analysis Output from principal component analysis (PCA).

To establish the content validity of the instrument, Eigen value of above 1 was used to select factor components into the PAT instrument. The factor matrix of all factors or components had to be rotated to determine the weight of each item within each of the components. The cumulative variance for all rotated sums of squared loading was estimated as 89.45%. This is an indication of the content validity of PAT. It revealed that PAT covered up to 89.45% of the domain of Physics Achievement Variable with a total of unexplained variance of 10.55%. Therefore, on the whole, the cumulative Eigen value of 89.45% is above 50% and hence the PAT was considered content valid.

In establishing the construct validity, the factor matrix of all factors or components had to be rotated using Varimax to determine the weight of each item within each of the components. Eigen value of above 1 was used to select factors that genuinely measure similar construct. From the observed scores, latent variables were identified with the number of items measured by the construct. Since rotated factor loading matrixes range between 0.29 and 0.81, it can be concluded that PAT has construct validity.

The Physics Achievement Test, after selection, was given to three experts in science education to establish the face validity of the instrument.

3.4 Treatment procedure

- Training of instructors

The physics teachers used for the experimental group were trained on the skills of using 5E learning cycle for teaching. This exercise lasted for three days (a day was devoted to a teacher in a school). The process started with explaining the meaning of 5E learning cycle, its origin, modifications and applications in the teaching-learning process. The next stage was done using the 5E training manual adopted from SEDL (2012), explaining the role of teachers and students on every stage of the model.

- Treatment proper

The researcher obtained an official permission from the heads of the six (6) secondary schools for the study. The Physics teachers in each school were used as research assistants. Where the school has more than one Physics teacher, the research assistant was the teacher assigned to Senior Secondary 11 (SSII). The researcher gave them orientation on the purpose of the study to ensure uniformity and ensure that each conformed to the content and method assigned. For the research assistants in the experimental groups, the researcher explained how to follow up the students at every stage and guide their transition to the next stage of instruction based on the 5E learning cycle. The researcher explained the kind of questions to be asked at different stages of the cycle. The level of their involvement at every stage was also explained to the teachers. The researcher embarked on several supervision visits to the schools to check on the effectiveness of the teachers with respect to the treatment procedure assigned to that school.

At the end of the orientation exercise, the research assistants for the experimental groups were handed a copy of the instructional lesson plans based on the 5E instructional model and the necessary instructional materials for the instructions to commence. In addition, copies of lesson plans for the control groups as well as the necessary instructional materials were given to the assistants in order to commence teaching.

The researcher visited each school before the commencement of instruction, and administered the test (Physics Achievement Test, PAT) with the assistance of the research assistants in each school. The students were given the necessary instructions and asked to answer the questions within specified time frame. After the test, the researcher retrieved the answer sheets and the question papers from them and marked. This accounted for the pre-test scores. Also, the researcher handed the test (PAT) to the research assistants a week to the end of the program to administer to the students within three days after instruction has been concluded. They were collected back by the researcher for marking. This accounted for the post-test scores.

3.5 Method of data analysis

The data obtained were analysed using descriptive statistics (mean and standard deviation) while hypothesis 1, 2 and 3 were tested using the Analysis of Covariance (ANCOVA).

4 Results and Discussions

Data gathered were analysed and the results are presented below.

4.1 Test of assumptions

The following tests of assumptions were established to justify the use of ANCOVA.

1. The covariate was measured before treatment
2. The Crombach alpha using the reliability procedure was established
3. More than one covariate were used
4. The linearity was established. The straight line shows a linear relationship with each other.
5. The significant interaction between the covariate and the treatment is 0.4. This is above 0.05 level of significance, thus, establishing the homogeneity of regression slope.

4.2 Test of hypotheses

- Result on hypothesis 1

The hypothesis examined whether there is no significant difference in the mean achievement scores of students in Physics between groups taught by using 5E learning cycle and those taught by using the lecture method.

From table 1, the experimental group had a mean achievement score of 25.50 while that of control group had a mean achievement of 15.25 in post-test score over 50. The difference in mean achievement is 10.25. Also, the standard deviation for post-test experimental group is 5.16 while that of control is 3.19. This shows that there is a difference in the mean achievement in favour of the experimental group.

An inferential statistics will be used to establish whether the difference is statistically significant.

Table 1. Mean and standard deviation (SD) of both experimental and control groups.

Test	Teaching Method	N	Mean	SD
Pretest	5E model	113	8.86	3.15
	Lecture	133	8.64	3.18
Posttest	5E model	113	25.50	5.16
	Lecture	133	15.25	3.19

Analysis of covariance (ANCOVA) was carried out to determine the difference is significant and the result is presented in table2.

The result of the ANCOVA gives an F (246) =360.591 which is significant at 0.05 level of significance. This implies that there is a statistically significant difference in the achievement of the groups. From the result, there are enough reasons to reject hypothesis 1. Therefore, there is a significant difference in the mean achievement scores of students taught using 5E learning cycle and those taught by using the lecture method in Physics. Furthermore, the value of Adjusted R Squared is 0.596. This implies that the 5E learning cycle contributed 59.6% to the achievement of students.

Table 2. Summary of analysis of covariance (ANCOVA) for the significance of difference in physics test scores between students exposed to 5E learning cycle and lecture method.

Dependent variable: post-test

Source	Type III Sum of Squares	*Df*	Mean Square	*F*	Sig.
Corrected Model	6441.352[a]	2	3220.676	182.041	.000
Intercept	10724.767	1	10724.767	606.193	.000
Pretest	25.904	1	25.904	1.464	.227
Treatment	6379.578	1	6379.578	360.591	.000
Error	4299.156	243	17.692		
Total	108701.000	246			
Corrected Total	10740.508	245			

a. R Squared = .600 (Adjusted R Squared = .596)

- Result on hypothesis 2

Hypothesis 2 examined whether there is no significant difference in the mean achievement scores in Physics between urban and rural students taught by using 5E learning cycle.

Table 3 shows the mean achievement of rural and urban students in pre-tests are 9.18 and 8.87 with a standard deviation of 3.25 and 2.85 over 50 respectively. The post test scores are 25.57 and 25.32 while the standard deviations are 5.88 and 4.64 respectively. From the result of the discipline statistics, there is a difference in the mean achievement scores in Physics between rural and urban students taught by using 5E learning cycle method.

Table 3. Mean and standard deviation between urban and rural students taught using 5E learning cycle.

Test	School location	*N*	Mean	*SD*
Pre-test	Rural	44	9.18	3.25
	Urban	62	8.87	2.85
Post-test	Rural	47	25.57	5.88
	Urban	62	25.32	4.64

When subjected to inferential statistics using ANCOVA, the result is presented in table 4. The ANCOVA result reveals that F (113) = 0.004 is not significant at 0.05 level of significance. This result shows that the F – ratio of 0.004 is not statistically coefficient. This implies that there is no statistically significant difference in the achievement of urban and rural students taught by using 5Elearning cycle.

Table 4. Result of analysis of covariance (ANCOVA) of test scores between urban and rural students on achievement in groups taught using 5E learning cycle

Dependent variable: post-test

Source	Type III Sum of Squares	*Df*	Mean Square	*F*	Sig.	Partial Eta Squared
Corrected Model	7.571[a]	2	3.785	.140	.870	.003
Intercept	6622.269	1	6622.269	244.723	.000	.690
Prettest	7.350	1	7.350	.272	.603	.002
Location	.097	1	.097	.004	.952	.000
Error	2976.624	110	27.060			
Total	76590.000	113				
Corrected Total	2984.195	112				

a. *R* Squared = .003 (Adjusted *R* Squared = .016)

- Result on hypothesis 3

Hypothesis 3 examined whether there is no significant difference in the mean achievement scores in Physics between urban and rural students taught by using the lecture method. Table 5 shows the mean achievement scores for rural and urban students in pre-test are 9.74 and 8.48 respectively, while the post test scores are 13.75 and 15.67 over 50 respectively. From the result of the description statistics, there is a difference in the mean scores in Physics between rural and urban students taught by using the lecture method.

Table 5. Mean and standard deviation between urban and rural students taught using lecture method.

Test	**School location**	***N***	**Mean**	***SD***
Pretest	Rural	34	9.74	3.57
	Urban	84	8.48	2.92
Posttest	Rural	34	13.71	3.58
	Urban	84	15.67	3.04

In determining whether the difference was significant, ANCOVA was employed. The result of F (133) = 2.914 is not significant at 0.05 level of significant. This implies that there is no statistically significant difference in the achievement of urban and rural students taught with the lecture method. Therefore hypothesis 3 is accepted.

Table 6. Result of analysis of covariance (ANCOVA) of test scores between urban and rural students on achievement in groups taught using lecture method

Dependent Variable: Post-test

Source	**Type III Sum of Squares**	**Df**	**Mean Square**	***F***	**Sig.**	**Partial Eta Squared**
Corrected Model	36.258[a]	2	18.129	1.790	.171	.027
Intercept	3017.837	1	3017.837	298.013	.000	.696
Prettest	10.246	1	10.246	1.012	.316	.008
Location	29.508	1	29.508	**2.914**	.090	.022
Error	1316.449	130	10.127			
Total	32398.000	133				
Corrected Total	1352.707	132				

a. R Squared = .027 (Adjusted R Squared = .012)

- Result on hypothesis 4

In determining the interaction between methods and location, ANCOVA interaction table was employed and the result is presented in table 7.

Table 7 shows F (228) = 2.178; P>0.141 which reveals that there is no interaction effect between method and location on students achievements in Physics. This implies that the factors could not interact to affect the achievement of students in Physics.

Table 7. Summary of ANCOVA for significant interaction effect between the methods used and school location on achievement in Physics.

Tests of Between-Subjects Effects
Dependent Variable: post-test

Source	Type III Sum of Squares	*Df*	Mean Square	*F*	Sig.
Corrected Model	6511.513[a]	4	1627.878	92.769	.000
Intercept	9305.457	1	9305.457	530.295	.000
Pretest	47.818	1	47.818	2.725	.100
Treatment	6361.421	1	6361.421	362.522	.000
Location	30.128	1	30.128	1.717	.191
treatment * location	38.219	1	38.219	2.178	.141
Error	4228.995	241	17.548		
Total	108701.000	246			
Corrected Total	10740.508	245			

R Squared = .606 (Adjusted R Squared = .600)

The graph of interaction in figure 1 shows an ordinary interaction because the lines does not cross each order. From table 7, F-cal (2.178)> F-crit (3.84) shows that there is no significant interaction effect between method and location on student achievement in Physics. Thus, the null hypothesis of non – significant interaction effect was accepted.

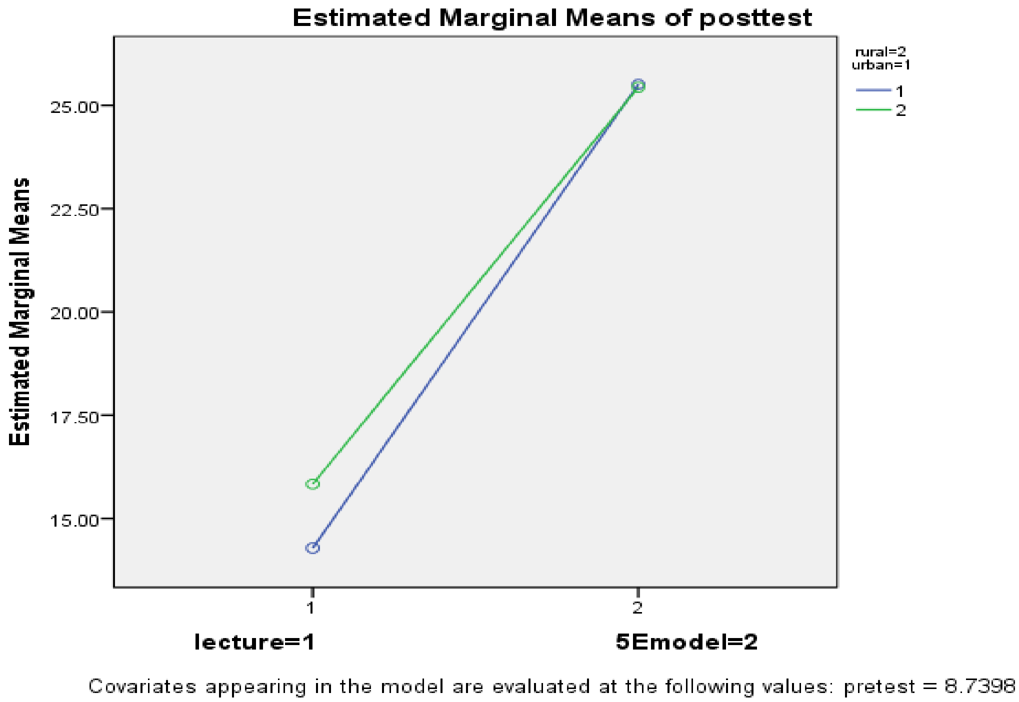

Figure 1. Graph illustrating significance interaction effect between method and location on students' achievement in Physics.

4.2 Test of hypotheses

This work examined the effects of school location on students' academic achievements based on the 5E learning cycle. The study also examined the interaction between the methods employed and school location.

Hypothesis 1 stated that there is no significant difference in the mean achievement scores of students in Physics between groups taught by using 5E learning cycle and those taught with lecture method. The mean score of the experimental group is higher than the mean score of the control group. This implies that the experimental (5E learning cycle) group achieved better than the control group. The *t*-test result (*t*-cal. (18.315) > *t*-crit (1.960), p>0.005) shows difference in performance is significant. A confirmatory test using ANCOVA (*F* (246) =360.591, p>0.005) also confirms that the difference is significant in favour of the experimental group. Therefore, the achievement scores of students taught by using 5E learning cycle and those taught by using the lecture method in Physics is significantly different. This result is in consonance with Cardak *et al.* (2008), Cepni, Sahin & Ipek (2010), Tuna & Kacar (2013), Ajaja & Eravwoke (2012), Qarareh (2012), Balci, Cakiroglu & Tekkaya (2006) who reported a significant difference in students' achievement in favour of 5E learning cycle.

Hypothesis 2 stated there is no significant difference in the mean achievement scores in Physics between urban and rural students taught by using 5E learning cycle. The result of the descriptive statistics showed the mean achievement for rural and urban students in pre-test which were 8.87 and 9.18 respectively differ from the post-test scores of students which were 25.32 and 25.25. The descriptive statistics showed a difference in the mean achievement scores between rural and urban students. This implies that the rural students performed better than their urban counterparts. The finding establishes the fact that rural students are not disadvantaged when the 5E learning cycle is employed. However, when subjected to ANCOVA (*F* (133) =0.004, p>0.005), the difference was not significant. This led to the acceptance of the null hypothesis. This implies that the achievement of rural and urban students do not differ significantly when they are taught Physics by using the 5E learning cycle. The result of Adjusted R Squared of 0.016 shows that the effect of school location on students' achievements based on 5E learning cycle is 1.6%. The result on hypothesis 2 is at variance with the results of Adesoji and Olatunbosun (2008), Adepoju (2001), Ogunleye (2002), Ndokwu (2002), Owoeye (2002), Yusuf and Adigun (2010), Ajayi (1999) and Owoeye and Yara (2011). These researchers have observed that the achievements of students in urban and rural areas are significantly different.

In examining whether there is a significant difference in the mean achievement scores in Physics between urban and rural students taught by the lecture method, post-test scores of students showed that rural students scored 13.71; while urban students scored 15.67. On the basis of this result, it was established that there is a difference in the mean achievement scores in Physics between rural and urban students taught the use lecture method. The mean score of the urban students was higher than that of the rural students. This implies that urban students achieved better when the lecture method is employed. However, on exposure to ANCOVA (F (113) = 0.05, P> 0.05), the difference was not significant. This implies that there is no significant difference in achievement scores between rural and urban students when they are taught by the using lecture method. The result on the hypothesis is at variance with results of Adesoji and Olatunbosun (2008), Adopoju (2001), Ogunleye (2002), Ndokidu (2002), Owoeye (2002), Yusuf and Adigun (2010), Ajayi (1999) and Owoleye and Yara (2011) who reported significant difference in the achievement of students in urban and rural areas. This show with equal treatment, there will be no significant difference in the achievement between rural and urban students. Also, irrespective of the school location, the 5E learning circle improved the mean achievement of students.

The research also showed that there is no interaction effect between method and school location in students achievements in Physics. F cal (2.178) < F-cal (3.84); p>0.141 show no significant interaction effect. Graphically, a dis-ordinal interaction between the lines crossed each other. This implies that the two factors did not interact to cause the desired test scores of the students. Thus, the null hypothesis of the non-significant interaction effect is established.

5 Conclusion and Recommendation

5.1 Conclusion

On the basis of the findings of this study, it is concluded that both methods (the lecture method and the 5E learning circle) improved students' achievement in Physics. However, the group exposed to 5E learning cycle achieved significantly better than the one taught with the traditional method. This study has established that the achievement of students taught by using both the lecture and the 5E learning circle did not differ significantly with school location (urban and rural). From the Adjusted R Square, the effect of school location on students' achievements in Physics based on the 5E learning cycle is just 1.6%. Also, there is no interaction effect between method and school location in determining the achievements of students in Physics when the 5E learning cycle is employed.

5.2 Recommendations

On the basis of the findings and conclusions of this study, the following recommendations are made:

1. The findings of the study have proved statistically the effectiveness of 5E learning cycle in enhancing better achievements in the learning of Physics. Thus, Physics teachers are encouraged to adopt the method in the teaching of the subject with a view to improving students' achievement in it. The method enables students to cooperate with one another and individual acquisition of knowledge instead of being spoon-fed. Using 5E learning cycle provides teachers with the opportunity to discover for themselves the individual problems of students and the general weakness of the students in the class.
2. Faculties of education of various institutions of higher learning should ensure that the 5E learning cycle is included as a method of teaching Physics and other science subjects. This will acquaint would-be teachers with the knowledge of the method and its advantages.

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Energy as a multidisciplinary concept in K-12 education – a case study

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Although one of key ideas behind the introduction of STEM (or STEAM) as a unifying concept, is to emphasize the interdisciplinary character of many real world problems, there is a non-trivial educational challenge in exceeding existing subject boundaries and implementing multidisciplinary activities into the classroom. The present case study covers the design, implementation and evaluation of a multidisciplinary unit in a Finnish lower secondary school, grade 9. The entire multidisciplinary unit was organized around the central concept of energy, and the present study focuses on an activity within that unit that explored how energy can be used to analyze both living and non-living systems. Evaluation of the activity was done with pre and post student questionnaires, analyzing the students' written poster presentations and focus group interviews done with a voluntary group of students after the whole unit. The aim of the study was to explore how students understand the multidisciplinary character of the energy concept. Our results show both challenges and possible gains of working in a multidisciplinary way. However, to succeed serious thought has to be invested in both identifying core concepts that gain by being analyzed in an interdisciplinary way, and in the design of appropriate learning activities around these core concepts. Our study is as an effort in this direction.

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1 Introduction

To meet the societal and educational challenges of the future, a set of so-called 21st-century skills have been identified as vital for students to thrive in a rapidly changing, digital society. As a direct response to this trend, educational policy developers and schools in many countries have tried to formulate goals in line with the 21st century skills (e.g. McPhail, 2018; Voogt & Roblin, 2012). In some countries the answer to the question of how to design a curriculum that better supports the development of these skills, is a push towards an increased integration among different school subjects and disciplines, or towards an increased interdisciplinarity (Czerniak & Johnson, 2014). The underlying idea is that by emphasizing the interdisciplinary features, of until now mostly STEM related subjects, the curriculum will better reflect the real world and therefore provide students with a more authentic context in which learning can take place. This will, according to the proponents of interdisciplinarity, not only sharpen students' critical thinking and problem-solving skills, but also have a positive effect on student interest and motivation for school (Czerniak & Johnson, 2014). The concept of energy, which is the focus of the present study, is by its very nature interdisciplinary.

The new Finnish curriculum formulates a set of transversal (generic) competences, similar to the set of 21st century skills mentioned above, as a way of meeting future challenges (Finnish National Board of Education [FNBE], 2016). The learning objectives of these competences include: thinking and learning to learn; cultural competence, interaction and self-expression; taking care of oneself and managing daily life; multiliteracy; information and communication technology (ICT) competence; working life competence and entrepreneurship; and participation, involvement and building a sustainable future (FNBE, 2016). One key idea in the new national core curriculum is to encourage closer cooperation between different school subjects, what in the curriculum documents is referred to as a multidisciplinary way of working. The use of the term "multidisciplinary" in this context is not one universally adopted internationally. We will use the term multidisciplinary in this study. For a deeper discussion of how the concept of multidisciplinarity is related to interdisciplinarity see (Braskén et al., 2019). The stated goal of the multidisciplinary approach is to support the development of the transversal (generic) competences and provide students with a more realistic context in which learning can take place. By offering students an opportunity to both identify, formulate, and investigate problems that are of interest to both them and their peers, the hope is that a more positive attitude towards both school and school subjects will result.

These are the hopes and ambitions put forward in curriculum documents, but how does one as a school, teacher and subject expert, turn the ideas of multidisciplinarity into actual practice? The present case study is an example of an attempt to do this. The context of our study is a set of STEM related activities, spanning three 60-minute long sessions in total, that were part of a larger multidisciplinary unit in grade 9 with the overall theme of "Energy". The entire multidisciplinary unit consisted of two weekly 60-minute long sessions for a total of seven weeks and all students in grade 9 participated in the unit. In spring 2017, 44 students participated in the unit. The content related goal of the unit was to give the students a broader understanding of the concept of energy, by exploring the energy concept through the lens of different subjects (science, technology and social sciences).

The teachers involved in the unit, representing the subjects, mathematics, physics, chemistry and biology, developed the majority of the activities making up the multidisciplinary energy unit. However, the activity that is the focus of the present case study was developed in collaboration with the researchers. This activity is described in detail in Appendix A (translated from Swedish) and was designed to let students explore the relevance of the energy concept to understanding the human body. This activity will hereafter be designated "the energy-body lab". Our goal as researchers in developing the energy-body lab, was to engage the students in a set of three, short explorations designed to highlight how the concept of energy can be used to analyze both physical (non-living) and biological (living) systems. By exposing students to the multidisciplinary character of the energy concept, the ambition was that they would gain a deeper understanding of the concept of energy, and specifically as it relates to biological systems and their own body. It should be noted that no other activity in the energy unit emphasized this connection. Out of the 44 students participating in the entire multidisciplinary unit, eight students chose to participate in the energy-body lab.

To probe the participating students' attitudes towards and perceptions of the whole unit, all students were given a questionnaire before and after the unit. The questionnaire was designed to probe possible shifts in their perceived importance of the energy concept in general, and how they perceived the multidisciplinary aspects of the energy concept. Two of the students participating in the energy lab were also interviewed after the whole unit, recording their experiences of the unit. Finally, the energy-body lab posters produced by the students, and documenting their lab results, were also gathered and analyzed. By using this data and these artifacts, we strived to answer the following research question:

Research question: *How is the students' understanding of the multidisciplinary character of the energy concept, as relating to living and non-living systems, visible in their responses, presentations and discussions?*

2 Theoretical framework

One of the motivations for integrating different subjects into a multidisciplinary unit is that some key concepts in science and technology are by their very nature interdisciplinary. Energy is one such concept and it is identified as one of seven so called crosscutting concepts in the Next Generation Science Standards (NRC, 2012). From a student perspective, a solid understanding of the concept of energy is deemed as foundational for being able to understand and participate in discussions concerning such vital topics as sustainability, climate change and possible future energy solutions (e.g. Nordine, 2016). The choice made by the teachers of focusing on energy as a central theme for the multidisciplinary unit is thus theoretically well motivated. However, the concept of energy is both a deep and difficult one. Much research has been done on both students' misconceptions of the concept of energy and how and at what age different aspects of energy should best be thought (Duit, 2014). Adding to this challenge is the fact that different disciplines use the term "energy" somewhat differently. While physicists view energy primarily as a quantity that is conserved in interactions, a biologist may focus more on how energy is transferred across system boundaries and an economist on it being a scarce resource. To add to this challenge, the word "energy" has various (non-scientific) uses in everyday language. We speak of our body "using" energy from food to keep us alive, that we are exhausted or "out of energy", and that the batteries of our cellphone are "empty". All these everyday uses depict energy as a material substance that can be stored in batteries, poured into us when we drink a sugary drink and that is forever lost when the cars gasoline tank is empty (Millar, 2005). In principle, there should be no problem with words having multiple meanings, if all participants in an exchange are aware of what meaning is being used in the present context. At this point, however, a final challenge appears, namely the fact that no simple and clear definition of the term "energy" exists, or to be more exact, the mathematical definition is too abstract to serve as a starting point in K-12 education (Constantinou & Papadouris, 2012).

Students (and teachers) are thus faced with the challenge; "*to learn about energy even though we have no knowledge of what energy is*" (Eisenkraft, 2014). Is the struggle worth the effort? The answer made by most researchers is yes, as the concept of energy is essential for understanding both the living and the non-living world. To appreciate the broad scope of the energy concept, students need to learn about the concept in the context of physics, chemistry, biology, health science, and physical education. Only by given the opportunity to explore and recognize that the energy concept can be applied to both living and non-living systems, students can deepen and broaden their understanding of the concept. To quote one leading researcher *"[...] the energy concept is perhaps best learnt the way we learn a new language, not through definitions, but through repeated exposure and active use"* (Constantinou & Papadouris, 2012).

The obvious next question is then how do one design learning activities that promote student learning and deepening the understanding of the energy concept? Traditional energy instruction many times involves only simple energy calculations on idealized systems, leaving the everyday, non-idealized (interesting?) systems for more advanced courses. While there is no one, unique road to a deeper understanding of such a broad concept as energy, and studying simplified systems have a clear role in any energy related course, there is also research that indicates that there is much to be gained from taking a broader, interdisciplinary approach to energy instruction (Nordine, Krajcik & Fortus, 2011). The energy-body lab described in our case study should be understood as an effort in this direction, providing an example of what a STEM activity could look like, when organized around a central, multidisciplinary concept (energy in this case). However, due to the broad scope of the energy concept, it is also important to be clear about what aspects of the concept one as an educator wants to highlight when designing a multidisciplinary teaching-learning sequence.

In designing the energy-body lab, our focus has been on the use of the principle of conservation of energy as an analytical tool for students to understand both non-living and living (carbon-transforming) systems (Dauer, Miller & Anderson, 2014). Our choice of placing the students' own bodies at the center of each activity is grounded in an embodied learning approach that emphasizes the use of action to support learning goals. This approach posits that the transition from action to abstraction is supported by bodily actions (Weisberg & Newcombe, 2017). Therefore, although kinetic and potential energy, as well as work, are concepts covered in the secondary physics curriculum, our hypothesis is that by letting students encounter the energy concept via its connections with the less abstract concept of work, and involving their own bodies in the process, a deeper integration of the physical and biological aspects of the energy concept will result.

3 Educational setting of the study

As stated in the national core curriculum, all students in the comprehensive school in Finland should be involved in (at least) one multidisciplinary unit during the school year (FNBE, 2016). However, the national core curriculum does not specify which subjects, or how many different subjects, should form a multidisciplinary unit, leaving this decision up to each, individual school and teachers. As mentioned earlier, the school in our study chose "Energy" as their theme for the multidisciplinary unit in grade 9. The structure of the multidisciplinary unit is shown in Figure 1. The unit was in large parts developed by the teachers involved in the implementation and consisted of two weekly 60-minute long sessions for a total of seven weeks. All students in grade 9 participated in the unit. In spring 2017, 44 students participated in the multidisciplinary unit.

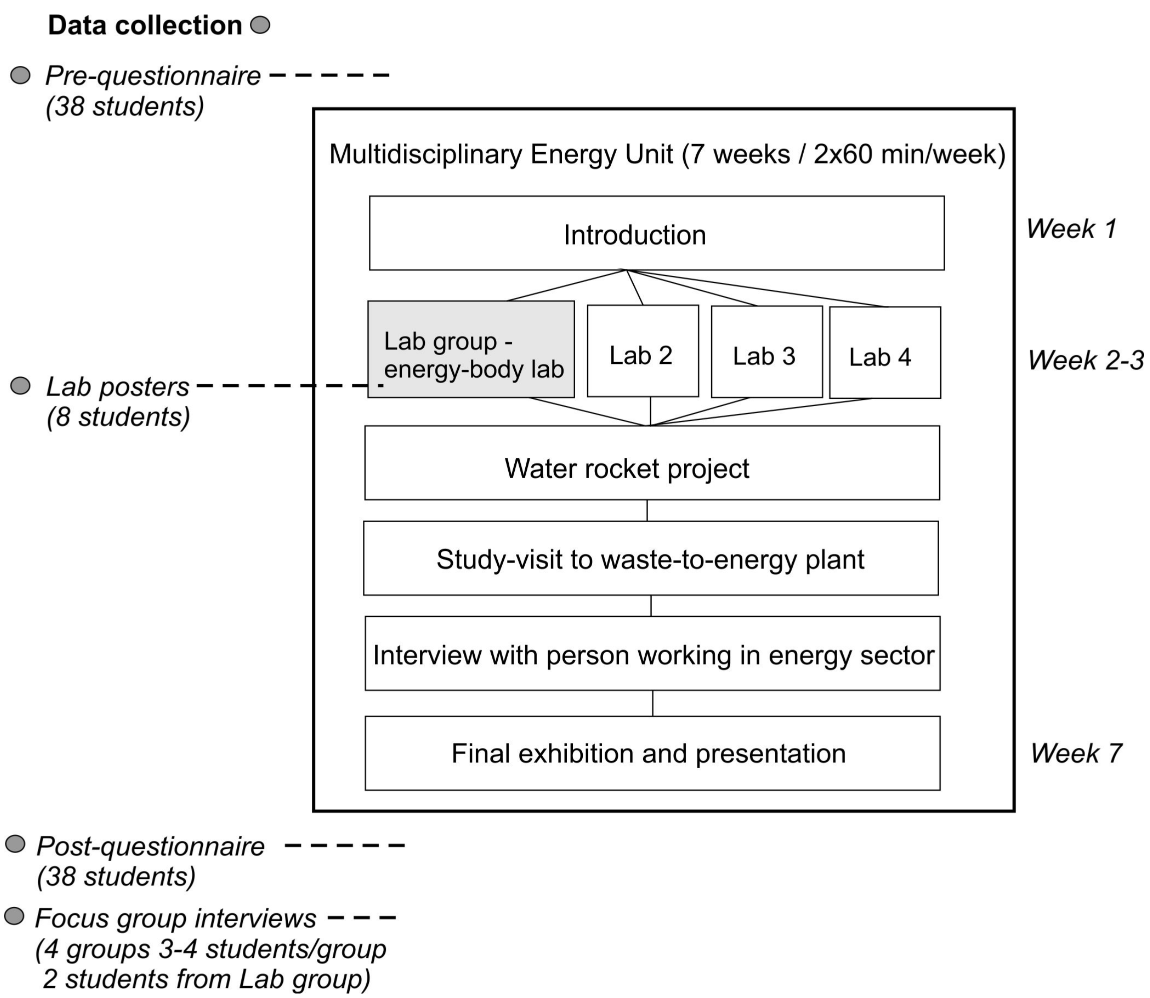

Figure 1. The structure of the multidisciplinary energy unit and timing of data collection.

The multidisciplinary unit was divided into six parts (Figure 1). The first part consisted of a general introduction to the energy theme, followed by a set of four laboratory sessions (where the students were to choose one). In the third part, the students built and optimized their own water rocket, while the fourth part consisted of a visit to a local waste-to-energy plant. The fifth part involved conducting an interview with a non-native speaker working in the local energy sector. The whole unit ended with an exhibition, to which pupils in grade 6 were invited. For the exhibition, each group of (3-4) students planned and gave a presentation (oral, poster, or audio-visual) and interacted with the 6-th grade exhibition attendees.

The four energy-related activities that the students could choose among were: building and optimizing the blades of a model wind-power plant, building and investigating a chemical battery, exploring the photosynthesis of plants, and exploring the connections between work, energy, chemical energy and the human body (what we call here the "energy-body lab"). Eight students out of the 44 chose the energy-body lab. The worksheet given to the eight students before the energy-body lab is found in Appendix A. The time reserved for the energy-body lab spanned three sessions (each 2 x 60 minutes) and all activities and measurements were done during the first session. The two following sessions were reserved for group discussions and documenting the results in the form of a poster. Both the activities, measurements and the following documentation were done in groups consisting of four students. The exact form of the poster documentation, and what each group wanted to emphasize in relation to the energy-body lab, was left up to each lab group to decide.

4 Material and methods

To answer the research question, and increase the validity of our results, we have used the research method of methodological triangulation (Denzin, 2006) and three ways to gather data: questionnaires, documents and student interviews. The timing of the data collection is summarized in Figure 1. The data collected in connection with the seven-week multidisciplinary energy unit, consisted of the following:

1. Two student questionnaires, one done before the start of the multidisciplinary unit and one after the unit (44 students participated in the whole unit and 38 students answered both questionnaires).
2. Poster presentations (PowerPoint) of the two student groups that participated in the energy-body lab.

3. A set of semi-structured focus group interviews (Stewart, 2015) with a voluntary group of students (four group interviews where done, with 3 – 4 students in each group). All interviews were audio-recorded and fully transcribed.

It should be noted that the persons interviewing the students where neither involved in designing the energy-body lab, nor in the actual implementation of the multidisciplinary unit. All the interviews where done after the end of the whole unit, and more than three weeks after the energy-body lab. The questions asked all student groups were the same and designed to probe their attitudes towards the multidisciplinary unit as whole. Each student group were asked about their experiences of this partly new way of working, what they saw as positive or negative with the unit and how they experienced the role of the different subjects making up the unit (physics, chemistry, biology and mathematics). Finally, they were asked if they thought they had gained a deeper understanding of the energy concept through working this way and if so, how. No questions were specifically about the energy-body lab.

For the present case study, we have selected the subgroup of eight students (of 44 students) that chose to participate in the energy-body lab. We have further focused only on the statements in the pre- and post-questionnaires that specifically relate to the multidisciplinary aspects of the energy concept (see Table 1). In answering the questionnaires, the students could choose between 5 alternatives on a Likert scale: Strongly disagree, disagree, neutral, agree and strongly agree, in response to the statements listed in Table 1 and Table 2. The alternatives are quantified from 1 (strongly disagree) to 5 (strongly agree). The four selected statements from the pre-questionnaire are listed in Table 1 and labeled SI to SIV. The eight selected statements from the post-questionnaire are labeled as S1 to S8 and listed in Table 2. The eight students are coded using capital letters A to H. Before the start of the energy-body lab, the eight students were divided into two groups, group I with five students (A-E) and group II with three students (F-H). Out of the group of students interviewed after the whole unit, two students (student A and H) also participated in the energy-body lab.

The questionnaires, the transcribed recordings of the interviews, as well as the posters produced by the students, have been analyzed to examine shifts in students' perceived importance of the energy concept and detect signs of a broadening of their energy concept after the multidisciplinary unit. In the written documents, as well as in the transcribed interviews, we have applied the selection criteria that the students should, within the same paragraph or sentence, use the term "energy" within at least two different contexts (for instance applying it to both living and non-living systems, i.e. their own body).

5 Results

The results are presented and summarized in this chapter. First, the results of the pre- and post-questionnaires are presented and discussed. Then, the student poster presentations are analyzed, and central quotes are extracted and highlighted. Finally, the results from the two focus group interviews (two students out of eight) are presented.

Questionnaire

A total of 44 students participated in the questionnaires, and 38 students gave complete answers (answered both the pre- and post-questionnaire). Eight of the 38 students participated in the energy-body lab. In the result presentation below, the 38 students are split into the group of 8 students that participated in the energy-body lab and the remaining 30 students. The column labeled Lab refers to the eight students participating in the energy-body lab and column Rest are the rest of the students in the multidisciplinary unit. The pre- and post-questionnaires differ, so the labelling of the selected statements between Table 1 and Table 2 differs.

Table 1. Statements selected from questionnaire before the multidisciplinary unit (N=38). Averages and standard deviations for statement responses are shown (AV/SD).

Label	Statement	Lab (n=8)	Rest (n=30)
SI	All people should have knowledge of energy issues	4,0 / 0,8	3,5 / 0,9
SII	Knowledge in biology is not needed to understand energy issues	3,8 / 1,2	3,6 / 0,9
SIII	Knowledge in chemistry is not needed to understand energy issues	4,4 / 1,1	3,7 / 1,0
SIV	I want to learn more about future energy sources	4,4 / 0,5	3,1 / 1,1

The average and standard deviation for the response on each statement in the pre-questionnaire is listed in Table 1, where the response values for the negated statements SII and SIII have been reversed. The eight students from the Lab group scored higher on all statements than the rest of the students. Due to the small sample size of the Lab group and non-normal distributions, no significance test is conducted.

On average, students agree that knowledge of energy issues are important and that they want to learn more about future energy sources. They also recognize that knowledge in biology and chemistry is needed to understand energy issues.

The average and standard deviation for the response on each statement in the post-questionnaire is listed in Table 2. Due to the small sample size of the Lab group and non-normal distributions, no significance test is conducted.

It should be noted that all students (N=38) participating in the multidisciplinary unit have had the energy concept as a part of their regular physics courses (i.e., heat, kinetic and potential energy), therefore energy is usually connected to physics in the minds of many students.

Table 2. Statements selected from questionnaire after the multidisciplinary unit (N=38). Averages and standard deviations for statement responses are shown (AV/SD).

Label	Statement	Lab (n=8)	Rest (n=30)
S1	Mathematics is needed to work on energy issues	4,1 / 0,6	3,7 / 1,0
S2	Knowledge of physics is needed when working with energy issues	4,8 / 0,5	4,2 / 0,8
S3	Knowledge of chemistry is needed when working with energy issues	4,1 / 1,1	4,0 / 0,7
S4	Knowledge in biology is needed when working with energy issues	3,8 / 0,9	3,7 / 1,0
S5	All people should have knowledge of energy issues	4,5 / 0,9	3,6 / 1,0
S6	The multidisciplinary unit has given me a greater understanding of how energy is produced	4,0 / 1,4	3,6 / 1,3
S7	The multidisciplinary unit has given me a greater understanding of the body's energy consumption	3,1 / 1,2	2,6 / 1,1
S8	The multidisciplinary unit has shown me that different school subjects are needed to better understand energy issues	3,1 / 1,6	3,3 / 1,0

The eight student responses to the selected statements from questionnaire after the multidisciplinary unit are shown in Figure 2. The average response for the eight students and the rest of the students are also visualized. The blue shaded columns in Figure 2 refers to the five students (A-E) that constituted Lab group I and the brown shaded bars correspond to the three students (F-H) in Lab group II. Student E did not respond to statement S3. Statements S1 to S4 are all related to if students agree that different subjects (mathematics, physics, chemistry and biology) are needed for the understanding of energy issues. Most students, both in the Lab group and overall, agree that knowledge of different subjects is needed in order to understand energy issues. Most students also agree with the statement that all people should have knowledge of energy issues (S5) and the multidisciplinary unit had given them a greater understanding of how energy is produced (S6).

One of the specific goals of the energy-body lab was to illuminate the use of energy concept to understand the connections between food, oxygen and bodily work. Although, despite the slightly higher average score for the Lab group on statement S7, they did not agree that the multidisciplinary unit had given them a much greater understanding of the body's energy consumption. They also remained neutral to the statement that the multidisciplinary unit had showed them that different school subjects are needed to better understand energy issues (S8).

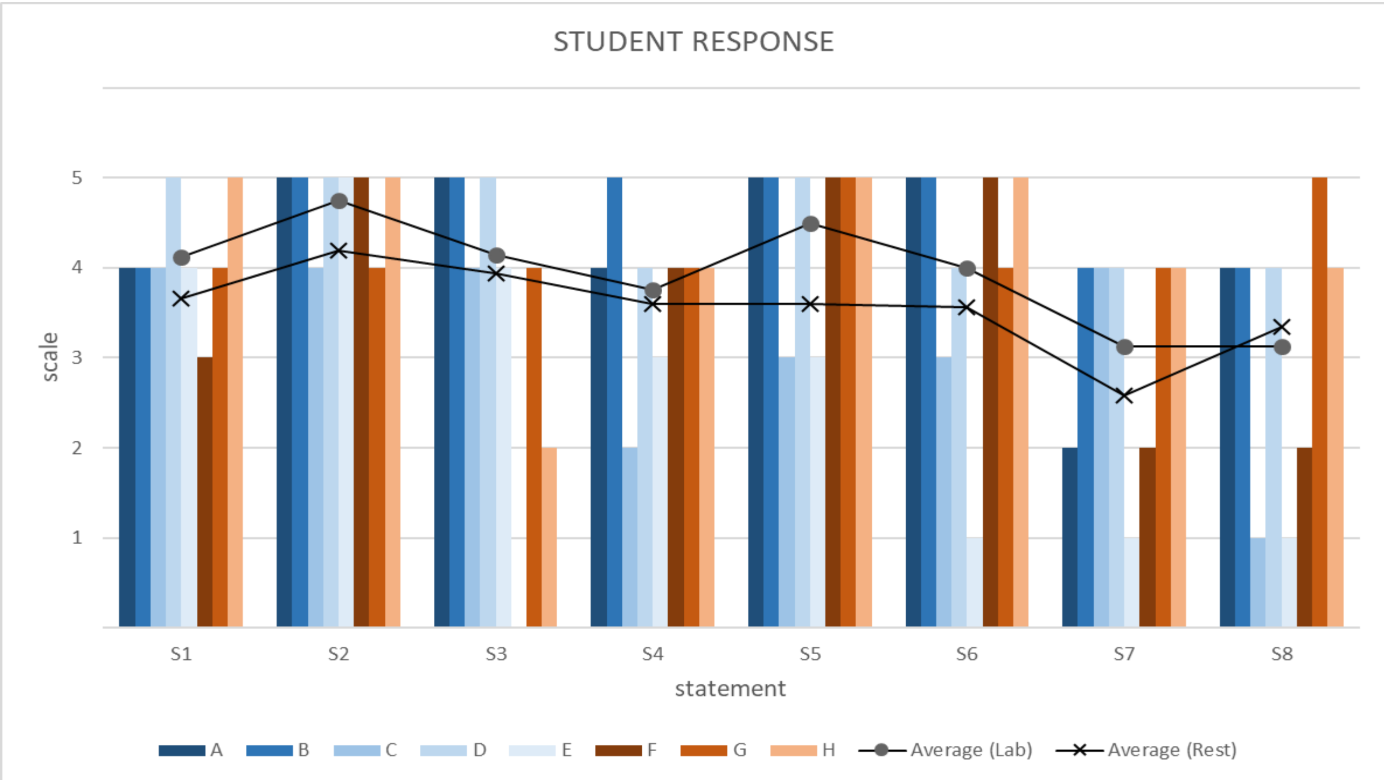

Figure 2. Student response to selected statements in the post-questionnaire.

Poster excerpts

Eight students participated in the energy-body lab. Each of the two lab groups produced a poster as their lab report after the activity (Appendix A). Excerpts directly related to the research question where identified and are reproduced (translated to English) below. The criteria for our selection were that at least two different aspects and uses of the energy concept should be visible in the text. This selection criterion is used as a visible sign of students' understanding of the multidisciplinary character of the energy concept.

> Our leg muscles converted first the chemical energy to kinetic energy. With the kinetic energy we moved up on the bench. (Lab group I, excerpt 1, activity 2)

This excerpt highlights chemical energy conversion by muscles to movement (kinetic energy).

> Then you get the amount of energy that has been 'used' during this period. Some of us used almost as much energy as a light bulb during the lab. So in -other words, we used quite a lot of energy during a short time. (Lab group I, excerpt 2, activity 2)
>
> [...] but then again much closer to the light bulb, as it is not likely that you can do work as the same pace as a kettle, which reaches a relatively high temperature. (Lab group II, excerpt 1, activity 2)

The body's energy conversion is also highlighted in these two excerpts. However, in this case the energy conversion rate is compared to that of inanimate objects (light bulb and a kettle).

> But with 1 kilojoule, you could theoretically lift an apple (ca 100 g) 1000 meters up in the air or two apples to the top of the Eiffel tower. (Lab group I, excerpt 3, activity 2)

This excerpt compares the total energy used by the body in the activity with the potential energy gained by lifting an inanimate object (an apple) to a certain height.

> We measured the body's outer temperature both before and after the workout to see how it changed and also thought about why these changes happened.
>
> Regarding our body temperatures, the most natural explanation was that it went up as we exerted ourselves and sweated. This is the conclusion that the majority probably would draw as you get incredibly warm during hard exercise. (Lab group II, excerpt 2, activity 2)

This excerpt states that a result of bodily work results in an increase of body temperature and sweating; kinetic energy is converted into heat.

> That the oxygen level should go down and the carbon dioxide up was the most likely hypothesis we had before doing the actual experiment and sure enough it did! This because the body during physical exertion needs more oxygen and so the body also gets rid of larger amounts of carbon dioxide than usual. (Lab group II, excerpt 3, activity 3)

This final excerpt highlights the bodily response to physical exertion; the body needs more oxygen to handle increased physical movement, resulting in more carbon dioxide being produced.

Interview excerpts

In this section results from the two focus group interviews are presented. Many things were discussed during the interviews, such as student motivation, working methods, etc. We have only included excerpts directly related to the multidisciplinary aspects of energy. Two of the eight students (student A and H) participated in the interview. Excerpts from the interview connected to the research question are listed below.

> [...] energy is like hugely important in the future [...] like this with non-renewable and renewable energy sources [...] because we will be affected by all oil running out, so we should get like more knowledge about these things. (Student A)

> [...] so energy is not only about physics, not only about chemistry, it is not focused on only one subject, but on them all cooperating. (Student A)

Student A states that a thorough understanding of the energy concept is important for a sustainable future and also that this knowledge is a result of the cooperation of several different subjects.

> [...] and then you could vary. Now it was about these science subjects, especially chemistry, maths and physics. That for instance subjects like history, geography and languages and such got completely left out. [...] Yes, you could have done the history of energy. (Student H)

> Yes but it was also that we worked with energy in physics while we were doing the multidisciplinary unit, so it built upon talking about kinetic energy in the unit, then we started with it in physics and that supported each other so it helped quite a bit. (Fellow student together with student H)

Student H reflects on the lack of other possible subjects in the multidisciplinary unit; in this case the historical aspect of energy development. The student also makes connections to the content of "ordinary" physics lessons and it clearly supported the ambitions of the multidisciplinary unit.

Summary of results

All students were initially relatively positive to the multidisciplinary unit (pre-questionnaire). The group of the eight energy-body lab students were above average on most statements, both in the pre- and post-questionnaire (on 11 out of 12). They were well motivated and saw the relevance of an interdisciplinary approach to better understand energy. The two student groups could freely choose what to document in their poster presentations. In the documentation of activity 2 and 3, the students were able to make connections between different aspects of energy; i.e. living and non-living systems. However, activity 1 proved to be too challenging for both groups. The written documentation on activity 1 contained several errors and the documentation showed a lack of deeper insights. The interview highlighted that the students realized that a thorough understanding of the energy concept needs the cooperation of different subjects.

6 Discussion and conclusions

The question of what exactly a multidisciplinary STEM unit should be in a school context, and what the roles of the individual school subjects should be within that unit, is the natural starting point before planning such a unit (cf. McPhail, 2018). Although, having several subjects, and thus several teachers, cooperate around a theme or a problem, has been advocated by some researchers (e.g. Brand & Triplett, 2012), it can also become very time consuming for the teachers involved (Braskén et al., 2019). It takes time to identify themes and develop criteria for the assessment of those themes, especially if a theme requires a multidisciplinary approach and, as is often the case, no existing teaching material emphasizing this approach is available to support the teachers (for an overview of various interdisciplinary science efforts, see Czerniak & Johnson (2014)).

One approach, and the one chosen by the teachers in our study, is to organize a multidisciplinary STEM unit around a so called crosscutting concept (NRC, 2012). By letting an interdisciplinary, crosscutting concept become the organizing element of a unit, much of the hard work of integrating various subjects into a meaningful whole is eased. However, due to the (by definition) very broad scope of such a concept, one needs to focus and emphasize only on certain aspects of a concept at any given time. There is also a tension in the order that various supporting concepts should be thought. In our case study, the students had encountered the concepts of work, kinetic and potential energy in their physics class, before they participated in the energy-body lab. It could be convincingly argued that trying to introduce several new concepts, at the same time as trying to facilitate students to forge interdisciplinary connections between these concepts, would result in an excessive cognitive load and thus minimal learning (Sweller, 1988). We can partly see this in the students' poster discussions relating to activity 1 (see Appendix A). This activity combined the concepts of force, work, potential energy and kinetic energy. Although the activity and measurements were made without difficulties, the students' poster documentation clearly shows that not even the most able student groups were able to bring all these concepts together to form a meaningful whole. No poster quotes, nor interview excerpts, relating to activity 1 could be identified that shows any signs of a deepened understanding of the multidisciplinary character of the energy concept.

This problematic activity 1 should be contrasted with activities 2 and 3, which contained fever calculations and seemingly fever challenging concepts. The poster quotes connected to these two activities are richer and deeper. In the quotes connected to activity 2, the students discuss and compare the energy conversion rate of a lightbulb with the energy conversion of their own body. The students also make the connection between energy being lost in the conversion and ending up as heat, i.e. an increase in body temperature. Thus, the students are able to use the energy concept to compare physical and biological systems, revealing an understanding of the multidisciplinary character of energy and work.

Based on the poster documentation related to activity 3, this was a more challenging activity than activity 2. Only one of the two groups showed a deeper understanding of the energy conversions involved, from chemical energy stored in the muscles to the carbon dioxide produced and measured, which was one of the main goals of the energy-body lab. Connecting to our research question, this shows the challenges of developing activities that promote a unified understanding of a complex concept, such as energy.

The student posters were all written as part of the energy-body activity, and the worksheets given to the students before the lab contained both scaffolding and leading questions pointing in the desired direction. However, both the post-questionnaire and the student focus group interviews were done several weeks after the energy-body lab. The general, open-ended interview questions served as initial conversation starters and were not intended to probe the energy-body activity or the different aspects of the energy concept. The multidisciplinary character of the energy concept is clearly visible and present in the interview excerpts. But, the students do not spontaneously bring up the connections between energy used to describe living and non-living systems. The results of the pre-questionnaire shows that the eight energy-body lab students were well aware of energy being an important and a multidisciplinary concept (SI-SIII), and that they especially wanted to know more about future energy sources. The post-questionnaire shows that they still agree with the statements that energy is a multidisciplinary concept and that it is important to have knowledge about it (S1-S4). However, their response to statement S7 shows that they did not consider the multidisciplinary unit had given them a greater understanding of the body's energy use. The individual variation in the response to statement S7 is especially large, suggesting that the energy-body lab was perhaps too difficult for some students. The low score on statement S8 also indicates that bringing different subjects together in a multidisciplinary unit, does not automatically enhance the students' understanding of energy.

Designing an integrated STEM activity can be a challenging task. One way to reduce the challenge is to build the activity around a crosscutting concept, which in our case study was energy. Due to the broadness of a crosscutting concept, it is necessary to focus on a few well-defined aspects of the concept, which in our case was to relate the use of energy to describe changes in both living and non-living systems. It is very important to be clear on which concepts are assumed familiar to the students and which concepts are to be learned and deepened in the activity. It is not realistic to assume that individual teachers nor schools can (or have the time to) develop high quality, integrated STEM material, without professional support. Therefore, one way forward is to systematically develop, test and make available research-based material for interesting and challenging STEM activities for different grades.

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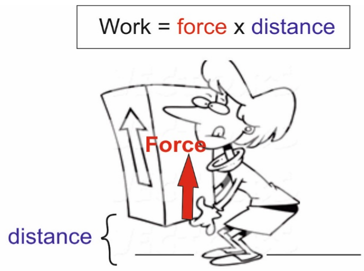
Appendix A: ENERGY-LAB WORKSHEET

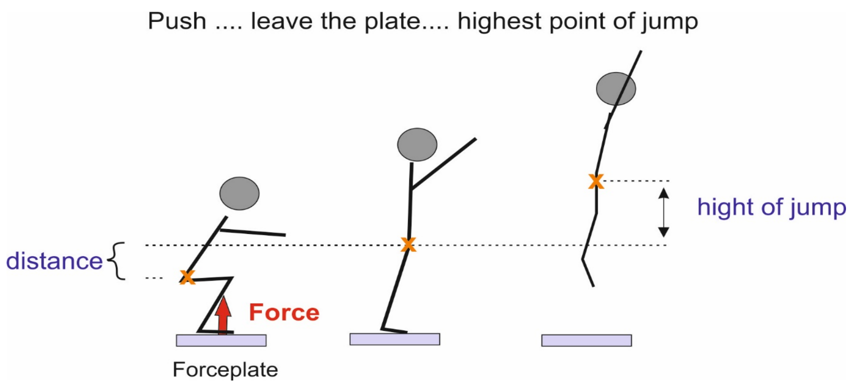
Activity 1. JUMP! HOW MUCH DO YOUR LEG MUSCLES HAVE TO WORK?

When you lift a heavy box, you can calculate the work you do by measuring how high you lift the box (= the distance) and how much force you use. As you climb the stairs, your leg muscles must work to lift your entire body upwards, which is why climbing stairs is so heavy. Your leg muscles also need to work when you jump with both feet together. You investigate these things in this lab.



Lab

In this lab, you will investigate how much force the leg muscles develop when you jump straight up with both feet together. You have a force plate to measure the force of take-off and a measuring tape to measure the height of the jump. Feel free to video record the entire jump with the measuring tape in the background. To calculate the work done by the leg muscles you have to measure the distance from the start of the jump (bent knees) to the point when the legs are completely straight (see picture below).



Analyze your measurement results and video clip and write down the results in the table below.

Maximum force during take-off (Newton)	Average force during take-off (Newton)	The distance from bent to straight legs * (meter)	The height of the jump (meter)

** To measure the distance and the height of the jump, you must follow the same point on the body. The easiest way is to follow how the hip joint moves, for example by attaching a clearly visible piece of tape to the hip*

Questions and calculations

How much work do your leg muscles do? (The unit for work is joule)

____

The work, calculated above, that your leg muscles do is used to give your body a certain initial speed. The height of the jump depends on the magnitude of the initial speed. Calculate the height (h) of jump when you know the work and the mass (m) of your body *

Calculated height of jump = _______________ m

Measured height of jump = _______________ m

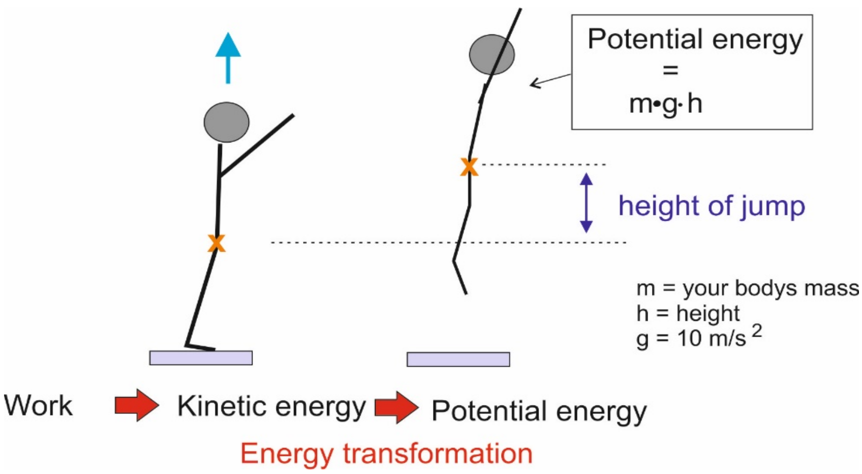
Did the calculated height agree with the measured height? If not, what could be the reason/cause?

____

____

Think about what you can do to jump higher. Suggestions:

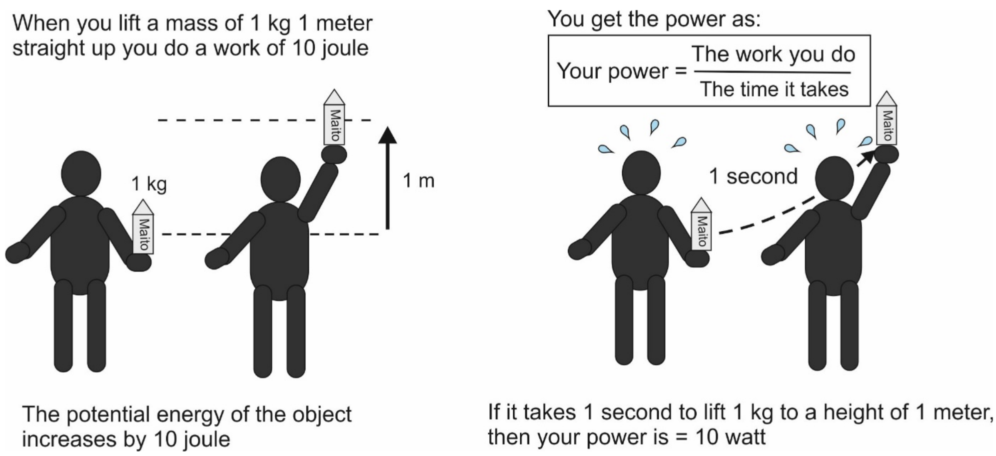
____



** The law of conservation of energy states that the work done by your leg muscles = the potential energy of your body when it is at its highest point (see picture above).*

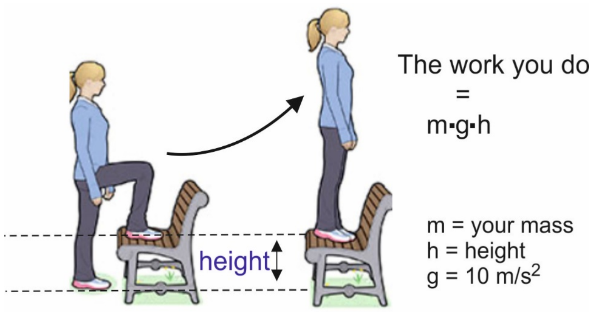
Activity 2. HOW EFFICIENT ARE YOU?

Doing a job requires energy. To compare different types of work, it is not enough just to look at the amount of energy required. We must also need to look at how long it takes to do the work. You already know this because you know that there is a big difference between working fast (high power) or working slowly (low power).



Lab

In this lab you will investigate the power your body develops when you as quickly as possible, climb/step up and down on a bench. For your help you have a tape measure and a watch. Measure how high your body is lifted when you climb/step up on the bench. Decide how many times you should climb/step up and down the bench (at least 30). Measure the temperature of your skin (e.g. at the neck) before and after the workout. Measure the time it takes to complete the workout and record your results in the table below.



Height of step (meter)	Number of steps	Time (seconds)	Total work (joule)

Skin temperature before = ________________ ^{0} C

Skin temperature after = ________________ ^{0} C

Questions and calculations

My power was = ______________________ watt

What is a typical (electrical) power of a light bulb and a kettle?

____

How large is your measured power compared to the power of these electrical devices?

____

Does the temperature of your skin change with effort? In what way?

____

The energy you used to do the work you just did comes from the chemical energy of the glycogen stored in your muscles *. Can you use all the chemical energy stored in the glycogen to do work? If not, where does the rest of the chemical energy end up?

____

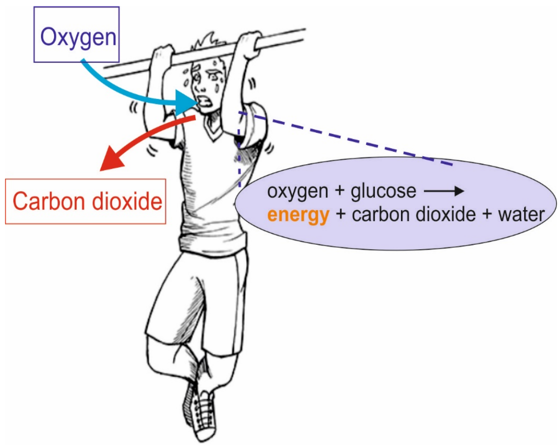
____

____

** To quickly increase the amount of glycogen you can e.g. eat carbohydrates (= sugar)*

Activity 3. FROM WHERE DO YOU GET YOUR ENERGY?

Energy is needed to do work and to keep you alive. You get the energy from the food you eat (carbohydrates, proteins and fat). The chemical energy from the food can be stored as glycogen (fast energy) or as fat (slower). The process that energizes your muscles to do their job is called cell respiration and that is the process of the oxygen in the air you breathe in + glucose (sugar) that turns into energy, with carbon dioxide and water as residual products (see picture to the right).



Lab

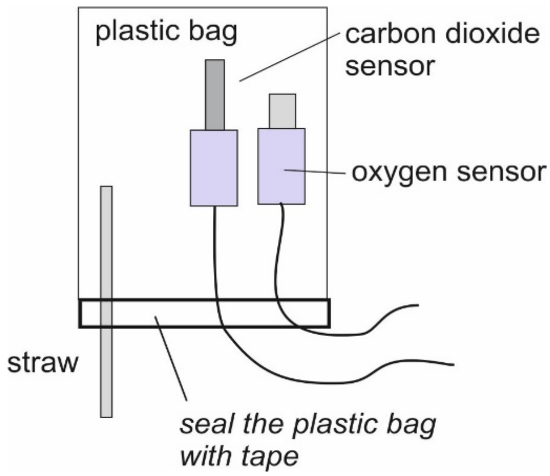
1. Start by measuring how much oxygen and carbon dioxide there is in the classroom.

Oxygen (O2) _______%

Carbon dioxide (CO2)__________ ppm *

** The proportion of carbon dioxide is so small in the atmosphere that it is stated in the unit ppm (parts per million ie millions of parts). The relationship is 1 ppm = 0.001 promille = 0.000001 percent.*

a) Insert the carbon dioxide and oxygen meter into a 2 liter sealable plastic bag. Also, insert a straw into the plastic bag so you can inflate the bag using your exhaled air (see picture). Exhale through the straw to fill the plastic bag. Measure and record the amount of oxygen and carbon dioxide in your exhaled air.



b) Make a new plastic bag similar to the first one. Increase your heart rate by jumping, running, ... for at least 5 minutes. Again measure the amount of oxygen and carbon dioxide in your exhaled air.

	Amount of oxygen in exhaled air (%)	**Amount of carbon dioxide in exhaled air (%)**
Before exertion		
After exertion		

Questions

Was there any difference in the amount of oxygen and carbon dioxide in the exhaled air before and after exertion?

____

Can you, with the help of how the body convert the chemical energy of glucose into energy, explain the results you obtained?

____

___________________________________ ___________________________________

____

Do you think there will be any difference in the results if you have good or bad fitness/condition? In what way, if so?

____

____

____

A systematic review on teaching fraction for understanding through representation on Web of Science database using PRISMA

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Through a search executed on Web of Science database with general keywords pertaining to 'teaching fractions' or 'understanding fractions' and 'representation', this study utilized PRISMA's procedure in analysing previously published articles. This review reveals seven articles in inclusion criteria and seventeenth articles in exclusion criteria with reasons. The included articles were reviewed for (a) studies characteristics, (b) instructional focus, (c) representation elements: real-world situation, manipulative aids, pictures, spoken and written symbols, and (d) the outcomes of each study. The metadata was analysed to organise the outcomes. Most of these articles focus on grade 3 and above and Western countries' urban area. The result indicates most studies emphasize both conceptual and procedural understandings. Multi representations utilize sequential or parallel concept related to fractions improve students' knowledge, particularly in understanding fractions. Meanwhile, developing fraction learning through multiple explicit representations at the initial Grade level of fraction instruction is for elementary school. However, less attention has been given to explicit representations in learning fractions at such a level.

Keywords: teaching fractions, representation, review, elementary school, PRISMA

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1 Introduction

Understanding fractions has been a global issue until today due to its importance as a foundation for mathematical knowledge and skills (National Council of Teachers of Mathematics [NCTM], 2007). Fractions' understanding supports the knowledge of advanced concepts and procedures at higher levels (Aliustaoğlu, Tuna, & Biber, 2018; Bailey et al., 2015; Siegler, Thompson, & Schneider, 2011). Notably, the most recent challenges in understanding fractions are closely related to cognitive bias (Hacker, Kiuhara, & Levin, 2019; Krowka & Fuchs, 2017; Liu, 2017). Provided that failure of some students in understanding the concepts and procedures of fractions indicates diverse cognitive abilities between the students, it becomes the factor for students' difficulties in solving problems, especially those involving fraction magnitudes. As a result, they fail to differentiate between the whole number and ratio representations (Hoch et al., 2018) during problem-solving.

As highlighted by Behr, Lesh, Post, and Silver (1983) almost 40 years ago, strong fraction knowledge is vital for algebraic understanding (Booth, Newton, & Twiss-Garrity, 2014). This argument remains relevant today, attracting current researchers' attention (Braithwaite et al., 2019; Fitzsimmons, Thompson & Sidney, 2020), mostly when fractions frequently misunderstood with whole numbers (Siegler et al., 2011). For example, students mistakenly believe that $\frac{1}{8}$ is large and $\frac{1}{2}$ is small (Hamdan & Gunderson, 2017). Therefore, effective strategies are essential to enhance their foundation on fractions (Siegler et al., 2011) that emphasize symbol for fraction means a single number and not two different whole numbers (Reinhold, Hoch, Werner, Richter-Gebert, & Reiss, 2020). For example, the following list of numbers: 2, $\frac{2}{7}$ means there are two numbers whereby students frequently assume that as three numbers.

Teaching fractions at early Grade level begins with part-whole and measurement interpretations. The part-whole is a combination of two single words; part and whole. The whole refers to all equal parts of a single object; for example, a cake has six equal parts) or all subsets of a set of objects; for example, three cakes with the same size and shape); whereby, the part refers to one or more than one equal parts. Therefore, the part-whole is defined as one or more equal parts of a single object; for example, two of six equal parts of a cake, or a set of a group of objects; for example, one of three cakes. Typically, understanding the part-whole is represented using an area model whereby one or more parts of a 2-dimensional shape is shaded, coloured or pasted with small pieces of paper to distinguish the part/ parts from other parts. This area model strategy is commonly used by most teachers from Western or Eastern countries to interpret part-whole through the story of sharing (Fuchs et al., 2016).

Various representation methods executed in previous studies (Flores et al., 2018; Simon et al., 2018) were conducted in the United States which include materials manipulation (e.g., fraction block, fractions disc, fractions cards, sheets of paper or virtual task), real-life context (e.g., problem situation, problem scenario), visual representation using pictures of various dimensional objects (e.g., one-dimensional number line, two-dimensional circle and rectangle, and three-dimensional cylinder, cube, cuboid, and sphere), verbalization of written form whether using words (e.g., one over two, half, quarter), numbers (e.g., 0, 1, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$), or even spoken (e.g., group discussions, read aloud word problems).

The studies involve various representations not restricted to the topic of fractions. For example, Tajudin and Chinnappan (2016) emphasized the relationship between real-world problems, manipulative, pictures and symbolic representations with higher-order tasks. Whereas, Supandi, Waluya, Rochmad, Suyitno, and Dewi (2018) highlighted the connection between spoken, written symbols and visual representations for different topics. On the other hand, a large-scale study conducted by Van Steenbrugge, Remillard, Verschaffel, Valcke, and Desoete (2015) in Flanders for 342 Grade 4 students focuses on the connection between the representation for fractions topic. Meanwhile, Flores, Hinton, and Taylor (2018) in the United States studied 17 Grade 3 to Grade 5 students using Concrete-Representational-Abstract (CRA), in which a synchronised use of different approaches was considered to improve students' understanding on fractions. Additionally, the students were provided with the opportunity to utilize manipulated object at a concrete level and subsequently at representation level by drawing a picture of previously used object to solve numerical or word problems (Flores et al., 2018). However, inadequate emphasis was placed on explicit representation of fractions in elementary school. Therefore, this study focuses on analysing multiple fraction learning representations among elementary students.

The purpose of this review article is to extend previously reviewed recommendations (Roesslein & Codding, 2019) and specifies how representation elements originated by Behr et al. (1983) and Zhu and Fan (2007) are interpreted and defined for elementary school level. The research questions are:

1. What are the study characteristics adapted in each study?
2. What is the instructional focus of fractions adapted in each study?
3. What are the representation elements of fractions adapted in each study?
4. What are the outcomes of each study?

2 Theoretical framework

Classical perspective of various representations originated by Bruner's theory begins with enactive (manipulative skills) followed by iconic (visual representations); then, to symbolic (using mathematical formulas) learning. These three levels are also known as discovery learning. Firstly, the enactive level is a concrete operation level; whereby, students learn by touching, feeling and manipulative skills. Secondly, the iconic level is the visualization stage; whereby, students develop the ability to formulate and explain concrete situations. On the other hand, symbolic or abstract level allows students to organize information in mind and relate the concepts together (Bruner, 1971).

Later, Lesh (1979) expanded Bruner's (1971) hierarchical idea; whereby iconic mode corresponds with manipulative aids and picture, whereas symbolic mode involves spoken and written symbols. Manipulative aids, picture, spoken and written symbols are important elements to explain real-life situations. Different elements may be used in several different ways depend on the situations. These elements support students' meaningful transformation from concrete learning operation into abstract level.

It is impossible to determine manipulative material is appropriate for all types of students in all situations since they have different intelligence and abilities (Behr et al., 1983). Therefore, Behr et al. (1983) proposed interactive representational system model. The model does not only represent all element representations such as manipulative aids, picture, spoken, written symbols and real-world situations, but also emphasizes interactions between elements. Through this model, mathematical problems can be solved in several ways; (a) translate real situations into multiple representations, (b) change or control representational systems to make decisions or predictions, (c) redirect decisions to real situations (Behr et al., 1983).

Later, Miller and Hudson (2007) renamed Bruner's idea on enactive, iconic and symbolic representation to concrete, representational and abstract (CRA) instructional sequence utilised by recent studies (Flores et al., 2018; Hwang et. al., 2018). Zhu and Fan (2007) introduced problem representation in pure mathematics (symbolic number), written, verbal and visual representations. However, they did not mention about manipulative aids; even when they discussed real-life situations. Therefore, both Zhu and Fan's (2007) and Behr et al.'s (1983) ideas were synthesized in current study to form framework for analysing fraction representations.

3 Methodology

This section elaborates five main sub-sections such as PRISMA, resources, systematic review process for article selection, data abstraction, analysis employed in current study and coding procedures.

3.1 Preferred Reporting Items for systematic reviews and Meta-Analyses (PRISMA)

Preferred Reporting Items for systematic reviews and Meta-Analyses (PRISMA) was utilised to identify PRISMA's characteristics and its utilization. There are four hierarchical phases (Liberati et al., 2009), and they are organized in flow diagram as presented in Figure 1.

3.2 Resources

Most common database, Web of Science (SCI) was utilized for it provides wide coverage of published articles in Science and Social Sciences fields. SCI is published by Clarivate Analytics and has indexed over 8700 journals (Burnham, 2006).

3.3 Systematic review process for the articles' selection

The first phase is identification process of which the authors enrich main keywords using several steps so that articles from the database could be retrieved as many as possible. Using Web of Science formatting in April 2020, the following search strings were generated:

TS= (("teaching fractions" OR "teaching rational number" OR "learning fractions" OR "learning rational number" OR "fraction instruction" OR "fraction intervention" OR "understand* fraction" OR "fraction knowledge" OR "fraction pedagogy" OR "fraction abilit*" OR "fraction skill") AND ("representation" OR "modelling" OR "manipulative" OR "real life" OR "picture" OR "symbol"))

The second phase is screening. At this phase, articles were included or excluded based on criteria agreed by the authors before generating the articles using the database. After the articles were generated, eleven non-related articles were identified and then removed. Hence, only thirteen articles were attained for review. Further screening was conducted, and five more articles were removed as they did not match the criteria for the included articles. Eventually, eight articles with eligible articles were included in the review.

The third phase is eligibility, the process where the authors thoroughly examined those eight articles by reading the titles, abstract, result and discussion to ensure they met the inclusion criteria; thus, serve current research objectives. It was unanimously agreed that one article needed to be rejected as the article focuses on textbooks analysis. As a result, only seven articles were deemed suitable to further data abstraction and analysis. Figure 1 shows a flow diagram of identification, screening, eligibility and included criteria as suggested in Shaffril, Abu Samah, Samsuddin, and Ali (2019).

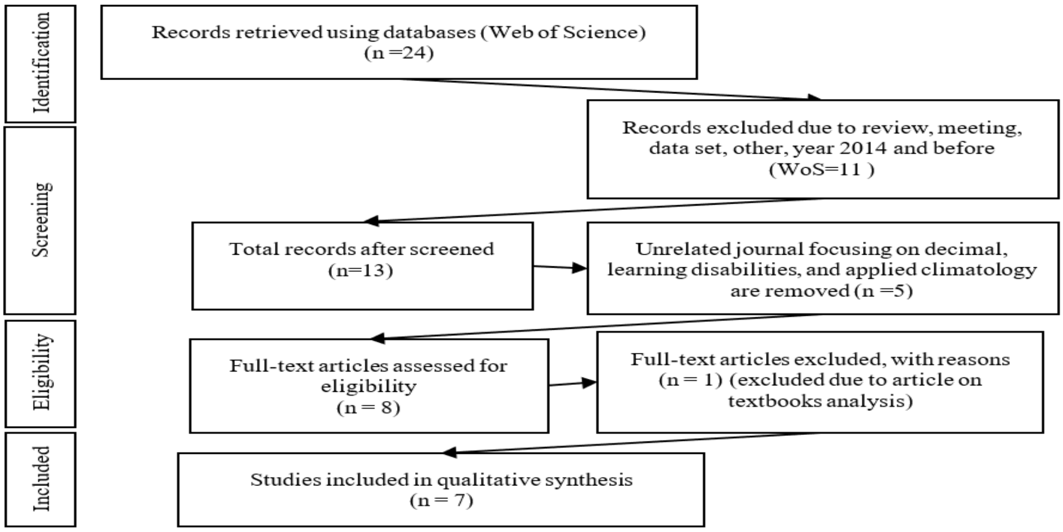

Figure 1. Flow Diagram of identification, screening, eligibility and included criteria.

3.4 Data abstraction and analysis

Thematic analysis was used to seek suitable themes and categories through several stages. First, the authors analysed seven selected articles on its abstract, methodology, results and discussions in order to extract important ideas in answering the research questions. Next, possible codes were listed according to study characteristics, instructional focus, representation elements and the outcomes of each study.

Then, five categories of study characteristics were identified such as participants, grade, ethnicity, urbanicity and location. Meanwhile, two categories of instructional focus were distinguished, namely procedural knowledge and conceptual knowledge. Five more categories of fraction representations were found and those were real-world situations, manipulative aids, pictures, spoken and written symbols. Study outcome categories were coded as concrete, representational and abstract (CRA), representational and abstract (RA) and concrete, representational (CR) (Table 5).

Model representational system by Behr et al. (1983) is referred to develop themes and categories for representation of fractions (inductive process). Under the category of written symbols, the author developed two sub-categories, namely words and numbers. The intention is to specify how representation elements originated by Behr et al. (1983) were interpreted and defined in this review article. Therefore, the operational definitions for each category of instructional focus and representations elements are listed in Table 2.

The authors closely read the articles' full text to identify the codes for study characteristics, instructional focus, representation elements and the outcomes for each study. Any arguments regarding the suitability of coding for each category is finalized with the second and third author as they are experts and senior lecturers in Mathematics and Science Education. Appropriate terms used for each category and coding arrangement in the respective categories were also established. Those categories and codes are highlighted in Table 2 to Table 5. Metadata analysis of the instructional focus and representation elements were re-coded dichotomously, 0 and 1 (Chan, Leu & Chen, 2007; Ni, 2000) whereby 1 stands for appear and 0 for does not appear in the article (Table 5).

3.5 Coding procedures

Characteristics of the studies

Characteristics of the studies were coded into five categories including participants, grade, ethnicity, urbanicity and location. For certain studies of which the characteristics are ambiguous or not clearly presented, dash symbol '-' was utilized.

Participants and Grade.

A total of 1606 students participated across those seven studies involving elementary students with different abilities from Grade 1 until Grade 6. Five studies utilized multi-Grade students, (Begolli, Booth, Holmes, & Newcombe, 2020; Degrande, Van Hoof, Verschaffel, & Van Dooren, 2017; Flores et al., 2018; Hamdan & Gunderson, 2017; Resnick et al., 2016) and two studies included single Grade students (Kaminski & Sloutsky, 2020; Liu, 2017). One article employed longitudinally study on mathematical development; whereby students' progress were monitored through Grade 3 to Grade 6 (Resnick et al., 2016).

Ethnicity, urbanicity and location.

Four studies encompassed of ethnically or racially diverse student samples (Begolli et al., 2020; Flores et al., 2018; Hamdan & Gunderson, 2017; Resnick et al., 2016) and two studies applied ethnically or racially homogeneous student samples (Kaminski &

Sloutsky, 2020; Liu, 2017). Study from Degrande et al. (2017) provides no explicit information about ethnically or racially student samples used. In addition, five studies provide information about school setting urbanicity; whereby suburban (n=1) (Kaminski & Sloutsky, 2020), urban (n=3) (Begolli et al., 2020; Liu, 2017; Hamdan & Gunderson, 2017) and rural (n=1) (Flores et al., 2018). Five studies took place in United States and one study was conducted in Belgium and China, respectively. Generally, the studies were carried out mostly in the Western countries. Table 1 presents the characteristics of the studies.

Table 1. Characteristics of the studies

Study	Participants	Grade	Ethnicity	Urbanicity	Location
Kaminski & Sloutsky (2020)	413 teachers 29 (students)	1st	White	suburban	United States
Begolli, Booth, Holmes, & Newcombe (2020)	565 students	1st -6th	62% White, 28% Black, 5% Hispanic, 3% Asian, and 2% multiracial	urban	United States
Flores, Hinton & Taylor (2018)	17 (7F, 10M) students LA (score ≤65%)	3-5th	6 white, 9 African American, 2 Latino	rural	United States
Liu (2017)	1 teacher 75 students	4th	Chinese	urban	China
Degrande, Hoof, Verschaffel, & Van Dooren (2017)	279 students	5th and 6th	-	-	Belgium (Flanders)
Hamdan & Gunderson (2017)	114 students	2nd and 3rd	67% African American, 23.9% Caucasian, 4.7% Asian, 1.9% Hispanic, 2.8% other or multiple races	urban	United States
Resnick, Jordan, Hansen, Rajan, Rodrigues, Siegler & Fuchs (2016)	517 students	3rd, 4th, 5th, 6th	51.9% White, 40.0% Black, 5.7% Asian/Pacific Island, and 2.5% American Indian/Alaskan Native; 17.7% of children identified their ethnicity as Hispanic.	-	United States

Instructional focus

For instructional focus, studies were coded into two categories such as procedural knowledge and conceptual knowledge. The followings are operational definitions for conceptual and procedural knowledge according to Anderson et al. (2001, p. 46), Misquitta (2011) and Roesslein and Codding (2019):

1. Conceptual focus involves the effort of making inter-relationships (Anderson, et al., 2001, p. 46) by comparing fractions on number line and utilizing skills to answer word problems (Misquitta, 2011; Roesslein & Codding, 2019).
2. Procedural focus encompasses strategy to aid problem solving (Anderson, et al., 2001, p. 46) by employing algorithms (Misquitta, 2011) computation (Roesslein & Codding, 2019), counting, writing, reading, drawing, shading, matching or labelling.

Representation elements

The representation elements were coded according to five categories namely real-world situation, manipulative aids, pictures, spoken and written symbols. Meanwhile, written symbols were also coded into subcategories which are words and numbers. These categories are based on Behr et al. (1983) model of interactive representational system. The definition for each category was applied based on various literature as follows:

1. Real-world situation indicates that contextual problems allow the application of real-world context in mathematical tasks, assessments or activities to represent the problems (Masingila & Moellwald, 1993); and can be explained in terms of educational, personal, occupational or public domain (Council of Europe., 2011, p. 48).
2. Manipulative aids indicate that concrete object can be manipulated or utilized without any manipulation (Istiandaru, Istihapsari, Prahmana, Setyawan, Hendroanto, 2017) to demonstrate the procedure to solve problem (e.g., sheets, task paper, fractions cards, blocks) (Zhu & Fan, 2007) in terms of educational, personal, occupational or public domain (Council of Europe., 2011, p. 48).
3. Pictures indicate that the application of monochromatic or colourful visual representation can also be applied such as modelling or diagram based on the information given in the problem (e.g., circle, rectangle, square, triangle and number line) (Zhu & Fan, 2007); and are connected to fraction interpretations of part-whole, measurement and part of a set.
4. Spoken indicates that teacher's, students', experimenter's or researcher's reading, speaking, talking or utterance which involves thinking aloud, reading

problem situation, teaching or discussing for problem comprehension or solving (Roesslein & Codding, 2019)

5. Written symbols are words that suggest students to use words representing quantities or problem situation (e.g., one over two, one part and two parts) (Zhu & Fan, 2007; Pimm, 1995). Number specifies that students use numbers to represent quantities in the problem (e.g., 0.5, 0, 1 and $\frac{1}{7}$) (Zhu & Fan, 2007; Pimm, 1995).

Table 2 summarises operational definitions of instructional focus and representation elements.

Table 2. Operational definitions of instructional focus and representation elements.

Procedural	Conceptual	Real-world situation	Manipulative aids	Pictures	Spoken	Written symbols (words)
Make Cut Fold Explore pattern Label Shade Glue Follow Make Name Writing Draw Matching Use arrow keys Slide cursor Press key Fill in empty space, etc.	Solving problems Compare fraction	chocolate bars, cookies, egg cartons, kite, fractions blocks, coins, cake, Snake picture, etc.	Constructivism manipulative Virtual manipulative	Part whole (area/ length model) Part of a set Number line	Fraction instructions (researcher) Read aloud (researcher/ students)	Word problems/ questions Verbalize in written form Crossword puzzle Written symbols (words) Fraction Whole number Decimals Operations Percentage

Outcomes of the study

The outcomes or result of the study was coded according to three categories, namely concrete, representational and abstract (CRA), representational and abstract (RA) and concrete and representational (CR). The definition for each category was applied as follows:

1. Concrete, representational and abstract (CRA) acts as an indicator for narrative language as it links verbal and non-verbal representations (Mergenthaler & Bucci, 1999). The sequence begins with physical objects; then continues with math symbols are transformed into pictorial representations (Fyfe & Nathan, 2018)
2. Representational and abstract (RA) involves merely representational and abstract (RA) strategy (Flores et al., 2018) which is commonly compared to CRA in previous study (Butler, Miller, Crehan, Babbitt, and Pierce, 2003). Butleret al., (2003) study indicates that both RA and CRA intervention groups made significant progress.
3. Concrete and representational (CR) involves only concrete and representational (CR) methods which is similar to concreteness fading when concrete transforms to representation in Ching and Wu (2019).

4 Result

4.1 Instructional focus

All except one study (i.e., Hamdan & Gunderson, 2017) emphasized on both conceptual and procedural knowledge. Conceptual knowledge was identified in the studies as students were required to compare fractions, find fractions magnitude using the number line, respond to fractions label correctly, use reasoning and answer word problems. On the other hand, procedural knowledge in these studies focused on the strategies of labelling, making, shading, arranging, writing, drawing, matching, naming, and comparing as the aids for solving problems. The procedures of computation and fractions operations were emphasized in Flores et al. (2018)'s study.

4.2 Representations elements

Real-world situation

In all except two studies (i.e., Hamdan & Gunderson, 2017; Resnick et al., 2016), real-life scenarios were utilized to present the problems. For the educational domain, pencil scenario was utilized to determine fractions (Kaminski & Sloutsky, 2020). For the public domain, eating chocolate (Begolli et al., 2020) and two snakes story with different lengths (Degrande et al., 2017) were employed. Whilst, personal domain involves sharing a cake with a friend (Liu, 2017); the occupational domain includes making cupcakes (Flores et al., 2018).

Manipulative aids

Out of seven studies utilizing manipulative aids, five studies applied multiple domains such as educational domain (e.g., paper, fraction blocks, pre-cut pieces of paper, jelly beans), occupational domain (e.g., technology tools), or public domain (e.g., kite, coins, cake, chocolate bar, egg carton, cookies) (Flores et al., 2018; Kaminski & Sloutsky, 2020; Liu, 2017; Hamdan & Gunderson, 2017; Resnick et al., 2016). Other two studies employed a single domain particularly educational domain (e.g., task, assessment, pencil and paper) (Begolli et al., 2020; Degrande et al., 2017).

Pictures

Meanwhile, five studies applied number line representations (i.e., Begolli et al., 2020; Flores et al., 2018; Hamdan & Gunderson, 2017; Liu, 2017; Resnick et al., 2016). In those studies, at least two picture representations were utilized consisting monochromatic pictures (e.g., circle, chain shape circle, square, linear bar diagram and snake) or colourful pictures (e.g., paper cut of pizza crusts, sauce, cheese and toppings), part of a set representations and a picture of number line or linear bar diagram which employed measurement representations. Both measurement and part of set representations were used in four studies (Begolli et al., 2020; Degrande et al., 2017; Flores et al., 2018; Resnick et al., 2016). Whereas, for part-whole representations, pictures of pizza, circle, chain shape circle and square were utilized.

Spoken

A study by Degrande et al. (2017) did not explicitly mention the utilization of spoken verbalization either from the students, teacher or researchers. Nevertheless, spoken words appear in the article since a one-to-one interview session was conducted between the researcher and the students to identify students' verbalization and reasoning. One study required the teacher to give limited fraction instructions (Liu, 2017). Meanwhile, four studies highlighted that the researchers verbalized the experiments in spoken words (Begolli et al., 2020; Flores et al., 2018; Hamdan & Gunderson, 2017; Kaminski & Sloutsky, 2020). Therefore, five studies employed students verbalization in the spoken form such as reading aloud the words, reading the instructions or repeating the information and counting together with the researchers (Begolli et al., 2020; Flores et al., 2018; Hamdan & Gunderson, 2017; Liu, 2017; Resnick et al., 2016).

Written symbols

First is the words. Five studies applied word problems for students' intervention (Begolli et al., 2020; Degrande et al., 2017; Flores et al., 2018; Kaminski & Sloutsky, 2020, Liu, 2017). However, all studies required students to use written symbols for words in problem-solving questions, verbalize the words in written form, perform crossword puzzle: match the words provided and list written words.

Second is the number. All seven studies used symbolic numbers such as fractions, decimals, percentages and whole numbers. Two studies specifically utilized a single form of symbolic numbers such as fractions (Kaminski & Sloutsky, 2020) or whole numbers (Degrande et al., 2017). While, the other five studies employed at least two representations of symbolic numbers consisting of whole numbers, fractions, decimals or percentages. It was also noted that four studies associated whole numbers and fractions using number lines especially to assess students' magnitude understanding (Begolli et al., 2020; Flores et al., 2018; Hamdan & Gunderson, 2017; Liu, 2017). Additionally, two studies involved fraction operation such as multiplication, division and addition of fractions in the studies (Flores et al., 2018; Resnick et al., 2016). The instructional focus and representations elements are highlighted in Table 4 and Table 5.

4.3 Outcomes of each study

Concrete, representational and abstract (CRA)

Two studies employed CRA method (Flores et al., 2018; Degrande et al., 2017). First, Flores et al., (2018) utilized different materials for each level of concrete-representational-abstract (CRA) instruction. It was discovered that the graduated sequence of CRA is an effective strategy for developing students' conceptual understanding. Second, Degrande et al., (2017) identified children's preference reasoning strategy that is additive or multiplicative by asking which of two snakes shown had grown the most. The children were requested to verbally explain their reasoning twice; first, after picture presentation of two snakes with different lengths for the first time and later, the presentation of the same picture after a certain period. In comparing this continuous type, children's answers much explicitly verbalized the discrete items (Degrande et al., 2017).

Representational and abstract (RA)

Using pretest-training-posttest design, Hamdan and Gunderson (2017) examined children's fraction learning in three ways: non-numerical control, the number line training and the area model training.

The results indicate that only number line training led children to correctly answer tasks related to magnitude understanding. Even at the initial training stage, the number line group improved at representing fractions with a number line and area model group improved at representing fractions with area models (Hamdan and Gunderson, 2017).

Meanwhile, Liu (2017) examined two different types of fractions instruction, namely limited and primary formal. Whilst, limited instruction class taught about $\frac{1}{2}$, name fraction, concrete and real-life situation and pictorial visual aids; primary instruction class introduced the concepts, definitions and meanings and fractions comparisons using symbolic number. The result indicates fraction representation among children with limited fraction instruction, was linear and understanding of fraction magnitude was related to both whole number knowledge and approximate number system (ANS).

The third is a longitudinal study by Resnick et al. (2016), who assessed the development of fraction number line estimation between 4^{th} and 6^{th} grades, identified number line estimation is essential to mathematical development (Resnick et al., 2016). Therefore, Hamdan and Gunderson (2017), Liu (2017) and Resnick et al., (2016) contributed to representational and abstract representation.

Concrete and representational (CR)

Two studies explored concrete and representational (CR) strategy (Begolli et al., 2020; Kaminski & Sloutski, 2020). Begolli et al. (2020) indicated that discretized formats were more challenging than the continuous ones; whereas discrete formats were harder. Their intervention was executed by assigning children with either continuous, discretized, or discrete spatial representations tasks. Whilst, an example of continuous format is a picture of liquid in a beaker; an example of discretized format is a picture of a beaker with unit marking (Begolli et al., 2020).

On the other hand, Kaminski and Sloutski (2020) examined two groups of students; the first is contextualized-then-generic group and the second is generic-then-contextualized group. The result indicates that the initial instruction should begin with a simple, generic and pre-made material followed by colourful and contextualized representations including those made by the students (Kaminski & Sloutsky, 2020). Even though, Kaminski and Sloutsky's (2020) study identified the representational method first followed by concrete methods (RC), it was considered as utilizing CR method. The outcomes of the studies are highlighted in Table 3.

Table 3. The outcomes of the study.

Multiple representations				
Lesh (1979)	Flores et al. (2018), Hwang et. al. (2018), Miller and Hudson (2007)	CRA	RA	CR
Real world	Concrete	Graduated sequence of CRA is effective for developing students' conceptual understanding (Flores, Hinton & Taylor, 2018) Children's answers were more often explicitly verbalized in discrete than continuous items (Degrande, Van Hoof, Verschaffel, & Van Dooren, 2017)	In children with limited fraction instruction, fraction representation was linear and fraction magnitude understanding was concurrently related to both approximate number system (ANS) and whole number knowledge (Liu, 2017) Number line (NL) training led to correct answer in magnitude task which implies NL is important for children magnitude understanding (Hamdan & Gunderson, 2017) Fraction magnitude understanding through number line estimation is found to be central to mathematical development (Resnick et al., 2016)	Initial instruction should begin with a simple, generic and pre-made material followed by colourful and contextualized representations including those made by the students (Kaminski & Sloutsky, 2020) Discretized formats were more challenging than the continuous ones; as predicted, discrete formats were harder (Begolli, Booth, Holmes, & Newcombe, 2020)
Manipulative aids and picture	Representational			
Spoken and written symbols	Abstract			

Table 4. The instructional focus and representation elements

Study	Year	Instructional focus		Real world situation	Manipulative aids	Pictures	Spoken	Written symbols	
		Procedural	Conceptual					words	number
Kaminski & Sloutsky, 2020	2020	Make Cut Fold Explore pattern Label Shade Glue	Solve word problems	Real-world situation: two chocolate bars, sets of cookies, empty egg cartons, kite	Make a kite of different coloured sections, make a number line. Cut strips of paper. Fold the strips into different proportions. Explore pattern blocks. Make pizza out of paper, make paper quilts by colouring (or pasting) equal-sized geometric. Parts of a square. Egg cartons, chocolate bar and cookies. Label the proportion of pizza remaining. Label proportion of the circle that is shaded. Art activity- In the student-made art condition, participants assembled and glued pre-cut pieces of paper resembling pizza crusts, sauce, cheese, and toppings. In the pre-made generic condition, participants assembled and glued pre-cut geometric shapes onto rectangular paper. Labelling proportion that matched the fraction.	Fraction models: colourful, contextualized student constructed material (paper pizza), simple pre-made material (monochromatic paper circles). The order was counterbalanced across questions. the picture of pizza/circles divided into equally sized slices	The researcher read the questions one at a time	Word problems	Fraction knowledge. Labelling proportion; there were four types of responses: correct answer, correct numerator/ incorrect denominator, correct denominator/ incorrect numerator and incorrect numerator/ incorrect denominator.
Begolli, Booth, Holmes, & Newcombe, 2020	2020	Shade	Fraction words (half, a quarter, two quarters, a third, three quarters, and an eighth), which were displayed at the halfway point on top of the number line Indicate fraction's location on a number line. Compare fractions	Real-life situation	Paper-and-pencil-based proportional equivalence task and mathematics assessment. Shade an area based on a fraction. Shaded area with symbolic fraction notation. Fraction subtraction (6 items), division (1 item), comparison (5 items), and part–whole. Picture --→symbol representation (1 item).	Part–whole and number line, number line was labelled at each end point with 0 and 1. Picture of rectangle. Chain shape circle.	Children read the instructions together with the researcher	Word problems	Whole number Fractions
Flores, Hinton & Taylor, 2018	2018	Follow CRA activity	Solve word problems at abstract level	Real-life problems	Manipulative: Concrete(C)=Sheets, fraction tiles, fractions blocks, number lines, and coins Representation (R)= Sheets with equipartitioned shapes and number lines, number lines with pictures of coins;	Area model Length model Number line	The researcher gave explicit instruction, students repeated information	Word problems	Fractions Whole number Computation Decimals Equivalent and

					Abstract (A)=Sheets using numbers only		and count with the researcher. think aloud about parts of the problem		Multiplication Addition
Liu, 2017	2017	Make Name	Solve word problems, Compare fractions	Real-life scenarios	Concrete: Cake, whole number line estimation task and number line estimation task. Make one clear mark on number line	Linear fraction representation, picture, visual aids, number line square	Limited fractions instructions (researcher). Name and compare fractions. Students' reading achievement	Problem solving questions	Fraction Decimal Percentage Whole number
Degrande, Van Hoof, Verschaffel, & Van Dooren, 2017	2017	Writing	Additive and multiplicative reasoning	Problem situations	Non-symbolic snake task Pencil-and-paper test	Snake picture in discrete and continuous tasks		verbalize in written form	Whole number
Hamdan & Gunderson, 2017	2016	Draw Shade Writing Matching			Draw number line and circle segment, shade the parts and writing fractions Online supplement materials for the training script. Number line estimation task and area model estimation task	The number line and circle, rectangle	Children read subsequent clues aloud. Systematic instruction, Direct instructions (experimenter explanation)	Cross word puzzle: match the words provided	Fractions Whole number on number line
Resnick et al., 2016	2016	Use arrow keys Slide cursor Press key Fill in empty space	Solve multiplication problems Whole numbers and fractions estimation on a number line to assess fraction magnitude understanding		Paper and pencil presentation and response. Estimated the locations of 28 fractions and mixed numbers on a laptop: use arrow keys, slide the cursor along number line and press different key. Fill in the empty space. Shaded sections of a polygon or set of polygons. Computer-based multiple-choice test. Teachers rated children's attention during mathematics classes.	Number line presented on laptop screen, Polygon or set of polygons	Reading fluency: read aloud	Written words	Multiplication fraction Proper fractions, improper fractions and mixed numbers. Symbolic division problem

Table 5. Metadata analysis of the instructional focus and representation elements

Study		**Instructional focus**			**Real world situation**		**Manipulative aids**				**Pictures**			**Spoken**		**Written symbols**					
			Conceptual														number				
	Year	Procedural	Solve word problem	Compare fraction	Realistic context	Authentic context	Constructivism manipulative	Virtual manipulative	Informative manipulative	Game-based manipulative	Part whole (area model/ length model)	Part of a set	Number line	Researcher/ experimenter	Students read	words	fraction	Whole number	decimal	operations	percentage
Kaminski and Sloutsky, 2020	2020	1	1	0	1	0	1	0	0	0	1	1	0	1	0	1	1	0	0	0	0
Begolli et al., 2020	2020	1	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
Flores et la., 2018	2018	1	1	0	1	0	1	0	0	0	1	0	1	1	1	1	1	1	1	1	0
Liu, 2017	2017	1	1	1	1	0	1	0	0	0	1	0	1	1	1	1	1	1	1	0	1
Degrande et al., 2017	2017	1	1	0	1	0	1	0	0	0	1	1	0	0	0	1	0	1	0	0	0
Hamdan and Gunderson, 2017	2016	1	0	0	0	0	1	0	0	0	1	0	1	1	1	1	1	1	0	0	0
Resnick et al., 2016	2016	1	0	1	0	0	0	1	0	0	1	1	1	0	1	1	1	0	0	1	0
TOTAL		7	5	3	5	0	6	1	0	0	7	4	5	5	5	7	6	5	2	2	1

***Note:* 1 stands for appear and 0 for does not appear in the article.**

5 Discussion

The purpose of present study is to identify the characteristics, instructional focus, representation elements and the outcomes of each study. The selected studies emphasize on procedural knowledge, symbolic words, and part whole representations especially the area and length models. However, most previous studies focused on both procedural and conceptual knowledge of which most representations use realistic context and constructivism manipulative aids. In fact, the students were encouraged to speak through reading or verbally respond to the researcher's one-to-one task-based interview. Finally, the three highest consecutive symbolic representations employed were symbolic words, symbolic fraction numbers and symbolic whole numbers.

Nevertheless, comparing fractions using part of a set (Begolli et al., 2020; Resnick et al., 2016), learning aids that encourage student's virtual manipulative (Resnick et al., 2016) and connecting with either symbolic decimals (Flores et al., 2018; Liu, 2017), operations (Flores et al., 2018; Resnick et al., 2016), or percentage numbers were lacking (Liu, 2017). According to Tsai and Li (2016), connecting fractions with decimals and percentages is important to help students moving among the concepts flexibly and efficiently in dealing with daily life situation. Besides, none of the selected studies were related to authentic problem context such as problem- or project-based learning (PBL) which was consistent with the result by Minarni, Napitupulu and Husein (2016). They discovered that conventional approach was still practiced for Indonesian students and class engagement activity was low as well as the students' mastery achievement and performance were low. Furthermore, none of the studies carried out informative manipulative method, which implies that those studies paid serious attention to the students' construction meaning (Fyfe & Nathan, 2018). In addition, game-based manipulative was not utilized to teach fractions for understanding.

More specific discussions for each focus question are presented accordingly. First, the study characteristics were summarized into participants' context and settings. Both demographic characteristics are important indicators for effective teaching mathematics; whereby students' various backgrounds and characteristics with teachers' different set of plans are demonstrated (Brophy & Good, 1986). Almost all studies explicitly demonstrate the participants' characteristics, grade, ethnicity, urbanicity and location which mostly directed from Western countries. It was discovered that those elementary schools emphasize on teaching fractions through representations with various targeted urbanicity. However, the location should be wider. East Asian countries may highlight different approaches for representations of fractions and specific instructional focus for students with various cultures and demographic backgrounds.

Second, most mathematics education researchers paid serious attention to connecting procedural with conceptual knowledge in teaching fractions (Begolli et al., 2020; Degrande et al., 2017; Flores et al., 2018; Kaminski & Sloutsky, 2020; Liu, 2017; Resnick et al., 2016). Roesslein and Codding (2019) emphasized both to develop a strong foundation on fraction concept and problem-solving ability. Therefore, emphasis should be given on both procedural and conceptual knowledge of fractions at the early years of schooling (Agrawal & Baker, 2013; Turner, 2011) including kindergarten level.

Third, the present review analyzed the representation elements adapted in these studies related to instructional fractions extended from Roesslein & Codding's (2019) review on instructional components and Behr et al.'s (1983) interactive representational model. The articles were coded for the representation elements applied such as (a) real-world situation; (b) manipulative aid; (c) pictures; (d) spoken; and (e) written symbols (words and number). All seven studies utilized multiple representations with different approaches whereby manipulative aids and pictures were the compulsory elements. Multiple representations not only limited between representation elements such as simple, generic, pre-made material (picture) and colorful, contextualized (realistic context) representations (Kaminski & Sloutsky, 2020) but also within representations which emphasized on explicit characteristics of the picture such as discrete, discretized or continuous format (Begolli et al., 2020). Although two studies (Hamdan & Gunderson, 2017; Resnick et al., 2016) did not utilize real-life problems in their intervention, the remaining five studies emphasized real-life situations in problems presented to students. This shows that real-life problem is an important representation element for teaching fraction through representations.

Furthermore, real-world situation and manipulative aids are critical in shaping learner's performance and understanding (Penalvo, 2008, p. 134) of fractions by offering a deep-set and large of sensory experience (Bartolini & Martignone, 2014). Besides, both elements are also related to the choices of educational domain, personal domain, public domain and occupational domain in problem solving task (Council of Europe, 2011). When students are familiar with the selected domain such as personal domain and educational domain, the students feel encouraged to express ideas in their own words and teacher could ask the students to respond (Patahuddin, Usman, & Ramful, 2017). If the domain is unfamiliar to the students such as occupational domain and public domain, it benefits them in the opposite as it prepares them for future job demand when they apply conceptual knowledge in novel situation (Kaminski & Sloutsky, 2020) and enhance their higher thinking skills. However, it still depends on the students' abilities and the outcomes that the teacher or researcher aims to develop.

Spoken representation is an important element to encourage students' response and develop interpersonal skills (Vygotsky, 1978). Typically, elementary students are trained with communication skills before they are asked to write in words. However, in current study spoken representation is emphasized less than the verbalization in written form. Additionally, it is suggested for future researches to observe the effects of both elements on students' communications skills and understanding fractions. Therefore, it is hoped that fractions difficulty and misconception can be overcome not only among elementary students but also among upper-grade level and up to higher education students as it has been highlighted for the past three decades (Behr et al., 1983; Hoch, Reinhold, Werner, Richter-Gebert, & Reiss, 2018).

One of the reasons for fractions difficulty is due to its various definitions. It is shown in this review that the representations of fractions appear in a symbolic number of fractions (numerator over denominator), decimals or percentages. However, the symbolic numbers were utilized in most of the reviewed studies also involve whole numbers, especially when a number line was applied. Utilized number line could overcome whole number bias in fractions interpretation. It is evident in five studies as demonstrated by the usage of symbolic number of fractions and whole numbers through a number line.

Fourth, the outcomes identified in each selected study are related to CRA method by Miller and Hudson (2007), Flores et al. (2018) and Hwang et. al. (2018). The emphasis is given on gradual sequence between concrete, representational and abstract, whereby students personally were given an opportunity to first-hand experience using hands-on manipulative task (Flores et al., 2018). The sequence also occurs from representational phase as in a simple, generic and pre-made material to concrete phase as in the colourful and contextualized representations (Kaminski & Sloutsky, 2020). In addition, the sequence begins from representational phase to abstract phase as in using a number line and estimating the value on a number line

(Resnick et al., 2016), utilizing number line to magnitude tasks (Hamdan & Gunderson, 2017) and using representations for approximate number system (ANS) and whole number knowledge (Liu, 2017). This shows that the transformation of representations is a flexible process (Deliyianni, Gagatsis, Elia & Panaoura, 2015) because it does not necessarily begin with the concrete phase; instead begins with representations phase to concrete phase.

The results also imply that at the initial phase of teaching fractions, students should be taught explicit translation methods from concrete or representational or abstract representations, explicit use of number line to compare fractions and integration use of representational characteristics such as discretized, discrete, and continuous formats. Therefore, multiple representations are not necessary to be used in sequence or connected to students' age (Hoch et al., 2018). As supported by Flores, Inan, Han and Koontz (2018), students learning is enhanced when multiple representations are parallelly utilized. However, the priority is to expose students to multiple representations of the same concept Fyfe and Nathan (2015).

5.1 Limitations and recommendations

The primary limitation of current review is the sole use of Web of Science database which was accessed via Universiti Sains Malaysia platform. The utilization of a single database restricts the researchers from obtaining articles from different languages, fields and journals available around the world (Falagas, Pitsouni, Malietzis, & Pappas, 2008). Next, small sample (n=7) was included. Nevertheless, the content analysis was conducted with care particularly in interpreting the results and developing confidence of the findings.

Since future research on teaching fractions among elementary students is important, instruction coding focuses on operational definitions of conceptual and procedural knowledge adapted from previous studies (Anderson, et al., 2001, p. 46; Misquitta, 2011; Roesslein & Codding, 2019). Therefore, the definitions may differ in different studies. The authors in current review coded the criteria for the knowledge accordingly but did not explicitly relate how the codes affect fractions understanding. Hence, this springs out the suggestion for future study. In coding representation elements, the authors relied on the descriptions in abstract, methodology and results in particular the procedures, measures and figures related to fractions. Figures and verbalization in written form are useful guidance to obtain necessary codings for the respective representation elements in the review.

5.2 Practical implications

The findings from this current review suggest several instructional implications for educators, teachers, curriculum developers, textbooks writers and researchers. First, fraction interventions that utilize multiple representation elements should be sequentially or parallelly employed to enhance students' understanding of the same concept related to fractions. Moreover, it provides choices of selected measures and procedures available to future researchers interested in representations for fraction learning. In fact, it can be an alternative strategy for teachers to overcome the misconception among students by exposing students to explicit use of representational characteristics. In addition, variations within representation such as part of a set, part-whole of an object and measurement representation are essential in developing students' understanding pertaining to fractions. All types of picture representations are closely related. Whilst a set is a group of an object; a number line is a measurement representation which is significant in assessing students' magnitude understandings. It assists students to differentiate fractions with the whole numbers. Therefore, more researches should explore the combination of all types of representations for teaching and learning fractions.

5.3 Conclusions

The current review provides the most recent evidence in the representation elements for teaching fractions in terms of study characteristics, instructional focus, representation elements and fractions research outcomes. While considering the limitations, the analysis outcomes offer teachers or researchers guidance on how to decrease misconception towards fractions from as early as elementary level students; while, providing a strong foundation for future challenging topics in Mathematics. This review provides a systematic PRISMA procedure as a direction for future researches; in particular for those interested in the review about fractions and possibly those in other fields.

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Oppilaiden tuen tarpeet luonnontieteiden opetuksessa opettajaopiskelijoiden näkökulmasta

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Oppilaiden diversiteetti on inkluusioajattelun lähtökohta. Opettaja tarvitsee tietoja ja taitoja diversiteetin havaitsemiseen ja oppilaiden tukemiseen. Tutkimuksessa selvitämme, millaisia ennakkokäsityksiä opettajaopiskelijoilla on luonnontieteiden opetukseen liittyvistä tuen tarpeista. Käsityksiä kartoitettiin e-kyselyllä monialaisten opintojen luonnontieteiden opintojaksolla vuosina 2015–2018. Kyselyyn vastasi 491 luokan- ja erityisluokanopettajaopiskelijaa, joista tutkimusluvan antoi 468. Aineisto analysoitiin sisällönanalyysillä. Enemmistö opiskelijoista tarkasteli oppilaiden diversiteettiä ongelmakeskeisesti, jolloin tuki nähtiin ratkaisuna diagnosoituun oppimisvaikeuteen. Opettajaopiskelijat käsittelivät oppilaiden tuen tarvetta myös oppiaineiden sisältöihin ja opetustapahtumaan liittyen. Luonnontieteiden opetustilanteet tulisi rakentaa diversiteettia huomioivaksi, jolloin diversiteetti-käsite kattaa kaikki perusopetuksen oppilaat monipuolisesti ymmärrettyinä yksilöinä.

Asiasanat: inkluusio, oppimisen tuki, opettajankoulutus, luonnontieteiden opetus, ympäristöoppi

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Pupils needs for support in science education: Teacher students' perceptions

Student diversity is the starting point for inclusive thinking. The teacher needs knowledge and skills to detect diversity and support students. In this study, we will investigate what kinds of preconceptions students have about the need for support in science education. Perceptions were surveyed in an electronic questionnaire during the 2015–2018 Interdisciplinary Studies in Natural Sciences. Totally, 491 class and special class teacher students, of whom 468 were granted research permission, answered the questionnaire. The data was analysed by content analysis. Most students looked at student diversity in a problem-oriented manner and support was seen as a solution to diagnosed learning disability. Teacher students also looked at the need for support in terms of subject content and teaching activities. Science teaching situations should be built with diversity in mind, so that the concept of diversity includes all students in basic education as diverse individuals.

Keywords: inclusion, support in learning, teacher education, science education, environmental studies

1 Johdanto

Perusopetuksen opetussuunnitelman perusteet edellyttävät inkluusioperiaatteiden huomioimista koulutyön kehittämisessä (Opetushallitus 2014, s. 8). Inklusiivinen kasvatus on valittu myös koulutuksen kansainvälisten kehittämislinjausten tavoitteeksi (European Agency for Special Needs and Inclusive Education 2014; UNESCO, 2015). Inkluusio määritellään kuitenkin eri yhteyksissä eri tavoin. Kapeimmillaan inkluusiokäsite kohdentuu oppilaisiin, joilla on diagnosoituja oppimisvaikeuksia, tai se voi asteittain laajentua koskemaan vähemmistöjä ja erityisen haavoittuvassa asemassa olevia ryhmiä. Inkluusio nähdään myös diversiteettiä kunnioittavana yhteiskunnallisena periaatteena, jolloin koulutuksen yhteydessä lähtökohtana on kaikkien oppilaiden yhteinen koulu (Messiou & Ainscow, 2020; Kiuppis & Hausstätter, 2014). Inklusiivisella toimintakulttuurilla laajimmillaan viitataankin diversiteetin arvostamiseen kaiken koulutoiminnan lähtökohtana (Kugelmass, 2001). Tällöin inkluusio näyttäytyy koulujen halukkuutena vastata kaikkien oppijoiden opetuksesta heidän lähikoulussaan poistamalla oppimisen esteitä ja edistämällä kaikkien oppilaiden osallistumista kouluyhteisön toimintoihin yhdenvertaisesti (Ainscow, Booth, & Dyson, 2006).

Oppijoiden diversiteetti on inklusiivisen kasvatusajattelun lähtökohta. Oppilaiden diversiteetti on haastava käsite, jota on jopa mahdoton määritellä yksiselitteisesti (Kousa & Aksela, 2019; ks. myös Goethe & Colina, 2018). Oppilasdiversiteettiin sisältyvät esimerkiksi oppilaan kulttuurinen, etninen, maantieteellinen ja sosioekonominen tausta, sukupuoli ja seksuaalinen orientaatio, kyvyt ja älykkyys, oppimisen ja kielen ongelmat, omat kokemukset, näkemykset ja asenteet sekä persoonallisuus (ks. esim. Goethe & Colina, 2018; Wassel, Kerrigan, & Hawrylak, 2018; Kousa & Aksela, 2019). Yhteiskunnassamme diversiteetti onkin määrittymässä uudelleen entistä moniulotteisempana hyperdiversiteettinä (Doucerain, Dere, & Ryder, 2013).

Oppilaan erityisyyksiin painottuvan näkökulman korostaminen on kuitenkin johtanut siihen, että käytännön opetustyössä yksittäisten oppilaiden monenlaiset tuentarpeet nimetään ja usein ne myös päällekkäistyvät (*co-morbidity*), jolloin oppilaasta muodostuva mielikuva rakentuu tuentarpeiden varaan ja jää yksipuoliseksi (Slee, 2001). Oppilaan määrittyessä vain omien tuentarpeidensa kautta voi syntyä kapeakatseisia stereotypioita (Zembylas & Isenbarger, 2002), jotka rajoittavat oppilaan oppimismahdollisuuksia. Esimerkiksi useilla autismin kirjon oppilailla on havaittu erityislahjakkuutta, jolloin koulutyössä tuetaan sosiaalisten taitojen kehittymistä ja lahjakkuuden potentiaaleja. Foley-Nipcon, Assouline ja Colangelo (2013) ovat käsitteellistäneet ilmiötä englanninkieli-sellä termillä *twice exeptional*, jolloin oppilaalla voi olla yhtä aikaa tuen tarvetta ja lahjakkuutta; lahjakkuus yhdellä osa-alueella ei siis itsessään poista tuentarvetta muissa taidoissa. Oppimisen esteeksi voikin nousta joustamaton ja kaavamainen opetus, joka ei tunnista eikä tunnusta oppijoiden diversiteettiä (esim. Ainscow & Sandill, 2010).

Koulun arkityössä yksittäisten ja erillisten tarpeiden huomioiminen yhden opettajan erillisillä toimenpiteillä voi olla haastavaa, ellei jopa mahdotonta. Siksi inklusiivisessa opetuksessa joustavan ja tukea tuottavan toimintakulttuurin rakentaminen on ensiarvoisen tärkeää. Jotta opettaja voi organisoida opetustaan oppilaiden tuentarpeita huomioivaksi, hänen tulee havaita oppilaidensa erityispiirteitä (Ainscow, Booth, & Dyson, 2006) sekä kehittää opetuskäytänteitä oppilaiden ja opettajien yhteisprosessina (Messiou & Ainscow, 2020). Oppilaiden tuen tarpeisiin vastaaminen tulee huomioida myös opettajien peruskoulutuksessa. Tutkimustehtävämme onkin selvittää, millaisia ennakkokäsityksiä opettajaopiskelijoilla on luonnontieteiden opetukseen liittyvistä tuen tarpeista.

1.1 Opettajien ja opettajaopiskelijoiden käsityksiä erilaisista oppijoista luonnontieteiden opetuksessa

Opettajat suhtautuvat inklusiiviseen opetukseen periaatteellisella tasolla myönteisemmin kuin käytännössä (Avramidis & Norwich, 2002). Esimerkiksi suomalaisten opettajien (Savolainen, Engelbrecht, Nel, & Malinen, 2012) ja irlantilaisten luokanopettajien (Young, McNamara, & Coughlan, 2017) on havaittu suhtautuvan kriittisesti mahdollisuuksiinsa toteuttaa inklusiivista opetusta.

Kousan väitöstutkimuksen (2019) mukaan suomalaisilla kemianopettajilla on ongelmia havaita oppilaiden yksilöllisiä eroja, eikä heillä ole riittävästi tietämystä erilaisia oppilaita tukevista opetusmenetelmistä. Erityisen haastavina opettajat pitävät yksilöllisiä oppimisen haasteita, kuten oppilaiden eritasoisia oppimisen ongelmia, kulttuurisia eroja, kieli- ja käytösongelmia sekä oppilaiden välisiä konflikteja. Lisäksi oppilaiden motivaation puute ja negatiivinen suhtautuminen kemian opiskelua kohtaan nähdään problemaattisena. (Kousa & Aksela, 2019; Kousa, 2019) Kemian opettajien mielestä heidän oppilaansa jakautuvatkin viiteen ryhmään: 1) hyvin menestyviin lahjakkaisiin oppilaisiin, 2) ei-motivoituneisiin heikosti menestyviin oppilaisiin, 3) heikosti menestyviin oppilaisiin, joilla on usein oppimisen tai kielen ongelmia, 4) ahkeriin oppilaisiin, jotka muistavat kaiken, mutta eivät välttämättä ymmärrä kemiaa sekä 5) käytännön töissä pärjääviin oppilaisiin, joilla on heikko käsitteellinen ymmärrys asiasisällöissä. Kemian opiskelua pidetään haastavana myös oppiaineen luonteen vuoksi: se on abstraktia, teoreettista ja vaikeasti ymmärrettävää. Kemian opiskeluun liittyy muun muassa paljon termejä ja käsitteitä, jotka ovat haastavia oppilaille. (Kousa & Aksela, 2019) Opettajien suurin huolenaihe on vastata erilaisten oppilaiden tarpeisiin. He kokivat tarvitsevansa enemmän kouluhenkilökunnan tukea ja resursseja sekä taitoja tunnistaa oppilaidensa tuen tarpeita. Opettajat kaipasivat myös oppimista tukevia opetus-metodeja ja -materiaaleja sekä taitoja yhdistää teoreettisiin asioihin käytäntöä oppilaita inspiroivalla ja mielekkäällä tavalla (Kousa & Aksela, 2019; Kousa, 2019).

Tutkimukset ovat osoittaneet, että opettajaopiskelijat suhtautuvat myönteisesti inklusiiviseen opetukseen (Beacham & Rouse, 2012; Sharma, Forlin, & Loreman, 2008; Varcoe & Boyle, 2014), kuten esimerkiksi oppilaiden oppimisvaikeuksien huomioimiseen osana yleisopetusta (Woodcock & Vialle, 2016). Toisaalta Varcoen ja Boylen (2014) tutkimus osoitti, että ennen inklusiivista harjoittelua hankittu opetuskokemus on negatiivisesti yhteydessä opettajaksi opiskelevan asenteisiin inkluusiota kohtaan. Tämän vuoksi opettajankoulutuksessa tulee kiinnittää huomiota opiskelijoiden ennakkokäsityksiin jo ennen opetusharjoittelujaksoja.

Jos opettajaopiskelijat eivät tule opiskelujensa aikana tietoiseksi oppijoiden diversiteetistä ja siihen liittyvistä ilmiöistä, tulevat he kohtaamaan haasteita tulevassa työssään (Tolsdorf, Kousa, Markic, & Aksela, 2018; ks. myös Kousa, 2019). Kousan (2019) tutkimuksessa tulikin esille, että oppilaiden diversiteettiin liittyvien aiheiden opiskelu ja STSE-opetukseen (tiede, teknologia, yhteiskunta ja ympäristö –opetus) perehtyminen sekä käytännön harjoittelu kemian aineenopettajien koulutukseen kuuluvalla opintojaksolla kehitti opettajaopiskelijoiden näkemyksiä diversiteetistä ja erilaisten oppilaiden opettamisesta. Opintojakson jälkeen opettajaopiskelijat kokivat olevansa itsevarmempia opettamaan erilaisia oppijoita. Oman tutkimuksemme kohteena olevan opintojakson kehittäminen lähtee opettajaopiskelijoiden ennakkokäsitysten tarkastelusta. Mielestämme opiskelijoiden ennakkokäsitykset tulee ottaa huomioon ennen kuin tarkastellaan inklusiivista luonnontieteiden opetusta.

1.2 Oppilasdiversiteetti luonnontieteiden opetuksessa

Luonnontieteiden opetuksessa oppilaille voi tuottaa vaikeuksia ymmärtää teoreettisen ja käsitteellisen tiedon suhde käytännölliseen tietoon ja tiedon tuottamisen prosesseihin (Osborne & Dillon, 2010; Kousa, 2019), mikä kuvastaa luonnontieteellisen tiedon ja toiminnan luonteen ymmärtämisen haasteita. Luonnontieteellisen lukutaidon oppiminen ja opettaminen ovat käytännön tasolla jännitteisiä ja opettajan näkökulmasta vaativia prosesseja (Kokkonen & Laherto, 2018).

Oppilaille vaikeuksia voivat tuottaa myös luonnontieteissä käytetty kirjallinen, kirjoitettu ja puhuttu kieli (Wellington & Wellington, 2002; McCarthy, 2005; Evagorou & Osborne, 2010; Mason & Hedin, 2011; Kousa, 2019; Sormunen, Lavonen, & Juuti, 2019). Luonnontieteille ja niiden opetukselle ominaiset matemaattiset ja numeeriset esitysmuodot voivat aiheuttaa ongelmia osalle oppilaista (Brigham, Scruggs, & Mastropieri, 2011). Akateemiseen suoriutumiseen vaikuttavat myös työmuisti, käyttäytymisen haasteet ja psyykkiset oireet (Aronen, Vuontela, Steenari, Salmi, & Carlson, 2005).

Kulttuurimme monipuolistuessa on tärkeää havaita tuentarpeet, jotka perustuvat oppijoiden sosiaaliseen, kulttuuriseen ja etniseen taustaan. Lee ja Buxton (2010) käyttävät tällöin oppijoista käsitettä *diverse learners*. Haasteet voivat liittyä esimerkiksi uuteen opiskelukieleen (Lee & Fradd, 1998) tai kulttuuritaustoihin perustuvien maailmankuvien eroavuuksiin (Lee & Buxton, 2010; Piliouras & Evangelou, 2012). Tuen tarpeet ja heikko koulusuoriutuminen voivat liittyä myös alueellisesti keskittyneeseen väestön heikkoon sosioekonomiseen asemaan (Bernelius, 2011) tai vanhempien vähäisiin mahdollisuuksiin tukea ja kannustaa lapsiaan luonnontieteiden opinnoissa (Smith & Hausafus, 1998). Lisäksi on havaittu, että sosioekonominen tausta (Gorard & See, 2009) ja sukupuoli (Greenfield, 1997) ovat yhteydessä oppilaan suhtautumiseen ja asenteisiin luonnontieteiden opiskelua kohtaan. Sukupuoli vaikuttaa esimerkiksi luonnontieteelliseen urasuuntautumiseen (Kupari, Vettenranta & Nissinen, 2012; Sikora & Pokropek, 2012).

Myös Suomessa vähävaraisuudella on yhteyttä luonnontieteiden osaamiseen (Kupari ym., 2012; Vettenranta, Hiltunen, Nissinen, Puhakka, & Rautopuro, 2016). Sosioekonomisen taustan ja sukupuolittuneisuuden vaikutus osaamiseen Suomessa on todettu myös kansainvälisen TIMMS-tutkimuksen tuloksissa vuodelta 2015: verrattuna vuoteen 2011 suomalaisten neljäsluokkalaisten poikien oppimistulokset ovat heikentyneet ja alempi sosioekonominen asema ja maahanmuuttajatausta ovat yhteydessä vähäiseen osaamisen tasoon (Vettenranta, Hiltunen ym., 2016). Alueelliset erot maassamme korostuvat ensimmäistä kertaa PISA2015-tutkimuksen tuloksissa: pääkaupunkiseudun oppilaiden menestys luonnontieteissä, lukutaidossa ja matematiikassa oli muuta maata korkeampaa (Vettenranta, Välijärvi ym., 2016).

Luonnontieteiden opetuksessa oppimiseen liittyvät haasteet voivat siis liittyä oppiaineen tai oppijan piirteisiin. Oppilaiden diversiteetin kohtaaminen luonnontieteiden opetuksessa edellyttää opettajalta uudelleen tulkittuja sekä yleisiä että oppiaineen erityispiirteet huomioivia pedagogisia taitoja.

1.3 Luonnontieteiden oppimisen tukeminen osaksi opettajankoulutusta

Luonnontieteiden opettajilla on yleensä vähän kokemusta tai taitoja huomioida tukea tarvitsevia oppilaita (Norman, Caseau, & Stefanich, 1998; Kousa & Aksela, 2019). Tukea tarvitsevien oppilaiden oppimista luonnontieteissä on tutkittu melko vähän eikä inklusiivisen koulun edellyttämiä muutoksia ole tarkasteltu riittävästi opettajankoulutuksen luonnontieteiden opetuksessa (Asghar, Sladeczek, Mercier, & Beaudoin, 2017). Luokanopettajaopiskelijoiden kiinnostuksen luonnontieteiden opetukseen on huomattu vähenevän entisestään (Gilbert & Byers, 2017) ja heidän minäpystyvyytensä luonnontieteiden opetuksen suhteen on havaittu matalaksi (esim. Velthuis, Fisser, & Pieters, 2014). Toisaalta erityisopettajilla on vähän luonnontieteisiin ja niiden opettamiseen liittyviä tietoja (Villanueva, Taylor, Therrien, & Hand, 2012). Opettajien eritasoinen osaaminen voi johtua ensinnäkin siitä, että opettajankoulutuksen luonnontieteiden opintojaksoissa ei yleensä tarkastella teoreettiskäsitteellisellä eikä käytännöllisellä tasolla tukea tarvitsevien oppijoiden huomioimista (vrt. esim. Norman ym., 1998). Toisekseen luokan- ja aineenopettajakoulutus järjestetään yleensä erityisopettajakoulutuksesta erillään, jolloin esimerkiksi aineenopettajilla ei ole valmiuksia tuen antamiseen eikä erityisopettajilla riittävää oppiaineen sisältöosaamista (McCarthy, 2005; Villanueva ym., 2012).

Opettajankoulutusten erillisyydellä ja tukimuotojen vähäisellä omaksumisella voidaan ajatella olevan vaikutusta myös opettajien minäpystyvyyden tunteeseen, esimerkiksi suhteessa oppilaiden käyttäytymisen ohjaamistaitoihin (Malinen & Savolainen, 2016). Kuitenkin juuri tarve ohjata oppilaiden käyttäytymistä aiheuttaa hämmennystä työuran alkuvaiheessa ja ensimmäiset työyhteisöt ovat opettajalle merkittäviä eriyttävän opetuksen harjoitteluympäristöjä (esim. Mulholland &

Wallace, 2001). Opettajien minäpystyvyyttä inklusiivisessa opetuksessa on kehitetty keskittymällä esimerkiksi oppimis- ja opetusstrategioihin sekä luokanhallintataitoihin (Chao, Sze, Chow, Forlin, & Ho, 2017).

Näkemys oppijoiden tukemisesta eri oppiaineiden opetuksen ja oppimisen kontekstissa on jäänyt pinnalliseksi; oppijan ilmeisimmät ominaisuudet ja taustatekijät, kuten äidinkieli, sukupuoli tai oppimisvaikeus todetaan, mutta opettajaopiskelijat ohittavat tunnistamattomat kulttuuriset mallit ja ennakkoasenteet, jotka säätelevät suhtautumista itselle uudenlaisiin ihmisiin (Moore, 2008). Tällaisten käsitysten pohdinta osana opettajankoulutusta voi nostaa esiin luonnontieteiden opetustilanteisiin sisältyviä ja vuorovaikutukseen vaikuttavia oletuksia, jotka olisi syytä reflektoida tarkemmin. Perinteiseen ja segregoivaan opetukseen liittyvät uskomukset olisi tärkeää työstää, jotta tietoisuus oppilasdiversiteettiä hyödyntävästä inklusiivisesta opettajuudesta (Kugelmass, 2001) voisi saada eksplisiittisiä muotoja. Oppilasdiversiteetin kasvaessa tulee opettajankoulutuksessa löytää tapoja, joilla voidaan vahvistaa opettajaksi opiskelevien taitoja toimia inklusiivisessa koulussa.

2 Tutkimuksen toteuttaminen

Opettajankoulutuksessa voidaan inklusiivista opetusta ja oppimisen tukea tarkastella erilaisin opintojaksototeutuksin: a) erityispedagogisella opintojaksolla (vrt. esim. Sharma & Sokal, 2015), b) käyttämällä läpäisyperiaatetta (vrt. esim. Lambe, 2011), jolloin teema on kirjattu opettajankoulutuksen opetussuunnitelmaan, mutta vastuu inklusiivisen opetuksen huomioimisesta on yksittäisen opintojakson kouluttajalla, tai c) siten, että opettajankouluttajat tuovat sitoutuneesti yhteistyössä kyseistä teemaa esiin ”diffuusisti” eri opintojaksoilla useamman opiskeluvuoden aikana (vrt. Sharma ym., 2008). Itä-Suomen yliopiston perusopetuksessa opetettavien aineiden ja aihekokonaisuuksien monialaisissa opinnoissa sovelletaan edellä mainittua diffuusiomallia, jolloin eri oppiaineiden opintojaksoilla erityispedagoginen näkökulma tuodaan esiin oppiainepedagogiikan ja erityispedagogiikan asiantuntijoiden välisenä yhteistyönä eri oppiaineiden opintojaksoilla.

Tutkimuksemme on osa praktista toimintatutkimusta, jonka tarkoituksena on kehittää ”Tutkiva oppiminen luonnontieteiden opetuksessa ja oppimisessa, A-osa, 3,5 op” –opintojaksoa. Siihen kuuluu luonnontieteiden sisällön ja pedagogiikan ohella myös oppimisen tuen näkökulma (Itä-Suomen yliopisto, 2016): *”...Opintojaksolla* *tarkastellaan oppijan ennakkotietämystä ja asenteita luonnontieteiden oppimisen lähtökohtana sekä mahdollisia erityisen tuen tarpeita…*”Opintojakso kuuluu keväällä ensimmäisen lukuvuoden opintoihin, eikä opiskelijoilla ole ollut opettajankoulutuksessa muita luonnontieteiden tai erityispedagogiikan opintojaksoja ennen sitä. Ennen opintojakson alkua opintojaksolle ilmoittautuneille opiskelijoille annetaan heidän ennakkotietämystään aktivoiva, sähköisessä muodossa oleva opintotehtävä, jossa heitä pyydetään ottamaan kantaa kysymykseen: ”Missä tapauksissa luonnontieteiden opiskelussa ja oppimisessa erilaiset oppijat mielestäsi tarvitsevat tukea? Perustele ja anna esimerkkejä.” Seuraava esimerkki on lainaus eräästä vastauksesta:

> ”Tukea voi tarvita missä vain, riippuen oppilaasta. Esim. pyörätuolissa istuvan oppilaan saaminen metsään voi olla hankalaa, lukihäiriöisen on hankala lukea myös luonnontieteiden oppikirjaa ja tehdä tehtäviä kirjallisesti. Jos esim. fykessä tehdään kokeita, pitää turvallisuuteen kiinnittää erityistä huomiota, jos luokassa on ylivilkkaita/käytöshäiriöisiä lapsia, ettei oppimistilanne muutu vaaralliseksi. Oppilaiden huomio pitäisi saada pysymään aiheessa, koska erilaisissa kokeiluissa huomio voi kiinnittyä helposti epäolennaisuuksiin.” (22.15)

Olemme koonneet vuosina 2015–2018 opintojaksolle osallistuneiden 491 opiskelijan opintotehtävän vastauksista aineiston, jonka tutkimuskäyttöön on vapaaehtoisesti antanut luvan (Tutkimuseettinen neuvottelukunta, 2019) 468 luokanopettaja- ja erityisluokanopettajaopiskelijaa. Tutkimukseen osallistuneiden opiskelijoiden vastaukset koodattiin juoksevin numeroin ja aineistonhankintavuoden mukaan; koodit ovat esillä myös jäljempänä analyysin kuvaukseen ja tulosten esittämiseen liittyvissä esimerkkilainauksissa. Aineistossa opiskelijoiden vastaukset vaihtelivat suppeasta yhden lauseen luettelomaisesta vastauksesta laajaan 18 lauseen vastaukseen. Suuri aineistokoko muodosti kuitenkin kokonaisuutena rikkaan aineiston, koska se sisälsi monipuolisesti erilaisia ennakkokäsityksiä. Aineistoa hyödynnetiin ja hyödynnetään opintojakson sisältöjen kehittämisessä.

Tehtävänannossa olevaa ilmaisua ”erilaiset oppijat” on käytetty kuvaamaan oppilaiden diversiteettiä. Inklusiivisen opetuksen yhteydessä tällä ilmaisulla voidaan viitata esimerkiksi tiettyyn ryhmään, joilla ajatellaan olevan erityisiä tarpeita opiskelussaan (*students with special educational needs*, ks. Villanueva ym., 2012). Tällöin aihepiirin tutkimuksissa käsitellään esimerkiksi oppimisvaikeuksia (Krull, Wilbert, & Hennemann, 2014), vammaisuutta (Round, Subban, & Sharma, 2016) tai muutoin diagnosoitavissa olevia neurobiologisia kehityshäiriöitä (Young ym., 2017). Toinen näkökulma, jonka olemme edellä luonnehditun ”erityisoppilaat”-näkemyksen lisäksi halunneet sisällyttää ”erilaiset oppijat” -ilmaisun alle, on aiemmin mainitsemamme kulttuurinen erilaisuus. Tällöin tuen tarpeiden tarkastelu liittyy oppijoiden sosiaaliseen, kulttuuriseen ja etniseen taustaan (ks. Lee & Buxton, 2010: *diverse learners*) sekä sukupuoleen, seksuaaliseen orientaatioon, lahjakkuuteen, oppijan kokemuksiin, näkemyksiin, asenteisiin ja persoonallisuuteen (ks. esim. Goethe & Colina, 2018; Wassel ym., 2018; Kousa & Aksela, 2019).

Tunnistamme kysymyksen asettelun luovan inkluusiota kategorisoivasti luonnehtivaa todellisuutta, mutta olemme pyrkineet käyttämään opiskelijoille ilmiötä arjessa avaavaa ja ymmärrettävää kieltä. Sana ”erilaiset” johdattavat vastaajaa myös ottamaan kantaa siihen, millaisia oppijoita he vastauksissaan tarkoittavat.

Aineiston analyysi perustui laadulliseen induktiiviseen sisällönanalyysiin (Hsieh & Shannon, 2005; Schreier, 2014). Kaksi tutkijaa luki tekstiaineistot huolellisesti läpi siten, että toinen heistä keskittyi vuosien 2015 ja 2016 aineistoihin ja toinen 2017 ja 2018 aineistoihin. Tämän jälkeen ensiksi mainittu tutkija luokitteli aineiston kahteen pääteemaan: tuki, joka kohdistuu yksilöön (Taulukko 1) - *”Jos on oppilas, jolla lukivaikeus, keskittymisvaikeus tai dysfasia, täytyy tämä huomioida opetuksen järjestyksessä...”* (14.16) - ja tukeen, joka perustuu ympäristöopin ominaispiirteisiin (Taulukko 2) - *”... Joskus luonnontieteessä tulee vastaan sellaisia sanoja ja käsitteitä, joita voi olla vaikea ymmärtää tai ne voi ymmärtää väärin, esim. kitka ja kasvukausi...”* (165.15). Tämän jälkeen tutkija muodosti kummastakin teemasta alustavat alakategoriat (ks. esim. Taulukko 1: ”Kielelliset vaikeudet”), alakategorioita yhdisteltiin yläkategorioiksi (Taulukko 1: “Oppimisvaikeuksien kautta nimetyt tuentarpeet”) ja yläkategorioita pääkategorioiksi (Taulukko 1: “Oppilaan oppimis- ja toimintakykyyn liittyvät tuentarpeet”). Toinen tutkija sovelsi edellä mainittua luokittelua vuosien 2017 ja 2018 aineistoihin, minkä jälkeen analyysiä tarkennettiin ala- ja yläkategorioiden nimeämisen suhteen käyttäen tutkijatriangulaatiota (vrt. Schreier, 2014).

Tiedostamme, että jo aineiston luokittelu sinänsä sekä kategorioiden nimeäminen tuottavat osaltaan todellisuutta, joka on inkluusiokäsitteen näkökulmasta vierasta. Tätä ei voida kuitenkaan aineiston analyysissä välttää, kun tavoitteena on tunnistaa ja tarkastella erilaisia opiskelijoiden käsityksiä tuen tarvitsijoista ja tuen tarpeista luonnontieteiden opetuksessa.

3 Tulokset

Tuen tarpeen käsittely jakautui karkeasti ottaen seuraavasti. Ensinnäkin havaittiin, että tuentarvetta perusteltiin oppilaskohtaisten ja yksilöihin liitettyjen ominaisuuksien kautta. Tämä viittaa käsitystapaan, jossa oppimisen tuentarve liitetään lähtökohtaisesti oppilaaseen henkilönä ja tuki mielletään usein myös ratkaisuna ongelmaan tai heikkouteen. Toisaalta havaittiin, että opiskelijat liittivät oppilaiden tuentarpeet myös oppiaineen eli ympäristöopin ominaispiirteisiin, kuten sisältöihin ja opetustilanteisiin.

3.1 Oppilaskohtaiset tuen tarpeet

Kun tuentarve perustui oppilaan ominaisuuksiin, jakautuivat opiskelijoiden käsitykset kolmeen pääluokkaan. Tuentarvetta kuvailtiin oppilaan oppimis- ja toimintakykyyn, opiskelumotivaation ja -orientaation vaihteluun sekä kulttuurisiin eroihin ja oppilaan taustatekijöihin liittyvien näkökohtien avulla (Taulukko 1).

Oppimis- ja toimintakykyyn sisältyi nimettyjä ja esimerkiksi neurologisesti ja psykologisin testein tutkittavissa ja diagnosoitavissa olevia oppimisvaikeuksia. Oppimisvaikeuksista opettajaopiskelijat mainitsivat yleisimmin lukemisen vaikeudet, keskittymis- ja tarkkaavaisuusvaikeudet, hahmottamisongelmat sekä matematiikan oppimisvaikeudet - *”… lukivaikeudesta kärsivällä lukijalla voi olla vaikeaa lukea teoriaa kirjasta ja sisäistää asiaa samaan tahtiin kun muiden. Hän saattaa tarvita saman tekstin lukemiseen paljon enemmän aikaa. ”* (103.16). Osin asian käsitteellistäminen pitäytyi ylätasolla, jolloin kuvauksena olivat kognitiiviset vaikeudet ja yleensä oppimisen ongelmat. Käsitteellistämisen täsmentyessä käytössä olivat myös neurologiset ja lääketieteelliset termit, kuten dysfasia, ADHD, dysleksia ja autismi - *”…Esim. ADHD lapsi tarvitsee selkeitä toimintaohjeita ja tarpeeksi virikkeitä esim. metsässä selkeät ohjeet etukäteen, että keskittyminen ei herpaannu…”* (27.17).

Opiskelijoiden mukaan tuentarve luonnontieteiden opetuksessa voi perustua myös pedagogisiin havaintoihin, kuten opettajan havaitsemaan lasten kehitysnopeuden vaihteluun, toiminnan hitauteen, oma-aloitteisuuden vähäisyyteen tai erityiseen lahjakkuuteen - *”Jotkut asiat voivat olla toisille hankalia ja toisille todella nopeita ja helppoja. Tämä tuo ongelman silloin kun tehdään esimerkiksi havainnollistavia kokeita luokassa. Osa ryhmästä saattaa saada tehtävän todella nopeasti valmiiksi, kun osa oppilaista ei saata olla vielä edes kerennyt hakea* *välineitä tarvittavan testin suorittamiseen."* (126.16). Lisäksi opiskelijat mainitsivat fyysisten rajoitteiden ja terveyden kautta nimettyjä tuentarpeita, kuten fyysiseen toimintakykyyn, aistitoimintoihin ja terveydentilaan liittyviä rajoitteita - *"...liikunta rajoitteinen taas tarvitsee erityistä huomiota esimerkiksi metsäretkien aikana."* (144.15).

Taulukko 1. Opiskelijoiden käsitykset oppilaiden ominaisuuksiin perustuista tuen tarpeista

Tuen tarpeet	Käsitys oppilaiden tuentarpeen syystä
1. Oppilaan oppimis- ja toimintakykyyn liittyvät tuentarpeet (268)	1.1 Oppimisvaikeuksien kautta nimetyt tuentarpeet (218) • kielelliset vaikeudet: eri asteiset lukemisen vaikeudet (59), dysleksia (2), dysfasia (3), kommunikointiongelmat (1) • matematiikan oppimisvaikeudet (29) • eriasteiset neurobiologiset ja neuropsykiatriset vaikeudet ja häiriöt: ADHD (3), autismi (1) • perustason kognitiiviset toiminnot: muistiongelmat (6), keskittymis- ja tarkkaavaisuusvaikeudet (43), hahmottaminen (32), toiminnanohjauksen haasteet (7), hienomotoriikan poikkeavuudet (4) • sosio-emotionaaliset vaikeudet (3) • yleismaininnat "oppimisvaikeuksista" tai "kognitiivisista vaikeuksista" (25) 1.2 Pedagogisten havaintojen kautta nimetyt tuentarpeet (25) lasten kehitysnopeuden vaihtelu (2), toiminnan hitaus (11), oma-aloitteisuuden puute (2), lahjakkuus ja nopeus suoriutua tehtävistä (10) 1.3 Fyysisten rajoitteiden ja terveydentilan kautta nimetyt tuentarpeet (25) fyysisen liikkumisen haasteet (13), allergiat (1), aistitoiminnot (11)
2. Opiskelumotivaation ja -orientaation vaihteluun liittyvät tuentarpeet (106)	2.1 Erilaiset kiinnostuksen kohteet ja motivaatio (44) 2.2 Ennakkotiedot ja -taidot (32) 2.3 Erilaiset oppimistyylit (29) 2.4 Minäpystyvyys (1)
3. Kulttuurisiin eroihin ja oppilaan taustaan liittyvät tuentarpeet (12)	3.1 Opiskelukieli (suomen kieli) (7) 3.2 Katsomukselliset erot (3) 3.3 Sukupuoli (1) 3.4 Sosioekonominen tausta (1)

Oppilaan oppimis- ja toimintakyky -pääluokka sisältää kuvauksia vaikeuksista, viivästymisestä ja ongelmista. Vain lahjakkuus rikkoi oppilaan ominaisuuksiin liitetyssä luokittelussa ongelmakeskeistä näkökulmaa – *"... Lahjakkaat ja tiedonjanoiset oppilaat tarvitsevat HAASTETTA ja MAHDOLLISUUDEN KOKEILLA OPPIMISEN RAJOJA."* (5.17).

Selkeästi erityyppisen tulkintakehyksen muodosti opiskelumotivaatioon ja -orientaatioon liitetty kuvailu. Opiskelijat kiinnittivät huomiota erilaisiin kiinnostuksen kohteisiin ja motivaatioihin, eritasoisiin ennakkotietoihin ja -taitoihin - *"Uuden oppiminen vaatii monesti aiemmin opetettavien asioiden sisäistämistä. Jokainen lapsista ei ole välttämättä oppinut aiemmin opetettuja asioita. Luokan oppilaiden "Lähtötason" selvittäminen onkin mielestäni tärkeää ja opetuksen tulee lähteä tästä liikkeelle. Myös esimerkiksi kiinnostus aineeseen ja motivaatio oppimiseen voi olla monella kadoksissa. Kiinnostuminen ja motivaatio on kuitenkin oppimisen taustalla, joten innostava ja havainnollinen opetustyyli on tärkeää."* (43.15). Myös oppilaiden oppimistyylien huomioimista pidettiin tärkeänä luonnontieteiden opetuksessa - *"…Toiset oppivat mm. kuuntelemalla ja toiset lukemalla, kolmas puolestaan käytännön esimerkeillä ja tutkimuksilla. Tällöin pitää mahdollisesti tarjota kaikkia mahdollisia oppimismetodeja, josta löytyy mahdollisesti jokin sopiva tapa/ sopivien tapojen yhdistelmä kaikille."* (156.15).

Vähiten tuentarvetta liitettiin kulttuurisiin ja oppilaan kasvun taustoihin liittyviin näkökulmiin. Vain seitsemän opiskelijan mielestä opetustilanteissa tulisi huomioida suomen kielen osaamiseen liittyviä ymmärtämisen ongelmia - *"… maahanmuuttajaoppilailla voi olla vaikeuksia luonnontieteiden opetuksessa käytettävän sanaston kanssa, sillä monet sanat voivat olla heille täysin uusia…"* (99.16). Lisäksi opiskelijat huomioisivat arvoihin, elämänkatsomukseen ja uskonnolliseen taustaan liittyvät seikat - *"Aatteellisesti erilainen oppija, jonka kotona ei esimerkiksi uskottaisi tieteisiin tai evoluutionteoriaan, tarvitsee varmasti paljon tukea asioiden monipuoliseen ymmärtämiseen ja vaihtoehtojen tarkasteluun…"* (50.18). Yksittäisten opiskelijoiden mielestä opettajan tulisi huomioida opetustilanteissa sukupuolisuus tai oppilaan sosioekonominen tausta ja kodin mahdollisuudet tukea lapsen koulunkäyntiä - *"…Kotiolot ja asuinympäristö rajoittavat esimerkiksi kasvien keräämistä kotitehtävänä."* (100.17).

3.2 Ympäristöopin ominaispiirteisiin liittyvät tuen tarpeet

Opiskelijat tarkastelivat oppilaiden tuen tarpeita myös oppiaineen eli ympäristöopin ominaispiirteiden näkökulmasta. Opettajaopiskelijoiden mielestä oppilaat tarvitsevat tukea luonnontieteiden oppisisältöjen oppimiseen. Heidän mukaansa tukea tarvitaan yleensä käsitteellisiä ja teoreettisia sekä vaikeita käsitteitä ja asioita opiskeltaessa (Taulukko 2).

Taulukko 2. Opiskelijoiden käsitykset ympäristöopin ja sen opetuksen ominaispiirteisiin liittyvistä tuen tarpeista

Tuen tarpeet	**Käsitys tuen tarpeesta ympäristöopin opetuksessa**
1 Oppiaineiden sisältöihin liittyvät tuentarpeet (136)	1.1 Käsitteelliset ja teoreettiset asiat yleensä (23)
	1.2 Vaikeat käsitteet ja asiat (113) •Abstraktit käsitteet (28) •Mikrotason asiat (9) •Syy- ja seuraussuhteet (4) •Looginen päättely (1) •Oppiainekohtaiset vaikeat käsitteet ja asiat (58)
2 Opetustapahtumaan liittyvät tuen tarpeet (263)	2.1 Opiskeluprosessi (193) •Käsitteiden oppiminen ja ymmärtäminen (57) •Luetun ymmärtäminen (34) •Käsitteiden muistaminen ja ulkoa opettelu (30) •Laajojen kokonaisuuksien hahmottaminen (28) •Symboliikka ja kaavat (17) •Laskutehtävät (10) •Tietojen soveltaminen (6) •Kirjalliset tehtävät (5) •Luonnontieteellisen kielen ymmärtäminen (2) •Itsenäinen työskentely (2) •Ongelmanratkaisu (1) •Tehtävien alkuun- ja loppuun saattaminen (1)
	2.2 Opetusmenetelmät (64) •Käytännön harjoitukset, oppilaiden tutkimustyöskentely ja projektityöskentely (39) •Yhteisöllinen oppiminen (16) •Opettajajohtoinen opetus (6) •Opetusmenetelmästä toiseen siirtyminen (3)
	2.3 Erilaiset oppimisympäristöt (6) •Luonnon ympäristöt (5) •Uudet ympäristöt (1)

Opiskelijoiden mielestä luonnontieteissä on erityisen paljon abstrakteja käsitteitä ja mikrotason asioita, joita ei voi konkreettisesti nähdä - *”… Luonnontieteissä on paljon abstrakteja "ei-käsinkosketeltavia" käsitteitä, kuten sähkö, painovoima ja mikrobiologia.”* (86.16). Lisäksi luonnontieteiden opiskeluun liittyvien kognitiivisten prosessien, kuten syy- ja seuraussuhteiden ymmärtämisen ja loogisen päättelyn arveltiin tuottavan ongelmia oppilaille - *”Tukea tarvitaan erilaisten käsitteiden ja ilmiöiden yhdistämisessä, syy-seuraus suhteiden jäsentämisessä mm. mannerlaattojen liike joka johtaa maanjäristykseen ja tätä kautta mahdollisesti tsunamiin tai tulivuorenpurkaukseen…”* (115.17).- Opiskelijoiden mainitsemia vaikeina pidettyjä biologian ja maantieteen asioita olivat muun muassa lajintuntemus, kartanluku, yhteyttäminen ja ravintoketjut. Vastaavasti vaikeita fysiikan ja kemian aihealueita olivat esimerkiksi voima, reaktiot, valo, planetaarisuus, avaruus, molekyylit, ilma, kitka ja sähkö.

Opettajaopiskelijoiden mukaan oppilaiden tuentarpeet liittyvät myös opiskeluprosessiin sekä luonnontieteiden opetukselle ominaisten opetusmenetelmien että oppimisympäristöjen käyttöön. Haasteellisiksi opiskeluprosessin osiksi nimettiin käsitteiden, luonnontieteellisen kielen sekä luetun tekstin oppiminen ja ymmärtäminen - *"Luonnontieteissä on paljon käsitteitä ja ilmiöitä, joiden ymmärtäminen on oppimisen perusta. Näissä asioissa jotkin oppilaat voivat tarvita tukea."* (5.16). Vaikeina asioina pidettiin myös käsitteiden muistamista ja ulkoa opettelua sekä laajojen kokonaisuuksien hahmottamista - *"Luonnontieteiden opiskelussa on tyypillisesti hankalia ja pitkiä oppikirjan tekstejä, jotka voivat tuottaa hankaluuksia. Nimien opettelu ulkoa voi myös olla vaikeaa."* (39.15) - Lisäksi luonnontieteiden opiskelulle ominaisten symbolien ja kaavojen, laskutehtävien sekä tiedon soveltamisen arveltiin tuottavan oppilaille ongelmia - *"Tukea tarvitaan erityisesti erilaisten kaavojen omaksumisessa ja niiden hyödyntämisessä käytäntöön. Ei siis opeteltaisi kaavoja vaan siksi, että ne täytyy osata, vaan pyrittäisiin syvällisempään ymmärtämiseen ja opittaisiin, mikä niiden idea ja tarkoitus on…"* (99.15). Yksittäisten opiskelijoiden mukaan haastavia opiskeluprosessiin kuuluvia asioita ovat muun muassa kirjalliset ja ongelmanratkaisuun pohjautuvat tehtävät sekä itsenäisen työskentelyn - *"…Kokeet voitaisiin tehdä muuten kuin kirjallisina jos se tuottaa ongelmia, ettei omien taitojen osoitus jäisi siitä kiinni."* (53.16).

Luonnontieteiden opetukselle ominaisten opetusmenetelmien käyttö voi lisätä tuen tarvetta. Tällaisia opiskelijoiden mainitsemia opetusmenetelmiä olivat käytännön harjoitukset (kokeelliset tehtävät, maasto-opetus, oppilaiden tutkimustyöskentely) ja projektityöskentely - *"Erilaiset oppijat tarvitsevat apua esimerkiksi kokeiden tekemisessä. Se, että jokainen pääsee perille käsiteltävästä aiheesta ja tajuaa tehtävänannon, on tärkeää. Esimerkiksi tutkittaessa jotakin kemiallista reaktiota jokainen oppilas tulisi perehdyttää välineisiin ja saada tehtävän alkuun. Esimerkiksi henkilöllä, jolla on keskittymisen kanssa ongelmia, voi olla vaikea kuunnella ohjeita ja siksi tehtävänano voi mennä aluksi ohi."* (84.16). Edellä mainittujen opetusmenetelmien käytössä tukea arveltiin tarvittavan erilaisten aineiden ja tutkimusvälineiden käsittelyssä, havaintojen tekemisessä, työskentelyturvallisuudessa, ohjeiden noudattamisessa sekä keskittymisessä - *"… Jos esim. fykessä tehdään kokeita, pitää turvallisuuteen kiinnittää erityistä huomiota, jos luokassa on ylivilkkaita/käytöshäiriöisiä lapsia, ettei oppimistilanne muutu vaaralliseksi. Oppilaiden huomio pitäisi saada pysymään aiheessa, koska erilaisissa kokeiluissa huomio voi kiinnittyä helposti epäolennaisuuksiin."* (30.15).

Myös tiedon hankinta, tutkimusongelmien asettaminen, tutkimusmenetelmien valinta, mittaaminen, tulosten tulkinta, itsenäinen työskentely ja monista työvaiheista suoriutuminen nähtiin oppilaille haasteellisina prosesseina - *"Tukea tarvitaan mietittäessä mikä on ongelma, millä eri keinoin sitä voisi ratkaista ja mihin lopputuloksiin päädytään. Mitä voimme päätellä loppuratkaisusta? Avustavat kysymykset ja johdattelu." (151.15).* - Edellisten lisäksi yhteisöllisen oppisen käytön - *"… Luonnontieteiden opetuksessa oppilas voi tarvita tukea sosiaalisissa taidoissa. Opetuksessa tehdään paljon ryhmätöitä ja tutkitaan yhdessä…"* (34.17) - ja opettajajohtoisen opetuksen - *"Luentomuotoinen opetus (opettaja opettaa taululla edessä ja oppilaat kirjoittavat) voi olla monille oppilaille haastavaa. Tällaisessa tilanteessa esim. Oppimisvaikeuksiset lapset tarvitsevat tukea, apua ja vaihtoehtoisia opiskelumuotoja…"* (85.15) - arveltiin lisäävän oppilaiden tuen tarvetta.

Opiskelijoiden mukaan myös erilaisissa ja uudenlaisissa oppimisympäristöissä opiskelu vaatii oppilaiden opiskelun tukemista. Eniten opiskelijoiden vastauksissa mainittiin koulun ulkopuolisia oppimisympäristöjä, kuten luonto ja metsät - *"… Jos ei ole helppoa löytää havaintoja ympäristöstä, tai esimerkiksi muuttuva oppimisympäristö vain häiritsee oppimista (esim. metsässä liikkuminen ja tutkiminen mikä ei sisällä rutiineja)."* (4.15)

4 Pohdinta

Opiskelijoiden vastauksista ilmeni, että enemmistö opiskelijoista tarkasteli oppilaiden diversiteettiä ongelmakeskeisestä näkökulmasta, jolloin luonnontieteiden opetuksen tuen tarpeet nähtiin liittyvän johonkin diagnosoituun oppimisvaikeuteen. Monet oppimisen esteet eivät kuitenkaan ole ensisijaisesti oppilaan persoonaan sidottuja eikä edes diagnosointi aina johda suoriin pedagogisiin johtopäätöksiin. Luonnontieteiden opetuksen näkökulmasta onkin tärkeää, että opettaja tunnistaa mahdollisemman laajasti oppimisen esteet ja pystyy pedagogisesti lapsen omassa toimintaympäristössä tukemaan kaikkien oppilaidensa oppimista (vrt. Aronen ym., 2005; Opetushallitus 2014; Villanueva ym., 2012). Opiskelijoiden käsityksistä ei vielä tässä vaiheessa opintoja löytynyt laajemmin kuvauksia tuentarpeen moniulotteisuudesta. Esimerkiksi lahjakkaan lapsen mahdollinen tuentarve näkyi aineistossa vain hyvin kapeasti (vrt. Foley-Nipcon ym., 2013.) Opettajakoulutuksessa olisikin syytä varoa yksipuolista ongelmakeskeistä oppilasdiversiteettitarkastelua, ja koulutuksen lähtökohtana tulisi olla laajan diversiteetin tunnistaminen ja kaikenlaisten oppilaiden oppimisen tukeminen (Ainscow ym., 2006; Kiuppis & Hausstätter, 2014, Kousa & Aksela, 2019; Slee, 2001).

Opiskelijoilla oli myös varsin kattava käsitys siitä, että oppilasryhmässä voi olla eritavoin orientoituneista oppilaita. Opiskelijoiden mukaan luonnontieteiden opetuksessa tulisi huomioida esimerkiksi oppilaiden motivaation tukeminen ja ennakkotietämys. Luonnontieteiden opetuksen tuleekin tarjota oppilaille mahdollisuuksia tuoda esiin omaa osaamistaan ja arkielämän kokemuksia sekä tarjota myönteisiä kokemuksia kaikille oppilaille (Villanueva ym., 2012). Opiskelijoiden melko laajasti mainitsemat oppimistyylit olivat jossain määrin hämmentävä havainto. Oppimistyyli voi toki olla arkikieltä ja siten sisältää kuvausta oppimisen monenlaisuudesta, mutta sillä voidaan myös viitata oppimistutkimuksessa kritisoituun käsitteistöön, jonka toimivuus on vähintäänkin ristiriitaiseksi osoitettu (Pashler, McDaniel, Doug Rohrer, & Bjork, 2009; An & Carr, 2017).

Opiskelijoiden vastauksissa oli vain yksittäisiä mainintoja sukupuolesta ja sosioekonomisesta asemasta. Koska näillä on havaittu olevan merkitystä luonnontieteiden opintomenestykseen, opiskeluasenteisiin ja urasuuntautumiseen sekä ulkomailla (Gorard & See, 2009¸ Greenfield, 1997) että Suomessa (Bernelius, 2011; Kupari ym., 2012; Vettenranta, Hiltunen ym., 2016) tulisi mainittuja näkökulmia käsitellä myös opettajankoulutuksessa. Opettajaopiskelijat eivät myöskään osanneet nimetä kattavasti kulttuurisia ja arvopohjaisia tuentarpeita. Heidän oma koulukokemuksensa on voinut olla hyvin kansalliseen valtakulttuuriin perustuva ja toisaalta monikulttuurisuutta käsittelevät opintojaksot eivät vielä olleet heidän opinnoissaan alkaneet. Kulttuuritaustoihin perustuvien maailmankuvien eroavuuksiin tunnistaminen on kuitenkin tärkeä huomioida monikulttuurisessa kouluyhteisössä (vrt. Lee & Buxton, 2010; Opetushallitus, 2014; Piliouras & Evangelou, 2012).

Opettajaopiskelijat tarkastelivat oppilaiden tuen tarvetta myös oppiaineiden sisältöihin ja opetustapahtumaan liittyen. Opiskelijat mainitsivat useita erilaisia opiskeluprosesseja (käsitteiden oppiminen ja muistaminen, luetun ymmärtäminen, symboliikka jne.) ja opetusmenetelmien käyttötilanteita (kokeellisuus, maasto-opetus, oppilaiden tutkimustyöskentely jne.), joissa oppilaat voivat tarvita tukea. Opiskelijoiden havaitsemat tuentarpeita paljastavat tilanteet ovat juuri niitä, joissa heidän tulisi oppia tarjoamaan tukea lapsille. Opiskelijoiden käsitykset olivat kuitenkin osin ristiriidassa aiempien tutkimustulosten kanssa. Opiskelijat esimerkiksi nimesivät tuentarvetta tuottaviksi samoja opetusmenetelmiä, joita tutkimus (ks. esim. Mastropieri ym., 1998; Therrien, Taylor, Hosp, Kaldenberg & Gorsh, 2011) esittää ratkaisuksi oppilaiden tukemiseen. Opetussuunnitelman ja tutkimusten mukaan luonnontieteiden oppimista tukee opiskelu- ja opetusmenetelmien sekä oppimisympäristöjen monipuolinen käyttö (vrt. Ainscow, Booth, & Dyson, 2004; Kousa, 2019; Opetushallitus, 2014; Sormunen ym., 2019; Varelas, Kane, & Wylie, 2011; Villanueva ym., 2012). Opettajaopiskelijat eivät vastauksissaan käyttäneet käsitettä ”tutkiva oppiminen”; tätä selittänee se, ettei heillä vielä opintojensa alkuvaiheessa ollut ymmärrystä kyseisestä käsitteestä. Toisaalta syynä voi olla se, että he pysyttäytyivät vastauksissaan melko traditionaalisissa ja oman osaamisen kannalta ”turvallisissa aktiviteeteissa” (ks. Appelton & Kindt, 2002). Opettajakoulutuksessa tulisikin tarkastella luonnontieteiden opettamista positiivisesti ainepedagogisesta näkökulmasta kysymällä, millaisia mahdollisuuksia oppiaineelle tyypilliset opiskeluprosessit, opetusmenetelmät ja oppimisympäristöt voivat tarjota erilaisille oppijoille (vrt. Varelas ym., 2011). Lisäksi koulutuksen tulisi sisältää tutkimuksellista tietoa siitä, millaisten asioiden ja taitojen oppiminen ovat haastavia luonnontieteiden opetuksessa: esimerkiksi luonnontieteellisen tiedon luonne (Osborne & Dillon, 2010), kieli (Evagorou & Osborne, 2010; Mason & Hedin, 2011) ja matemaattinen esitystapa (vrt. Brigham ym., 2011).

Opiskelijat käyttivät runsaasti terminologiaa ja käsitteitä erilaisten oppimisvaikeuksien nimeämisessä. Osin terminologia oli toisaalta hyvinkin epätarkkaa, mutta käytössä oli myös määriteltyjä diagnoosinimikkeitä. Tästä ei kuitenkaan voida tehdä suoraan johtopäätöksiä siitä, tuntevatko he käyttämiensä käsitteiden tai diagnoosinimikkeiden eksakteja sisältöjä ja määritelmiä. Myös opiskelijoiden pedagogisten ilmaisujen ja käsitteiden käyttö vaihteli. Kaiken kaikkiaan tutkimuksemme osoitti, että opiskelijoiden käsityksien kirjo oli laaja. Tulosten perusteella opiskelijoiden käsitys oppilasdiversiteetistä on melko laaja pitäen sisällään perusopetuksen oppilaat monipuolisesti ymmärrettyinä yksilöinä. Luonnontieteiden ainedidaktiikan tulee sisältää myös oppiaineen kannalta keskeisten opiskelutaitojen ohjausta, kun tarkastellaan oppilaiden oppimista luonnontieteiden tunneilla.

Tutkijoina asetimme itsellemme tutkimustehtävän, jonka mielekkyyttä pitää myös arvioida eettisesti. Jos tavoitteenamme opettajankoulutuksen tutkijoina on kehittää opettajuutta ja tutkimusta kohti inklusiivisen koulun käytänteitä, niin miksi ylipäätään tarvitaan tuen tarpeen luokittelua tai edes abstrakteja kuvitteellisiin oppilaisiin liitettäviä luokittelunimikkeitä (vrt. Ahtiainen, Lintuvuori, Hienonen, Jahnukainen, & Hautamäki, 2017)? Pidämme kuitenkin tärkeänä opiskelijoiden pedagogisten käsitysten kartoittamista. Näiden käsitysten tunnistaminen ja nimeäminen on opettajankoulutuksen näkökulmasta tarpeellista, jotta opiskelijoiden havainnoista päästään keskustelemaan. Opiskelijoiden tulisi ymmärtää opetuksen lähtökohdaksi myönteinen suhtautuminen oppilasdiversiteettiin (Ainscow ym., 2006; Tolsdorf ym., 2018; Kugelmass, 2001).

Luokittelu on ongelmallista myös tutkimuksellisesti: Luokitellessaan ja luokkia nimittäessään tutkijat myös tuottavat käsityksiä. Kun käsitteitä tuotetaan, niiden tulisi olla lähtökohtaisen inklusiivisen viitekehyksen mukaisia. Toisaalta asiasisällön jakamisen näkökulmasta käytettyjen käsitteiden tulisi olla ymmärrettävissä myös inklusiivisen viitekehyksen ulkopuolelta asiaa tarkasteleville. Tässä aineistossa opiskelijoiden tuottamat oppilaaseen liitetyt käsitykset pohjautuvat osin medikalistiseen ajatteluun ja tuen syy sijoitetaan oppilaaseen tai hänen ominaisuuksiinsa. Toisaalta käsitystä avartavat opiskelijoiden vastauksista ilmenevä käsitteiden moniulotteisuus sekä oppiaineeseen liitetyt näkökulmat. Opettajaopiskelijoiden edellä mainitut käsitykset tuen tarpeista ovat samankaltaisia kuin Kousan (2019) opettajia koskevassa tutkimuksessa.

Tuen tarpeita on tärkeää tutkia eri oppiaineiden opetuksen ja oppimisen konteksteissa, koska oppiaineet ovat luonteeltaan erilaisia – tuen tarpeiden ja tukimuotojen voidaan siis ajatella olevan erilaisia eri oppiaineissa. Eri oppiaineissa käytetyt opetusmenetelmät ja oppimisympäristöt sinänsä voivat tuottaa tukea inklusiivisessa koulussa. Opetuksen inklusiiviset lähtökohdat edellyttävät opettajaopiskelijoilta yhteistyö- ja verkostoitumisvalmiuksia viimeistään työelämässä. Tällöin opetusmenetelmät laajenevat kohti yhteisopettajuuden eri toteutustapoja (esim. Bešić, Paleczek, Krammer, & Gasteiger-Klicpera, 2017) ja koulun arki laajemminkin muuttuu lasten kasvua eri tavoin tukevien ammattilaisverkostojen jakamaksi (Ben-Yehuda, Leyser, & Last, 2010). Tämän vuoksi on tärkeää aloittaa eri opettajaryhmien keskinäinen yhteistyö jo opettajankoulutuksen aikana (ks. Malinen, Väisänen, & Savolainen, 2012).

Opettajankoulutuksen näkökulmasta olisikin tärkeää, että erityispedagoginen näkökulma ja inklusiivinen kouluajattelu ovat ”diffuusisti” mukana opettajankoulutuksen opinnoissa (vrt. Sharma & Sokal, 2015). Yksi keskeinen jatkotutkimusalue olisikin inkluusioperiaatteiden pedagoginen soveltaminen eri oppiaineiden opetuksessa ja opettajankoulutuksessa.

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Project-based learning in integrated science education: Active teachers' perceptions and practices

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Project-based learning (PBL) is a promising teaching method for integrated science education that has gained momentum in educational research and curriculum reforms, especially as a method to enhance 21st century skills and connected worldview. How teachers implement PBL greatly affects students' content understanding and development of skills. The purpose of this qualitative study is to highlight active teachers' PBL practices and their perceptions of the advantages and challenges of implementing PBL to better promote the implementation of PBL in teacher education programs and in integrated science education. This study consisted of two parts: (1) a qualitative-led survey and (2) a case study. First, the data for the survey was collected from January to March 2017 through an online reporting form of an international StarT programme. This programme supports the implementation of interdisciplinary and collaborative PBL in science, mathematics and technology education. 244 teachers from early childhood education to upper secondary school participated from 28 countries. Second, 12 PBL units reported by the teachers were chosen for a case study. The teachers exploited PBL practices that were theme- and inquiry-based, collaborative and engaging to students. However, closer inspection revealed variation and defects in the practices particularly in relation to assessment, using reflection and student-centred approach. In addition, teachers reported several challenges relating to the implementation of PBL. The results indicate that teachers see PBL as beneficial but need support with the implementation. Science teachers' pedagogical competence in PBL could be promoted through collaborative learning in which students, teachers and other participants are learning from each other.

Keywords: project-based learning, science education, teachers' practices, STEAM

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1 Introduction

Project-based learning (PBL) has a lot of potential to enhance 21st century skills and engage students in real-world tasks (e.g. Bell, 2010; Han et al., 2015; Kingston, 2018). It promotes interconnected worldview, links among disciplines and presents an expanded view of subject matter (Blumenfeld et al., 1991; Kingston, 2018). Therefore, PBL is a promising teaching method for integrated science education that can be defined as an effort to organize or integrate science curriculum content into a meaningful whole by a constructive and context-based approach that crosses subject boundaries and links learning to real world (Åström, 2008; Beane, 1997; Czerniak & Johnson, 2014). Integrated science education has traditionally meant integration with mathematics, and/or technology, such as STS (science–technology–society) or STEM (science–technology–engineering–mathematics) education (Bennett et al., 2007; Czerniak & Johnson, 2014). In recent years, there has been an increase in discussion of a wider approach to integrated science curriculum by including other discipline areas, for example, a move to STEAM education by including the Arts to STEM (Lyons, 2020).

The successful implementation of PBL in a classroom is dependent on teacher's ability to effectively motivate and guide students' learning (Kokotsaki et al., 2016) as well as on teacher's understanding of the criteria for effective PBL (Han et al., 2015). In relations to integrated science education, there exists evidence that when PBL is implemented and instructed properly by teachers, students' learning increases, whereas teachers who ineffectively implement PBL have a negative effect on students' learning (Han et al., 2015; Kingston, 2018). However, the lack of a uniform vision of PBL complicates efforts to determine the fidelity of a PBL unit and to evaluate its effects (Condliffe et al., 2017; Hasni et al., 2016). This imposes a concern as many current national curricula (Finnish National Agency for Education [EDUFI], 2016; National Research Council, 2013) are urging teachers to implement more integrated and inquiry-based approaches, such as PBL (Hasni et al., 2016). How are science teachers supposed to assess the quality of their implementation or to know how to improve their practices, if there is no consensus on what a PBL approach in integrated science education should look like?

The purpose of this study is to describe teachers' perceptions and practices of PBL to understand how it can be implemented with fidelity as an integrated approach to science education. The teachers participating in this study are considered active and motivated to develop their teaching as they have voluntarily taken part in the international StarT programme (LUMA Centre Finland, n.d.), which supports the implementation of interdisciplinary and collaborative PBL in science, mathematics and technology education.

2 Theoretical background

2.1 Project-based learning

Project-based learning (PBL) is a student-driven, teacher-facilitated pedagogical approach that organizes learning around clearly defined projects (Han et al., 2015; Kokotsaki et al., 2016; Thomas, 2000). PBL has roots in constructivist theories of learning: learning is context-specific, learners actively construct their understanding by engaging in meaningful real-world issues, and they achieve their goals through social interactions and sharing of knowledge and understanding (Kokotsaki et al., 2016; Krajcik & Shin, 2014; Savery, 2019).

Similar instructional strategies exist such as problem-based learning, inquiry-based learning and "Learning by Design" (Savery, 2019), and there is some debate in the literature, especially about the distinction between project- and problem-based learning. Scholars acknowledge that the two concepts have different histories (Condliffe et al., 2017), but have argued for seeing problem-based learning as a type of project-based learning (Boss & Larmer, 2018; Thomas, 2000). On the contrary, Savery (2019) argued that it is important to clarify the differences between the two concepts since, unlike problem-based learning, project-based learning requires constructing a concrete artefact as an answer to the driving problem or a question.

Many attempts have been made to clarify the PBL design principles that describe the essential components of a PBL approach. There exist a wide agreement that PBL is a process of learning; including activities and inquiry that results in artefacts or final products that address the driving question or a problem set at the beginning (Blumenfeld et al., 1991; Boss & Larmer, 2018; Condliffe et al., 2017; Thomas, 2000). However, there is still no consensus on what constitutes PBL (Condliffe et al., 2017). For example, PBL can be emphasized as interdisciplinary (e.g. Han et al., 2015) or according to others (e.g. Savery, 2019) projects may also be disciplinary specific. Furthermore, it is unclear whether PBL design principles should address the content of learning, to what extent students' choice or collaboration needs to be included in PBL approach or how learning should be assessed (Condliffe et al., 2017). In Table 1 is a synthesis on design principles adapted from three reviews (Condliffe et al., 2017; Kokotsaki et al., 2016; Savery, 2019) discussing the issue and with recommendations for the essential elements to be considered when designing and implementing PBL.

Table 1. The project-based learning (PBL) design principles adapted from Condliffe et al. (2017), Kokotsaki et al. (2016) and Savery (2019).

Element	**Description**
Student learning goals	The content of a PBL curriculum or study unit that ensures the successful implementation of PBL as a part of science teaching. The project should be focused on teaching students (1) key concepts and understanding derived from national curriculum or standards, and (2) subject matter content as well as 21st century skills (e.g. critical thinking, problem solving, collaboration and self-management).
Centrality of the project	This feature distinguishes PBL from other instructional approaches: project is not the culmination of learning as it often is in standard classrooms, but instead in PBL approach the project is seen as a process through which learning takes place.
Context	Projects should be authentic, meaningful, related to a real-world context or an important issue, and be connected to students' own concerns and interests. Furthermore, projects require a well-designed and open-ended driving question or a problem, at the appropriate level of challenge for students, that serves to organize all the project activities.
Project artefact	Project activities should involve the creation of a final tangible product that addresses the driving question and offers representation of students learning.
Collaboration	PBL requires social negotiation of knowledge, working collaboratively in groups, to develop possible solutions to the project. Collaboration should be a feature of all project stages.
Construction of knowledge	PBL involves students in a process of constructing knowledge. This can be achieved through in-depth inquiry, critical thinking, the use of problem-solving, and by revision of what is currently known and what needs to be understood before proceeding.
Student engagement	Teachers should foster student engagement from the beginning of the project to the end. Students' freedom to generate project artefacts and their active participation is vital for the construction of knowledge. Although encouraging student choice align with student-centred approaches, it is not explicitly clear what the extent of student autonomy should be in a PBL unit.
Scaffolding instruction	Scaffolding instruction refers to any method or a resource (e.g. teachers, peers, learning materials and technologies) used by teachers to help learners to accomplish more difficult tasks than they otherwise are capable of completing on their own. Two key elements of scaffolding: (1) scaffolds need to be tailored to a student's current level of understanding and (2) scaffolds should be faded over time as students learn to apply their new knowledge or skills on their own.
Assessment	Emphasis should be on formative assessment that aims at supporting students learning. This includes reflection, self and peer evaluation, and teachers' feedback throughout the project process. Assessment should include a specific end-of-project phase that ensures reflection on what was learned as well as the creation of a project artefact.
Publicity	A public presentation of the project supports students' communication skills, can motivate students, and presents an opportunity for feedback. Instead of a presentation, the product itself can be public. This element includes the PBL criterion of authenticity.

2.2. PBL in integrated science education

PBL has a lot of potential to enhance 21st century skills and engage students in real-world tasks (Bell, 2010; Han et al., 2015; Kingston, 2018). The 21st century skills or transversal competences (EDUFI, 2016) are common denominators for various skills necessary for success in daily life, such as critical thinking, problem-solving, collaboration, communication, and self-management skills (EDUFI, 2016; Viro & Joutsenlahti, 2020). Of these skills, for example, problem-solving is closely linked to mathematics and inquiry is an essential part of science education. However, skills alone are not enough as objectives of a PBL unit in integrated science education; students need to develop their knowledge and understanding of the key concepts of science, mathematics and other integrated subjects (Viro & Joutsenlahti, 2020). Research evidence indicates that PBL can promote student learning in acquiring deeper content knowledge and skills in science and mathematics (Condliffe et al., 2017; Kingston, 2018; Viro & Joutsenlahti, 2020). In addition, some studies have reported increased attendance, self-reliance, and improved attitudes towards learning on the part of students (Kingston, 2018; Thomas, 2000). Furthermore, evidence suggests that teachers regard PBL as beneficial for both teachers and students (Condliffe et al., 2017).

Researchers have identified common implementation challenges that relate to the design principles of PBL. These include teachers' knowledge, skills and attitude related issues such as (1) teachers' resistance to student-centred learning, (2) confusing inquiry-based instruction with hands-on activities, (3) inability to motivate students to work in collaborative teams, (4) scaffolding instruction, and (5) the development of authentic assessment (Condliffe et al., 2017; Mentzer et al., 2017; Viro et al., 2020). Furthermore, melding required curriculum with PBL is one of the most important but difficult aspects of designing a project-based approach (Condliffe et al., 2017). Other challenges relate to students' resistance to employing critical thinking (Mentzer et al., 2017), unsatisfactory group working (Condliffe et al., 2017), lack of motivation (Condliffe et al., 2017; Marshall et al., 2010) and readiness for student-centred approaches in integrated science education (Han et al., 2015). In addition, teachers struggle with time constraints and inadequacy of resources to support in-depth student investigations needed for constructing knowledge (Condliffe et al., 2017; Viro et al., 2020).

Externally developed PBL curricular units for science education, such as Project-Based Inquiry Science (Kolodner et al., 2015) and Investigating and Questioning our

World through Science and Technology (Shwartz et al., 2008), have been developed in recent years to answer the challenges. Both were inspired by the project-based science design principles of Blumenfeld et al. (1991) and Krajcik and Shin (2014), which emphasize driving questions, collaborative student-led inquiry, the use of technology to scaffold student learning, and the creation of authentic artefacts. The common thread of PBL elements that runs through these two middle school science curricula demonstrates the importance of connecting concepts, research, and practice (Condliffe et al., 2017; Kolodner et al., 2015; Shwartz et al., 2008). However, the externally developed curricula can be overly prescriptive, and most teachers do not have access to them (Condliffe et al., 2017). As a result, PBL continues to be mostly designed and implemented by teachers on their own.

Teachers' self-perception and conceptualization of teacher roles have a fundamental impact on teachers' implementation of PBL (Habók & Nagy, 2016). Han et al. (2015) state that teachers' role in implementing STEM related PBL must differ from the traditional classrooms. Changing teachers' beliefs about their role in a classroom from that of director to facilitator is one of the main implementation challenges for student-centred pedagogical approaches like PBL (Ertmer & Simons, 2006). In addition, teachers' beliefs about their students' potential can also influence PBL implementation (Condliffe et al., 2017). In relation to integrated science education, the evidence suggests that teachers' understanding and implementation of PBL affects learning outcomes (Han et al., 2015; Kingston, 2018).

Viro et al. (2020) investigated teachers' views on PBL in mathematics and science. The results were somewhat varied. The development of teamwork skills and the connection between theory and practice were both deemed highly important characteristics of PBL. Other elements of PBL, that teachers perceived positively, were its contribution to students' motivation and mathematics learning. However, teachers expressed contradictory views on PBL: (1) it was irrelevant for mathematics, and (2) it hinders organizing, scheduling and teamwork. Furthermore, teachers perceived PBL negatively because it was an unfamiliar method to them (Viro et al., 2020).

3 Methodology

The main question that guided this qualitative study from the start was: How can PBL be implemented in integrated science education? This question was further divided into three sub-questions:

1. What are teachers' perceptions on the advantages of implementing PBL?
2. What are teachers' perceptions on the challenges of implementing PBL?
3. What kind of elements of PBL are incorporated in teachers' practices reported to the StarT programme?

The study consists of two parts. First, a qualitative-led survey (Braun et al., 2017) was administered through an online reporting form of the StarT programme. Second, a case study (Cohen et al., 2007) was made to have a more in-depth understanding of teachers' practices reported to the StarT programme.

3.1. Context of the study

StarT is an international programme organized annually by LUMA Centre Finland and for the first time in the 2016–2017 school year. The aim of StarT is to support integrated science, mathematics and technology education by collaborative PBL from early childhood education to upper secondary school. The PBL approach of StarT includes broad themes (e.g. Mathematics around us, Nature and environment, Well-being, and Stars and space) to help teachers and students focus their project activities. There are five requirements and a recommendation for StarT projects:

1. The project is multidisciplinary and linked to science, mathematics or technology.
2. The project is carried out in a team of students.
3. The project is a product of the students' work, showing their expertise and making use of their own interest and creativity.
4. The project includes a learning diary that outlines what students have learned during the project.
5. The project results in a final artefact that is visualized by a short video.
6. It is recommended that students are given a chance to present their project publicly. In addition, project descriptions, videos and diaries are published as examples on the webpage of StarT.

In other respects, the StarT programme gives teachers autonomy to design their own PBL units (e.g. the form of the artefact, the length of the project or the subjects to be incorporated).

3.2. Data collection

Data was collected from January to March 2017 through the StarT online reporting form in Finnish and in English. The participants are considered active teachers who are motivated to develop their teaching, as they have voluntarily taken part in the international StarT programme and designed and implemented their own PBL unit in science, mathematics and technology education. For StarT programme, participants were asked to report PBL practices and activities implemented during 2016 or 2017. In addition, participating teachers were asked to answer the survey questionnaire. Out of 275 teachers, 244 participated in the research: 99 Finnish teachers and 145 teachers from 27 other countries, mainly from Europe. Teachers represented various levels of education, from early childhood education to upper secondary schools. Teacher distribution, according to the taught level of education, for Finnish teachers was 13% in early childhood education, 57% in primary schools, 24% in secondary schools and 6% in upper secondary schools. The taught level of education was lacking in many reports of international teachers.

Based on the reports, twelve PBL units were chosen for the case study to examine how the design principles of PBL were incorporated in teachers' practices. The data included project descriptions, videos, photographs, and diaries as well as teachers' descriptions of their practices related to carrying out StarT projects. The selection of cases was done according to two criteria: 1) the PBL unit had a comprehensive report and 2) the sample would include various project examples with different StarT themes and from different education levels and countries. A short description of the chosen cases is given in Table 2.

Table 2. The twelve project-based learning (PBL) units included in the case study. Description includes the country, the level of education, the StarT theme, the length of the PBL unit and the number of projects done by student groups.

CASE	Country	Level of education	Number of projects	StarT theme	Length of the PBL unit
1	Lithuania	Primary school (4th grade)	1–5	Everyday mathematics	Time spent on project was spread across the school year
2	Indonesia	Lower secondary school (8th grade)	1–5	Technology around us	4 weeks
3	Greece	Lower secondary school	1–5	Programming and robotics	Not specified
4	Romania	Upper secondary school	6–10	Stars and space	Not specified
5	Portugal	Upper secondary school	6–10	Well being	From December 2016 to March 2017
6	Turkey	Upper secondary school (16-year-old)	1–5	Nature and environment	Not specified
7	Spain	Secondary school	1–5	Everyday mathematics	Not specified
8	Hungary	From early childhood education to secondary schools	1–5	Nature and environment	A week, working daily
9	Belgium	Primary school (5th grade)	1–5	Programming and robotics	Six or seven 'sessions' during a month
10	Finland	Primary school (5th grade) and lower secondary school	1–5	Stars and space	Time spent on project was spread across the school year
11	Finland	Lower secondary school (9th grade)	1–5	Nature and environment, Technology around us	Two or three lessons (45 minutes) per integrated subject; approximately 10 lessons
12	Lithuania	Upper secondary school	over 15	Everyday mathematics	Multiple project events organized during the school year

3.3. Data analysis

As the aim of this study is to explore and understand how PBL can be implemented in integrated science education, a qualitative content analysis (Drisko & Maschi, 2015; Mayring, 2014) was chosen as an analysis method both for the qualitative-led survey and the case study.

A qualitative content analysis with an explorative design (Mayring, 2014) was used in the survey. It included a data-based, inductive category formation of teachers' (n=244) answers to open-ended questions mapping, (1) teachers' experiences with PBL in StarT, (2) teachers' perceptions of the main advantages of implementing PBL, and (3) teachers' perceptions of the main challenges of implementing PBL. Teachers' answers were first coded with a partial sample (99 Finnish teachers) and tested with two inter-raters. Cohen's kappa coefficient was chosen to test the reliability as it is used for assessing agreement between two raters on a nominal scale. Once the reliability was considered good (>0.60), the coding was done for the whole sample. The material not relevant for answering the research questions were omitted from the analysis. The final inter-coder reliability was k=0,79 for the categories of advantages of PBL and k=0,82 for the challenges of PBL.

The analysis technique followed in the case study was an interpretive, theory-driven content analysis (Drisko & Maschi, 2015; Mayring, 2014). It aimed at describing the elements of PBL incorporated in the 12 PBL units chosen as cases (see Table 2). An effort was made to ensure reliability of the case study through a careful analysis of the versatile data reported by teachers, and by checking the intra-coder reliability throughout the analysis process. The reliability could be improved by an inter-coded reliability.

4 Results

4.1. Advantages of PBL in practice

Teachers viewed PBL as having multiple advantages that are shown in Table 3. Especially, teachers (60,7%) valued PBL for its possibilities for learning. Often these answers referred to learning in general such as "we learned a lot more than we initially thought we would" (teacher F3), or the answers related to students' increased skills (e.g. group working, social interaction and problem-solving skills) as well as to students learning how to use equipment or programs; often related to making videos. Fewer comments related to the learning of subject content knowledge, and only a couple of teachers mentioned students having a more interconnected view as a learning advantage. Mostly learning was regarded from student's point of view, but in some instances learning included everyone involved in the project, students and teachers alike.

Table 3. Teachers' views on the advantages of project-based learning (PBL).

Advantages of PBL	Teachers					
	Finnish (99)		Other (145)		All (244)	
	n	n (%)	n	n (%)	n	n (%)
Learning outcomes	62	62,6	86	59,3	148	60,7
Skills	37	37,4	43	29,7	80	32,8
Increased awareness	2	2,0	27	18,6	29	11,9
Collaboration	57	57,6	74	51,0	131	53,7
Motivation	56	56,6	41	28,3	97	39,8
Student-centredness	47	47,5	44	30,3	91	37,3
Versatility for education	36	36,4	49	33,8	85	34,8

Many teachers (53,7%) valued collaboration and a sense of community generated by the practice. Collaboration with other teachers or classes was found useful in practice:

> More experienced teachers oriented the less experienced teachers and always supported them. (Teacher I29)
>
> Projects unified the whole school and added communality and "we" atmosphere. (Teacher F2)
>
> Collaboration between classes of different age students was enjoyable and important. (Teacher F24)

In addition, the wider collaboration possibilities offered by StarT or collaboration with other interest groups were found fruitful as a support for implementation or because of the opportunities for public presentation of the projects:

> The idea [of StarT] is interesting because it is an opportunity for our high school to highlight our activities and share them with others. (Teacher I2)
> Belonging to a bigger unity has given structure to our project. The educators have had an opportunity to get peer support and ideas to own project. (Teacher F103)
>
> Students demonstrated their work to their parents. The parents are proud of them. (Teacher I21)

> The greatest experience was that the students have learned to work in teams not only with mates but also with parents, grandparents, teachers. (Teacher I84)

> Kids share their knowledge and tell everyone (to friend, father or mother, kids from other class and all schools community) about their project and what they have learned during this StarT project. (Teacher I52)

Some reported collaboration as a benefit because it created more joy or positive attitudes that can have an effect on the motivation of students and teachers. This was evident in the answers highlighting motivation as the main benefit. Motivational aspects of PBL related to positive attitude change, building self-esteem, relevance, enthusiasm and getting excited or engaged in project working. Most answers related to enthusiasm.

> The enthusiasm for project-based work was very infectious and initiated an actual snowball effect as the idea to pick Aronia berries for juice developed into a diverse market day! (Teacher F11)

In the student-centred learning category, most cases were about students being active learners. In addition, comments related to group working and taking different learners or students' interests into account. Versatility in education was a more heterogenic category compared to the others. This category included all cases with new possibilities for implementing curriculum and using versatile teaching methods and learning environments.

Finnish teachers' views differed somewhat from the teachers in other countries. Finnish teachers regarded the student-centred nature of PBL as one of the main benefits or even the most useful element of PBL, whereas a minority of teachers from other countries mentioned this element as a benefit. On the contrary, they seemed to view the usefulness more from the perspective of teaching and regarded the versatility of education as one of the main benefits.

4.2. Challenges of the PBL in practice

Teachers' views on the challenges (see Table 4) of implementing PBL were more coherent than views on the advantages of PBL.

Table 4. Teachers' views on the challenges of implementing project-based learning (PBL).

Challenges of PBL	**Teachers**					
	Finnish (99)		Other (145)		All (244)	
	n	n (%)	n	n (%)	n	n (%)
Facilitating PBL	62	62,6	81	55,9	143	58,6
Time management	33	33,3	26	17,9	59	24,2
Project facilitation	30	30,3	51	35,2	81	33,2
Teachers skills	12	12,1	9	6,2	21	8,6
Structural issues						
Technical	35	35,4	16	11,0	51	20,9
Resources	26	26,3	10	6,9	36	14,8
Interactional issues						
Student-related	23	23,2	30	20,7	53	21,7
Collaboration	20	20,2	10	6,9	30	12,3

Facilitating PBL was considered the main challenge in most responses. Besides facilitating the project work, this included notions relating to time management or laborious planning. In addition, responses were linked to teachers' self-efficacy or their perceived skills to facilitate PBL, even if this was not explicitly voiced.

> Believing in yourself [was a challenge for me]. I took this as a big challenge to experience and learn something new and I exposed myself to learning a new teaching method. (Teacher F90)

> Doing [projects] raises feelings of insecurity on whether this is away from something important and have we fulfilled the subject content required by curriculum. The most difficult part was to manage the time. We had lots of thoughts to discuss and sort out the information to improve and get the best project in two months. (Teacher I99)

Technical issues include not only the challenges faced with using different technological tools, but also issues with the documentation for StarT such as "making video with non-existent ICT skills" (Teacher F30). The second structural issue related to resources or the lack there of; mainly teachers were lacking space, ICT equipment and time.

Student-related challenges revealed that teachers were having motivational issues with students, as either it was difficult "getting different learners engaged into active learning and working" (Teacher F34), or students were so engaged that they "were working much more than was needed to fit into the curriculum" (Teacher I152). In some cases, students lost their interest during the project work, as was reported by Teacher F71: "The schedule was too heavy, and some students got tired and started to go it alone. It has taken the time of the adults involved in the project to motivate these students shirking their duties." Furthermore, teachers reported issues with scaffolding instructions "in balanced proportions, so that you don't restrict students too much but give opportunities and offer tools" (Teacher F52) and with students' inadequate skills and knowledge.

> The most challenging was to find suitable action that suited the students' skills. (Teacher F35)
>
> The most challenging part of our project was that it took time for the students to realize their potential because they had never taken part in similar projects before. (Teacher I13)
>
> Working in pre-set groups is not easy for everybody. (Teacher F55)

Possibilities for collaboration were reported as limited, mainly because teachers had trouble with finding the time for planning with colleagues.

Overall, Finnish teachers reported more challenges with collaboration, time management and technical issues than teachers from other countries. Language was not a challenge to Finnish teachers. However, this most likely is because Finnish teachers could report in Finnish or Swedish. Teachers from other countries all reported in English.

4.3. Teachers' PBL practices

Teachers' practices analysed in this study seemed initially to meet most of the PBL design principles. All projects featured the elements specified in StarT project requirements. However, in closer inspection, the variation and shortcomings of the implementations became evident. The overall results are gathered in Appendix 1.

Student learning goals. Mainly the learning outcomes set for the projects related to 21st century skills such as communication, collaboration, problem-solving and thinking skills. However, in 75% of the cases, teachers reported learning goals related to subject contents or cross-curricular concepts, especially socio-scientific issues (SSI).

> We develop a pro-active, dynamic, open-minded attitude, valuating the creative potential and the personal experience of each individuals involved in the project (students, teachers), as well as their high order abilities and cross-curricular capacities, their learning, research, thinking, communication, cooperation, working, adaptability competences, namely the 21st century competence. (Teacher, case 4)

> The main objective is to study the mathematical properties of the mosaics (tessellations of the plane), in addition to the way in which they have been constructed (using twirls, symmetries, translations, ...). (Teacher, case 7)

Centrality of the project. All reported practices and projects had elements indicating PBL being viewed as a learning process rather than a simple project product to indicate previously learned. Some teachers specified the process steps and others emphasized the link between project practice and subject content in design. However, teachers' responses were contradictory. For example, in case one the learning goal focused on skills and "applying the acquired knowledge of mathematics", indicating the project is perhaps seen more like a rehearsal. This was also a very teacher-led project: the teacher set all specific learning activities that accumulated into workshops and assignments to be used at a special event for the school and parents.

Contextual. This category was divided into three subcategories specifying how the context had been taken into account in the practices:

1. Projects that had a driving question or a problem
2. Projects based on a common theme or a topic
3. Projects linked to real-world

In most cases (11 out of 12), context was created by a theme that could be directly taken from the themes of StarT programme such as "Mathematics around us" or themes related to real-world issues such as climate change or gender equality. Less than half of the cases (5 out of 12) had set a driving question. Some questions were driven from students' interest and engaged in inquiry and investigation such as "How can we solve the problem of not being seen as a pedestrian on the dark roads?" (case 2). However, some of the driving questions were not open-ended or engaged in inquiry. For example, "Why is Mars called a red planet?" (case 3) enables copying the answer directly from Wikipedia. In this case, teachers give students the autonomy to choose their own questions. However, with some guidance, the question could have been revised into one that engages in inquiry and focuses the project activities towards the main goal of learning programming.

Project artefact. As it was a requirement of StarT programme, in all projects a product or products were made. These were for example booklets, posters, written reports, crafted products with electronics, videos, songs, exhibitions, and workshops for other students and parents. In most cases, students had the freedom to choose what was created. In four projects, the artefact to be created was set by the teachers in preplanning phase.

Collaborative learning. This category was divided into three subcategories to specify the nature of collaboration. In all projects, the students worked in groups, as this was a requirement of the StarT programme. 66,7 % of the projects were interdisciplinary and included collaboration with teachers of different subjects. Furthermore, collaboration was done with other classes and different education levels as well as with experts and organizations.

> Our Science Festival was called “What About Geology?”... As the teams investigated and studied the subject, the group of cicerones, with my help, drew the space that would be our festival (four in total). The collaboration of the community was essential: the city council provided transportation for kindergarten students [to the festival], the military provided and set up four tents and two large awnings, the school staff helped in the construction of some scientific models and in the placement of large structures, gym teachers assisted in the supervision... National geoparks, science centres and biosphere reserves were present; the university experts trained the students in the areas that were being investigated. (a teacher, case 5)

Constructive nature of PBL was featured in all projects except one. This was mainly concluded from the aims teachers and students had set for their working. To support the construction of knowledge, teachers’ practices included for example lecturing, using assignments or mind-maps, and collaborating with experts. In many cases, projects were based on inquiry and investigations that included gathering of information from various sources. Discussions and brainstorming sessions were used within the group, with the teacher or with the whole class. Furthermore, with projects focused on building a concrete artefact, students tested possible solutions and built prototypes.

The full potential of the learning diary was not utilized, as it was evident that in most cases the diaries were written after the project was finished, therefore, serving more as reports. Only one project (case 9) clearly used the diary as a tool for reflection and assessment with students writing daily about the progress of the project and their group working.

Student engagement was taken into consideration in all cases. The practices varied from teacher-led projects (four out of 12), to student-centred projects (three out of 12). Most cases had elements of both with teachers setting the aims and frame for working, and students making their choices within the frame. Teachers' practices to engage and activate students included discussions, brainstorming, hands-on activities, quizzes and study visits. In addition, participation in contest and events, such as offered by StarT, can be seen as an engaging practice.

Scaffolding instructions were not specifically mentioned by teachers, but project reports included various elements of teachers' practices to support and guide student working. For example, in case four, teachers set a clear timeframe with deadlines for the students' project working, and in case twelve, a Facebook group was set up to ask questions and give project updates.

Assessment. Only a few teachers mentioned aspects of assessment in their reports. From the data gathered, it was mostly impossible to draw any conclusion about the assessment of the project.

Publicity was mentioned in the recommendations of StarT programme and all projects were presented publicly. Mainly this was done in schools with other classes, teachers, and school staff as audience, but some projects participated in local events or organized one such as an exhibition in library by themselves. Furthermore, the project products in five cases were public by nature. For example, publicly distributed videos and websites were created.

5 Discussion

5.1. Teachers' perceptions of the advantages of PBL

National curricula, standards and many researchers promote PBL as a potential method for integrated science education and for learning the 21st century skills. The class, science and mathematics teachers participating in this study share this positive perception of the advantages of PBL. Especially for learning science and mathematics-related skills such as problem-solving, inquiry and critical thinking. Teachers regarded increased motivation, collaboration and educational versatility among the main advantages of PBL. These results relating to teachers' perceptions of the advantages of PBL are consistent with earlier findings (e.g. Han et al., 2015; Kingston, 2018; Viro et al., 2020). Interestingly, few teachers mentioned as an advantage the promotion of interconnected worldview that has been highlighted in literature (Blumenfeld et al., 1991; Czerniak & Johnson, 2014), especially in relation to PBL being an integrated approach to science education.

Finnish teachers' perception on the advantages of PBL varied from teachers from other countries. Finnish teachers regarded as a major benefit the student-centred nature of the PBL, whereas international teachers emphasized more the versatility to education. This can perhaps be explained by the nature of StarT programme, as in Finland it includes collaborative events, science and technology festivals, for both students and teachers to share their learning and practices. Whereas international StarT programme was focused on the competition.

5.2. The PBL design principles of teachers' practices

To have successful outcomes, PBL implementations must meet design principles (see synthesis in Table 1) that are still under some debate. This lack of a uniform vision of PBL still continues to complicate the efforts to determine the quality of a PBL unit and to evaluate its effects (Condliffe et al., 2017; Hasni et al., 2016). Results of this study indicated that teachers' PBL practices seemed to meet most of the key elements. However, in closer inspection the inadequacy and variation of the implementations became clear.

First, the amount of student autonomy varied from teacher-led activities with little student choice to complete student autonomy in relation to the execution of projects. In general, students' involvement was minimal or even absent in setting the learning aims, overall theme, schedule and assessment of the project. This raises the question whether all PBL practices meet the criteria for successful PBL in the first place, as student choice is a key element of the PBL approach (e.g. Bell, 2010; Boss & Larmer, 2018; Kokotsaki et al., 2016).

Second, half of the cases in the case study had not set a clear driving question or a problem to focus students' inquiries and motivate learning. Instead, the PBL activities and artefact created were based on a common theme. It should be noted that the broader theme allowed in some cases more freedom for the students to choose over the direction of their own project, and it is possible students set specific questions or problems even though this was not brought out in the material teachers or students shared with the StarT programme. In any case, a clearly set driving question is argued as an essential criterion by many researchers (e.g. Blumenfeld et al., 1991; Boss &

Larmer, 2018; Condliffe et al., 2017) and a lack thereof can have an effect on the learning outcome.

Third, the construction of knowledge was further lacking as many projects seemed to lack the critique and revision phases. Partly this can be because of the difficulties to manage time that was mentioned by many teachers as one of the main challenges. The critique or revision was mainly done in two phases of a project process: (1) in the beginning to assess what is known and what needs to be learned, and (2) while presenting the project and artefact at the end of the project process.

Fourth, perhaps relating to the previous, only a couple of cases referred to formative assessment as being a part of the PBL unit, even though most teachers had set specific learning goals for students' projects. PBL should not merely be a supplementary activity that supports learning; the project should be central in the learning process (Boss & Larmer, 2018; Condliffe et al., 2017; Thomas, 2000), and assessment should be formative by nature to include students' entire learning process. In addition, the full potential of project diaries as a learning aid and an assessment tool was not taken advantage of as many reported to have written the diaries after the project process, only as a part of the reporting to StarT programme. However, one should not generalize this observation, as there was not enough evidence about the assessment included in the PBL units. It is possible that teachers did not feel the need to report about their assessment to the StarT programme, as assessment was not stated in the guidelines for StarT projects nor in the assessment criteria for the StarT competition. Could it be that teachers regarded participating in StarT as being only motivational, adding versatility to their education, and not as being part of the science education curriculum? Regardless, to be feasible in science education PBL should include the learning of curriculum concepts through a project (Bell, 2010; Savery, 2019; Viro & Joutsenlahti, 2020) and these curriculum-related contents should be included in the assessment of the project.

5.3. Teachers' perceptions of the challenges of PBL

Teachers' practices and professional competence for implementing PBL have an effect on the challenges teachers face while implementing PBL. Major challenges, according to this study, are facilitating PBL and the lack of time (similar to Mentzer et al., 2017). Often teachers referred as a challenge the planning time with colleagues or the time-consuming nature of project work in general. The latter is an issue teacher can facilitate, as are many of the challenges teachers reported (see Table 3). Thorough and careful planning is essential to the flow of the project and the success of students (Bell, 2010). Unfortunately, teachers are reporting that they do not have sufficient time for planning, and it can have a direct effect on the implementation as well as on teachers' and students' experiences during the PBL unit.

Interestingly, Finnish teachers reported more challenges compared to teachers from other countries. Internationally, StarT is mainly a competition, and this could have had an effect on the reports by international teachers and their desire to portrait their own work in as positive light as possible. However, PBL is a new approach to Finnish teachers; can it be that their inexperience with PBL is showing in these results? On the other hand, based on the cases analysed in this study, the Finnish projects were more collaborative and student-centred, which could explain the greater amount of faced challenges. Earlier research has indicated that the culture and educational system has an influence on teachers and their teaching approaches. Further research comparing different countries, cultures and educational systems is needed to answer these questions.

6 Conclusions

Teachers are in a pivotal position in transferring PBL into integrated science classroom practices that are commended by many national science curricula and reforms. How teachers perceive and implement PBL greatly affects learning outcomes. The aim of this qualitative study was to explore and understand teachers' perceptions and practices. The results are based on teachers' reports to the StarT programme. Efforts were made to ensure the reliability of the results through a careful analysis of versatile data as well as checking the intra- and inter-coder reliability. However, the researchers had minimal opportunity, only through videos and photographs, to observe the actual implementations of the PBL units analysed in the case study. Even though the results cannot be generalized, they add to our understanding of teachers' perceptions of PBL and PBL design principles for integrated science education.

The results of this study indicate that teachers have a general idea of PBL and its advantages. Nevertheless, even the implementations of active teachers who voluntarily share their practice and participate in a competition seem to be lacking in certain key elements, such as assessment or the critique and revision phase. In addition, the results indicate and support earlier findings on the challenges teachers face when implementing PBL. The structural challenges reported in this and earlier studies (e.g. Viro et al., 2020; Mentzer et al., 2017) are hindering schools and teachers' efforts to implement PBL in integrated science education and should, therefore, be taken into account on a national level, when reforming curriculum or standards recommending integrated approaches such as PBL. Teachers can partly overcome the challenges relating to facilitating PBL with more experience and a deeper understanding of the PBL method. To this end, we have two recommendations. One, the academic discussion and research to further clarify the PBL design principles should continue to achieve a consensus on PBL as a method for integrated science teaching. For example, PBL design principles should address the content of learning to guarantee the inclusion of core concepts and skills of integrated subjects. Second, teachers need education programmes that support their pedagogical competence in executing PBL in integrated science education.

The results could be taken carefully into account in preparing teacher education for pre-service and in-service teachers. Without adequate attention to ways of supporting teachers, these innovative educational approaches will not be widely adopted (Blumenfeld et al., 1991; Mentzer et al., 2017). Integrated approaches such as PBL also require substantial changes in teachers' thinking about and dispositions toward classroom structures, activities, and tasks (Han et al., 2015). Furthermore, as it can take even two to three years for teachers to shift their understanding and learn to use PBL in practice (Mentzer et al., 2017), there is a need for developing long-term or even continuous and collaborative models for teacher education. Some teachers in StarT found collaborative learning and being a part of an international community professionally useful. Therefore, StarT in itself could be seen as a novel model for continuous teacher education programme in which:

1. Teachers' pedagogical development occurs while facilitating PBL and working together with the students, other teachers and other collaborators.
2. Teachers have access to tested models for PBL and good teaching practices from other teachers as well as online instructions and training.
3. Participating teachers and schools are a part of the StarT community, where learning is shared through workshops, science fairs and online voting for best projects as well as best teaching practices.

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Appendix

Appendix 1: The Elements of project-based learning (PBL) per case and examples of teaching practices related to the elements.

Elements of PBL	Cases												Science teaching practices related to the element
	1	2	3	4	5	6	7	8	9	10	11	12	
Learning goals	1	n/a	1	1	1	1	1	1	1	1	1	1	To apply and learn subject knowledge; to understand the relationships between phenomena; to improve thinking, communication and team-working skills; to train creativity and rigour; to improve self-esteem and motivation; to raise awareness of an issue related to the project (e.g. climate change and gender equality); to grow up to be responsible citizens.
Subject content	1	n/a	1	0	1	0	1	1	1	1	1	1	
Skills	1	n/a	n/a	1	1	1	0	1	1	0	0	1	
Centrality of the project	1	n/a	1	1	1	1	1	1	1	1	1	1	Setting project framework or steps of the process; integrating project working into subject teaching by choosing a convenient theme and allocating sufficient lesson time for projects; including versatile teaching activities to bridge theory and practice; supporting inquiry
Contextual	1	1	1	1	1	1	1	1	1	1	1	1	Teachers set a theme or a problem beforehand that can be incorporated into subject teaching; setting a common theme or problem together with students; deriving a theme or a problem from the local context; giving students freedom to choose their own driving question or a problem from a common theme.
Driving question	0	1	0	0	1	1	0	n/a	1	0	1	0	
Theme-based	1	1	1	1	1	1	1	1	1	1	1	1	
Real world	0	1	0	0	1	1	1	1	1	1	1	1	
Project artefact	1	1	1	1	1	1	1	1	1	1	1	1	Booklets, brochures, posters, written reports, crafted products with electronics, videos, quizzes, songs, workshops for younger students and exhibitions.
Collaborative learning	1	1	1	1	1	1	1	1	1	1	1	1	Whole class discussions and brainstorming, working in small groups of 3 to 5 students or in pairs, collaboration of multiple classed (same grade or different graded) and different subjects, collaboration with organizations or companies (special material, expertise), public presentation for larger audience (whole school, parents, at events).
Group work	1	1	1	1	1	1	1	1	1	1	1	1	
Interdisciplinary	1	0	1	1	0	n/a	0	1	1	1	1	1	
Other	1	0	1	1	1	1	0	1	1	1	1	1	

Constructive Investigation Critique and revision	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 1	0 0 0	1 1 1	1 0 0	1 1 1	Establishing the aim and tasks, gathering information by student (using schoolbooks and online resources) or given by teachers or outside experts (lectures, demonstrations, and assignments), discussing and analysing the problem (within group, with teacher and/or the whole class), students test possible solutions (building a prototype, taking measurements), writing project diaries, making mind-maps.
Student engagement Student-centred Teacher-led	1 0 1	1 1 0	1 1 1	1 1 1	1 1 1	1 1 0	1 0 1	1 1 1	1 0 1	1 0 1	1 1 1	1 1 0	Group formation by teachers or students; students involvement in choosing a theme, aims, project artefact and how to work and create the artefact varied from teacher-led to autonomous group work by the students; often teachers set the frame for the project and students work autonomously inside the frame. Teachers engage students by discussions, brainstorming, activities (hands-on, games or quizzes), participating in contests and study visits.
Scaffolding instruction	1	n/a	0	1	n/a	n/a	0	1	1	n/a	1	1	Diversifying learning assignments and projects, giving theory lessons related to the project topic, setting the project phases and schedule, guiding towards a source of information (books, online material, experts), providing needed resources, asking guiding questions, making it possible for students to help each other and ask questions.
Assessment Project artefact Student reflection Feedback	1 n/a 1 n/a	n/a n/a n/a n/a	n/a n/a n/a n/a	1 1 1 n/a	n/a n/a n/a n/a	n/a n/a n/a n/a	n/a n/a n/a n/a	1 n/a 1 1	1 n/a 1 1	n/a 1 n/a n/a	n/a 1 n/a n/a	1 1 1 1	Students reflect on their work (what was successful, what did not work, what they learned) for example in project diaries or during presentations. Presentations, class discussion and forms are used as opportunities for peer and teacher feedback, school teachers and other experts asked to evaluate project presentations, quizzes relating to project theme, voting for best project artefacts, participating in contest
Publicity Public presentation Public product	1 1 0	1 1 0	1 1 1	1 1 0	1 1 0	1 1 1	1 1 0	1 1 0	1 1 1	1 1 0	1 1 1	1 1 1	Organizing fairs or exhibitions (school, library, local events) with oral presentations, posters and stands to present the project and artefact, oral presentations in classroom, building a website for the project, making a short video and other online applications, writing a magazine, making a brochure.

Primary education degree programs in Alicante, Barcelona and Helsinki: Could the differences in the mathematical knowledge of incoming students be explained by the access criteria?

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This perspective paper draws on the interest in ensuring that students who enter primary teacher training programs have a solid background knowledge of mathematics. We describe the access criteria and requirements for admission to the primary education degree programs at the Universidad de Alicante and Universitat Autònoma de Barcelona, in Spain, and the University of Helsinki, in Finland. We present the results of an evaluation of the mathematical knowledge that students bring to their education as teachers at these three institutions. The results show that in each program, the subgroup of students who had followed the longer track of mathematics courses scored significantly higher on the mathematical test, although this was no longer as clear when we compared across universities. We also found that the students who had taken the mathematics section of the entrance examination or the matriculation examination scored higher on the test than those from the same program who had not, but this tendency broke down when cross-university comparisons were made. We also explored how the cap set on the number of students admitted to the three programs – this being the most striking difference in the admission policies – could be an explanatory variable for these discrepancies. The comparison between universities leads us to hypothesize that expecting applicants to have met certain requirements in their academic trajectories prior to university entrance and adjusting the cap set on the number of places could ensure a better mastery of mathematical knowledge among those students admitted to the Spanish programs.

Keywords: primary teacher education, university access criteria, university admission requirements, mathematical knowledge, background mathematical knowledge

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1 Introduction

It is the responsibility of education degree programs to prepare quality teachers to serve society. Given this responsibility, such programs must have valid and reliable measures for screening candidates. The admission requirements of these programs should include an assessment of the following: the candidates' prior knowledge of the content of the different subjects they will later have to teach; their reasoning, problem-solving and critical thinking skills; their adherence to social ethics; and their dispositions to teaching (Miller-Levy, Taylor, and Hawke, 2014).

As educators of mathematics teachers, we have no doubt about the importance of primary teachers being sufficiently prepared to help children start growing mathematically. However, several studies have shown that many prospective teachers are admitted to and graduate from teacher education programs with insufficient knowledge to teach mathematics effectively (Beswisk and Goos, 2012; Hine, 2015; Ingram and Linsell, 2014; Lo and Luo, 2012; Norton, 2018; Tatto et al., 2008 and Qian and Youngs, 2016). In our paper, we focus on the evaluation of students' mathematical content knowledge at the beginning of primary education degree programs, taking as variables the distinct criteria and admission requirements of the programs.

Few studies address the characterization of the mathematical knowledge taken as a starting point on primary education degree programs or how far the admitted students' background mathematical knowledge is from this starting point. Furthermore, we have no knowledge of studies, except those carried out by our own research group (Gorgorió and Albarracín, 2019; Gorgorió, Albarracín, and Laine, 2019), that analyze possible relationships between the background mathematical knowledge of those enrolling on a primary education degree program and the screening criteria under which they were admitted to the programs. Our research at two universities in Spain, the Universidad de Alicante and the Universitat Autònoma de Barcelona, and one university in Finland, the University of Helsinki, was intended to provide an initial contribution to this analysis. The purpose of this study was to explore whether we can build a hypothesis about how changing admission criteria and requirements could lead to an improvement in the initial mathematical knowledge of students entering primary teacher training degrees in Spain.

Little has been accomplished in terms of determining how the institutional characteristics and national context in which teacher education takes place, together with the students' individual characteristics (Blömeke and Delaney, 2012), could influence the development of mathematical knowledge for teaching during teacher education. Comparative international studies focus on the organization, curriculum, processes, and results of teacher education (Li, 2012; Tatto et al., 2008), examining the structure and characteristics of programs and the knowledge and beliefs of student teachers at the end of their preparation (Blömeke and Delaney, 2012). Even less attention has been paid to the criteria and requirements for admission to teacher education programs, even though these are factors – together with the quality of learning opportunities and the methods by which student teachers are taught – that influence the development of their mathematical knowledge for teaching.

One of the characteristics to which little attention has been paid is the mathematical knowledge that students bring to teacher education programs, and this despite the fact that their background mathematical knowledge should be considered a powerful predictor of their success on such programs in terms of the development of mathematical knowledge for teaching. While studies like the *Teacher Education and Development Study in Mathematics* TEDS-M (Tatto et al., 2008) inform us about the quality of teacher education programs, and the mathematical knowledge that students have at the end of their teacher education, they provide us with less information about the entry requirements for teacher education programs, particularly regarding prior mathematical knowledge.

In their analysis of how opportunities to learn on teacher education programs influence future primary teachers' mathematics knowledge, Qian and Young (2016) suggest the need to account for selection effects within and across countries when investigating candidates' knowledge. However, the assessment of the mathematical knowledge of program applicants, either as a precondition for admission or for diagnostic and formative purposes, has rarely been the subject of research and, for the most part, the few studies that examine the topic have essentially limited themselves to describing the knowledge that students demonstrate (Linsell and Anakin, 2012). Moreover, exactly what prior mathematical knowledge should be considered desirable and what theories and methods might be used to characterize it has so far only been spelled out rather tentatively.

The lack of theoretical and methodological support implies that the evaluation of the mathematical background of students by teacher educators is currently a rather challenging undertaking (Linsell and Anakin, 2012), which is further hampered by a scarcity of research on the relationship between institutional criteria and admission requirements and the mathematical knowledge with which students enroll on the programs. This absence of a thorough theoretical debate about what constitutes desirable prior knowledge in students entering a teaching program runs parallel with the coexistence of multiple models for admission.

Admission criteria and entrance requirements, which differ across countries, reflecting both their social and cultural contexts, contribute to delineate candidates' individual characteristics, thus conditioning the opportunities to learn from which they can benefit. Comparative studies between countries that include an assessment of the prior knowledge that students bring to teacher education programs could help to explain the differences in the subsequent development of their knowledge during and upon completion of these programs. The same would be true of any comparison of how universities in different countries go about deciding how many and which applicants will be granted admission. International comparisons focusing on the background mathematical knowledge of students initiating a primary education degree program and how it relates to admission criteria and requirements should make it possible to identify variables and issues that may remain unquestioned in national studies, providing the opportunity to go beyond the familiar.

In this study we explored the mathematical knowledge evidenced by a test administered to students admitted to three teacher-training programs, taking as variables different factors linked to the admission criteria and requirements. Our data came from two universities in Spain, the Universidad de Alicante – in the region of Valencia – and the Universitat Autònoma de Barcelona – in the region of Catalonia, and one university in Finland, the University of Helsinki. In both Spain and Finland, initial teacher education is university-based. At the time we collected our data, none of these institutions had access requirements to their primary education degree programs that explicitly addressed the mathematical knowledge of candidates, but they did have different general admission criteria and requirements that could favor applicants with more or less solid background mathematical knowledge.

By admission criteria, we mean those aspects that are taken into consideration when establishing recruitment policies, either from the point of view of the desired characteristics of candidates admitted or from the perspective of the costs and benefits of teacher education (Davies et al., 2016). The criteria might refer, for example, to prior training, to the candidate's personal characteristics, and his/her academic trajectory or age, among others. For each of these admission criteria, different requirements could be established as compulsory. Moreover, it is important to analyze whether the way in which the number of places on each of the programs is decided generates groups with different mathematical background knowledge.

The purpose of this research was to explore whether we could generate a hypothesis about how the modification of admission criteria and requirements might lead to an improvement in the initial mathematical knowledge of students enrolling for teacher training degrees in Spain. To this end, we engaged in a three-stage study where we worked on each stage to achieve a specific goal related to the general goal. It is interesting to note that the results obtained in each stage, by way of a response to the specific goal of that stage, suggested the need for the study undertaken in the following stage. The results corresponding to the third goal indicate the need for further work in this area of research.

With this idea in mind, the goals that guided the three stages were:

1. to study whether there were any significant differences between the three primary education degree programs as regards the background mathematical knowledge of admitted students;
2. to examine whether there were significant differences in the students' background mathematical knowledge between groups of students who have followed different academic mathematics trajectories – the mathematics courses selected during secondary school, or having taken the mathematics section of the matriculation examination or the university entrance examination – within or across the three institutions;
3. to explore whether changing the cap set on the number of students admitted to the Alicante and Barcelona programs, taking the program at Helsinki as a reference, could affect the observed differences between the background mathematical knowledge of the students at the different institutions.

To simplify the reading of the text, from now on, we will refer to the primary education degree programs at Alicante, Barcelona and Helsinki as ALI, BCN and HEL, respectively.

2 Why Alicante, Barcelona and Helsinki?

The proposal presented in this document derives from a real concern of a group of mathematics educators involved in the initial training of primary school teachers. The first two authors, lecturers on BCN, were members of the project "Estudi per a l'avaluació diagnòstica de les competències matemàtiques dels estudiants del grau en Educació Primària"[1] – *Study for the diagnostic evaluation of the mathematical competences of Primary Teaching degree's students*, developed as part of a program to improve teacher training in Catalonia. The results obtained within the framework of this project prompted them to seek the collaboration of the other two authors, lecturers on ALI and HEL, for the reasons detailed below.

[1] Funding body: AGAUR, Agència de Gestió d'Ajuts Universitaris i de Recerca, 2014 ARMIF-00041, Generalitat de Catalunya.

Over the last four decades, the training of primary school teachers in Spain has evolved towards their professionalization, abandoning an approach closer to vocational training in favor of university studies (Sancho-Gil, Sánchez-Valero, and Domingo-Coscollola, 2017). However, as a legacy of the period in which a craft-oriented vision of teaching was predominant, access to primary education degree programs remains, in practice, almost universal, as will be seen when we describe the candidate selection process. In Spain, all university degree programs have the same general framework of admission requirements, which is mandated by the Spanish central government. Students are allocated to the degree of their choice based on their ranking in the final list of access scores. However, the number of places available for incoming students on ALI and BCN differs, as does the number of applicants, which can result in successful applicants with different academic profiles at both universities. For this reason, we considered that it was worth exploring whether there were any differences in the background mathematical knowledge of those students entering ALI and BCN.

Since the results of the first PISA Study were published in 2000, there has been widespread international recognition of the success of Finland's educational model. Though many factors no doubt contribute to this success, the most important is believed to be the quality and competence of its teachers (Sahlberg, 2011). In Finland, where the teaching profession is highly valued (Niemi, 2016), there are specific selection processes for access to teacher education programs, during which candidates must demonstrate strong academic abilities and a passion for teaching. Darling-Hammond (2017) establishes that the recruitment of highly capable candidates into high-quality teaching degree programs is one of the Finnish practices that might be considered a strategy for improving teaching. In Finland, the policies and admission requirements of primary teaching programs are different from those in Spain. However, at the time the data was collected for this study, no specific requirement in relation to mathematics was included in either case. Thus, when comparing ways to access teacher education programs, it seemed interesting to include one from Finland. Moreover, places on HEL are considerably more restricted than on ALI and BCN. Thus, we found it interesting to include HEL in the study.

Below, we describe the access requirements and criteria for admission to the primary teaching degree programs in Spain and Finland, and we trace how mathematics is or is not part of the admitted candidates' academic trajectories.

3 Admission to the primary education degree programs and the role of mathematics in the process

In Spain, for admission to a primary teaching degree (4 years, 240 ECTS), students must have completed compulsory education (up to 16) – during which all of them will have received a minimum[2] of 1050 hours of formal preparation in mathematics. From this point on, they access the degree program by taking one of two paths that are different from the standpoint of their mathematical education: a) either the science and technology track or the social sciences track of the Bachillerato (the two years of non-compulsory, pre-university secondary education, hereinafter referred to as the baccalaureate), during which students complete a mathematics course each year – amounting to a minimum of 280 hours – under the rubrics Mathematics and Mathematics for Social Sciences respectively; or b) the humanities or arts tracks of the baccalaureate, or any vocational training cycle related to education, none of which include courses in mathematics. Therefore, only some of the students admitted to primary education degree programs will have studied mathematics beyond compulsory education.

Spanish universities admit candidates according to two factors: the number of places available to students on a particular degree program and their access score. To be admitted to a university degree program, baccalaureate graduates must pass the university entrance examinations (Pruebas de Acceso a la Universidad). Among other content, the Spanish university entrance examinations include four mandatory subject-area-specific papers and a fifth compulsory language paper in autonomous communities with their own language, which examines this co-official language. Examples are Catalan in Catalonia and Valencian in the Community of Valencia. For those who have completed a social sciences baccalaureate, one of the four compulsory papers is Mathematics for social sciences, while the mandatory exam for those completing a science and technology baccalaureate is entitled simply Mathematics. Students who have completed a vocational training cycle may also gain access to university, based exclusively on their grades, although they may decide to take some university entrance exam papers in the hope of raising their access score. Therefore, only some of the students admitted to primary education degree programs will have

[2] Catalonia and the Valencian Community have a certain degree of educational autonomy and this allows them some flexibility in defining their compulsory education programs, always on the basis of the national legal framework. This autonomy is reflected by the fact that the number of hours of mathematics taught may exceed the nationally established minimum.

taken one of the two university entrance mathematics exams. On the other hand, in Spain the number of course places on the different university teacher education programs bears no relation to the number of teaching jobs in the market, and at the time that data was collected for this study, there was no specific requirement for admission to teacher education programs outside the universal procedure described above.

Despite the inflexibility of the Spanish system, which establishes general access criteria for all degrees at a national level, there is a loophole that makes it possible to introduce complementary criteria for admission to a particular degree in one or more institutions – the so-called personal aptitude tests (PAP). In this way, a university, when offering a particular degree program, may require applicants to pass a personal aptitude test as a complementary admission requirement. Thus, the only way to influence the selection of applicants for a particular degree is that the competent authorities of the Autonomous Community and the central government accept the personal aptitude tests as an additional requirement established by the university. Degrees in translation and interpreting, sports sciences and some foreign languages have socially recognized and accepted personal aptitude tests. At the time the data was recorded for this study, none of the Spanish teacher training programs required a pass in a personal aptitude test. However, in Catalonia, by agreement among all the public and private universities, a personal aptitude test consisting of a mathematics exam and a Catalan language exam was introduced in the 2017-2018 academic year for admission to their teacher training courses.

Finnish teachers – both primary and secondary school teachers – are required to have completed a master's program (300 ECTS). This requirement was introduced for primary teachers in 1979 (Niemi, 2012). The structure of primary and secondary education in Finland is similar to that in Spain. During primary and lower secondary education (age 16) all students receive a total of 1216 hours of formal education in mathematics. In upper secondary school, students can choose either an intermediate mathematics track (for a total of at least 228 hours) or an advanced mathematics track (for a total of at least 380 hours).

At the end of upper secondary education, students take the Finnish matriculation examination (Ylioppilastutkinto), which is the only national-level assessment in Finland. At the time the data was recorded, each examinee was required to take at least four papers, only one of them – their mother tongue, Finnish or Swedish, depending on the vehicle language at the candidate's school – being compulsory. The remaining three were chosen by the student, with the option of one paper in mathematics, either intermediate or advanced. Vocational school graduates may also gain admission to university, the amount of mathematics they have studied depending on the specific vocational track they have pursued. However, enrolling from vocational school to study teacher education at university is an exception.

In Finland, students have to apply to enroll on specific programs at universities. Applicants interested in a primary education degree must go through a special two-phase application process. The first phase, for all applicants in Finland, involves taking an exam (the so-called VAKAVA test[3]) based on a set of education-related readings. Based on their performance in the VAKAVA test, some applicants pass on to the second phase, which involves taking a suitability test that assesses their appropriateness to work as a teacher, their motivation and study skills, and their commitment to completing the degree. The actual form and content of the suitability test may differ from one teacher education program to another. Each university in Finland decides on the number of new admissions in conjunction with the Ministry of Education. At HEL, about 10% of applicants are admitted annually to the primary teacher education program. Out of the students accepted in 2016-17, 48 students (40%) were selected on the basis of the suitability test alone, and 72 students (60%) were selected on the basis of the suitability test and their grades in the YLIO in four subjects – Finnish and three optional subjects.

From the perspective of our study, the Finnish matriculation examination and the Spanish university entrance examinations have comparable functions when considering students' academic trajectories in relation to mathematics. Neither the Spanish university entrance mathematics exams nor the Finnish matriculation mathematics exams are an unavoidable requirement for access to primary teaching degree programs. In both cases, taking the mathematics exam corresponds to a decision made at the time of completing upper-secondary school or entering the University, but this decision was not determined by any requirement of the system. Therefore, we regarded the Spanish university entrance examinations and the Finnish matriculation examination as equivalent variables when determining population groups in our study.

[3] See https://www.helsinki.fi/en/networks/vakava/about-vakava-0

4 The test

The instrument we used to collect our data was a mathematics test designed previously as the pilot version of the specific complementary entrance examination in Catalonia mentioned above, prior to its implementation. This complementary entrance examination had been created for the administrative purpose of limiting access to the primary education degree programs in Catalan universities to those applicants who possessed a minimum mastery of mathematical knowledge and competence in the Catalan language. The test consisted of 25 open-answer questions that evaluated mathematical content knowledge related to the prescribed curriculum of the Spanish compulsory education system, namely Numbers and Arithmetic, Space and Shape, Relationships and Change, Magnitude and Measure, and Statistics and Randomness (see the appendix for the English version of the test). The weight given to each content area was deliberately linked to the weight given to them in actual practice in the school system. For most of the questions, students had to generate their own answers in order to avoid guessing or the substitution of the given options strategy. Solving the questions required an understanding of the concepts and procedures involved, and the results had to be interpreted within each problem's context.

The content and structure of the test had been defined by a panel of experts from four Catalan universities – among them the two first authors – within the framework of the above-mentioned study at the beginning of the second section. It was then reviewed by experts from the University of Helsinki – the third author –, Oxford Brookes University, and Linköping University as well as by the fourth author, thus also ensuring the construct validity of the instrument for our research purposes. The instrument was then piloted on three separate occasions by BCN students starting the primary teacher education program in the 2013–2014, 2014–2015 and 2015–2016 academic years, respectively. This confirmed the instrument's test-retest reliability and provided the authors with valuable feedback from the students, allowing, for instance, the correction of troublesome wording. It also allowed us to confirm the instrument's criterion-related validity by studying the correlation of the test results with the grades obtained in the mathematics course by this sample of first-year students on BCN (for a full description of the process, see Gorgorió and Albarracín, 2019). The test form also included several questions in which the student gave us information about the academic trajectory they had followed up to that point – which allowed us to infer their prior exposure to formal mathematics preparation – and about whether they had taken or not the mathematics section of the Spanish university entrance examinations or the Finnish matriculation examination YLIO.

Though the original version of the test was written in Catalan, Spanish and Finnish language versions were prepared by the authors of this paper who are native speakers of these languages and know the target culture. People's names and the activities described in the questions were adapted to the different contexts where necessary. Since the instrument was designed with the specific purpose of assessing the candidates' mathematical knowledge in order to regulate their access to teaching programs in Catalonia, we felt that it would be a valid tool to measure students' background mathematical knowledge. Given the uniformity of schooling in Spain, we felt that it would be valid to also use it with ALI students. Moreover, the curricular content reflected in the test also responds to the Finnish school curriculum. Thus, it could also supposedly be used with HEL students.

To test the reliability of the data obtained with the mathematics test, we calculated the Cronbach's alpha, both overall and for each campus, as an internal consistency estimate of the reliability of the test scores (see Table 1).

Table 1. Cronbach's alpha of test scores for each university

	Cronbach's alpha
ALI	0.7428
BCN	0.8157
HEL	0.8106
Overall	0.7925

The results that we obtained were in all cases higher than the reference value of 0.7, a suitable indicator of the internal consistency of the instrument, meaning that the different test questions were consistent in their evaluation, from the perspective of the national curriculum, of the mathematical knowledge with which the students arrived at the primary education degree programs. Thus, we regard the instrument as a reliable measure of background mathematical knowledge.

We used the test to evaluate the background mathematical knowledge of all the students who had begun the first year of ALI, BCN and HEL – in the 2016–2017 academic year. The test was completed by a total of 756 students – 386 on ALI, 254 on BCN and 116 on HEL – before they had started any mathematics or mathematics education courses. None of them were under 18 years of age, and our statistical treatment of the data allowed us to anonymize their answers in the test. Furthermore, we asked them for permission to use the answers they gave in the test as research material.

At each university, a regular class period was set aside for students to answer the 25-item test on a worksheet, without using a calculator and in a maximum time of 90 minutes. The answers were subsequently marked by mathematics teachers at the respective universities, who assigned 1 point when the answer was completely correct (meanings, calculations, units, etc.) and 0 points if it was not.

5 Differences in performance between Alicante, Barcelona and Helsinki students

Table 2 presents the basic statistics that describe the centrality and distribution of the scores in the test by the students at the three universities, the maximum possible score on the test being 25 points. Table 3 presents the results of a series of means comparison t-tests across the universities.

Table 2. Centrality and distribution of the test scores on ALI, BCN and HEL

Variable	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
ALI	386	12.343	0.203	3.988	2	9	13	15	21
BCN	254	15.996	0.302	4.812	3	12	16	20	25
HEL	116	17.233	0.405	4.363	7	14	17.500	21	25

Table 3. T- tests comparison of score means across universities

	Estimate for Difference	Confidence Interval for difference (95%)	p-value
HEL vs. BCN	1.237	(0.242; 2.232)	0.015
HEL vs. ALI	4.889	(3.995; 5.784)	0.000
BCN vs. ALI	3.653	(2.938; 4.368)	0.000
HEL vs. BCN+ALI	3.440	(2.560; 4.319)	0.000

Thus, we can affirm (95% confidence) that HEL students obtained better results in the test than BCN students, who in turn performed better than ALI students.

6 Students' prior academic trajectories and background mathematical knowledge

6.1. Trajectories that include mathematics courses and performance in the test

As noted above, in Spain, after their compulsory schooling, and depending on the academic track they chose before entering university, students may well study no further mathematics at all. We, therefore, divided the two Spanish groups into two subgroups each, with students who had taken mathematics courses during their baccalaureate in one subgroup and those who had only studied mathematics during compulsory schooling in the other. In Finland, all students had taken mathematics courses during their compulsory education and also in upper secondary school, but they had to choose between either advanced or intermediate mathematics. We, therefore, split the HEL group into two subgroups, depending on whether they had taken advanced or intermediate mathematics.

Table 4 shows the percentage of students in each subgroup, the mean score on the test for each subgroup, and the standard deviation. Table 5 presents the results of the t-tests of comparisons of means of the scores of the two subgroups at each university.

Table 4. Percentage of students, at each university, according to the mathematics track they followed in post-compulsory secondary education, mean of test scores of each subgroup, and standard deviations

	Percentage of students in each subgroup	Mean score (max. 25)	Standard deviation
ALI only compulsory maths	40.67%	10.990	4.000
ALI baccalaureate maths	59.33%	13.271	3.715
BCN only compulsory maths	42.52%	13.611	4.488
BCN baccalaureate maths	57.48%	17.760	4.263
HEL intermediate maths	59.48%	15.261	4.065
HEL advanced maths	40.52%	20.128	2.960

Table 5. T-test comparing mean scores of subgroups within each university, depending on the mathematics track in post-compulsory secondary education

	Estimate for difference	Confidence interval for difference (95%)	p-value
ALI only compulsory vs. ALI baccalaureate maths	-2.281	(-3.073; -1.489)	0.000
BCN only compulsory vs. BCN baccalaureate maths	-4.149	(-5.248; -3.050)	0.000
HEL intermediate vs. HEL advanced maths	-4.867	(-6.160; -3.575)	0.000

It can be seen that the subgroup of students at each university who had followed the longer and/or more intensive track of mathematics courses scored significantly higher on the test. However, as the courses are structured differently in Finland and Spain, the only quantitative variable for which we can infer comparable values between the three universities is the number of hours of mathematics taken by the students before accessing the primary education degree programs.

Table 6 shows the total number of hours of mathematics lessons received by the six subgroups during both compulsory and non-compulsory education, and also the total.

Table 6. Hours of mathematics received during their education by each subgroup

		Hours of mathematics		
		Compulsory education	**Post-compulsory secondary education**	**Total**
ALI & BCN	ALI & BCN baccalaureate maths	1050	280 (2 years)	1330
	ALI & BCN only compulsory maths		0 (2 years)	1050
HEL	HEL advanced maths	1216	380 — 494 (3 years)	1596 —1710
	HEL intermediate maths		228 — 304 (3 years)	1444 —1520

Taking into account the information presented in Table 5 and Table 6, we observe that at each university the group of students who had taken more hours of mathematics did significantly better on the test.

However, this does not hold true if we compare across universities. Table 7, in which mean test scores by subgroup are ordered from highest to lowest and accompanied by the number of hours taken by this subgroup, shows that while the number of hours may justify differences in performance at each university, this assertion cannot be maintained across universities.

Table 7. Subgroups by hours of mathematics received during compulsory and post-compulsory secondary education, ordered by mean scores on the test

	Mean test score (max. 25)	**Number of hours of mathematics received**
HEL advanced maths	20.128	1596 — 1710
BCN baccalaureate maths	17.760	1330
HEL intermediate maths	15.261	1444 — 1520
BCN only compulsory maths	13.611	1050
ALI baccalaureate maths	13.271	1330
ALI only compulsory maths	10.990	1050

Moreover, as regards the difference in performance between the Spanish universities, ALI and BCN, the t-tests of comparison of means show that the difference is significant across analogous subgroups, as shown in Table 8, despite the fact that the number of hours of mathematics received was identical.

Table 8. T-test of comparison of means of test scores of analogous subgroups from the two Spanish universities depending on the number of hours of mathematics received

	Estimate for difference	Confidence interval for difference (95%)	p-value
BCN baccalaureate maths vs. ALI baccalaureate maths	-4.489	(-5.335; -3.642)	0.000
ALI only compulsory maths vs. BCN only compulsory maths	-2.621	(-3.680; -1.562)	0.000

At this point, our results show that the influence of whether or not academic trajectories contain mathematical courses on background mathematical knowledge is more complex than it might appear at first glance.

6.2 Having taken the mathematics section of the entrance or matriculation examination and performance in the test

We wanted to know whether students who had taken the mathematics section of the Spanish university entrance examinations or the Finnish matriculation examination scored differently in the background mathematics knowledge test from those who had not. Therefore, we divided each university group into two subgroups depending on whether or not they had taken the mathematics section of the corresponding examination. For each university, Table 9 shows the proportion of students in each group and their scores in the test.

Table 9. Proportion of students who had taken the mathematics section of the PAU or YLIO or had not, and the mean of scores each subgroup obtained on the test, with standard deviation

	Percentage of students in each subgroup	Mean score on the test (max. 25)	Standard deviation
ALI maths section of university entrance examinations	60.88%	13.204	3.842
ALIN no maths section of university entrance examinations	39.12%	10.931	3.940
BCN maths section of university entrance examinations	62.20%	17.623	4.830
BCN no maths section of university entrance examinations	37.80%	13.589	4.239
HEL maths section of matriculation examination	75.86%	18.420	3.868
HEL no maths section of matriculation examination	24.14%	13.500	3.717

Table 10 shows the results of a series of T-tests of comparison of means when looking for differences between these subgroups at each university.

Table 10. T-test comparing mean scores of subgroups within each university, depending on whether or not students had taken the mathematics section of the PAU or YLIO

	Estimate for difference	Confidence interval for difference (95%)	p-value
ALIN no maths section vs. no maths section of university entrance examinations	-2.273	(-3.072; -1.474)	0.000
BCN no maths section vs. no maths section of university entrance examinations	-4.033	(-5.175; -2.891)	0.000
HEL no maths section vs. maths section of matriculation examination	-4.920	(-6.559; -3.282)	0.000

The t-test results show (with a 95% confidence level) that at each university, students who had taken a mathematics paper as part of their university entrance or matriculation examination scored significantly higher on the test than students who had not.

However, the scores obtained by the ALI and BCN subgroups of students who took the university entrance mathematics exam are significantly different, and this discrepancy also holds for those who did not take the university entrance mathematics exam (see Table 11).

Table 11. T-tests of comparison of mean scores of analogous subgroups from the two Spanish programs depending on whether students had taken the mathematics section of the PAU or had not

	Estimate for difference	Confidence interval for difference (95%)	p-value
ALI math section vs. BCN maths section of university entrance examinations	-4.419	(-5.322; -3.516)	0.000
ALI no maths section vs. BCN no maths section of university entrance examinations	-2.658	(-3.491; -1.825)	0.000

Overall, the results obtained by grouping the students according to their previous academic trajectory show that we should look for some variable that better explains the differences in performance between students on ALI, BCN and HEL and especially between ALI and BCN.

7 On how changing the cap set on the number of students admitted would have changed the observed differences in performance across universities

There is a clear difference between the Finnish institution and the two Spanish ones regarding admission policies to primary education degree programs. While in Finland, the cap set on the number of students admitted is based on the number of teachers needed by the community, in Spain there is no explicit policy that regulates it. Moreover, the proportion of students admitted relative to the total number of applicants to ALI is clearly higher than the corresponding proportion to BCN. To understand this difference in the proportions, the concept of the *cut-off mark* generated during the admission process of students to Spanish university courses must be taken into account. Students who apply to enroll for any particular program are ordered according to their university entrance exam mark, and as many students as there are available places on that program are admitted to it. The cut-off mark is the entrance exam mark of the student who occupies the last place among those admitted. In this way, the greater the proportion between the number of admitted students and the number of students who apply, the lower the cut-off mark. For the 2016-2017 academic year, the cut-off mark was 7.904 out of 14[4] for ALI, while for BCN it was 8.758[5].

In the following paragraphs, we analyze how a hypothetical reduction in the number of admitted candidates might close the gap observed between the results obtained for HEL, BCN and ALI in the mathematics test that we set. Thus, we establish, ex post facto, a reduction in the number of students whose test scores we now take as data. This allows us to discuss the possible effects that restricting the number of places might have on the test scores.

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[4] Generalitat Valenciana. Informe 2016–2017. Servicio de Regulación Universitaria Dirección General de Universidad y Estudios Superiores. http://preinscripcionuniversitaria.edu.gva.es/docs/NotasCorte2016.pdf

[5] Consell Interuniversitari de Catalunya. Cut-off marks. 2nd selection June 2016, 22/07/2016. http://universitatsirecerca.gencat.cat/web/ca/03_ambits_dactuacio/acces_i_admissio_a_la_universitat/.content/acces_i_admissio_a_la_universitat/preinscripcio_per_a_les_universitats_publiques__i_universitat_de_vic/llistes_de_notes_de_tall_i_de_places_universitaries/documents/Notes-tall-2a-assignacio-21-07-16-JUNY-2016.pdf

We can get a rough idea of what the HEL cap set on available places means as compared to that for ALI and BCN by looking at the number of places in each institution relative to the total population of the area that the universities serve. The respective catchment areas are Finland for HEL and, in Spain, the Valencian Community for ALI and Catalonia for BCN. Table 12 shows this ratio for Finland, based on data from the 2016–2017 academic year, and compares it to analogous data for ALI and BCN and the two Spanish autonomous regions where they are located – Valencia and Catalonia, respectively.

Table 12. Total population, students admitted to primary teacher education programs in the region/country in the 2016-17 academic year, number of inhabitants per student admitted, and number of students admitted to ALI, BCN and HEL

	Population	Students admitted in 2016–2017 to the teaching programs in the region/country	Number of inhabitants per student admitted	Number of students on ALI, BCN and HEL
Valencia	4 932 302 [a]	1750 [b]	2818	386 (ALI)
Catalonia	7 453 957 [c]	1920 [d]	3882	254 (BCN)
Finland	5 509 717 [e]	841	6551	116 (HEL)

[a & b] Instituto Nacional de Estadística, June 2017: http://www.ine.es/dynt3/inebase/es/index.htm?padre=1894&capsel=1900

[b] Informe 2016_2017. Servicio de Regulación Universitaria. Dirección General de Universidad, Investigación y Ciencia. Generalitat Valenciana. http://www.ceice.gva.es/documents/161863209/163489417/Informe+General+de+Preinscripci%C3%B3n+.pdf/73dafb19-6132-432a-847a-6da2687df874;jsessionid=3B35D14C840604C5287EA6C7CD0F3964

[d] Informes i estadístiques de la preinscripció universitària. http://universitats.gencat.cat/ca/altres_pagines/informe_i_estadistiques/informes_i_estad_pre/

[e] Statistics Finland, August 2017: http://www.stat.fi/til/vamuu/2017/08/vamuu_2017_08_2017-09-21_tie_001_en.html

For comparison, we take HEL as a reference since it is the program among the three studied that admits the fewest students. If we consider the number of students admitted to teacher training programs in Finland (841) and the number of Finns (5,509,717), we obtain a ratio of 1: 6551, i.e. in Finland one student is admitted for every 6551 inhabitants. Similarly, the ratios for ALI and BCN are calculated in relation to their corresponding catchment areas, the Valencian Community and Catalonia. Therefore, in Table 12 we can see that far fewer students were admitted to the primary teaching program in Finland than in either Spanish region, with a student to population ratio of 1:6551 as compared to 1:2818 and 1:3822.

To simulate what would happen if an admissions policy as similarly restrictive as the one in Finland was applied at the Spanish institutions, we have to take the ratio of inhabitants per student admitted to the Finnish programs as a benchmark and adjust the number of students admitted to BCN and ALI to achieve the same proportion. To adjust the proportions to the Finnish ratio, we have to divide the number of students admitted to ALI by 6551/2818 = 2.32 and the number of students admitted to BCN by 6551/3882 = 1.69. Thus, hypothetically only 166 students (386/2.32) would have been admitted to ALI and 150 students (254/1.69) to BCN.

Since individual scores on the test are available, we then select the 166 and 150 highest-scoring ALI and BCN students, respectively, to make up our new hypothetical groups. Descriptive data for these groups' test scores are shown in Table 13, alongside those of the original HEL group.

Table 13. Number of students in each group, highest and lowest score per group, mean of scores in each subgroup obtained on the test, with standard deviations when creating new groups adjusting the cap set on the number of students admitted

	Number of students	Highest student score (max. 25)	Lowest student score	Mean score for the group	Standard deviation
ALI hypothetical new group	166	21	13	15.979	2.120
BCN hypothetical new group	150	25	15	19.320	2.586
HEL	116	25	7	17.233	4.363

That the standard deviations for the BCN and ALI hypothetical new groups are smaller is not surprising since the data was artificially trimmed. It is worth noting that the mean score achieved by the restricted BCN group is now significantly higher than the HEL mean score, the estimate for the difference being 2.087 and the confidence interval for the difference (95%) being (1.1916, 2.9824) while the ALI mean score is still lower than the HEL mean score.

To simulate what would happen if we adjusted the ratio of students admitted across the two Spanish institutions, the number of ALI places would have to be reduced by a ratio of 3882/2818 = 1,378, so that 280 students would be accepted (386 ALI students / 1.378). We present the descriptive data for BCN and ALI when adjusting the ratio in ALI to that of BCN, with 280 students, in Table 14.

Table 14. Test score data for the top 280 ALI students (ALI280) and BCN students

	Highest student score (max. 25)	Lowest student score	Mean score for the group	Standard deviation
ALI adjusting the ratio to that of BCN (280 students)	21	10	14.205	2.826
BCN	25	3	15.996	4.812

The table shows that despite this reduction, students in the reduced group of ALI still perform lower (1.791 out of 25) than BCN students, significant at 95% since the difference cannot be less than 1.111.

8 Discussion

Our research assumes that the background mathematical knowledge that students bring to primary education degree programs plays a key role in their completing such programs with sufficient mathematical knowledge for teaching. In our study, we used a 25-item test to measure the mathematical knowledge of students enrolled on three of these programs, two in Spain and one in Finland, which had not yet begun any courses related to mathematics or mathematics education. We, therefore, regard their scores on this test as a legitimate index of their background mathematical knowledge prior to admission.

8.2 Searching for explanatory variables for performance differences in the test

In our study, we found that the performance of Finnish students was significantly better than that of Spanish students. The superior performance of HEL students may not surprise the reader because these results would seem to be consistent with the current prestige of the Finnish education system. However, BCN students also performed significantly better than ALI students, despite the fact that they shared the same general admission criteria and requirements at that time, and with the school curriculum being the same all over Spain. Here we identified a difference between the two Spanish institutions that required an explanation.

Our results also show that at each of the three universities the subgroup of students who had followed the more intense track of mathematics courses scored significantly higher on the background mathematical knowledge test. We could not make a direct quantitative comparison of the results obtained for ALI and BCN and those obtained for HEL since the mathematical tracks at Finnish upper-secondary are structured differently from those at the Spanish baccalaureate. In fact, the only quantitative variable for which we can infer comparable values at the three universities is the number of hours of mathematics taken by the students up to the moment of accessing university.

When we take into account the number of hours of mathematics received on comparing performance in the test, the results show that at each university the group of students who had taken more hours of mathematics did significantly better on the test. This suggests that the absolute number of hours of mathematics that students receive during their education is what determines average scores on the test. However, our data does not support this assumption if we compare across universities. The number of hours of mathematics received by all HEL students, even those who had only taken intermediate mathematics courses prior to university, is higher than the number of hours of mathematics received by students on ALI and BCN who had taken mathematics during their two pre-university years. Nevertheless, the second-ranking subgroup by mean score is the group on BCN that had taken mathematics during their baccalaureate, in fact receiving fewer hours of mathematics preparation than the intermediate mathematics HEL group.

We could justify this apparent discrepancy by arguing that student scores in this test are not only related to the number of hours of mathematics preparation but also linked to whether the way of working in the mathematics classroom is aligned or not with the emphasis encountered in the test questions, the greatest divergence in this regard possibly being an interest in real-life problem-solving as opposed to an interest in more formal mathematics. The verification of this hypothesis requires access to information that is unavailable to us.

However, the students on BCN who studied mathematics during their baccalaureate performed significantly better than the analogous subgroup of students on ALI, even if they took the same number of hours of mathematics. The same is true for the subgroups on BCN and ALI that had not studied mathematics during the baccalaureate. This significant difference in the results obtained by analogous groups on ALI and BCN, with the programs both admitting students taught in a system in which school mathematics education is quite uniform, is a striking discrepancy, and it demands an explanation.

The other variable that we considered in relation to the academic trajectory of students is whether they had taken the mathematics section as part of their university entrance examination in Spain, or as part of the Matriculation Examination in

Finland. We observed that the students who had taken the mathematics section scored higher on our test than those from the same university who had not, though again, this tendency broke down when cross-university comparisons were made. Again, what is most striking is that, despite the uniformity of the requirements for admission to Spanish universities, the scores obtained by the ALI and BCN subgroups of students who took the PAU mathematics exam are significantly different, and this discrepancy also holds for those who did not take the PAU mathematics exam.

Therefore, our overarching conclusion regarding the students' academic trajectories in mathematics is that, although the variables considered in our study may explain the differences within each university, they do not explain the differences across universities. Furthermore, in the case of the two Spanish universities, there are clear discrepancies between our findings and what might be expected in a school system as uniform as the Spanish system is assumed to be.

The cap set on the number of students admitted as explanatory variable

As institutions that exist in different socio-cultural contexts, ALI, BCN and HEL are likely to exhibit important differences in their organization. Perhaps the most striking difference is the policy regarding the cap set on the number of students admitted. While in Finland this cap is based on the number of teachers that the community needs, in Spain there is no explicit policy that regulates it. Given the limited number of candidates admitted to HEL and the differences in admission rates between ALI and BCN, we decided that it was worth evaluating the interaction between this restrictiveness in admission policy and the students' results in the background mathematical knowledge test. We are not saying that this is the only factor that might influence performance on the test, but it certainly deserves consideration.

On adjusting the cap set on students admitted to ALI and BCN by taking the cap on HEL as a reference, we observe that the mean score achieved by the restricted BCN group is significantly higher than the HEL mean score. The new difference between HEL and the reduced BCN group could be explained not only by the restriction on the number of students admitted but also because the questionnaire we worked with was bound to the Spanish context.

When introducing a similar restriction between the two Spanish institutions, we observe that the distance between the mean scores is reduced. Even so, BCN students continue to perform significantly better. This continuing difference, together with the discrepancies that we have pointed out in the differences between the scores for ALI and BCN, could be explained by "competitiveness in the market". BCN, given its geographical location and its academic prestige, competes strongly with the other five primary teaching degree programs offered by universities in the area. As a result, it is the first choice for many of the students, producing a greater reduction in the proportion of students admitted than that artificially created by the restrictions we can apply to our data. The confirmation of this hypothesis requires further research in itself.

9 Conclusion

To date, most studies discussing the mathematics component of the curriculum of primary education degree programs seem to assume that incoming students have already mastered a certain body of basic mathematical knowledge, when there is increasing evidence, at least in some countries, that they have not (Beswisk and Goos, 2012; Hine, 2015; Ingram and Linsell, 2014; Lo and Luo, 2012; Norton, 2018; Tatto et al., 2008; and Qian and Youngs, 2016). The studies of Arce, Marbán and Palop (2017), Buforn and Fernández (2014), Gutiérrez-Gutiérrez, Gómez and Rico (2016), Montes et al. (2015), Nortes and Nortes (2013), and Sáenz (2007) are some examples of research that has gathered evidence of limitations in the mathematical knowledge of students on primary teaching degree programs in Spain.

Our findings regarding the performance of the different groups of students allowed us to identify variables that tend to go unquestioned at a national level but which may have a considerable impact on the quality of primary education degree programs and their graduates. Variables such as the criteria according to which students are admitted to these programs or the number of places available on a particular program are defined at national level and shaped by the social and cultural context of the teaching institutions. How these criteria are established may influence the success of primary education degree programs because they have an impact on the extent of the mathematical knowledge that successful applicants will bring to the program. This is yet one more way in which social context may impact teacher education, albeit with a broader interpretation than that offered by Blömeke, Suhl, and Dörmann (2013).

We are aware that our research only addresses student performance from a quantitative perspective, leaving unstudied other individual and social variables that undoubtedly influence the academic profile of applicants admitted to the programs. There are several qualitative variables that distinguish the Finnish and Spanish institutional contexts. Among them, at an individual level, we find the different value ascribed to the mathematics grades achieved by applicants taking the matriculation examination in Finland and the entrance tests in Spain. Also, at an individual level, the consideration of the candidates' interest in and attitude to becoming teachers is different, something that can provide strong motivation to learn the content they will have to teach, such as mathematics, among other subjects. Thus, in Finland at present, among the students who reach the second stage of the selection process, 60% are selected only on the basis of their grades in four subjects in the matriculation examination (mother tongue, mathematics, another language and one subject from the humanities and natural sciences) and 40% are selected only on the basis of their results in the VAKAVA, a test that reflects the candidates' attitude to studying to be a teacher and to the profession. The social status of teachers, which is very high in Finland, is a contextual social variable that may influence the previous ones.

In relation to mathematical knowledge, it is urgent to encourage a debate not only to establish what mathematical knowledge could legitimately constitute a prerequisite for admission to teacher education programs, but also to provide primary teaching programs with reliable instruments that can be used to determine whether applicants have a solid background knowledge of mathematics. While we wait for this debate to produce clear results, the findings of the present study suggest that two measures, in particular, might have a significant impact on the academic profile of students enrolling on the Spanish programs: on the one hand, changing the policy regarding the cap set on the number of students admitted to Spanish primary teaching degree programs and adjusting it to the number of teachers needed and, on the other hand, using screening criteria based on the results of a specific mathematics entrance test might have a significant impact on the academic profile of students enrolling on these programs. Furthermore, the pool of applicants could be limited by only allowing those who had passed the mathematics section of the university entrance examinations or completed a minimum number of mathematics courses at secondary school to apply in the first place. Any of these options would ensure a better mastery of mathematical knowledge at the start of the program.

In Spain, the number of places available on teacher training programs could be reduced, because nowadays there is an important segment of teacher training graduates who have an occupation other than teaching. In addition, the gap between the number of graduates and the number of new teachers needed each year could be narrowed down, as evidenced by the waiting lists for teaching positions. It would also be possible to introduce some additional selection criteria. We have already mentioned that, despite the inflexibility of the university entrance system in Spain, there is at least one way to influence the selection of applicants, which is through personal aptitude tests.

It could be argued that the introduction of a selective test of mathematical content as a requirement for admission to primary education degree programs would obviously have an economical cost. Nevertheless, it is clearly feasible to require applicants to have completed a minimum number of mathematics courses during secondary school or to have taken one or the other of the two options of the mathematics section of the national university entrance examination. Neither of these options would imply major changes in the university system since they would only affect admission requirements. Resistance to their implementation is likely to be less related to possible financial costs for the public education system than to a reluctance among administrators to raise what the general public might view as a new and specifically targeted obstacle to a teaching career. Overcoming such resistance to these measures may require highlighting the link between excellence in teacher education and the immense value that teachers can add to society as those in charge of the education of future generations.

We approached our research with the goal of finding out how the processes and criteria for the admission of candidates could have an impact on the mathematical knowledge of the students who enter teacher-training programs. Our study has identified variables that affect the profile of successful candidates. However, there is still a need to explore how these variables might influence the results obtained by students during their training as future teachers and, therefore, the quality of the programs. All told, this research only answers a few questions about admission to primary education degree programs, while raising many more.

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Appendix A

SURVEY OF BACKGROUND MATHEMATICAL KNOWLEDGE

Answer the following questions. Write in the empty space how you ended up to your answer. You can use drawings, text, calculations...

1. Which of the following numbers is the largest?
   - 0,625 ; - 4/10 ; - 0,375 ; - 1/2

*Answer:*_

2. How many 4.5 MB pictures can be stored on a 1 GB disc? (1 GB = 1024 MB)

*Answer:*_

3. A kilogram of cheese costs £15.50. How much does 700 g of cheese cost?

*Answer:*_

4. A product is on sale. According to the label, the normal price is £125. The sale price is £100. What percentage of discount has been applied?

*Answer:*_

5. A cookie contains 20 % butter. Write as a fraction the part of the cookie that is not butter. Your answer should be expressed in its simplest form.

*Answer:*_

6. The cost of a product is £36. There is a special offer on at the store: "Buy two and you get the second at half-price". If you did buy two, what would be the cost of each product?

*Answer:*_

7. Which prime number can be made by subtracting two multiples of 7?

*Answer:*_

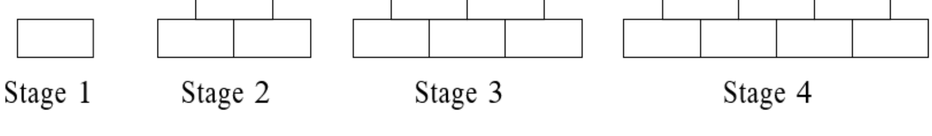
8. For the following expression to be correct, which number should replace Q?
   $Q \times Q = 3 \times 3 \times 7 \times 7$

*Answer:*_

9. What is the next number in the series n^2 : 1, 4, 9, 16, ...

*Answer:*_

10. Imagine you are building a brick wall. The sequence of images shows the first stages of the construction.



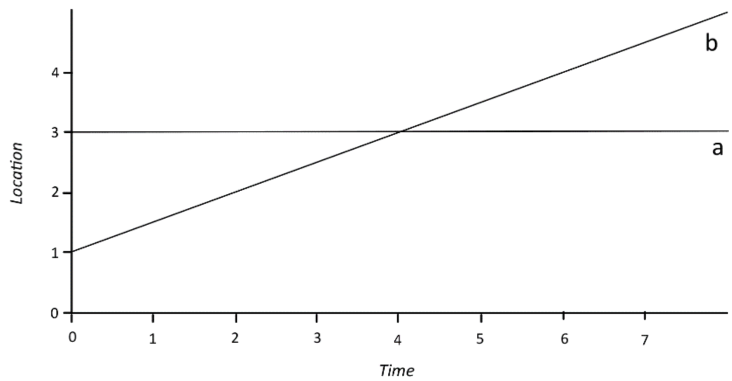
How many bricks will be in the wall in Stage 74?

*Answer:*_

11. When preparing an omelette, Lewis uses 2 egg whites and 1 yolk. To make a bowl of custard Nora needs 6 yolks. How many omelettes does Lewis have to prepare for there to be enough spare yolks for Nora to make a bowl of custard?

*Answer:*_

12. This graph shows the position of two objects (a and b) over the time. Are either of the objects stationary? If so, which one?



*Answer:*_

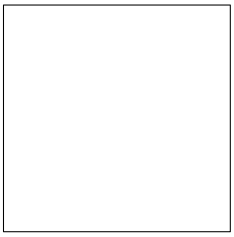
13. You have two boxes which contain the same amount of sweets. In one of the boxes there are 2 bags of sweets and 3 single sweets. In the other box there is 1 bag of sweets and 8 single sweets. The amount of sweets in each bag is the same. How many sweets are in each bag?

*Answer:*_

14. What is the surface area of a square with a perimeter of 32 cm?

*Answer:*_

15. How much is a half of a half of a half? Draw it using the square below and then write the resulting fraction.

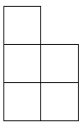


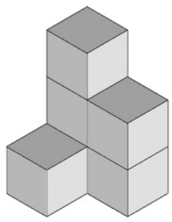
*Answer:*_

16. Which of the following figures must have equal diagonals?
   A. square
   B. rhombus
   C. rectangle
   D. trapezium

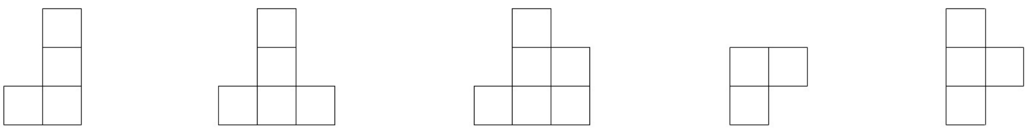
*Answer:*_

17. The figure on the left is a plane view of the object on the right.





Knowing there are no hidden cubes, which of the following figures are also a plane view of the object?



A B C D E

*Answer:*_

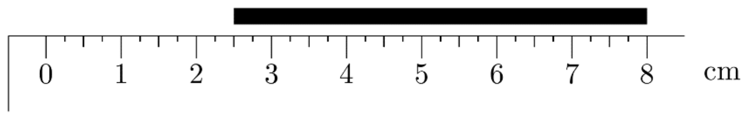
18. How many cm are in 7.8 km?

Answer: _

19. How many minutes are in 2.5 hours?

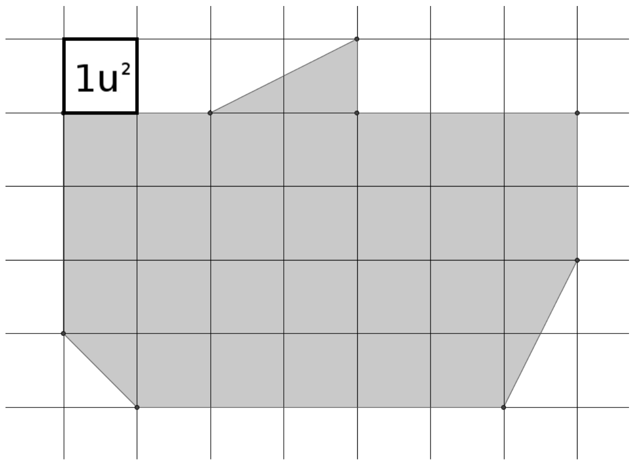
Answer: _

20. What is the length of the thick black line?



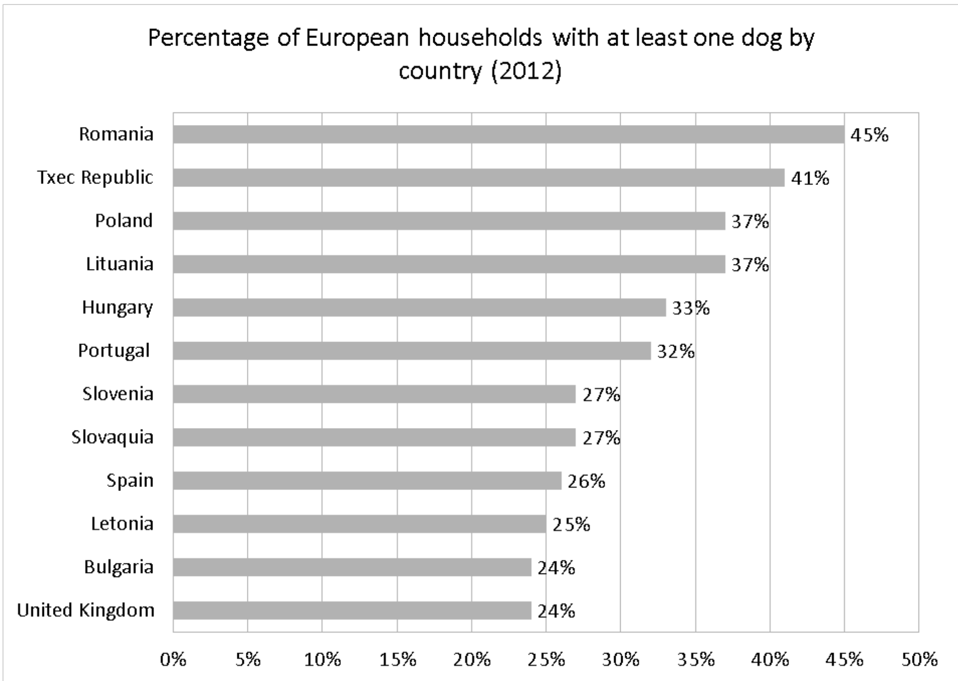
Answer: _

21. What is the area of the shape below?



Answer: _

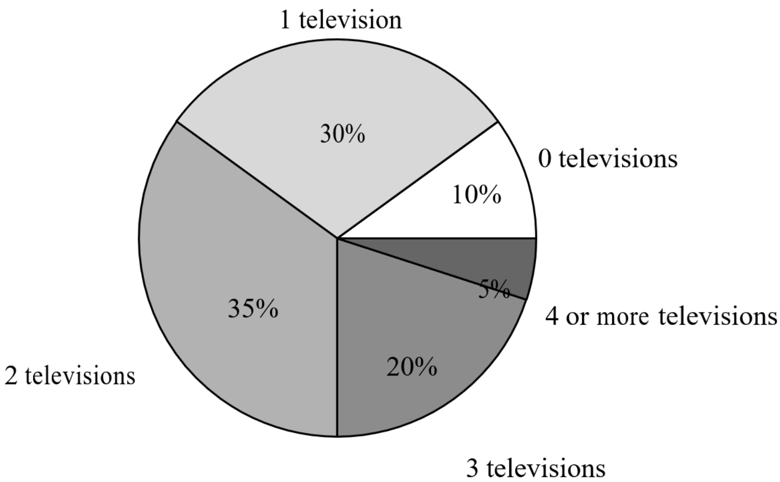
22. Use the graph below to answer the following question.



How many percentage points separate Lithuania and Romania?

*Answer:*_

23. 800 people took part in a survey concerning the number TV sets per household. The following pie chart shows the results of the survey.



How many people participating in the survey had 1, 2 or 3 TV sets?

*Answer:*_

24. What number would we add to the list below in order for the mean to equal 7?

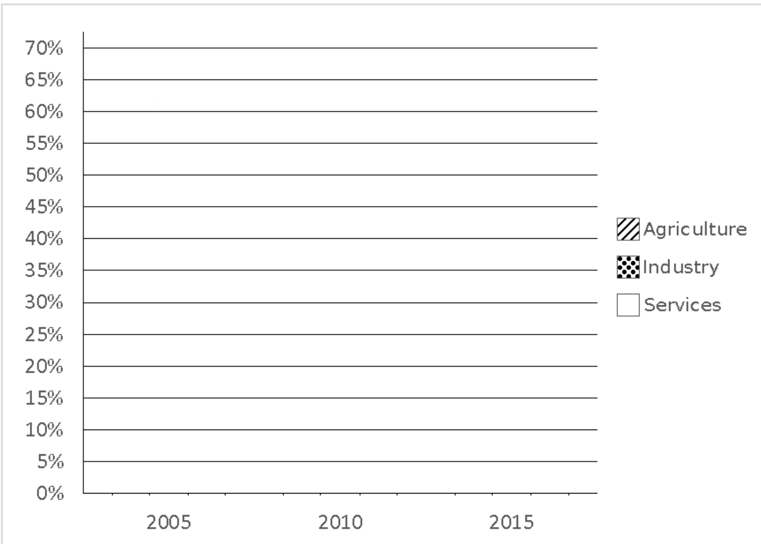
{2, 6, 7, 7, 8, 8, 9}

*Answer:*_

25. Create a bar chart using the data given in the table below.

Occupation by sector

	2005	2010	2015
Agriculture	15%	10%	5%
Industry	35%	30%	35%
Services	50%	60%	60%



A model for continued professional development with focus on inquiry-based learning in science education

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This is a qualitative study with a twofold aim. The first aim is to describe and analyse teachers' perceptions of advantages and challenges with a model for continued professional development (CPD) for primary school teachers. The CPD course was about inquiry-based learning (IBL) in science education. The second aim of the study is to analyse the teachers' thoughts after implementing inquiry-based methods in their own science teaching. The empirical data, in the form of video transcriptions, notes, interviews and results from electronic forms, were collected during and after four separate in-service courses for teachers (N=26). The analysis of the data is done through thematic analysis. As positive results, the teachers emphasised the implementation of IBL with their students and the individual mentoring, which were parts of the CPD model. Teachers highlighted the importance of students' possibilities to make investigations based on their own questions, which proved to have a positive impact on students' interest and motivation. Teachers also saw the importance of their own planning and goal setting. The results indicate that the teachers perceived a tension between having control and relinquishing control, which can become a challenge. Two other challenges were teachers' perceived lack of time for planning and implementing but also teachers' deficient subject knowledge.

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1 Introduction

This study focuses on a continued professional development (CPD) course for primary science teachers in first through sixth grades in Finland. The CPD course was completed with four different groups of teachers. The theme of the CPD course, which lasted one year for each group, was inquiry-based learning in science, with focus on open inquiry. The challenge today for teachers is to educate their students for future challenges, described in the 21st century skills (European Union, 2006). Lonka (2018, p. 68) describes this in the following way; "citizens need to understand the nature of scientific investigation and be able to interpret scientific evidence and conclusions". Inquiry-based teaching is a pedagogical approach where students can develop these skills. In the Finnish core curriculum (Finnish National Board of Education [FNBE], 2016), the objectives of research and working skills state that students should be able to formulate their own research questions and plan for small-scale research projects.

These objectives compel teachers to develop their own competences for guiding the students in the process of learning the above-mentioned skills. The teachers participating in this study took an active part in their own learning, e.g., by planning for their students to carry out inquiry-based projects in science. The teachers created their own learning path by trying out inquiry-based thinking in their teaching, with individual mentoring from teacher trainers. The two teacher trainers responsible for the course also acted as researchers. During the study sessions, the teachers presented their students' inquiry-based projects and received feedback from other participating teachers, which led to collective reflections on students' thinking and learning. The CPD was followed by research during and after the courses. To investigate whether the course led to lasting changes in the teachers' thinking and practice five of the participating teachers were interviewed about one to two years after they finished the course.

1.1 Toward science education for all

"Science education should be an essential component of a learning continuum for all, from pre-school to active engaged citizenship" (European Commission, 2015). Correlating with this and with the European Union's (2006) recommendations for teaching key competencies in the 21st century, seven central competencies are described and focussed on in the curriculum. Inquiry-based learning (see Section 2) is one of the principal elements of the key competency of thinking and learning how to learn (Lonka, 2018). The same competency also is one of the seven key competencies in the national core curriculum launched in Finland in 2016 for first through sixth grades (FNBE, 2016). These competencies are: Thinking and learning to learn; Cultural competence, interaction and self-expression; Taking care of oneself and managing daily life; Multiliteracy; Information and communication technology (ICT) competence; Working life competence and entrepreneurship; Participation, involvement and building a sustainable future (FNBE, 2016). According to Lonka "inquiry-based learning helps the teachers to prepare students for the future challenges of work. In future workplaces, the skills of asking the right questions, defining and solving the most important problems, creating new knowledge, and making changes will be the most crucial requirements." (Lonka, 2018, p. 65).

In first through sixth grades, science, geography and health care are combined into the subject environmental studies. One of the learning goals for environmental studies is students' development of scientific skills. Teachers should guide students to communicate their pre-understanding, formulate questions, plan and carry out small-scale research projects based on these questions, described in the core curriculum. (FNBE, 2016). Learning goals include the ability to recognise causal relationships, draw conclusions from research results and, ultimately, present and discuss the results. Primary teachers in Finland usually teach most school subjects, posing challenges regarding teachers' knowledge and understanding of science. The implementation of inquiry-based learning requires changes in classroom practice, as well as a new understanding of science. This understanding includes all three aspects of science: learning science, learning about science (the nature of science) and learning to do science (Hodson, 2014). Science is much more than learning facts, and important learning goals that have been stressed in the Finnish core curriculum is to acquire processing skills and an understanding of them in the context of science (FNBE, 2016).

Recent international studies, e.g., PISA (Organisation for Economic Cooperation and Development [OECD], 2018) and TIMSS 2019 (International Association for the Evaluation of Educational Achievement, IEA, 2019), have shown declining results from Finnish students in science, both in achievement and in motivation and attitudes. Pupils develop their attitudes toward science even before age 11 (Royal Society, 2004), and primary teachers play a central role in shaping these attitudes. Evoking pupils' curiosity about and interest in science and phenomena in the environment is important in science education (DeWitt & Osborne, 2007). In the Finnish core curriculum, this objective is expressed thusly: "The objective of teaching and learning is to attract and deepen the pupils' interest in various fields of knowledge of environmental studies. ...Problem solving and research assignment are utilised in deepening their interest in phenomena in their surroundings" (FNBE, 2016, p. 257). According to Henriksson (2016), it is not a challenge to arouse interest in science among primary children, but that the primary teachers in her investigation struggled to maintain students' interest as the students aged, and the subject matter became more difficult. Andersson (2008) disagrees with the idea that students' interest is the most important objective in science education. Students' interest, attitudes and process skills are important, but according to Andersson (2008) and Tobin (2006), teachers must raise ambitions and successively take in and use science concepts. Students' interest and motivation can develop concurrently while they acquire content knowledge and do not need to exist first (Berg, Löfgren, & Eriksson, 2007).

1.2 Research objectives and questions

The first aim of the study is to develop, try out and assess a CPD course model for primary teachers. The CPD course consisted of lectures, workshops, individual mentoring and teachers' implementing IBL with their students. The second aim of the study is to assess the teachers' perceptions of inquiry-based science learning in their own teaching.

Our research questions are:

1. What advantages and challenges did the teachers account for in the continued professional development course model?
2. What were the teachers' perceptions after the CPD concerning inquiry-based learning in primary science?

2 Theoretical background

2.1 Inquiry based learning

Inquiry-based learning (IBL) in science is a goal, as well as a tool, for learning (Harlen & Qualter, 2014). The term *inquiry* in science has been used and understood in many different ways (e.g., Abd-El-Khalick et al., 2004; Anderson, 2007; Crawford, 2014; Lunetta, Hofstein, & Clough, 2007; Minner, Levy, & Century, 2010; Rönnebeck, Bernholt, & Ropohl, 2016). Crawford (2014) interprets *teaching science as inquiry* as including both the pedagogy and learning outcomes of inquiry in her review article. As Crawford (2014, p. 515) describes it: "... pedagogy being the method of engaging students in designing and carrying out investigations and the learning outcomes referring to learning science subject matter by engaging in these investigations, in addition to learning "about" the nature of scientific inquiry". Inquiry-based teaching and learning describe a spectrum from teacher-led confirmatory inquiry, where students more or less follow a recipe, to open inquiry. In open inquiry, the teacher defines the context while the students formulate their own research questions and different student (groups) conduct different inquiry projects. Between these extremes, lies guided inquiry, where the starting point is the teacher's questions, while the students are given more responsibility to plan and conduct the inquiry. (NRC, 2000). Inquiry is an active learning process in which one learning goal is to learn about the process that, to some extent, can be compared with professional scientists' processes (Anderson, 2002). The process can be described in an inquiry cycle (Figure 1). The inquiry circle used is visualised in an article by Elo and Kurtén (2019) and is in line with a model described in Pedaste & al. (2015). In this study, we use the concept of IBL in line with Anderson's definition, and focus is on guided and open inquiry where a learning goal is the processes of science.

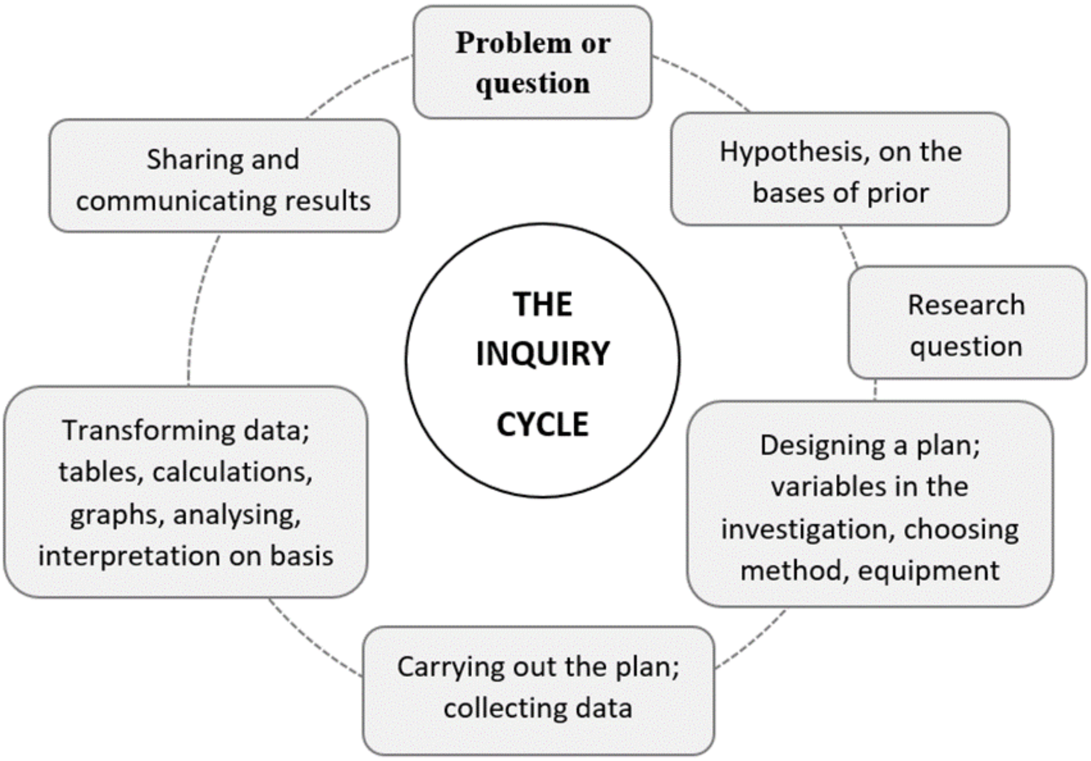

Figure 1. The inquiry cycle (Elo & Kurtén, 2019).

2.2 Primary teachers' subject knowledge in science

A change in how learning is viewed, from an empirical approach to a constructivist approach, has accentuated the need for teachers to develop their teaching (Lederman, Lederman & Bell, 2004; Skamp, 2011). A constructivist approach entails increased use of student-centred methods, open questions, discussions and group work. For example, the teacher no longer can rely on the textbook (Kikas, 2004). This, in turn, places higher demands on the teacher's subject knowledge, and Harlen and Holroyd (1997) have shown that many primary teachers have insufficient subject knowledge, which can complicate their use of inquiry-based methods in science. Several articles about the Finnish context show that Finnish primary teachers have a lack of subject knowledge in science, knowledge about working methods and about inquiry-based learning (Ahtee & Johnston, 2006; Herranen & Aksela, 2013; Palmberg, 2012).

Researchers also have found that the same misconceptions that students have about science concepts also are present among teacher students and in-service teachers (e.g., Burgoon, Heddle & Duran, 2011; Härmälä-Braskén, Hemmi, & Kurtén, 2020; Kikas, 2004; Palmberg et al., 2011; Papageorgiou & Sakka, 2000; Pine, Messer, & St. John, 2001; Schoon & Boone, 1998; Smith, 1997). To be able to challenge pupils to ask questions, scaffold them with relevant resources and assess their understanding in relation to learning goals, primary teachers need basic knowledge and a broad understanding of science.

2.3 Teacher development

A skilled teacher integrates successfully subject knowledge with pedagogical knowledge, technological knowledge and knowledge about his students, the core curriculum and the school (e.g., Shulman, 1986; Mishra & Koehler, 2006). Teachers constantly need to develop their own learning. According to Guskey (2002), teachers will change their perceptions and attitudes if they notice that changes in their teaching positively impact students' learning. Significant educational change comprises changes in beliefs and teaching styles, which can be developed through a process of personal development in a social context. Based on Guskey's model, Clarke and Hollingsworth (2002) developed their interconnected model of professional growth to visualise "the complexity of teacher professional growth through the identification of multiple pathways between the domains" (Clarke & Hollingsworth, 2002, p. 950), which are personal domain, external domain, domain of practice and domain of consequence. Professional growth also requires that teachers have opportunities to interact with each other, get technical help, and peer support (Fullan, 2016). Teachers are in a key position to effect changes in schools (Hewson, 2007). Their content knowledge is important, but according to Harlen et al. (1997), primary teachers, through in-service training, can get support to develop a deeper understanding of science. They say that teachers need opportunities to discuss their perceptions and develop these together with others.

Professional development can elicit new ways of looking at both the subject, the teaching process and students' learning. Teachers have pedagogical content beliefs that strongly correlate with their teaching and classroom practice (Levitt, 2001). Many reforms have failed because the importance of teachers' beliefs and attitudes has not been considered (Lumpe, Haney, & Czerniak, 2000). Anderson (2007) points out that professional development should be connected to teachers' own context and their own students. When problems arise, teachers should have access to support so they can handle problems.

Appleton (2008) describes how a supervisor, by assuming the role of a mentor and a critical friend, can help primary teachers develop increased pedagogical subject knowledge in science. For a professional development (PD) programme to be fruitful in the long run, according to Levitt (2001), teachers' educational approach should correspond with the individual programme's content and epistemology. In connection with various PD programmes within science education, researchers have found that teachers sometimes follow the programme throughout its duration, and then return to previous working methods when the programme ends (Levitt, 2001). Teachers need time for reflection, as well as support and resources, for professional development. Continued professional development is a prerequisite for teachers' fundamental changes (Appleton, 2008; Forbes & Skamp, 2014; Levitt, 2001; Peers, Diezmann, & Watters, 2003). Different teachers have different professional-development needs. In this vein, the school's headmaster or principal plays a central role (e.g., Lumpe et al., 2012; Fullan, 2016).

3 The design of the continued professional development

The CPD course in our study was addressed to primary teachers in first through sixth grades (ages 7–12) who teach environmental science (which includes biology, chemistry, geography, physics and health care). The course consisted of two study sessions, one at the beginning of the course and one concluding part. Between these days, the participants were supposed to complete one-two sequences where they scaffolded their students through guided/open inquiry-based projects in their own class. During this time, the participants also received individual mentoring with regard to their sequences (see Figure 2). The guidance model and the teachers' presentations of their students' projects made it possible for the researchers to follow the participants' development and understanding of IBL. The courses lasted for one year each and were conducted in four different regions of Finland during the 2014–2017 period, with four different groups of participating teachers. One request concerning participants was that at least two teachers from any one school participate, but this was not a requirement. A total of 34 teachers participated in the course. For various reasons (e.g., parental leave, illness, change of work), eight of the teachers only participated during the first half of the course. These eight teachers did not take part in the final evaluations discussion, nor did they answer the electronic evaluation after the course.

The purpose of the CPD course was to help teachers develop their teaching toward open inquiry-based science, while considering students' preconceptions. The focus should be on students' development of their scientific thinking by learning to ask investigative questions, form hypotheses, and plan and conduct their own investigations.

The structure of the CPD course is illustrated in Figure 2. The CPD course started with one day of lectures and workshops (or two afternoons). The participants received electronic material about IBL in advance. The first day focussed on the inquiry cycle (see Figure 1) and children's preconceptions. The inquiry cycle was presented and discussed. The participants worked according to the cycle in a workshop. Other topics presented were how to help students formulate research questions and how to conduct fair tests. At the end of the CPD course, one day was devoted to formative and summative assessment and the use of digital tools in science education. After the first study session, the participants' task was to choose one theme from their ongoing science course. The theme's starting point was that students should use IBL and plan and conduct their own investigations. For implementation, the participants received support in the form of individual mentoring, partly by e-mail and partly from one of the teacher trainers visiting the teacher at his or her own school. During the course, the participants were expected to complete two inquiry-based sequences with their students. During the final study sessions, the participants presented their students' projects and participated in the final evaluation discussion. The teachers answered the electronic evaluation sheet after the completed course. Based on feedback from the third CPD group, a study session was held in the middle of the last CPD course.

The participants were encouraged to keep a diary in which they could reflect on their sequences and their students' learning. However, the participants had difficulty finding the time and motivation to do this. Because of that, the diary element was excluded from the third and fourth CPD groups.

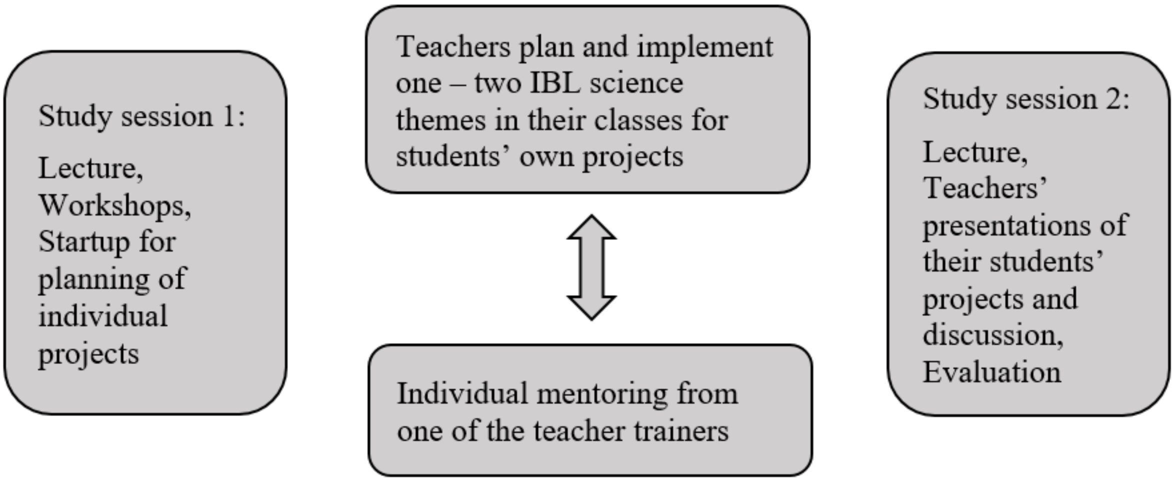

Figure 2. The structure of the CPD course.

4 Methods of data collection and analysis

This qualitative and inductive study investigates a CPD course, in inquiry-based learning in science for primary teachers, in line with the national core curriculum (FNBE, 2016). The CPD course was conducted four times with four different groups of teachers. The research data were collected from all parts in the list below and the analysis is based on all the collected data:

1. Video recordings from four final evaluation discussions (26 teachers in four groups, one after each completed CPD course).
2. The participants' electronic evaluations (open questions) of the CPD (N=26).
3. The participants' course diaries (from the first and the second CPD course) (N=8).
4. Recorded semi-structured interviews with five of the participants one to two years after they finished the CPD course (N=5).

During the final discussions, participants (N=26) were asked to reflect on the CPD course and on their own implementing IBL with their students within the course. The electronic evaluation form contained ready formulated open questions that the two researchers responsible for the course prepared. The questions concerned the participants' views on the CPD course, their view on teaching science and the role of teacher and student in IBL. Furthermore, the participants were asked whether they perceived that the CPD course affected their perceptions about these issues, and if so, in what way. To investigate whether the course made a more permanent impact on the participants' teaching in science, semi-structured interviews with five participants (from three different CPD groups) were carried out one to two years after they completed the course. The interviews took place by phone and were recorded and transcribed.

The data were organised and categorised using NVivo as a software tool. The transcribed and the written material was analysed inductively using two separate models (Braun & Clarke, 2006; Desimone, 2009). Keywords were identified via repeated reading of the transcribed material, course diaries and the answers from the electronic evaluations. The keywords were used to identify important themes in connection to our research questions.

Braun and Clarke (2006, p. 79) use the following concepts in the thematic analysis: "Data corpus refers to all data collected for a particular research project … data set refers to all the data from the corpus that are being used for a particular analysis … data extract refers to an individual coded chunk of data, which has been identified within, and extracted from, a data item." In this research the data corpus of teachers' experiences was analysed thematically on three levels: 1) a data set comprising all instances in which the teachers referred to the research questions; 2) data extracts on a personal level that were analysed and thematically coded; and 3) qualitatively different themes on a general level that were coded. Quotes are used to illustrate these themes' meaning. The analysis for the first research question is based on the participants' statements regarding the different elements and the content of the CPD model. The analysis follows the steps in the model of Braun and Clarke (2006) (see Table 1).

Table 1. Subthemes and themes created and used in the analysis of the collected data (video recordings, evaluation discussion, electronic evaluations, course diaries and semi-structured interviews) for the first research question.

Initial codes (The different elements of the CPD)	Subthemes	Themes ADVANTAGE (A) CHALLENGE (C)
Study sessions	Lectures and workshops: inspiration, insight in inquiry-based learning and in research in science education, practical skills	(A) Impact on participants' knowledge, attitudes and skills
	New ideas from other participants.	(A) Collegial learning
Personal coaching	Personal help by the teacher trainer at my own school, help with equipment and material	(A) Impact on participants' courage to test IBL
	Difficult to let go of old ways of thinking and acting	(C) To change one's mindset
Teachers' own planning and implementation	The requirement that you had to implement IBL in your own class -> you immerse yourself more, you challenge yourself	(A) Satisfaction of having completed projects
	Got to work with your own class with your own themes	(A) Flexibility in the course
	Completed models would be less time consuming Limit the projects in your class	(C) Time for the projects
Facebook / Cooperation with other participants	A strength to be two from the same school	(A) Collaboration between teachers from the same school
	FB: no sharing of questions, ideas	(C) Teachers not used to sharing material: do not think what they have is good enough to share
The whole course	Gave development ideas and started a process The CPD as a whole is a maturation process	(A) Starting point for a own reflections of one's teaching and a new mindset
	Asking questions without ready-made answers	(C) Sense of limited subject knowledge

For the analysis regarding research question two concerning teachers' perceptions on IBL we have used the model of Desimone (2009) (Figure 3). The outcome of the analysis is visualised in Figure 4 and discussed against the backdrop of previous research in section 6. The participating teachers' statements regarding IBL are analysed thematically to find qualitatively different themes on a more general level. The results from the analysis are collected in Figure 4.

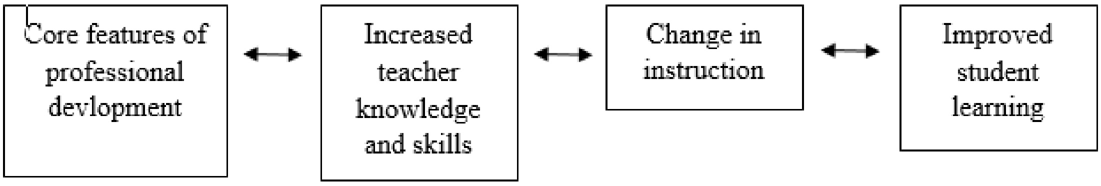

Figure 3. 'Proposed core conceptual framework for studying the effects of professional development on teachers and students' (Desimone, 2009, p. 185).

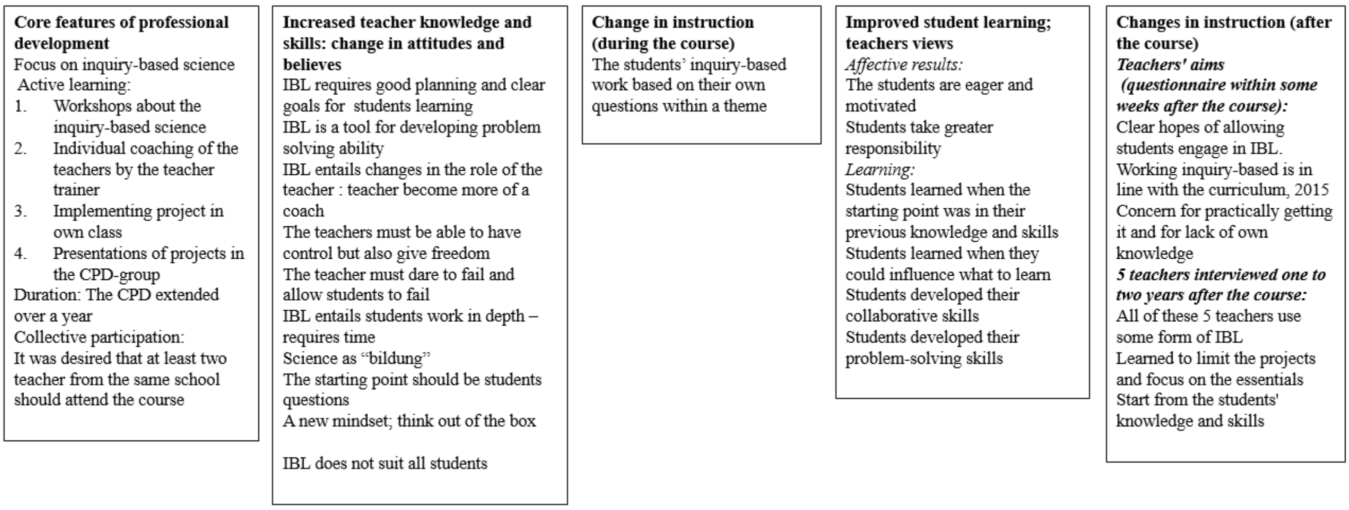

Figure 4. Results analysed and presented according to a modified version of a model of Desimone (2009).

The analysis was carried out in collaboration between the two researchers. The interpretations of the participants' comments in the evaluation discussions, diaries, interviews and in their responses to the questionnaire were discussed until a consensus was reached. The categories were iteratively developed and refined in a shared meaning-making process. The use of several data sources gave a broad basis for the analysis. Through the individual mentoring of the participating teachers in the CPD course, we got a close contact to these, which was important for the collection and interpretation of data. Our own long-term experience, as teachers in primary school and/or secondary school and within in-service teacher education, were valuable in the interpretation of the teachers' reflections. The fact that the researchers had a close relationship with the context is an advantage. According to Dalen (2007), the ideal is that the researcher can achieve a so-called Picasso profile, i.e. that the researcher is both inside and outside the research profile at the same time. Kvale (1997) states that the researcher's knowledge of the subject is a prerequisite for a valid interpretation. In reporting the results, we offer authentic extracts from the participants to make our analysis transparent. The aim of the study is to find and present critical features in the CPD model and in implementing IBL in science education, to consider for comparison with other similar cases.

5 Results

The outline in this section is based on the two research questions. The subtitles in section 5.1 relate to the advantages and challenges that the teachers identified in the CPD course, while the subtitles in section 5.2 are based on the results concerning teachers' views of IBL, compiled in Figure 4.

5.1 What advantages and challenges do the teachers account for in the continued professional development model?

Advantages with the CPD model

The study sessions with lectures, in combination with practical exercises, were valuable, according to many participants. One participant highlighted the link to research and the educational structure, as teachers usually have neither the time, nor the opportunity, to immerse themselves in current research: "I value the research and methods shared by the teacher trainers. The course has not only been about education; it had an educational structure, and that was valuable." The study sessions also instilled inspiration and courage to try new things. An important part of the sessions was the opportunity to take part in and reflect on other participants' presentations of their students' projects. This gave the participants new ideas for their own teaching. For example, two teachers working in the same school shared the experience of having he students make cartoons of a theme that they worked on. The cartoons were then used in the assessment of the students' work. This inspired other participants in the group. The participants served as sounding boards when they took part in the other teachers' presentations, and at the same time, they contributed to each other's development.

The individual mentoring that the participants received when developing their teaching sequences was important, according to the participants. This mentoring, that the teachers received, helped the teachers with both ideas and materials, and encouraged them to carry out their sequences: "The advisory coaching gave the support I needed to plan my project." The tutoring also elicited an affective effect: "Absolutely wonderful that someone takes time to help me, find electronic links and so on." At the same time, the guidance was not perceived as controlling: "You are pushed forward, but still you plan your own projects and develop your own thoughts." After the course, many participants expressed their satisfaction with their own efforts and sequences. The set-up in which the participating teachers should implement IBL, which was in focus during the first workshop day, was viewed as important:

> It is valuable when you, right away, must try out the new thoughts and ideas. To get supervision and help with materials at the same time is valuable. In this way, I think that the CPD also has a ripple effect and gives a lasting result.

The CPD course was, in this way, anchored to the teachers' own teaching, which was viewed as a basis for leading to lasting changes in their own teaching. When the participants had to plan and implement science as inquiry in their own teaching as part of the course, they developed a better understanding of inquiry learning. The struggle with a new way of teaching and experiencing success provided positive feelings and satisfaction.

In the invitation to the course, it was recommended that two or more teachers from the same school attend the course. Teachers who were from the same school highlighted this as being valuable in their evaluation: "I am grateful for having a colleague on the same course. We have been able to discuss our thoughts about and our reflections on the course content." Most participants did not have colleagues from their schools attending with them.

The course was carried out four times in different regions of Finland. Because of this, the number of participants in each region was relatively small (7-12 teachers). This was viewed as important: "It was good that we were not so many ... everybody became more active when it's a smaller group." The fact that the course lasted a long time and was not just a one-day course was viewed as significant: "You need this time to think and reflect." The number of study sessions also was viewed as appropriate. Too many sessions might have been discouraging and might have prevented the teachers from participating in the course. A longer-lasting course like this was also perceived as eliciting a more permanent effect on their own teaching.

Among comments after the course, some said it opened up new ways of thinking, built courage and opportunities to try new teaching methods or opened their eyes to teaching science in a manner that interests the students.

Challenges with the CPD course

Primary teachers' lack of time and fragmented daily work were problematic factors, according to the participating teachers in the CPD course. The teachers felt that they had neither the time nor the opportunity to focus on developing their inquiry-based sequences in their own classes as much as they wanted: "External factors … time does not stop … other factors continue with undiminished strength even though I would like to concentrate on this."

One challenge was to limit the students' own projects and prevent them from becoming too extensive and time-consuming. One of the teachers who carried out two inquiry-based sequences in her class during the CPD course noted that this gave her the opportunity to gain experience from the first sequence and make improvements on the second: “I’m pleased with my second project because it was more limited. The first one was very (educational) for me with many attempts and mistakes. It is important to limit!” To make IBL implementation more feasible and less time consuming, one participant suggested that there could be ready-made models.

One part of the course involved the participants’ own reflections on their own development during the course, recorded in a course diary. This turned out to be difficult to implement, mainly due to lack of time, but also due to lack of motivation. As course leaders, we had not succeeded in communicating the diary’s purpose.

From the start of the CPD, the hope was that the participants would collaborate with each other, share their ideas and keep in touch with each other. To facilitate this, a Facebook group was created for every separate course. The participants also had access to each other’s e-mail addresses. However, these tools were never used that often. The sharing of ideas was limited to the study sessions, when participants met with each other, except for those teachers who could cooperate within their own schools. One participant suggested that it perhaps would have been better if teachers from different schools had chosen the same themes for their sequences. It turns out that even when some teachers had chosen the same themes, they did not cooperate. A comment about the lack of cooperation was that teachers in the Finnish culture do not ask for help, and they do not believe that their own work is good enough to share with others.

5.2 What were the teachers' perceptions after the CPD concerning inquiry-based learning in primary science?

Increased teacher knowledge and skills: change in attitudes and beliefs

Several participants in the CPD said IBL in science requires that the teacher do much planning and possess good subject knowledge: "The teacher should have an 'iron grip' on the subject!" Well-thought-out planning, in which the teacher has clear goals for the students' learning, is a prerequisite for students' work in formulating researchable questions, then planning and conducting their own investigations: "To open up the task for the students without giving them total freedom." However, the teacher simultaneously should have the courage to relinquish control when students work with their own questions.

The teacher has an important role as the person who can awaken students' curiosity and interest: "It's easy to kill the students' interest from the first minute if the introduction is boring." One teacher commented on the fact that teachers most often answer questions that nobody has asked, but students are interested in getting answers to their own questions.

One teacher highlighted the problem of coping with all the core curriculum content when students use IBL. Another teacher emphasised the importance of teaching in depth and allotting time for this: "The most important is not to cover much, but to do thoroughly what you do." Many teachers highlighted their own perception of lack of time for planning, collaborating and reflecting.

Many of the participants perceived that IBL involves a change in the teacher's role. The teacher's task is to inspire, but also to act as a coach. For a teacher, it is challenging to have the courage to relinquish control and believe in the students' ability to learn, i.e., to believe that students will learn even if you, as the teacher, do not present everything to them. One teacher also stressed that he had developed more courage to relinquish control and make mistakes. It is acceptable to arrive at different solutions and different answers. In addition to being responsible for learning goals, the teacher is responsible for assessing students. The participants emphasised formative assessment as an assessment of the processes and of students' work with their projects. However, this assessment method was viewed as challenging and time consuming.

Improved students' learning: Teachers view

Many of the teachers in the CPD course stressed the affective factors that they observed when the students used IBL: "They work with joy and enthusiasm"; "I start to realise how fun the students think it is to investigate on their own"; "I think that students are more motivated when they work inquiry-based"; "It engages". However, the participants also emphasised students' learning and how it affects these when they studied according to IBL. One of the teachers highlighted the importance of students' previous knowledge and experience emerging when they use IBL: "Students learn when they can influence their learning situation on the basis of their own knowledge and experiences, which they do when they work inquiry-based.*"* The importance of student participation, responsibility and activity was emphasised: "The students should be more active in the planning, in setting goals and in the assessment." IBL also entails that the students must take greater responsibility for their work and develop collaborative skills when working in a group. According to teachers, it is important that the students feel like they are allowed to make mistakes and learn from them.

Some participants pointed out that IBL is not appropriate for all students and that not all students like this method of working: "Many students can work in this way, but for some of them, it is hard and challenging." One of the teachers saw the challenge of getting the students interested in learning for their own sake, not just because they must achieve for the teacher's sake.

Although the teachers viewed IBL as a way of teaching in which students take more responsibility and learn more independently, they also stressed the importance of guidance and support from the teacher. One teacher pointed out that IBL requires time to learn and that as a teacher; you should approach it in small steps. She thought that it might be easier if the students learned IBL in their early years in school.

Changes in instruction

Looking ahead, what did the participants think about their science teaching after the course? Would the CPD course have a lasting impact on their teaching? Teachers' comments show some changes in their thinking*: "*The course reopened my eyes, and I had the opportunity to reflect on my working methods"; "I have seen opportunities"; "I have already noticed that I change my way of teaching to get the students to solve problems and investigate more"; "I do not serve everything*"*. However, on the question of what impact the course would have on their future teaching, the participants remained cautious. Formulations like "I think I will", "I aim for", "I hope" and "I try" dominated the answers when asked about letting students use IBL. There were clear hopes, as well as insights into difficulties in terms of lack of time, stress, dividing up time for countless other tasks and shortcomings in their own knowledge. The teachers also saw difficulties in breaking their old patterns and changing their mindset. A mindset can affect what changes will be made and how big they will be. However, the teachers clearly saw that IBL was in line with the core curriculum, and they saw opportunities in this method of learning. IBL was an eye-opener for one teacher, who reflected on its benefits with science learning:

> It is interesting to wonder about things around us. It is important not only from national economic aspects (which are often emphasised), but as rewarding for the individuals, in a way that is difficult to measure, like all 'bildung'.

Semi-structured interviews were conducted one or two years after the CPD course with five of the attending teachers from three separate CPD groups. The purpose of these interviews was to investigate what perceptions these teachers have about teaching science after the course and what possible effects the course exerted on their teaching and attitudes.

All five teachers said they use IBL in some form in their teaching. According to all these teachers, a coherence exists between the national core curriculum's objectives and IBL. The teachers feel that the CPD course gave them an increased awareness about their teaching: "What questions should you start from?"; "Should the students find out more themselves?" According to one of the interviewed teachers, the most important thing that stuck in his mind is that the course gave him a new mindset. He feels that he has been able to free himself from old, inward models so that he can, as he expresses it, "think outside the box".

While working with various inquiry-based projects in their classes after the CPD course, the teachers gradually learned to limit the projects and focus on more relevant aspects. One teacher says that she now starts from the students' prior knowledge and competence levels in her teaching. The ability to solve problems of various kinds, individually or together with others, is described by another teacher as a competency that the students will need in different societal contexts. He sees IBL as a tool to help students develop problem-solving competencies. The importance of teachers having a firm grasp of subject knowledge, being well-prepared and linking projects to scientific concepts is emphasised.

6 Discussion and conclusions

6.1 Core features of the professional development model and teacher growth

Central in the CPD course was the connection between the theory and the themes that were addressed during the workshops and the teachers' own implementation of them in their own science teaching practice. Garet et al. (2001, pp. 925–926) stress the importance of teachers' active learning that "involves the opportunity to link the ideas introduced during professional development experiences to the teaching context in which teachers work." Participants experienced the study sessions as being important to gain theoretical knowledge of IBL and increase subject knowledge. The teachers also gained insight into current research in the field about e.g. children's pre-conceptions in science, the inquiry cycle and formative assessment. The teachers felt that they normally had neither the time nor the opportunity to take part in current research. Teacher trainers gave the teachers individual mentoring at their own schools. The teachers deemed this as being very important (cf. Kleickmann et al., 2015). The guidance that the teachers received encouraged them to develop and carry out their own sequences. The teachers did not receive any ready-made teaching materials. Instead, they had to design their IBL teaching sequences by themselves. For the trainer, it was challenging to find a balance in supervising, but not dictating, what each teacher should do (cf. Bjønnes & Kolstø, 2015). The same challenge applies to the teacher in teaching situations in which students are involved in IBL. This supervision situation can provide the teacher with a model for these situations (cf. van der Valk & de Jong, 2009).

This study's results confirm previous research (e.g., Garet et al., 2001; Peers, Diezmann & Watters, 2003; Desimone, 2009) that found longitudinal CPD courses provide teachers with opportunities to reflect on their own learning and develop sequences where student work with own IBL projects. The course in this study provided the participants with opportunities to discuss ideas with other course participants during the study sessions, which was viewed as important. If they took the course with colleagues from their school, the teachers continually could discuss and develop their sequences in relation to course content between study sessions. The participants who had colleagues from the same school highlighted the advantage of this. The significance of collaboration with colleagues also is stressed in previous research (Garet et al., 2001; Desimone, 2009; van der Valk & de Jong, 2009).

6.2 Implementing IBL – teachers' views

The teachers in this study expressed agreed that IBL entails new roles for both the teachers and the students, requiring time for reflection and for maturing into these roles (cf. Fullan, 2016; Åhman, Gunnarsson & Edfors, 2015). The role of the teacher becomes more that of a coach than a knowledge communicator. This entails challenges for teachers regarding their view of their own teaching role, learning goals for the students and the students' responsibilities in the learning process.

Teachers in the study emphasised the importance of having their own control over their teaching, clear goals for students' learning and a well-thought-out plan when students work inquiry-based (cf. Hodson, 2009; Smith et al., 2012). At the same time, the teachers saw the importance of the students working with their own questions, which means that they, as teachers, must have the courage to let go of some of their control. This is in line with what van der Valk and de Jong (2009) discuss as giving the students structure as well as space. Teachers who are accustomed to following a textbook may find it difficult to free themselves from the teaching pattern they usually use.

IBL is about having the courage to believe that students learn, even if the teacher does not "present everything to them"; it is about having the courage to open up to questions without having the answer. As the teacher, you must dare to work outside your comfort zone. The need for deep subject knowledge was highlighted. Our interpretation is that when you feel secure in your own subject knowledge, you feel safer to let your students engage in questions you do not have direct answers. The teacher's coaching and scaffolding role is important (Furtak et al., 2012). This way of working is new for many teachers. According to the teachers in the study, IBL requires time to learn and as a teacher, one should approach it in small steps.

The teacher's view of the subject itself and what is important for students to learn is also challenged when working more student-centered with IBL. While one teacher saw the problem of "coping with all the curriculum content" when students use IBL, another emphasised the importance of "doing thoroughly what you do". Letting students start from their own prior knowledge with questions that they formulate themselves takes more time than traditional teacher-centered instruction where the teacher follows the textbook. It is perhaps easy to forget that the objectives in the core curriculum consist not only of content knowledge but also of research and working skills (FNBE, 2016). These can be learnt by doing inquiry-based science.

A new insight and a new mindset in teachers after the course was expressed by one participant: "Science is about thinking outside the box and it means much more than factual knowledge." Another participant expressed her new insight "Science is about bildung", which is in line with the European Commission report (2015).

6.3 Teachers' views of students' learning with IBL

During the CPD there was an ongoing reflection and discussion within the group of teachers about how IBL affects different aspects of students' learning. Particular factors that emerged are the need to achieve deep learning by giving students enough time to work with their projects. Deep learning, which is about thinking and understanding, must according to Fullan (2016) be developed if one is to understand the world. One challenge that teachers highlighted was their own fragmented work picture and the perception that IBL takes more time than traditional teaching. The teachers were worried about not having enough time for teaching the subject matter. Van Aalderen-Smeets and Walma van der Molen (2015, p. 711) argue that "science should be taught as the process of acquiring scientific knowledge (inquiry-based learning approach) and should stimulate an understanding about the nature of scientific inquiry, rather than teaching science as a body of knowledge".

National (FNBE, 2016) and international steering documents (European Commission, 2015) stipulate that students shall develop their problem-solving skills and collaborative skills, among others. The teachers highlight both of these skills in the study as skills that the students develop by working with IBL. Several of the participating teachers also described IBL as motivators for students, exerting a positive influence on their learning (cf. Kurtén-Finnäs, 2008). According to the teachers, students felt that they could influence their own learning situation. The students' interest and motivation, in turn, inspired the teachers to continue to use IBL.

However, according to the teachers, IBL does not work with all students, though the teachers did not specify which group of pupils they were talking about. Research on the use of IBL and students with learning disabilities shows that investigative work makes a positive impact on the students' understanding, as well as on their scientific attitudes and their attitudes toward the subject (e.g., Aydeniz et al., 2012). A potential group that is negative toward IBL might be students who want clear instructions, as well as structured teaching. Eysink, Gersen and Gijlers (2015) have examined talented students who work with both structured inquiry and unstructured inquiry. Their results show that the gifted students experienced a lack of control of the task when it was unstructured. For them, it was important to get some guidance during the process. For teachers who have no or very little experiences of inquiry-based teaching, it can be difficult to know how much guidance they should give students when they are working inquiry-based.

6.4 Teachers' changes in instruction

IBL, as a working method, is entirely in line with intentions in the national core curriculum (FNBE, 2016). Immediately after the CPD course, the teachers had clear hopes of implementing IBL in their teaching. Simultaneously, concerns arose over lack of time, positioning of teacher-student roles and the ability to work toward learning goals without dictating what the students must do. There was also a fear that their own subject knowledge was insufficient.

Teachers who were interviewed one or two years afterwards completing the course stated that they use IBL in some way in their own teaching. They have learned to limit the sequences and the students' projects and focus on the essentials. They also stated that they now approach teaching based on students' knowledge and skills.

The model of a CPD course in which theoretical input, scaffolding, practical work and educational reflections are combined appears to result in positive changes in teacher instruction and, by extension, hopefully in students' science learning.

6.5 Critical reflections

Each CPD course ended with an evaluation discussion that was recorded. In this way, each teacher's thoughts about the course and the use of IBL in their teaching were collected. Teachers' diaries would have provided even more information about their thoughts during the course. For the continuous formative follow-up, some other type of tool for reflecting and reporting could have been used. The tool should be one that teachers experience as less work-intensive. For the researchers, this means a balance between exposing the teachers to extra work and collecting important information. Despite the fact that the teachers' diary entries became few, these still contributed to the information becoming more versatile.

The teachers who participated in the course volunteered to participate and were motivated. To introduce and implement the same programme for all the teachers in one school would be more challenging. The teachers who participated in the CPD course were interested in teaching science. They wanted to develop their subject knowledge and their pedagogical competence within the subject. For teachers who lack personal motivation, more personal guidance is needed, which involves guidance on subject knowledge as well as on IBL.

Only five teachers who participated in the CPD course were interviewed in the study, between one and two years after. Interviews with all participating teachers would have given more information about the long-term impact on their teaching.

6.6 Implications

In the future a CPD course about IBL could be directed toward all teachers within a school. IBL in Science education fits well into interdisciplinary themes where e.g. aspects from biology, physics and chemistry can be investigated. A new CPD course about IBL could be directed toward all science teachers within a school. This would create more opportunities for discussion between teachers. Such a CPD course about IBL, which is aimed at an entire teacher group, also presupposes that the principal of the school is familiar with the CPD's objectives and supports the teachers in the developmental work that the CPD entails. According to Fullan (2016), the principal is in a key position concerning teachers' professional development and development of the school community. In the Finnish school context the principals, parallel to the administrative leadership, also have the responsibility of being educational leaders. This demands that the principals are familiar with current educational research and with various opportunities for teacher development.

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Organizing the mathematical proof process with the help of basic components in teaching proof: Abstract algebra example

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The aim of this study is to identify the basic components of the mathematical proof process in abstract algebra and to organize the proof process into phases with the help of these basic components. A basic component form was prepared by arranging a draft basic component form, which was created as a result of a document analysis in accordance with the opinions of three academicians, who were experts in algebra. The data obtained as a result of both document analysis and expert examination were analyzed by the descriptive analysis method and are explained here in a detailed manner. It is believed that this basic component form will facilitate the step-by-step addressing of such a complex process as proofs in a non-compulsory and non-hierarchical order.

Keywords: teaching abstract algebra, basic components, mathematical proof process, teaching proof

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1 Introduction

The aims of mathematics are to understand the rules on which numbers, algebra, and geometry are based; to find new and non-routine ways in these systems; and to explain new situations that are encountered. The world of mathematics is a world of ideas, insights, and discoveries, which will not be realized by people who are only interested in how to separate a function, and it is necessary to use mathematical proof and abstraction to enter this world (Goldberg, 2002). A mathematical proof, which is far more than just one or several examples supporting a mathematical statement (Derek, 2011), is a logical explanation of why a mathematical statement is true (Altıparmak & Öziş, 2005). The best proof is the one that helps understand the proved theorem by showing not only that it is true but also why it is true (Hanna, 2000).

1.1 The purpose of the study

Proofs constitute the basis of mathematics (CadwalladerOlsker, 2011; Sarı, 2011). Therefore, many studies have been conducted on mathematical proofs. However, the mathematical proof process was not presented step by step in any study as in this study.

Abstract Algebra is one of the most important course of mathematics and mathematics education at the university level. According to Agustyaningrum, Husna, Hanggara, Abadi and Mahmudi (2020), abstract algebra is full of definitions and theorems which all require proof. Therefore, the students need to understand every definition and theorem they learn and be able to organize the concepts needed to proving theorems.

For that to be possible, students should have the ability to organize the required information for proving theorems in abstract algebra. This requires them to have an idea about the structure and stages of the proofs.

Due to these reasons, the mathematical proof process in abstract algebra should be examined and presented step by step. The aim of this study is to classify the mathematical proof process in abstract algebra with the help of basic components. Furthermore, it was also aimed to determine the knowledge that would help students facing problems and would also increase their understanding of mathematical proof. The following two research questions are posed for this purpose:

1. What are the difficulties students face during the mathematical proof process?
2. What are the stages of the mathematical proof process in abstract algebra?

The answer to the first research question was sought by reviewing the literature addressing the difficulties experienced in the mathematical proof process and analyzing lecturers and students' notes in order to see how the mathematical proof process in abstract algebra was experienced by students and teacher candidates and what kinds of difficulties they faced in this process. Similarly, the second research question was replied based on literature and the opinions of experts. Thus, the proof process was staged as basic components and then exemplified.

Unlike previous studies (Boero, 1999; Leron, 1983) in which the structure and stages of proofs were revealed, this study is inspired by the studies that deal with the difficulties students experience in the process of proof and the existing literature. In studies investigating the difficulties of proof, the content of the proof is also addressed, albeit not directly, and this gives us an idea of the nature of the proof.

2 Literature Review

The proof process is a complicated and hierarchical process in which more than one thinking process and stage are included, and mathematical information is used in an intertwined manner. Students come to a dead end when they do not know what to do when constructing proofs (Selden & Selden, 2008; Weber, 2001). In the proof process, students mentally go through steps of identifying the problem, making an assumption, testing the truth of the assumption by outlining the proof, conclusions, re-checking the proof that has been obtained, and making justification (Faizah, Nusantara, Sudirman, & Rahardi, 2020). Researchers state that pre-service mathematics teachers who encounter a mathematical proof for the first time face problems such as not knowing how or where to start, not understanding the logic of the proof, not being able to decide on what kind of method and conceptual information to use, and not being able to conclude the proof (Güler, 2013; Güler & Dikici, 2014; Moore, 1990; Moore, 1994; Polat & Akgün, 2016; Selden, Selden, & Benkhalti, 2018; Weber, 2001; Yeşilyurt Çetin & Dikici, 2020).

To understand the abstract and conceptual structure of mathematics, it is very important for students to understand the concept of proofs, what a proof is, why proofs are constructed, and the proof process (Sarı, 2011). If students do not know how to construct a proof, they try informal approaches, such as using examples or looking at a graph (Raman, 2002). At the undergraduate level, when mathematics students are expected to construct a proof, sufficient time is not allocated to helping students learn how to do so, and therefore the difficulties that students face cause many of them to quit studying mathematics (Selden & Selden, 2008). Pre-service teachers have difficulty in constructing long, acceptance-based proofs that they encounter for the first time (Polat & Akgün, 2016). Furthermore, most pre-service teachers do not know how to start a proof (Güler & Dikici, 2014; Polat & Akgün, 2016).

According to Weber (2001), the primary cause of failure in constructing proofs is the lack of strategic knowledge and it must therefore be ensured that students gain effective strategic knowledge. Pre-service teachers have difficulty in determining a proper proof method and strategy when constructing proofs (Doruk & Kaplan, 2015; Güler, 2013). According to Karakuş and Dikici (2017), students of secondary school mathematics teaching have difficulty in using proof methods effectively, although they think that proof methods play a significant role in the proof process. Demiray (2013) determined that pre-service teachers are highly successful in refutation and proof by contradiction, but they give incorrect answers to contrapositive proof questions since they have difficulty in realizing and understanding the equivalence of contrapositive expressions, and pre-service teachers further fail to see the difference between an empirical argument and a valid proof. According to Ceylan (2012), the use of examples in the proof process by pre-service teachers may mean that they do not have sufficient logical inferences. Furthermore, according to Güler, Özdemir, and Dikici (2012), pre-service teachers failed to understand the relationship between mathematical induction steps completely and perceived this proof method as a procedure to be followed. The results of that study showed that pre-service teachers understand mathematical induction but fail to generate the induction step by using the induction hypothesis. In another study, Jones (2000) pointed out that pre-service mathematics teachers do not have a sufficient level of skills to construct proofs; that they do not have the mathematical knowledge necessary for effective mathematics teaching, including those in advanced grades; and that they graduate this way. It was also shown that pre-service teachers in advanced grades can construct proofs more smoothly in technical terms, but this does not provide them any benefit in terms of their associated mathematical knowledge in conceptual terms.

Karaoğlu (2010) stated that sufficient conceptual knowledge is needed to complete a proof and to understand how to use such knowledge in the proof of a theorem. Pre-service teachers have difficulty with proofs that can be done by using definitions even at the most basic level (Şahin, 2016). A pre-service mathematics teacher's experience with proofs, sufficient conceptual understanding, and skill in following different methods are all important for directing towards conceptual images when encountering a mathematical problem and having the skills to begin a proof (Bayazit, 2009). When pre-service teachers do not have conceptual knowledge about the theorem they want to prove, they cannot start to construct the proof, and so they begin with the help of the concepts obtained by investigating all the concepts related to the theorem in their mind and try to find the one that works. However, if their available knowledge is insufficient, they cannot conclude the proof (Karaoğlu, 2010). Another difficulty encountered in the proof process is incomplete or incorrect preliminary knowledge (Polat & Akgün, 2016; Yeşilyurt Çetin, & Dikici, 2020). Even if the pre-service teachers know which property and definition to use in the proof, they have difficulty using them while constructing the proof (Güler, 2013; Yeşilyurt Çetin, & Dikici, 2020).

By using the relevant literature, Moore (1990) addressed the difficulties experienced by undergraduate mathematics students in understanding proofs and the construction of proofs within seven categories. Accordingly, these students:

- do not know or cannot express definitions;
- have an inadequate intuitive understanding of the concepts;
- are not competent in proving conceptual images;
- are unsuccessful or unwilling to create and use their own examples;
- do not know how to use definitions to create the whole structure of a proof;
- are unsuccessful in understanding and using mathematical language and notation; and
- do not know how to start the proof process.

According to Moore (1990), students also experience an inability to coordinate and use all the information simultaneously in constructing mathematical proofs. Güler (2014) addressed the difficulties faced by pre-service teachers in the proof process within five categories as follows: determining how to start the proof, using the mathematical language and notation, using definitions, forming the setup of the proof (fully determining the steps to be followed), and selecting the elements from the set.

Polat and Akgün (2016), who observed the proof process among pre-service teachers, stated that pre-service teachers could not decide on which definition to use when constructing proofs, do not make any plans before starting the proof process, and cannot decide on how to begin the proof. They examined the reasons for the difficulties experienced by pre-service teachers in the proof process under two headings, those originating from the individual and those originating from the mathematical subject, and they charted them as follows:

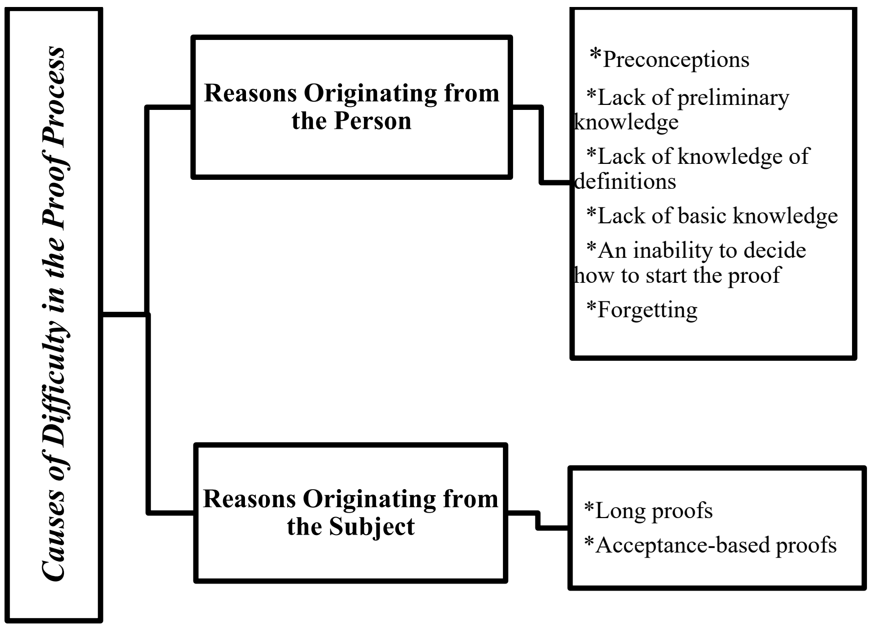

Figure 1. Reasons for experiencing difficulties in the proof process by pre-service teachers (Polat and Akgün, 2016).

According to Stewart and Thomas (2019), the reason why students face so many difficulties during the proof process is that each component in a proof is packed with different conceptual ideas. Therefore, it may be unrealistic to expect students to logically piece together the many definitions, other theorems, and various results that would help in completing a proof.

As can be seen from this literature review, the proof process is a complex process involving many difficulties that must be overcome step by step. In this study, each of these steps that make up a proof is called a basic component, and it is aimed to structure the mathematical proof process in abstract algebra into phases with the help of these basic components. This would make it possible to refine the proof process from relative complexity and address it within the framework of a certain order.

3 Method

A qualitative research method was used in this study, which aimed to phase the mathematical proof process in abstract algebra with the help of basic components by giving explanations about the process in accordance with expert opinions. Qualitative research methods are preferred when there is a theory gap in a subject, or when the existing theory is incapable of explaining the phenomenon. Moreover, the primary tool in data collection and analysis in qualitative research is the researcher himself or herself (Merriam, 2013). In this study, in which the proof process is partitioned into phases, the researchers themselves were the main elements in the collection and analysis of data.

Among qualitative research methods, a case study design was utilized in this study. A case study is a qualitative research method that offers the opportunity to gain in-depth knowledge of the meaning of situations and events (Merriam, 1998). In this study, basic components were created in order to understand the proof process in depth, and as these basic components were identified, relevant documents and literature were used together with expert opinions.

3.1 Participants

In determining the basic components that make up the mathematical proof process in abstract algebra, three professors who conduct academic studies in the field of algebra were asked to make an examination and provide their opinions. These experts had at least 10 years of experience in teaching abstract algebra, number theory, and linear algebra.

3.2 Data Collection

In this study, document analysis was performed, and expert opinions were obtained in order to find the answer to the following questions: "What are the difficulties students face during the mathematical proof process?" and "What are the stages of the mathematical proof process in abstract algebra?". Documents are important sources of information that must be used effectively in qualitative studies. A document analysis involves the examination of written materials about the cases or situations that are being investigated (Yıldırım & Şimşek, 2008).

Mathematical proofs are complex processes with many challenges that consist of several steps. Therefore, the proofs determined by the researchers were divided into steps so that the proof completion process could be addressed step by step with the complexity removed. In addition to literature review, the textbooks (Çallıalp, 2009; Karakaş, 2001; Taşçı, 2010) and course materials for abstract algebra, the notes of the lecturers teaching abstract algebra, the course notes of students were also examined and evaluated to identify the difficulties in the mathematical proof process. Accordingly, a draft basic component form involving those difficulties was constructed. The literature on the difficulties experienced in the mathematical proof process was briefly explained to three experts in algebra together with basic information about the study, and they were asked to share their views on the draft basic component form and evaluate it. And then this draft form was reshaped in the light of their opinions. The final form of the 'basic component form', which was shaped by the document analysis and expert opinions, is presented in detail below in the "Findings" section.

3.3 Credibility and Transferability

The credibility and transferability issues for the findings were pursued in the following way: After 'the draft basic component form' was prepared, two mathematicians checked 'the draft basic component form' and after 'the basic component form for proofs' was prepared, five mathematicians checked 'the basic component form for proofs'.

One of the methods used to increase the credibility of a study is to obtain expert opinions. While determining the basic components, the opinions of three experts were obtained, and the process was directed in this direction. In this study, textbooks and course materials for abstract algebra, the notes of the lecturers, the course notes of students, notes taken during the study, and the relevant literature were examined and evaluated in the light of expert opinions. In this way, the credibility of the study was increased through a diversity of methods.

3.4 Analysis of the Data

In case studies, the researcher must rely on his or her own instincts and skills (Merriam, 2013). The data for the determination of the basic components that make up the mathematical proof process were analyzed by descriptive analysis in line with the researchers' instincts and skills and are presented with a holistic approach. Purpose in descriptive analysis to present the obtained findings in an organized and interpreted manner (Yıldırım & Şimşek, 2008). In the descriptive analysis of the data, the expert opinions on the draft of the basic component form, which was created as a result of document analysis, were taken as a basis. The opinions of the three expert academicians on the draft of the basic component form are quoted in detail.

The literature addressing the difficulties experienced in the mathematical proof process is explored step by step and analyzed as follows:

Table 1. Difficulties experienced in the mathematical proof process according to literature review

The Challenges Encountered in the Mathematical Proof Process	The Role of the Challenges Encountered in the Mathematical Proof Process in the Draft of the Basic Component Form
Understanding the logic of the proof	Determine the hypothesis and determine the judgement
Knowing how and where to start the proof	Use of hypothesis
• Being able to decide what kind of method and conceptual information to use • Knowing how to use preliminary knowledge, definitions, properties and concepts to create the whole structure of the proof • Understanding and using the mathematical language and notation • Forming the setup of the proof (fully determining the steps to be followed)	Process steps
Being able to conclude the proof	Reaching proof

Figure 2, created based on Table 1, was submitted for expert examination. Figure 3 was created in accordance with the expert opinions.

3.5 Ethical Issues

In this study, the purpose and method used are presented to the reader with a detailed explanation. The experts whose opinions were used were informed about the research process and their personal information was kept confidential, but some academic information about the experts was included in order to ensure the credibility and transferability of the study. Furthermore, the researchers in this study did not act biasedly in the process of data analysis and have reported the findings that they obtained without making any changes to them.

4 Findings

In this section, it is aimed to determine the basic components of the mathematical proof process. To this end, the proof process was schematized as a draft basic component form after examining textbooks about theories and proofs in abstract algebra, the course notes of the lecturers and the course notes of students together with the literature revealing the challenges encountered in the mathematical proof process.

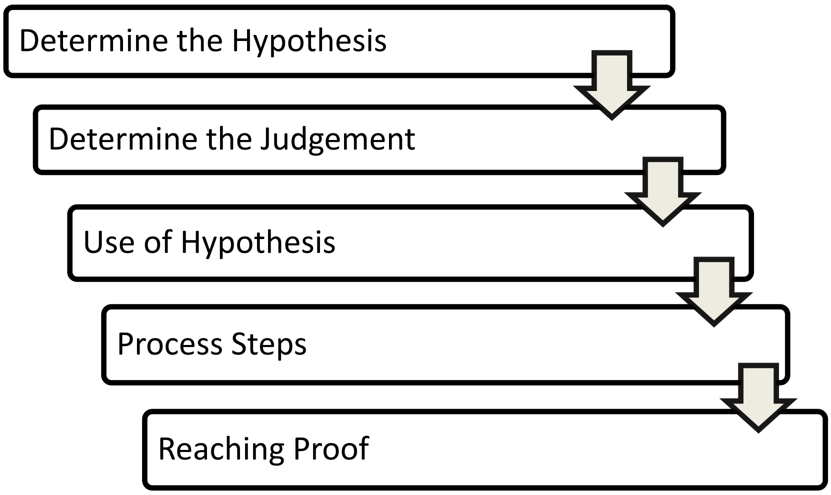

Figure 2. Draft basic component form.

The draft of the basic component form was submitted for the review of the three expert academicians. The literature on the difficulties experienced by students in the mathematical proof process was also provided to the experts, and their opinions on the draft form were obtained. One of the experts (P1) stated that the division of the process steps here should be expanded by being detailed in a non-hierarchical order, such as the use of definitions, use of properties, use of knowledge and theorems, and the performing operations. The second expert (P2) expressed similar opinions and stated that the step of using the hypothesis should be included in this non-hierarchical order. The third expert (P3) stated that the step of determining the method should be added after the step of determining the judgement.

The literature that reveals the difficulties experienced in the mathematical proof process and the expert opinions about the basic component form were analyzed as follows:

Table 2. Developmental process of the basic component form based on literature and expert opinions

The Challenges Encountered in the Mathematical Proof Process	**Expert Opinions**	**The Role of the Challenges Encountered in the Mathematical Proof Process in the Draft of the Basic Component Form**
Understanding the logic of proofs	No opinion	Determine the hypothesis and the judgement
Being able to decide what kind of method and conceptual information to use	*The step of determining the method should be added after the step of determining the judgement (P3),*	Determine the proof method
Knowing how and where to start the proof	*The step of using the hypothesis should be included in a non-hierarchical order (P2)*	Use of hypothesis
Knowing how to use definitions to create the whole structure of the proof	*The step of using definitions should be included in a non-hierarchical order (P1, P2)*	Use of definition
Knowing how to use properties to create the whole structure of the proof	*The step of using properties should be included in a non-hierarchical order (P1, P2)*	Use of property
Being able to decide what kind of conceptual information to use to create the whole structure of the proof	No opinion	Use of conceptual knowledge
Knowing how to use preliminary knowledge and basic information to create the whole structure of the proof	*The step of using knowledge and theorems should be included in a non-hierarchical order (P1, P2)*	Use of knowledge
Forming the setup of the proof (fully determining the steps to be followed)	*The step of performing operations should be included in a non-hierarchical order (P1, P2)*	Perform Operations
Being able to conclude the proof in accordance with mathematical language and notation	No opinion	Complete the proof

Based on the expert opinions and the challenges encountered in the mathematical proof process, the basic component form was reshaped as follows.

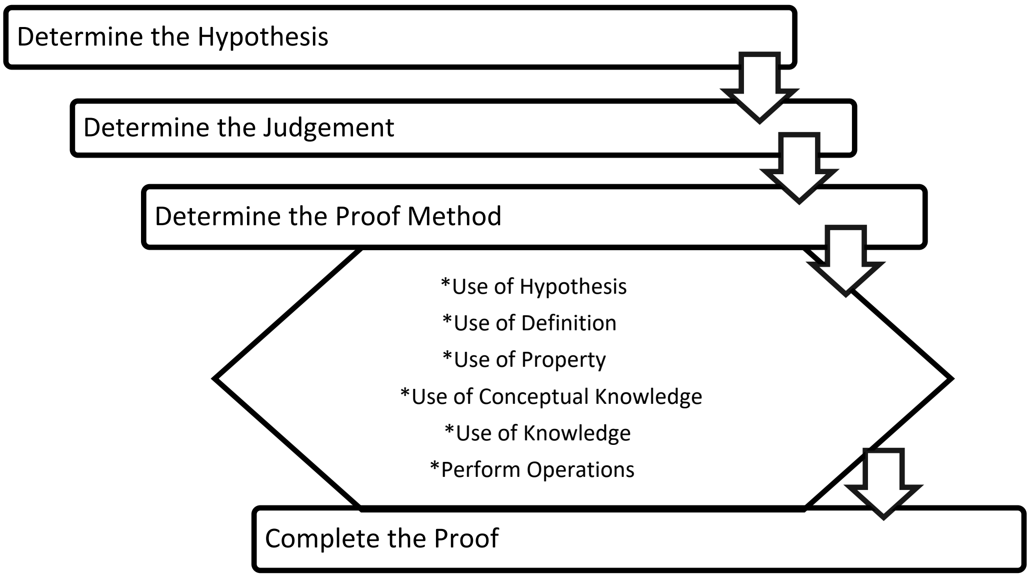

Figure 3. The basic component form for proofs.

These steps are non-hierarchical, and there is no obligation to apply every step in every proof. In other words, it is not necessary to determine the hypothesis in the proof of a theorem that only determines the judgement, such as "Prime numbers are infinite." Similarly, while no definition is used in certain proofs, there is no need to use any conceptual knowledge or properties in some others.

It is expected that an assumption that will provide the basis for a proof, or in other words that will lay the foundation of the proof, will be established in the stage of determining the hypothesis. For example, for the proposition $\Rightarrow q$, the expression p is the hypothesis and it is important to be able to determine hypothesis p and write it in mathematical language and notation in order to start proving this proposition.

At the stage of determining the judgement, it is expected that the judgement to be achieved based on the hypothesis will be determined, creating the basis of the proof in this way. For example, for the proposition $p \Rightarrow q$, the expression q is the judgement, and it is important to write judgement q in mathematical language and notation in order to be able to shape the proof of this proposition.

In the basic component form, comprising the process steps of the draft basic component form as finalized by expert opinions, the components are the use of hypothesis, use of definitions, use of properties, use of conceptual knowledge, use of knowledge, and perform operations. These are used according to the content of the proof and in a non-hierarchical order. Therefore, in some proofs, all these basic components are used, while only one or some of them are used in others.

The writing of expressions to help construct a proof based on the hypothesis is expected in the step of using the hypothesis, and it is expected that an auxiliary theorem will be used, which should be known at the time or else should be information gained during the flow of the proof in the step of using knowledge. The use of a property that helps to build the proof is essential in the step of using properties, while the use of an expression that is present in the hypothesis/ judgement or that is mentioned anywhere in the proof and expected to help in the construction of the proof is essential in the step of using definitions. It is expected that the appropriate concepts and the information related to these concepts will be selected and used correctly in the step of using conceptual knowledge. For example, using the definition of subgroups or prime numbers was addressed as the use of definitions, selecting and using the concepts of unit elements and inverse elements in accordance with their properties was addressed as the use of conceptual knowledge, and using group properties when performing an operation was addressed as the use of properties. The component of performing operations aims to reveal the state of taking the proof to a certain level with various algebraic operations. In order to perform an algebraic operation, it may sometimes be necessary to use a definition, sometimes a property, and sometimes conceptual knowledge. At this point, deficiency in one of these areas will make it impossible to successfully complete the proof process.

At the stage of completing the proof, students are expected to complete the proof according to mathematical language and notation by using all the data or information obtained.

It is believed that the basic proof components determined in this form will help students successfully undertake the proof completion process by following a specific order.

4.1 Examples of Proofs Divided into the Basic Components

Examples of proofs that are divided into the basic components of the mathematical proof process in abstract algebra are presented below. The proofs below have been taken from the textbooks (Çallıalp, 2009; Karakaş, 2001; Taşçı, 2010) and course materials for abstract algebra, the notes of the lecturers, the course notes of students. These proofs are divided into the basic components by researchers and confirmed by five mathematicians that checked ‘the basic component form for proofs’.

An example of a proof in which the components of “use of property”, “use of definition”, “perform operations”, “use of conceptual knowledge”, and “complete the proof” are used is given in the following theorem.

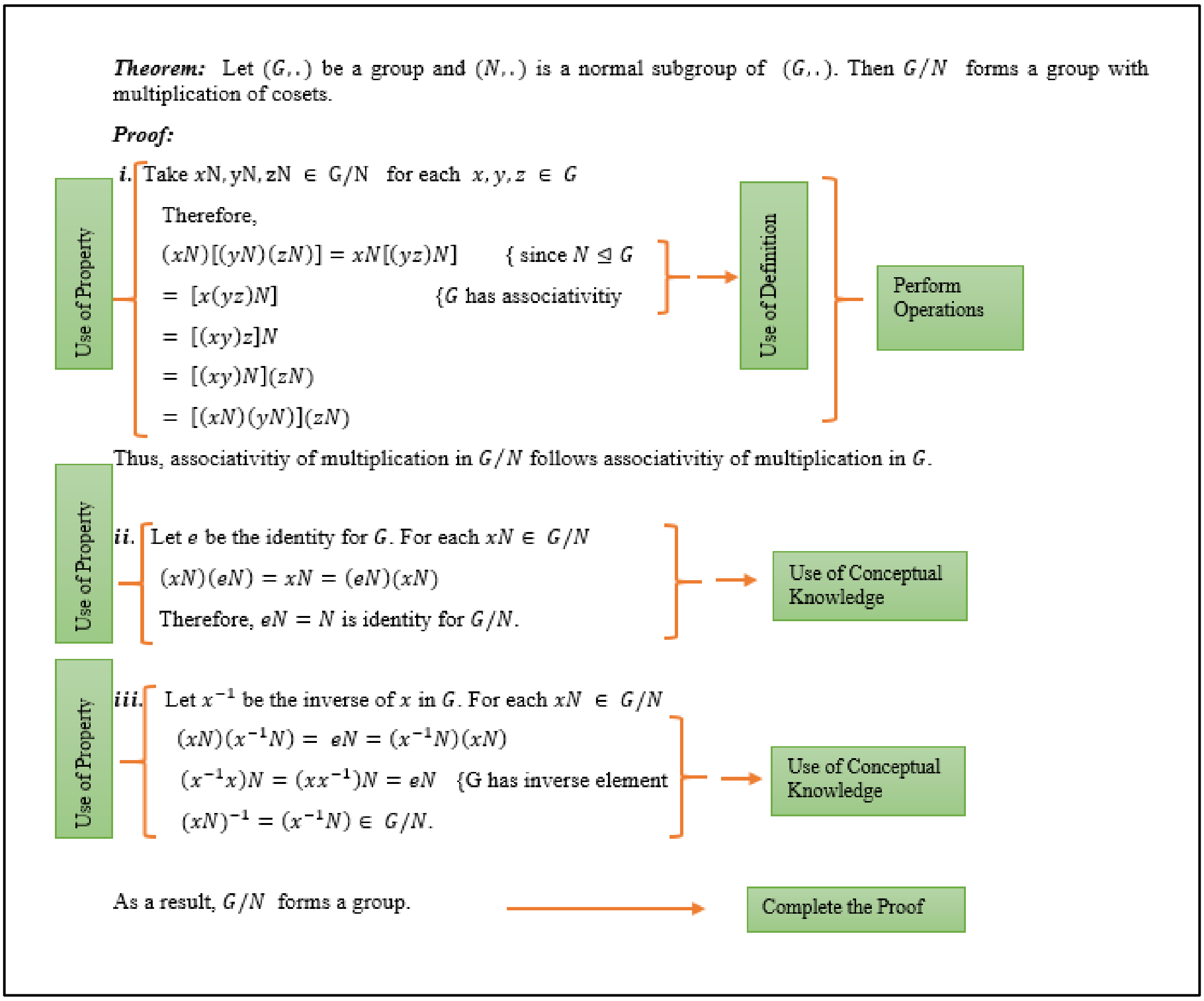

Figure 4. First example for a proof divided into basic components.

In the proof of the above theorem, in cases i, ii, and iii, the subgroup properties are called “use of property”. In step i, the process of performing operations is discussed using the definition of the normal subgroup. In step ii, knowledge about the concept of the unit element is used. In step iii, knowledge about the concept of the inverse element is used. Based on all these steps, the completion of the proof is written as the final sentence.

An example of a proof in which the components of “determine the proof method”, “determine the hypothesis”, “use of hypothesis”, “use of definition”, and “complete the proof” are used is given in the following theorem.

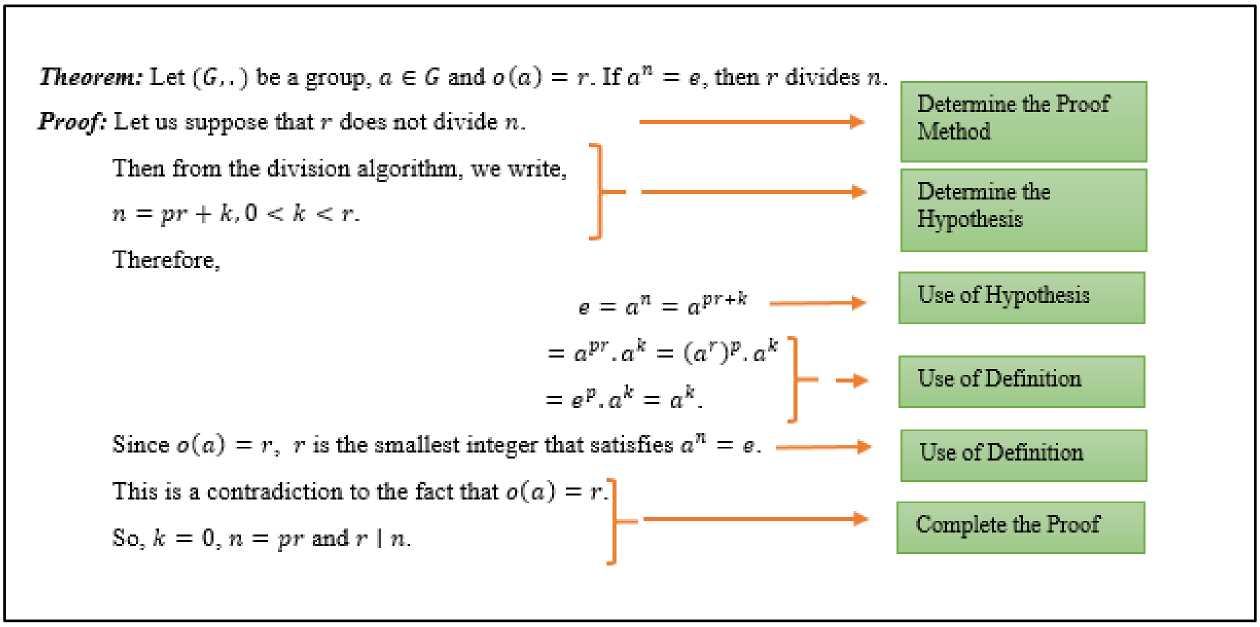

Figure 5. Second example for a proof divided into basic components.

The proof of the theorem presented above was started by determining the method, and then the hypothesis was determined, and an equation was established using this hypothesis. In the established equation, the proof was constructed using the definition of the unit element $o(a) = r \Rightarrow a^r = e$ and an inference was made by using the definition of order. All of this knowledge was interpreted together, and the proof was completed.

An example of a proof in which the components of "determine the hypothesis", "determine the judgement", "use of definition", and "complete the proof" are used is given in the following theorem.

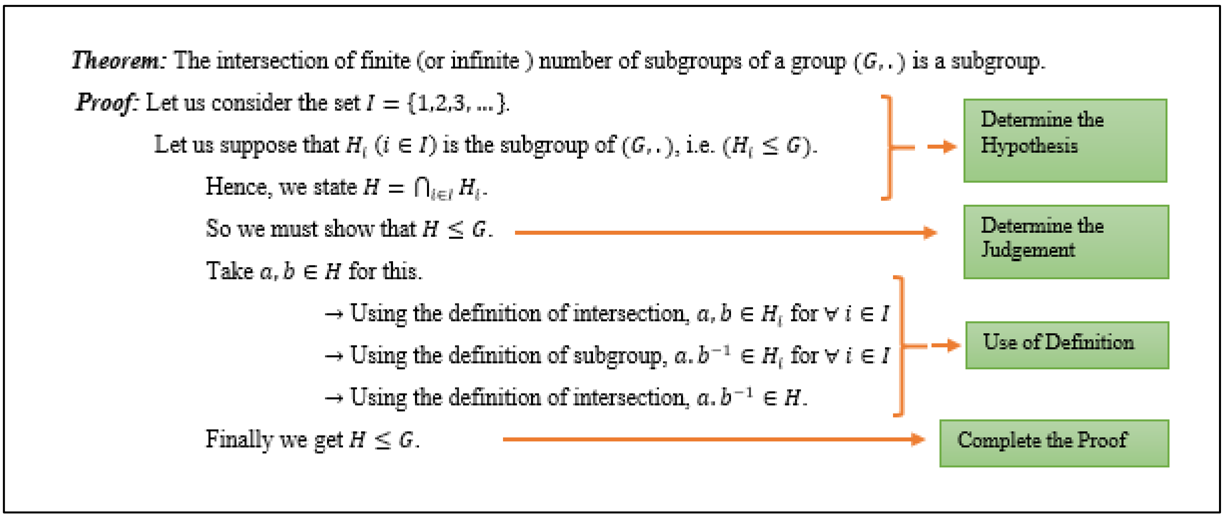

Figure 6. Third example for a proof divided into basic components.

In the proof of the theorem presented above, the hypothesis and judgement were determined based on the statement of the theorem and proof was started in that way. Necessary comments were made using the definitions of intersection and subgroup, and the proof was completed in light of these comments.

The example of a proof in which the components of "determine the hypothesis", "use of hypothesis", "use of definition", "use of conceptual knowledge", "use of knowledge", and "complete the proof" are used is given in the following theorem.

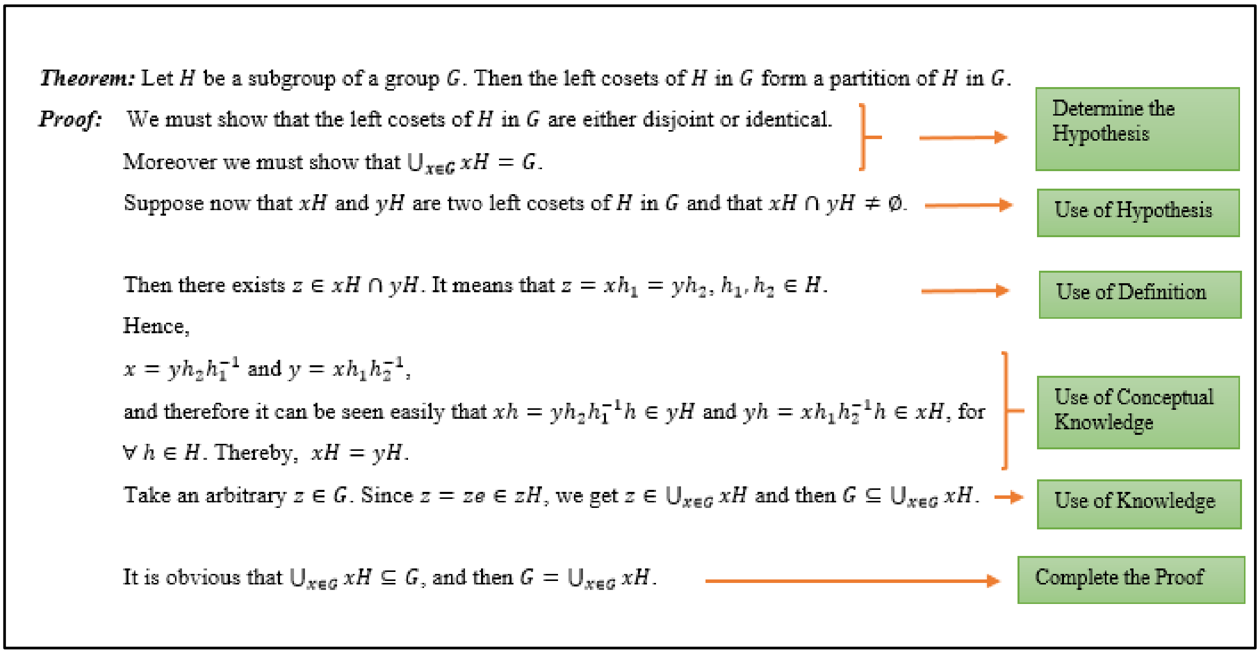

Figure 7. Forth example for a proof divided into basic components.

In the proof of the theorem presented above, the hypothesis was determined based on the statement of the theorem, and by using this hypothesis, left cosets xH and yH, the intersections of which are different from the empty set, were established. Then, using the definition of intersections, the proof was advanced by using conceptual knowledge about the concept of cosets. Finally, it was concluded that $G \subseteq \bigcup_{x\in G} xH$ by using the information achieved while determining the hypothesis and the flow of the proof, and the proof was completed using all this information.

5 Results and Discussion

The purpose of this study is to stage the mathematical proof process in abstract algebra according to its basic components. Therefore, a literature review was carried out regarding how the mathematical proof process was experienced by students and teacher candidates and what kinds of difficulties they faced in this process. The proof process was then staged as basic components based on the difficulties identified according to the literature and the opinions of experts.

Proofs were addressed step by step with a non-compulsory and non-hierarchical structure with the basic component form (Figure 3) that was created in line with the challenges that students encounter in the mathematical proof process. The final basic component form was established in accordance with the opinions of algebra specialists. It is not necessary for each of these components to be included in a proof. The basic components of a proof may vary according to the statement and the proof structure of the theorems. Similarly to this study, Boero (1999) and Leron (1983) dealt with proof generation with non-linear steps.

One of the difficulties encountered in the mathematical proof process is not knowing how and from where to begin the construction of the proof (Karaoğlu, 2010; Moore, 1990; Moralı, Uğurel, Türnüklü, & Yeşildere, 2006; Polat & Akgün, 2016; Yeşilyurt Çetin & Dikici, 2020). In the basic component form presented in Figure 3, the process of starting the proof is addressed in two sub-steps: determining the hypothesis and determining the judgement. The step of determining the hypothesis includes the establishment of an assumption that constitutes the basis of proof, while the step of determining the judgement includes the determination of the judgement to be achieved based on the hypothesis. Similarly, according to Boero (1999), the first two stages of mathematical proof construction are generating an assumption and formulating the statement according to shared textual conventions.

Another difficulty encountered in the proof process is determining the proof method and strategy (Doruk & Kaplan, 2015; Güler, 2013; Karakuş & Dikici, 2017; Weber, 2001), and it is necessary to check the accuracy of the selected method while examining the proof process. In this study, the step of determining the proof method in the basic component form involves determining the appropriate proof method based on the use of the theorem.

The proof process is found to be influenced by the fact that students have adequate conceptual knowledge (Karaoğlu, 2010; Moore, 1990) and understand mathematical definitions and how to use them (Bayazit, 2009; Moore, 1990; Polat & Akgün, 2016;

Şahin, 2016). Students' lack of preliminary basic knowledge also makes this process more difficult (Polat & Akgün, 2016). Therefore, whether students' preliminary knowledge and conceptual knowledge are sufficient and whether they can use their knowledge and the definitions are also factors that shape the mathematical proof process. The component of "use of definition" in the basic component form presented in Figure 3 involves the use of definitions for the purpose of the proof. The component of "use of knowledge" involves the use of preliminary knowledge that must be possessed or the information achieved in the proof process, while the component of "use of conceptual knowledge" involves accurately selecting and using mathematical concepts and information related to these concepts. The component of "use of hypothesis" involves the use of this hypothesis in the flow of the proof. In addition, the component of "use of hypothesis" in this study is similar to the third phase of Boero's (1999) study, "exploration of the content of the conjecture".

It is thought that the components of "use of property" and "perform operations", which were added to the basic component form in accordance with the opinions of three academicians, experts in the field of algebra, shape the proof process. The component of "use of property" involves using a mathematical property that is expected to help construct the proof, while the component of "perform operations" involves carrying the proof to a certain level with various algebraic operations.

The component of "complete the proof" requires that all the information obtained in the proof process is addressed in a certain order, that the necessary inference is made, and that the proof is completed with expressions suitable for mathematical language and notation.

It is thought that structuring the proof of a theorem into phases with the help of basic components and addressing the proof process of students step by step with these components will provide convenience in both the teaching and the investigation of the proof process. It is hoped that the teaching of proofs within a specific non-hierarchical order using the basic component form presented in Figure 3 will facilitate understanding and allow teachers to identify which step in the proof process is difficult for a student. Therefore, it is suggested that including the basic component form and the proofs prepared for this form in textbooks would provide convenience for students in the proof process. In addition, similar studies can be conducted on whether the basic component form revealed in this study can be applied to other mathematics courses other than abstract algebra.

As Selden, Selden, & Benkhalti (2018) suggests, if certain stages of the proof are requested from the students, the success in proving can be increased. Proofs can be staged with the basic components set out in this study. In addition, students may be asked to complete some missing components instead of completing a proper proof. In this case, the teacher can decide about which component will be missing according to the use of this component in the proof. For example, in a proof about homomorphism, the 'use of property' in which the homomorphism property is used is left incomplete and the student can be expected to complete. Or, while teaching such kind of proof, it can be emphasized that the most important thing in making such a proof is the 'use of property'. Thus, students focus on a major part of the proof but not a whole proof, and they are exposed to staged proofs rather than long and intimidating proofs.

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Innovation in the teaching-learning process of global climate change through the collaborative wall

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The aim of this mixed research is to analyze the students' perception about the use of the collaborative wall in the educational process of global climate change considering data science. The collaborative wall is a web application that allows the active participation of students and discussion of ideas in the classroom. During the face-to-face sessions, the students use mobile devices to share the information and images of the courses through the collaborative wall. The sample is made up of 74 students from the National Preparatory School No. 7 "Ezequiel A. Chávez" who took the Biology IV course during the 2019 school year. The results of machine learning (linear regression) indicate that the organization of ideas and dissemination of information in the collaborative wall positively influence the learning process of global climate change, motivation and interest of the students. Data science identifies 6 predictive models about the use of the collaborative wall in the field of Biology through the decision tree technique. In fact, the use of the collaborative wall in the Biology IV course facilitated the assimilation of knowledge about the global climate change and improved the active participation of the students in the classroom. Finally, the collaborative wall allows the creation of new educational spaces where students acquire the main role during the learning process.

Keywords: collaborative wall, learning, data science, machine learning, global climate change

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1 Introduction

Today, universities are using technological advances such as educational platforms, digital tools and web applications during the organization of courses in order to develop the students' skills (Almoeather, 2020; Zhang, 2018). In fact, teachers use Information and Communication Technologies (ICTs) to create new virtual spaces that facilitate the learning process at home and participation of students in the classroom (Almoeather, 2020; Benitez et al., 2020).

The incorporation of technological tools and software in the educational context increases the motivation and satisfaction of students during the performance of the school activities (Benitez et al., 2020; Elekaei, Tabrizi, & Chalak, 2020; Galimullina, Ljubimova, & Ibatullin, 2020). Also, teachers use technology to promote the active role of students inside and outside the classroom (Benitez et al., 2020; Shen et al., 2020; Whalley & Barbour, 2020).

In the 21st century, the Internet plays a fundamental role in the educational field (Bidarra & Rusman, 2017; Paepe, Zhu, & Depryck, 2018; Elekaei, Tabrizi, & Chalak, 2020; Salas-Rueda, 2018). During the innovation of the learning process, teachers are incorporating web applications in the school activities to encourage the active role of students (Appleton & Mackie, 2019; Manrique et al., 2017). For example, the collaborative wall is a web application that allows the active participation of students and discussion of ideas in the classroom through the organization, dissemination and use of images and information (Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021).

In the Biology IV course, the students have a passive role during the realization of the school activities and present difficulties to understand the topics. In fact, the exchange of ideas, collaborative work and discussion of contents in the classroom is rare in this course. Therefore, this mixed research proposes the incorporation of the collaborative wall in the teaching-learning process of global climate change. The research questions are:

- What is the impact of the use of collaborative wall in the learning process of global climate change, motivation and interest of the students?
- What are the predictive models about the use of the collaborative wall in the educational process about biology?
- What are the perceptions of students about the use of the collaborative wall in the school activities?

2 Use of technology in the educational field

Technological advances are changing the organization of school activities inside and outside the classroom (Benitez et al., 2020). In fact, the incorporation of ICTs in the educational field transformed the teaching-learning conditions and improved the courses of English Language (Benitez et al., 2020; Elekaei, Tabrizi, & Chalak, 2020), Geometric Representation Systems (Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021), Biology (Manrique et al., 2017; Poli et al., 2016; Shen et al., 2020), Financial accounting (Han, 2018), Costs (Zhang, 2018) and Computer science (Salas-Rueda, 2020).

In the field of Computer science, the use of MySQL software transformed the role of the students and facilitated the learning process about database issues (Salas-Rueda, 2020). In the classroom, these students worked as a team and used the MySQL software to link the theoretical concepts on the database with the productive field (Salas-Rueda, 2020). After the class, the students of the Database course solved the laboratory practices using the MySQL software (Salas-Rueda, 2020).

In the English language courses, the use of the Nota Bene software facilitated the collaborative work among the students and teacher through the dissemination of opinions and comments (Benitez et al., 2020). This software transformed the teaching-learning process through the presentation of multimedia resources and elaboration of comments (Benitez et al., 2020). The benefits of Nota Bene software in the educational field are the interaction between the participants of the educational process and increase of the academic performance (Benitez et al., 2020).

In the same way, the use of technology in Foreign Language courses facilitated the creation and organization of new school activities (Benitez et al., 2020; Elekaei, Tabrizi, & Chalak, 2020). In particular, the students of the English Language course developed their vocabulary skills through the consultation of podcast and review of animations (Elekaei, Tabrizi, & Chalak, 2020). Even the incorporation of ICTs in this course allowed the personalization of the learning process (Elekaei, Tabrizi, & Chalak, 2020).

Han (2018) built an educational web system in order to facilitate the teaching-learning process about financial accounting. In fact, the students actively participated by consulting the audiovisual contents about finance and holding the discussion forums in this educational web system (Han, 2018). Also, the incorporation of technology in the Introduction to Financial Accounting course caused an increase of the academic performance and motivation of the students (Han, 2018).

In the same way, Zhang (2018) designed and built a technological application in order to facilitate the learning process about accounting. The students used this technological application to facilitate the assimilation of knowledge about cost issues through the visualization of the mathematical procedure (Zhang, 2018). In addition, the use of technology in the Costs course allowed the development of mathematical skills (Zhang, 2018).

The collaborative wall allows transforming the behavior of teachers and the role of students during the performance of the school activities in the classroom (Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021). In the Geometric Representation Systems course, the students used the collaborative wall to exchange ideas with their classmates and discuss the information with the teacher (Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021).

ICTs are changing the roles of students and teacher inside and outside the classroom (Han, 2018; Wang & Fan, 2018). In particular, the creation of new virtual educational spaces encourages the active role of students at any time and place (Elekaei, Tabrizi, & Chalak, 2020; Han, 2018; Xia, 2018).

2.1 Technology in the educational process about Biology

Technology has improved the teaching-learning conditions in the field of Biology (Appleton & Mackie, 2019; Poli et al., 2016; Shen et al., 2020). For example, teachers used ICTs to create new educational spaces that facilitated the assimilation of knowledge about infectious diseases (Shen et al., 2020), anatomy (Manrique et al., 2017), protein evolution (Poli et al., 2016), biodiversity (Lysne & Miller, 2015) and biological evolution (Appleton & Mackie, 2019).

Shen et al. (2020) built a website called "Find the Source of Infection" in order to facilitate the exchange of ideas about the ways of transmission of diseases. This website facilitated the assimilation of knowledge about the origin of infections and improved the communication between the teacher and students through the use of mobile devices (Shen et al., 2020).

Augmented reality plays a fundamental role to improve the teaching-learning conditions in the field of Biology (Manrique et al., 2017). In the Anatomy course, the students used an augmented reality application to facilitate the learning process about the distribution of bones, muscles and organs in the human body (Manrique et al., 2017). The incorporation of technology in the Anatomy course increased the motivation, satisfaction and academic performance of the students (Manrique et al., 2017).

In the Biodiversity course, mobile devices facilitated the active participation of the students inside and outside the classroom (Lysne & Miller, 2015). In particular, smartphones allowed the capture of images about organisms and dissemination of this information (Lysne & Miller, 2015). The students of the Biodiversity course improved their academic performance on the morphology of organisms by analyzing the images obtained from mobile devices (Lysne & Miller, 2015).

Poli et al. (2016) proposed the use of MapBox Studio software in the field of Biology in order to promote the active role of the students during the teaching-learning process. In particular, this software facilitated the assimilation of knowledge about the evolution of the protein (Poli et al., 2016).

On the other hand, Appleton and Mackie (2019) explain that the incorporation of technological applications facilitated the assimilation of knowledge about biological evolution. For example, the Earth application improved the learning process related to evolution, population balance problems and natural selection (Appleton & Mackie, 2019). Likewise, the Biomorphs application facilitated the assimilation of knowledge about the gradual process of biological evolution through the use of simulations (Appleton & Mackie, 2019).

Lastly, the use of technology in the field of Biology allows innovating the courses, updating the school activities carried out inside and outside the classroom, increasing the motivation of students and improving the academic performance (Lysne & Miller, 2015; Manrique et al., 2017; Poli et al., 2016).

2.2 Use of the virtual wall in the educational field

Today, technological advances allow that teachers build new educational virtual spaces where students play a fundamental role during the learning process (Pardo-Cueva et al., 2020; Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021). For example, Padlet is a virtual wall that promotes the active role of students inside and outside the classroom (De-Witt et al., 2015; Pardo-Cueva et al., 2020; Rashid, Yunus, & Wahi, 2019; Sangeetha, 2016).

Virtual walls allow the organization of collaborative activities before, during and after the face-to-face sessions (Pardo-Cueva et al., 2020; Rashid, Yunus, & Wahi, 2019). In the Administration course, the use of Padlet favored the collaborative work, improved academic performance and increased the satisfaction of the students during the teaching-learning process (Pardo-Cueva et al., 2020). In the same way, the incorporation of the collaborative wall in the Geometric Representation Systems course facilitated the participation of the students during the face-to-face sessions (Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021).

Teachers use virtual walls to build new educational spaces that facilitate collaborative work and participation in the classroom (De-Witt et al., 2015; Rashid, Yunus, & Wahi, 2019; Sangeetha, 2016). In the Foreign Language course, the use of Padlet promoted autonomy, increased motivation and decreased the anxiety of the students during the teaching-learning process (Rashid, Yunus, & Wahi, 2019). Also, this technological tool allowed collaborative work during the face-to-face sessions and development of writing and communication skills about the English Language (Rashid, Yunus, & Wahi, 2019).

The benefits of the use of virtual walls in the educational field are the publication of information, storage of data and dissemination of school contents (Pardo-Cueva et al., 2020; Rashid, Yunus, & Wahi, 2019). According to Sangeetha (2016), the incorporation of Edmodo and Padlet facilitated the dissemination of information, improved the learning process about the English Language and developed their writing skills. Similarly, the collaborative wall and mobile devices facilitated the exchange of ideas in the Social Sciences course (Salas-Rueda et al., 2020).

Finally, virtual walls such as Padlet and the collaborative wall have transformed the teaching-learning process in the courses of Administration (Pardo-Cueva et al., 2020), Geometric Representation Systems (Salas-Rueda, Ramírez-Ortega, & Eslava-Cervantes, 2021), Foreign Language (Rashid, Yunus, & Wahi, 2019; Sangeetha, 2016) and Social Sciences (Salas-Rueda et al., 2020).

3 Methodology

The particular aims of this mixed research are (1) analyze the impact of the use of collaborative wall in the learning process of global climate change, (2) analyze the impact of the use of collaborative wall in the motivation of the students, (3) analyze the impact about the use of the collaborative wall in the interest of the students, (4) identify the predictive models about the use of the collaborative wall in the educational process about biology and (5) analyze the perceptions of the students about the use of the collaborative wall in the Biology IV course.

3.1 Participants

The sample is made up of 74 students (29 men and 45 women) from the National Preparatory School No. 7 “Ezequiel A. Chávez” who took the Biology IV course during the 2019 school year. This course belongs to the fifth year of high school. The average age is 16.89 years.

3.2 Procedure

The teacher of the Biology IV course took the Classroom of the Future 2019 Diploma offered by the National Autonomous University of Mexico (NAUM) with the purpose of training in the topics related to pedagogy and the use of technology. This diploma uses the pedagogical model proposed by Gamboa-Rodríguez (2015) and the collaborative wall to innovate the teaching-learning process (See Figure 1).

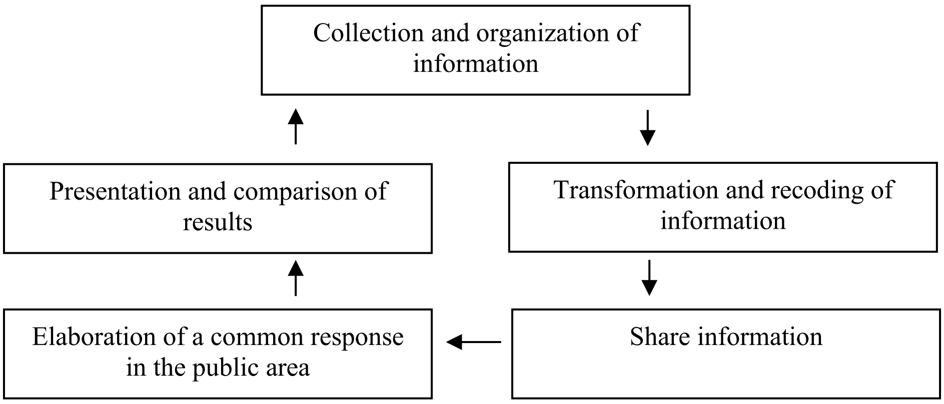
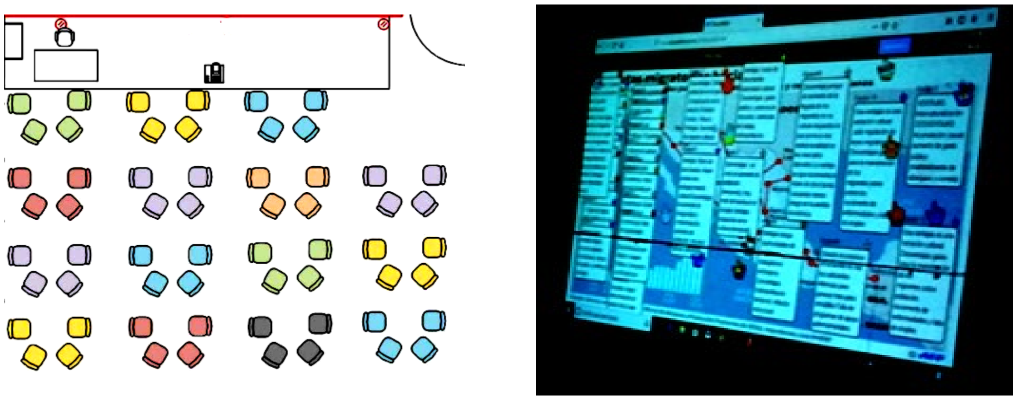


Figure 1. Pedagogical model proposed by Gamboa-Rodríguez (2015).

In the Institute of Applied Sciences and Technology of the NAUM, Gamboa-Rodríguez built the collaborative wall in order to improve the teaching-learning conditions and facilitate the active role of students during the face-to-face sessions (See Figure 2). The collaborative wall is a web application that allows the active participation of students and discussion of ideas in the classroom through the organization, dissemination and use of images and text.



(a) Distribution in the classroom (b) Images and text in the collaborative

Figure 2. Example about the use of the collaborative wall.

Table 1 shows the educational context of the Biology IV course at the National Preparatory School No. 7 “Ezequiel A. Chávez”.

Table 1. Educational context.

No.	Aspect	Item	Description
1	Analysis	Problem	In the Biology IV course, the students have a passive role during the realization of the school activities and present difficulties to understand the topics. In fact, the exchange of ideas, collaborative work and discussion of contents in the classroom is rare in this course.
		Characteristics of the students	The students attended the fifth year of high school at the National Preparatory School No. 7 "Ezequiel A. Chávez"
		Technology	The teacher of the Biology IV course does not use ICTs during the teaching-learning process on global climate change
2	Design	Learning objectives	Know the concept of global climate change Understand the factors that influence the global climate change Analyze the impact of global climate change on the society
		Use of technology	The collaborative wall is a web application that allows the active participation of students and discussion of ideas in the classroom During the face-to-face sessions, the students use mobile devices to share the information and images of the courses through the collaborative wall
3	Development	Before the face-to-face sessions	The students of the National Preparatory School No. 7 "Ezequiel A. Chávez" searched and analyzed the information about the global climate change at home
		During the face-to-face sessions	The students of the National Preparatory School No. 7 "Ezequiel A. Chávez" used mobile devices to enter the information about the global climate change in the collaborative wall and continue with the discussion of the topics in the classroom
4	Implementation	Face-to-face sessions	3 face-to-face sessions
		Duration	50 minutes in each face-to-face session

Before the face-to-face sessions, the students of the National Preparatory School No. 7 "Ezequiel A. Chávez" searched, selected and analyzed the information about the global climate change at home. In the classroom, the teacher of the Biology IV course used the computer and Internet to access the collaborative wall. Also, the video projector displayed the workspace of the collaborative wall.

The teacher of the Biology IV course started the discussion forum in order to promote the active role of the students. Later, the students of the National Preparatory School No. 7 "Ezequiel A. Chávez" used mobile devices to enter the information about the global climate change in the collaborative wall and continue with the debate of the topics (See Figure 3).

Figure 3. Use of mobile devices to enter in the collaborative wall.

Figure 4 shows the model used to analyze the impact of the collaborative wall in the Biology IV course. In particular, machine learning allows analyzing the use of this technological tool in the learning process of global climate change, motivation and interest of the students through linear regressions.

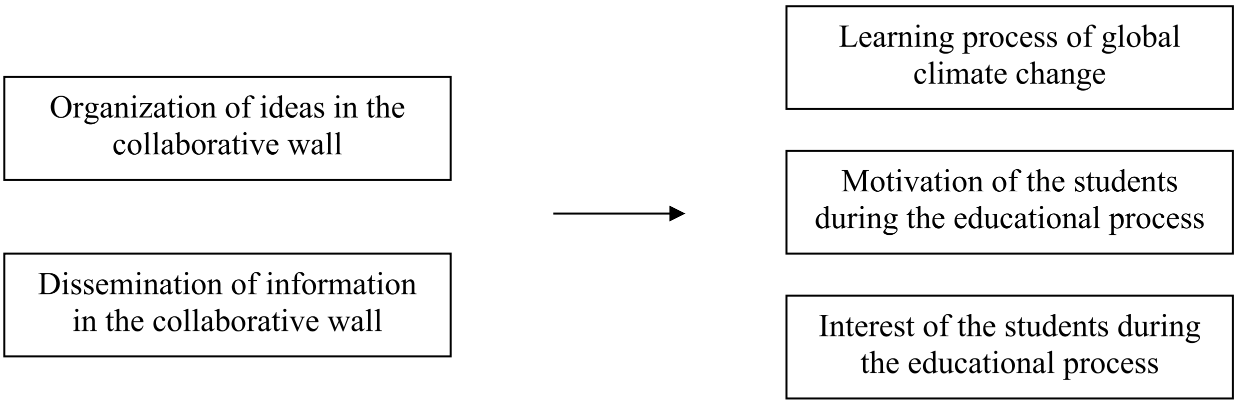


Figure 4. Model about the use of the collaborative wall in the Biology IV course.

The incorporation of technological advances in the educational field of Biology facilitates the learning process (Appleton & Mackie, 2019; Shen et al., 2020). Therefore, the hypotheses about the use of the collaborative wall and learning process are:

- Hypothesis 1 (H1): The organization of ideas in the collaborative wall positively influences the learning process of global climate change

- Hypothesis 2 (H2): The dissemination of information in the collaborative wall positively influences the learning process of global climate change

Teachers of the Biology courses use technology to increase the motivation of students (Manrique et al., 2017; Shen et al., 2020). Therefore, the hypotheses about the use of the collaborative wall and motivation of the students are:

- Hypothesis 3 (H3): The organization of ideas in the collaborative wall positively influences the motivation of the students during the educational process of global climate change
- Hypothesis 4 (H4): The dissemination of information in the collaborative wall positively influences the motivation of the students during the educational process of global climate change

In Biology courses, teachers use ICTs to build new educational spaces (Lysne & Miller, 2015; Shen et al., 2020). Therefore, the hypotheses about the use of the collaborative wall and the interest of students during the educational process are:

- Hypothesis 5 (H5): The organization of ideas in the collaborative wall positively influences the interest of the students during the educational process of global climate change
- Hypothesis 6 (H6): The dissemination of information in the collaborative wall positively influences the interest of the students during the educational process of global climate change

On the other hand, the predictive models about the use of the collaborative wall in the educational process of Biology are:

- Predictive Model 1 (PM1) about the organization of ideas in the collaborative wall and learning process of global climate change
- Predictive Model 2 (PM2) about the dissemination of information in the collaborative wall and learning process of global climate change
- Predictive Model 3 (PM3) about the organization of ideas in the collaborative wall and motivation of the students during the educational process of global climate change
- Predictive Model 4 (PM4) about the dissemination of information in the collaborative wall and motivation of the students during the educational process of global climate change

- Predictive Model 5 (PM5) about the organization of ideas in the collaborative wall and interest of the students during the educational process of global climate change
- Predictive Model 6 (PM6) about the dissemination of information in the collaborative wall and interest of the students during the educational process of global climate change

3.3 Data collection

Data collection was carried out at the National Preparatory School No. 7 "Ezequiel A. Chávez" during the month of November 2019. Table 2 shows the questionnaire used to retrieve the information about the use of the wall in the educational process of Biology.

Table 2. Questionnaire about the use of the collaborative wall.

No.	Variable	Dimension	Question	Answer	n	%
1	Profile of the students	Age	1. Indicate your age	15 years	5	6.76%
				16 years	48	64.86%
				17 years	14	18.92%
				18 years	4	5.41%
				>18 years	3	4.05%
		Sex	2. Indicate your sex	Man	29	39.19%
				Woman	45	60.81%
2	Collaborative wall	Organization of ideas	3. The collaborative wall facilitates the organization of ideas	Very much (1)	59	79.73%
				Much (2)	11	14.86%
				Little (3)	4	5.41%
				Very little (4)	0	0.00%
		Dissemination of information	4. The collaborative wall facilitates the dissemination of information	Very much (1)	57	77.03%
				Much (2)	12	16.22%
				Little (3)	5	6.76%
				Very little (4)	0	0.00%
		Learning process	5. The use of the collaborative wall facilitates the learning process of global climate change	Very much (1)	53	71.62%
				Much (2)	13	17.57%
				Little (3)	5	6.76%
				Very little (4)	3	4.05%
		Motivation of students	6. The use of the collaborative wall increases the motivation of the students during the educational process of global climate change	Very much (1)	49	66.22%
				Much (2)	13	17.57%
				Little (3)	5	6.76%
				Very little (4)	7	9.46%
		Interest of students	7. The use of the collaborative wall increases the interest of the students during the educational process of global climate change	Very much (1)	45	60.81%
				Much (2)	19	25.68%
				Little (3)	8	10.81%
				Very little (4)	2	2.70%
3	Students' perception	Use of the collaborative wall	8. What is your opinion about the use of the collaborative wall?	Open	-	-

The values of Load Factor (> 0.500), Alpha Cronbach (> 0.600) and Composite Reliability (> 0.700) are necessary to validate the measurement instrument. Table 3 shows that the values of Load Factor (> 0.519), Alpha Cronbach (> 0.780) and Composite Reliability (> 0.870) allow to validate the questionnaire about the use of the collaborative wall.

Table 3. Validation of the questionnaire about the use of the collaborative wall.

Variable	Dimension	Load Factor	Alpha Cronbach	Average Variance Extracted	Composite Reliability
Collaborative wall	Organization of ideas	0.742	0.790	0.581	0.871
	Dissemination of information	0.804			
	Learning process	0.520			
	Motivation of students	0.850			
	Interest of students	0.846			

3.4 Data analysis

The Rapidminer tool allows data analysis through the construction of predictive models and calculation of machine learning (linear regression). In particular, the information of the student's profile (sex and age) and use of the collaborative wall allows the construction of predictive models by means of the decision tree technique. Also, the criterion for creating these models is the Gini index.

In machine learning, the training section (50%, 60% and 70% of the sample) allows the calculation of linear regressions to evaluate the hypotheses about the use of the collaborative wall in the field of Biology, and the evaluation section (50 %, 40% and 30% of the sample) allows identifying the accuracy of these linear regressions by means of the squared error.

4 Results

The collaborative wall allows the construction of new educational spaces where students participates actively during the learning process. In fact, the collaborative wall facilitates very much (n = 59, 79.73%), much (n = 11, 14.86%) and little (n = 4, 5.41%) the organization of ideas (See Table 2). Likewise, the collaborative wall facilitates very much (n = 57, 77.03%), much (n = 12, 16.22%) and little (n = 5, 6.76%) the dissemination of information.

The results of machine learning (linear regression) indicate that the organization of ideas and dissemination of information in the collaborative wall positively influence the learning process of global climate change, motivation and interest of the students (See Table 4).

Table 4. Results of machine learning.

Hypothesis	Training	Linear regression	Conclusion	Error squared
H1: Organization of ideas in the collaborative wall → learning process	50% 60% 70%	y = 0.131x + 1.325 y = 0.160x + 1.163 y = 0.154x + 1.204	Accepted: 0.131 Accepted: 0.160 Accepted: 0.154	1.014 1.260 1.204
H2: Dissemination of information in the collaborative wall → learning process	50% 60% 70%	y = 0.199x + 1.168 y = 0.216x + 1.119 y = 0.246x + 1.038	Accepted: 0.199 Accepted: 0.216 Accepted: 0.246	0.661 0.775 1.024
H3: Organization of ideas in the collaborative wall → motivation of students	50% 60% 70%	y = 0.726x + 0.787 y = 0.629x + 0.911 y = 0.679x + 0.836	Accepted: 0.726 Accepted: 0.629 Accepted: 0.679	0.625 0.507 0.379
H4: Dissemination of information in the collaborative wall → motivation of students	50% 60% 70%	y = 0.584x + 0.955 y = 0.731x + 0.745 y = 0.766x + 0.694	Accepted: 0.584 Accepted: 0.731 Accepted: 0.766	0.558 0.549 0.461
H5: Organization of ideas in the collaborative wall → interest of students	50% 60% 70%	y = 0.675x + 0.826 y = 0.583x + 0.949 y = 0.594x + 0.907	Accepted: 0.675 Accepted: 0.583 Accepted: 0.594	0.498 0.338 0.276
H6: Dissemination of information in the collaborative wall → interest of students	50% 60% 70%	y = 0.678x + 0.804 y = 0.806x + 0.622 y = 0.800x + 0.611	Accepted: 0.678 Accepted: 0.806 Accepted: 0.800	0.355 0.345 0.298

Table 5 shows the Pearson's correlations about the use of the collaborative wall in the field of Biology.

Table 5. Pearson's correlations about of the collaborative wall.

	Organization of ideas	Dissemination of information	Learning process	Motivation of students	Interest of students
Organization of ideas	1	-	-	-	-
Dissemination of information	0.647	1	-	-	-
Learning process	0.087	0.160	1	-	-
Motivation of students	0.476	0.496	0.260	1	-
Interest of students	0.452	0.577	0.265	0.732	1

4.1 Learning process

The use of the collaborative wall facilitates very much (n = 53, 71.62%), much (n = 13, 17.57%), little (n = 5, 6.76%) and very little (n = 3, 4.05%) the learning process of global climate change (See Table 2). The results of machine learning with 50% (0.131), 60% (0.160) and 70% (0.154) of training indicate that H1 is accepted (See Table 4). Therefore, the organization of ideas in the collaborative wall positively influences the learning process of global climate change.

Table 6 presents the 9 conditions of the PM1 with the accuracy of 74.32%. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 17.5 years and is a woman then the use of the collaborative wall facilitates much the learning process of global climate change. On the other hand, if the student thinks that the collaborative wall facilitates very much the organization of ideas and has an age ≤ 17.5 years then the use of the collaborative wall facilitates very much the learning process of global climate change.

Table 6. Conditions of the PM1 about the use of the collaborative wall.

No.	Collaborative wall → organization of ideas	Age	Sex	Collaborative wall → learning process
1	Very much	≤ 17.5 years	-	Very much
2	Very much	> 17.5 years	Man	Very much
3	Very much	> 19.5 years	Woman	Very much
4	Very much	≤ 19.5 & > 17.5 years	Woman	Very little
5	Much	> 17.5 years	-	Very much
6	Much	≤ 17.5 & > 16.5 years	Man	Much
7	Much	≤ 16.5 years	Man	Very much
8	Much	≤ 17.5 years	Woman	Much
9	Little	-	-	Very much

The results of machine learning with 50% (0.199), 60% (0.216) and 70% (0.246) of training indicate that H2 is accepted (See Table 4). Therefore, the dissemination of information in the collaborative wall positively influences the learning process of global climate change.

Table 7 presents the 8 conditions of the PM2 with an accuracy of 77.03%. For example, if the student thinks that the collaborative wall facilitates much the dissemination of information, has an age ≤ 16.5 and is a man then the use of the collaborative wall facilitates very much the learning process of global climate change. On the other hand, if the student thinks that the collaborative wall facilitates very much the dissemination of information and has an age ≤ 17.5 then the use of the collaborative wall facilitates very much the learning process of global climate change.

Table 7. Conditions of the PM2 about the use of the collaborative wall.

No.	Collaborative wall → dissemination of information	Age	Sex	Collaborative wall → learning process
1	Very much	≤ 17.5 years	-	Very much
2	Very much	> 17.5 years	Man	Very much
3	Very much	> 19.5 years	Woman	Very much
4	Very much	≤ 19.5 & > 17.5 years	Woman	Very little
5	Much	> 16.5 years	-	Very much
6	Much	≤ 16.5 years	Man	Very much
7	Much	≤ 16.5 years	Woman	Much
8	Little	-	-	Much

4.2 Motivation of the students

The use of the collaborative wall increases very much (n = 49, 66.22%), much (n = 13, 17.57%), little (n = 5, 6.76%) and very little (n = 7, 9.46%) the motivation of the students during the educational process of global climate change (See Table 2).

The results of machine learning with 50% (0.726), 60% (0.629) and 70% (0.679) of training indicate that H3 is accepted (See Table 4). Therefore, the organization of ideas in the collaborative wall positively influences the motivation of the students during the educational process of global climate change.

Table 8 presents the 7 conditions of the PM3 with an accuracy of 71.62%. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is a woman, then the use of the collaborative wall increases much the motivation of the students during the educational process of global climate change. On the other hand, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is a man then the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change.

Table 8. Conditions of the PM3 about the use of the collaborative wall.

No.	Collaborative wall → organization of ideas	Age	Sex	Collaborative wall → motivation
1	Very much	-	-	Very much
2	Much	> 17.5 years	-	Very much
3	Much	≤ 17.5 & > 16.5 years	Man	Much
4	Much	≤ 16.5 years	Man	Very much
5	Much	≤ 17.5 & > 16.5 years	Woman	Very much
6	Much	≤ 16.5 years	Woman	Much
7	Little	-	-	Very little

The results of machine learning with 50% (0.584), 60% (0.731) and 70% (0.766) of training indicate that H4 is accepted (See Table 4). Therefore, the dissemination of information in the collaborative wall positively influences the motivation of the students during the educational process of global climate change.

Table 9 presents the 7 conditions of the PM4 with the accuracy of 72.97%. For example, if the student thinks that the collaborative wall facilitates much the dissemination of information and has an age > 16.5 years then the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change. On the other hand, if the student thinks that the collaborative wall facilitates much the dissemination of information and has an age ≤ 15.5 years then the use of the collaborative wall increases much the motivation of the students during the educational process of global climate change.

Table 9. Conditions of the PM4 about the use of the collaborative wall.

No.	Collaborative wall → dissemination of information	Age	Sex	Collaborative wall → motivation
1	Very much	-	-	Very much
2	Much	> 16.5 years	-	Very much
3	Much	≤ 16.5 & > 15.5 years	-	Very much
4	Much	≤ 15.5 years	-	Much
5	Little	> 16.5 years	Man	Much
6	Little	> 16.5 years	Woman	Little
7	Little	≤ 16.5 years	-	Very little

4.3 Interest of the students

The use of the collaborative wall increases very much (n = 45, 60.81%), much (n = 19, 25.68%), little (n = 8, 10.81%) and very little (n = 2, 2.70%) the interest of the students during the educational process of global climate change (See Table 2). The results of machine learning with 50% (0.675), 60% (0.583) and 70% (0.594) of training indicate that the H5 is accepted (See Table 4). Therefore, the organization of ideas in the collaborative wall positively influences the interest of the students during the educational process of global climate change.

Table 10 presents the 7 conditions of the PM5 with an accuracy of 70.27%. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is a woman, then the use of the collaborative wall increases much the interest of the students during the educational process of global climate change. On the other hand, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is man then the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change.

Table 10. Conditions of the PM5 about the use of the collaborative wall.

No.	Collaborative wall → organization of ideas	Age	Sex	Collaborative wall → interest
1	Very much	-	-	Very much
2	Much	> 17.5 years	-	Little
3	Much	≤ 17.5 & > 16.5 years	Man	Little
4	Much	≤ 16.5 years	Man	Very much
5	Much	≤ 17.5 & > 16.5 years	Woman	Very much
6	Much	≤ 16.5 years	Woman	Much
7	Little	-	-	Little

The results of machine learning with 50% (0.678), 60% (0.806) and 70% (0.800) of training indicate that the H6 is accepted (See Table 4). Therefore, the dissemination of information in the collaborative wall positively influences the interest of the students during the educational process of global climate change.

Table 11 presents the 6 conditions of the PM6 with an accuracy of 74.32%. For example, if the student thinks that the collaborative wall facilitates much the dissemination of information and has an age ≤ 16.5 years then the use of the collaborative wall increases much the interest of the students during the educational process of global climate change. On the other hand, if the student thinks that the collaborative wall facilitates very much the dissemination of information then the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change.

Table 11. Conditions of the PM6 about the use of the collaborative wall.

No.	Collaborative wall → dissemination of information	Age	Sex	Collaborative wall → interest
1	Very much	-	-	Very much
2	Much	> 17.5 years	-	Little
3	Much	≤ 17.5 & > 16.5 years	-	Very much
4	Much	≤ 16.5 years	-	Much
5	Little	-	Man	Little
6	Little	-	Woman	Much

4.4 Perception of the students

New technologies are promoting the construction of new virtual spaces that favor the learning process. For example, the collaborative wall facilitated the assimilation of knowledge about the global climate change.

> "Better learning and more reasoning" (Student 18, man, 16 years old).
>
> "It helped me to understand the topic more" (Student 20, woman, 16 years old).
>
> "It helped me to organize my ideas and understand the class in an easier way" (Students 40, woman, 16 years old).

According to the students of the National Preparatory School No. 7 "Ezequiel A. Chávez", the use of the collaborative wall allowed the analysis of ideas about the global climate change in the classroom.

> "In a group, we analyze the opinions" (Student 19, woman, 18 years old).
>
> "We analyze the issues with classmates to have a broader understanding and learn other opinions" (Student 23, woman, 16 years old).
>
> "It allows increasing the capacity for analysis and understanding" (Student 43, man, 17 years old).

Technology allows that students have a leading role during the realization of the school activities. In fact, the students of the Biology IV course actively participate in the classroom through the collaborative wall.

> "Share ideas and opinions with all my classmates, understand and have a motivation to know about the information of the course" (Student 46, woman, 16 years old).

> "Creativity, participation and collaboration" (Student 55, man, 16 years old).

> "It encourages the group participation" (Student 64, woman, 16 years old).

Teachers can use the collaborative wall to create new learning and teaching spaces. In particular, the students of the National Preparatory School No. 7 "Ezequiel A. Chávez" mention that the use of the collaborative wall is fun.

> "It helps us understand the topics. It's fun because it's interactive" (Student 21, woman, 16 years old).

> "Classes become more didactic and fun" (Student 42, woman, 16 years old).

> "A new way of using technology for education. It generates interest and is not boring" (Student 45, woman, 15 years old).

Advances in technology are transforming the roles of teachers and students in education. For example, the incorporation of the collaborative wall in the Biology IV course allowed the debate and exchange of ideas about the global climate change.

> "A debate is created and different opinions are presented that help to understand the information" (Student 9, woman, 16 years old).

> "Understand the ideas from different points of view" (Student 14, woman, 16 years old).

> "It allows the exchange of ideas and opinions. We reach the conclusions" (Student 74, man, 16 years old).

Finally, teachers can use technology to create and conduct creative school activities. In particular, the collaborative wall allowed that students work as a team during the learning process about the global climate change.

> "All work together" (Student 15, man, 16 years old).

> "Learning and teamwork" (Student 33, man, 16 years old).

> "Working as a team" (Student 44, man, 16 years old).

Figure 5 shows the word cloud about the perceptions of the students during the use of the collaborative wall. The words that present the highest frequency are understand, ideas, technology, change, students, fun, use, opinions, new, Biology, analysis, information, classmates, participation, debate, activities, help and classroom.

Figure 5. Word cloud about the use of the collaborative wall.

5 Discussion

The incorporation of technology in the educational field facilitates the planning, organization and implementation of new school activities (Almoeather, 2020; Benitez et al., 2020; Salas-Rueda, 2015). For example, Manrique et al. (2017) used digital tools in the Anatomy course in order to improve the teaching-learning conditions about Biology. Similar to Pardo-Cueva et al. (2020), the use of virtual walls in the Administration course facilitated the active role of the students. In particular, most of the students think that the collaborative wall facilitates very much the organization of ideas (n = 59, 79.73%) and dissemination of information (n = 57, 77.03%). Therefore, the students of the National Preparatory School No. 7 “Ezequiel A. Chávez” have a favorable opinion about the use of this technological tool in the Biology IV course.

5.1 Learning process

Various authors (e.g., Appleton & Mackie, 2019; Poli et al., 2016; Shen et al., 2020) mention that technological advances such as website, educational software and web applications allow the realization of new school activities that facilitate the learning process about Biology. As Rashid, Yunus and Wahi (2019) indicated, virtual walls allow the construction of new virtual spaces that favor the learning. Most of the students (n = 53, 71.62%) consider that the use of the collaborative wall facilitates very much the learning process of global climate change. As a result of the analysis performed, 17.57% of the students (n = 13) thinks that the use of the collaborative wall facilitates much the learning process of global climate change. Therefore, 89.19% of the students have a favorable opinion about the use of this technological tool in the Biology IV course.

This study shares the ideas of various authors (e.g., Pardo-Cueva et al., 2020; Rashid, Yunus, & Wahi, 2019; Sangeetha, 2016) about the use of virtual walls to innovate the school activities in the classroom. The results of machine learning on H1 are higher than 0.130, therefore, the organization of ideas in the collaborative wall positively influences the learning process of global climate change. Data science identifies 9 conditions of the PM1 with an accuracy of 74.32%. In this predictive model, the Age and Sex of the students determine how the collaborative wall influences the learning process of global climate change. The decision tree technique identifies 6 conditions where the use of the collaborative wall facilitates very much the learning process of global climate change. For example, if the student thinks that the collaborative wall facilitates very much the organization of ideas and has an age ≤ 17.5 years then the use of the collaborative wall facilitates very much the learning process of global climate change. On the other hand, the sex of the students allows identifying 5 conditions of the PM1. For example, if the student thinks that the collaborative wall facilitates very much the organization of ideas, has an age > 17.5 years and is a man then the use of the collaborative wall facilitates very much the learning process of global climate change.

The results of machine learning on H2 are higher than 0.190, therefore, the dissemination of information in the collaborative wall positively influences the learning process of the global climate change. Data science identifies 8 conditions of the PM2 with an accuracy of 77.03%. In this predictive model, the Age and Sex of the students determine how the collaborative wall influences the learning process of global climate change. The decision tree technique identifies 5 conditions where the use of the collaborative wall facilitates very much the learning process of global climate change. For example, if the student thinks that the collaborative wall facilitates very much the dissemination of information and has an age ≤ 17.5 then the use of the collaborative wall facilitates very much the learning process of global climate change. On the other hand, the sex of the students allows identifying 5 conditions of the PM2. For example, if the student thinks that the collaborative wall facilitates very much the dissemination of information, has an age > 17.5 and is a man then the use of the collaborative wall facilitates very much the learning process of global climate change.

5.2 Motivation of the students

In the field of Biology, ICTs allow the construction of new educational spaces that favor the motivation of students (Appleton & Mackie, 2019; Rashid, Yunus, & Wahi, 2019; Shen et al., 2020). According to Rashid, Yunus and Wahi (2019), the incorporation of the virtual wall in the Foreign Language course favored the motivation of the students. In the Biology IV course, most of the students (n = 49, 66.22%) think that the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change. Likewise, quantitative data reveals that the use of the collaborative wall increases much (n = 13, 17.57%) the motivation of the students during the educational process of global climate change. Therefore, 83.79% of the students have a favorable opinion about the use of this technological tool in the Biology IV course.

Similar to Rashid, Yunus and Wahi (2019), the use of virtual walls in the Foreign Language course facilitated the creation of educational spaces where the students actively participated and increased their motivation during the learning process. The results of machine learning on H3 are higher than 0.620, therefore, the organization of ideas in the collaborative wall positively influences the motivation of the students during the educational process of global climate change. Data science identifies 7 conditions of the PM3 with an accuracy of 71.62%. In this predictive model, the Age and Sex of the students determine how the collaborative wall influences the motivation of the students. The decision tree technique identifies 4 conditions where the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is a man then the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change. On the other hand, the sex of the students allows identifying four conditions of the PM3. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is a woman, then the use of the collaborative wall increases much the motivation of the students during the educational process of global climate change.

The results of machine learning on H4 are higher than 0.580, therefore, the dissemination of information in the collaborative wall positively influences the motivation of the students during the educational process of global climate change. Data science identifies 7 conditions of the PM4 with an accuracy of 72.97%. In this predictive model, the Age and Sex of the students determine how the collaborative wall influences the motivation of the students. The decision tree technique identifies 3 conditions where the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change. For example, if the student thinks that the collaborative wall facilitates much the dissemination of information and has an age > 16.5 years then the use of the collaborative wall increases very much the motivation of the students during the educational process of global climate change. On the other hand, the sex of the students allows identifying 2 conditions of the PM4. For example, if the student thinks that the collaborative wall facilitates little the dissemination of information, has an age > 16.5 years and is a woman then the use of the collaborative wall increases little the motivation of the students during the educational process of global climate change.

5.3 Interest of the students

Technological advances are transforming the functions and attitudes of students during the performance of school activities in the field of Biology (Appleton & Mackie, 2019; Salas-Rueda & Vázquez-Estupiñán, 2018; Shen et al., 2020). Most of the students (n = 45, 60.81%) consider that the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change. Also, 25.68% of the students (n = 19) think that the use of the collaborative wall increases much the interest of the students during the educational process of global climate change. Therefore, 86.49% of the students have a favorable opinion about the use of this technological tool in the Biology IV course.

In the English Language course, the use of the virtual wall improved the interaction between the participants of the educational process and facilitated the active role of the students in the classroom (Sangeetha, 2016). The results of machine learning on the H5 are higher than 0.580, therefore, the organization of ideas in the collaborative wall positively influences the interest of the students during the educational process of global climate change. Data science identifies 7 conditions of the PM5 with an accuracy of 70.27%. In this predictive model, the Age and Sex of the students determine how the collaborative wall influences the interest of the students. The decision tree technique identifies 3 conditions where the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is man then the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change. On the other hand, the sex of the students allows identifying 4 conditions of the PM5. For example, if the student thinks that the collaborative wall facilitates much the organization of ideas, has an age ≤ 16.5 years and is woman then the use of the collaborative wall increases much the interest of the students during the educational process of global climate change.

The results of machine learning on the H6 are higher than 0.670, therefore, the dissemination of information in the collaborative wall positively influences the interest of the students during the educational process of global climate change. Data science identifies 6 conditions of the PM6 with an accuracy of 74.32%. In this predictive model, the Age and Sex of the students determine how the collaborative wall influences the interest of the students. The decision tree technique identifies 2 conditions where the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change. For example, if the student thinks that the collaborative wall facilitates very much the dissemination of information then the use of the collaborative wall increases very much the interest of the students during the educational process of global climate change. On the other hand, the sex of the students allows identifying two conditions of the PM6. For example, if the student thinks that the collaborative wall facilitates little dissemination of information and is a man, then the use of the collaborative wall increases little the interest of the students during the educational process of global climate change.

5.4 Perception of the students

Technological advances such as web applications are causing that teachers organize and carry out new school activities. In particular, the use of the collaborative wall in the Biology IV course facilitated the assimilation of knowledge about the global climate change, the active participation of the students in the classroom and the construction of fun educational spaces.

Teachers have the opportunity to use virtual walls during the school activities in order to improve the teaching-learning conditions because this technological tool allows the communication between the participants of the educational process through the organization of ideas and dissemination of information in the classroom.

Even the incorporation of the collaborative wall in the National Preparatory School No. 7 “Ezequiel A. Chávez” facilitated the analysis, debate and exchange of ideas about the global climate change in the classroom. Finally, the collaborative wall allowed that the students work as a team during the learning process.

6 Conclusion

Technological advances are changing the way of organizing and conducting the school activities in the 21st century. In particular, the collaborative wall allows the active participation of students in the classroom. The results of machine learning indicate that the organization of ideas and dissemination of information in the collaborative wall positively influence the learning process of global climate change, motivation and interest of the students. Also, data science identifies 6 predictive models about the use of the collaborative wall in the field of Biology through the decision tree technique.

This research recommends the incorporation of the collaborative wall in the school activities because this web application allows the active role of the student in the classroom through the collaborative work. In fact, the use of the collaborative wall in the Biology IV course facilitated the assimilation of knowledge about the global climate change, participation of the students during the face-to-face sessions and construction of educational virtual spaces.

The limitations of this research are the use of the collaborative wall in the field of Biology, size of the sample and analysis of this technological tool in the learning process about global climate change, motivation and interest of the students. Therefore, future research can analyze the impact of the collaborative wall in the areas of engineering, education, medicine, computer science, management and marketing. Also, teachers can analyze how the collaborative wall influences the development of skills and the satisfaction of students in high schools and universities.

The implications of this study are related to the incorporation of virtual walls in school activities in order to improve the teaching-learning conditions and organize new school activities focused on students. In conclusion, technology allows the construction of new spaces for learning and teaching. In particular, the collaborative wall is a web application that allows the active participation of students and facilitates the discussion of ideas in the classroom through the organization, dissemination and use of images and information.

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Programmering i svensk skolmatematik

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När programmering skulle inkorporeras i skolans arbete valde Sverige i sin läroplansrevidering 2017 att skriva in det i matematikämnet, med stark koppling till algebra. Samtliga matematiklärare ställdes då inför utmaningen att undervisa i programmering. Vi undersöker här resultatet av 32 lärargruppers gemensamma arbete med att planera och genomföra lektioner i programmering i matematik i grundskolan. För att få insikt i hur lärare tolkar uppdraget och transponerar läroplananes beskrivning av programmering till klassrumspraktiken analyserar vi det matematiska innehållet i dessa lektioner samt vilken syn på relationen mellan matematik och programmering som framträder i lärarnas beskrivning av syfte, lärandemål, aktivitet och reflektion. Vi finner att programmeringsaktiviteter i 1/3 av lektionerna inte kopplas till något traditionellt matematiskt innehåll. I övriga lektioner är det främst aritmetik eller geometri som utgör det matematiska innehållet. Få explicita kopplingar görs till algebra förutom till begreppet variabler, men då är det främst variabler inom programmering som avses. I materialet framträder fyra olika relationer mellan matematik och programmering: 1) enbart programmering; 2) matematik som en kontext för programmering; 3) programmering som ett verktyg för att effektivisera beräkningar; 4) programmering som ett verktyg för att utforska matematik. Resultaten diskuteras i relation till matematikämnets syfte och innehåll i den svenska läroplanen.

Nyckelord: programmering, matematik, algebra, grundskolan

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Programming in Swedish school mathematics

When incorporating programming in the school curricula, Sweden decided to integrate it with mathematics, and specifically within the core content of algebra. As a result, all mathematics teachers at all levels were faced with the challenge of teaching programming. In this study we analyse documentation from 32 lesson studies where groups of teachers have planned, conducted, and revised lessons on programming within the school subject mathematics. To gain insight into how the teachers interpret the new task and thus contribute to the transposition of knowledge from the curriculum level to the classroom level, we analyse the mathematical content in these lessons and the relations between mathematics and programming that emerge in the way the teachers describe the aim, the activites and the learning outcomes of the lessons. We find that 1/3 of the lessons do not connect to any traditional mathematics content, and the rest of the lessons mostly focus on arithmetic or geometry. Few explicit connections are made to algebra, except for the variable concept, but when variables are treated the focus is on variables in the programming sense rather than algebra. Four different relationships between mathematics and programming emerge in the data: 1) programming without connecting to mathematics; 2) mathematics as a context for programming, 3) programming as a tool for efficient calculations; 4) programming as a tool for exploring mathematics. The results are discussed in relation to the transposition of knowledge of mathematics as a school subject.

Keywords: programming, mathematics, school algebra, transposition of knowledge

1 Introduktion

Under de senaste fem åren har flera länder infört programmering i skolundervisningen. I debatten framhålls att programmering numera ses som en nödvändig kunskap för alla medborgare (Mannila m fl., 2014; Nouri m fl., 2020). Programmering brukar även lyftas fram som ett pedagogiskt verktyg för utveckling av elevers datalogiska tänkande.[1] Implementeringen av programmering i skolans läroplaner har gjorts på olika sätt i olika länder, se t ex Australien (Falkner m fl., 2014), England (Brown m fl., 2014), Danmark (Misfeldt m fl., 2020), Finland (Manilla m fl., 2014), Sverige (Kilhamn & Bråting, 2019) och USA (Fisher, 2016). I England och Danmark har programmering blivit en del av ett nytt ämne, medan exempelvis Finland och Sverige har fört in programmering i redan befintliga ämnen, för Sveriges del i matematik och teknik. Något som gör Sverige unikt i sammanhanget är att de grundläggande principerna för programmering skrivits in i kursplanen som en del av det centrala innehållet i algebra (Kilhamn & Bråting, 2019; Skolverket, 2017).

Den forskning som finns kring programmering i matematik har tidigare främst handlat om matematikämnet på gymnasiet eller universitetet (Grover & Pea, 2013). Den pågående världsvida implementeringen av programmering i skolans lägre årskurser har lett till en efterfrågan på kunskap om den roll programmering har eller skulle kunna ha i grundskolans matematikundervisning (Hickmott m fl., 2018; Nouri m fl., 2020). Vi önskar bidra till detta fält genom att analysera dokumentation från lärargrupper som har planerat, genomfört och utvärderat lektioner i programmering i matematik i grundskolan. Lektionerna har arbetats fram inom ramen för så kallade lesson studies, en form av kollegialt fortbildnings- och utvecklingsarbete. Studien är en del av ett mer omfattande forskningsprojekt om den pågående implementeringen av programmering i grundskolans matematik som tar avstamp i Chevallards (2006) teoretiska ramverk om hur kunskap transponeras mellan olika instanser i skolans verksamhet (Bråting m fl., 2021).

Syftet med den studie som rapporteras här är att belysa hur lektioner i programmering som sker inom ramen för matematikämnet interagerar med den traditionella skolmatematiken i det inledande skedet av implementeringen av programmering som ett nytt kunskapsinnehåll i grundskolans matematikämne.

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[1] Datalogiskt tänkande, kallas internationellt för ”computational thinking”. Begreppet kan förklaras som en problemlösningsprocess för att beskriva, analysera och lösa problem så att datorer kan hjälpa till, eller som Aho (2012) definierar det ”de tankeprocesser som är involverade i att formulera problem så att dess lösningar går att representera som stegvisa instruktioner eller algoritmer” (p. 832).

Samtliga lektioner i vårt datamaterial har ett givet programmeringsinnehåll helt i linje med läroplanens formulering (se avsnitt 3.1). I analysen ställer vi två frågor:

1. Vilket matematikinnehåll väljer lärare till sina programmeringslektioner i matematik i grundskolan?
2. Hur ser relationen mellan matematik och programmering ut i dessa lektioner?

Forskningsfrågorna fokuserar på de val lärarna gör och den tolkning av relationen mellan programmering och matematik som framkommer av de syften och lärandemål de formulerar relativt programmeringslektionerna.

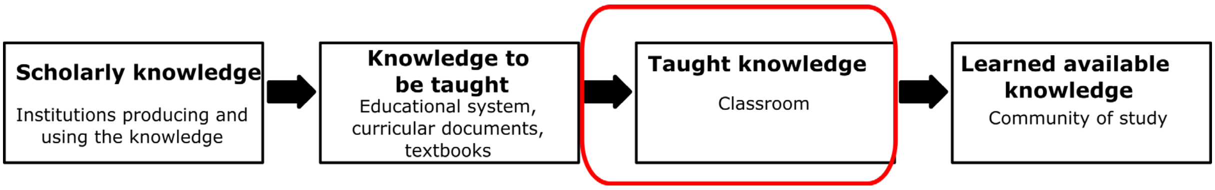
2 Lärares transposition av kunskap

Eftersom vi är intresserade av den kunskap som formuleras av lärare när ett nytt innehåll införs av institutioner utanför skolan är Chevallards (2006) teori om didaktisk transposition av kunskap en lämplig inramning. I avsnitt 2.1 beskrivs den teoretiska inramningen och i 2.2 redogör vi för tidigare forskning om hur svenska lärare ser på programmering i matematik.

2.1 Didaktisk transposition från avsedd till undervisad kunskap

Figur 1 illustrerar hur kunskap formas och förändras när den införlivas i skolans verksamhet. I den första transpositionen behöver den akademiska kunskapen *(scholarly knowledge)* brytas ned i mindre delar och anpassas för att göras undervisningsbar. Resultatet blir vad som kallas *avsedd kunskap (knowledge to be taught)* (Bosch & Gascon, 2006; Kang & Kilpatrick, 1992). Den kunskapen legitimeras i samband med att olika aktörer beslutar vad som ska ingå i skolans läroplaner och i vilket syfte. Exempelvis tas beslut om var i läroplanen i relation till andra ämnen som programmering ska integreras, vilka aktiviteter och begrepp som ska ingå, samt i vilken ordning och i vilka årskurser olika aspekter ska tas upp. I Sverige skedde detta i samband med revideringen av läroplanen 2017, då programmering infördes i kursplanerna för matematik och teknik. Den beskrivning av programmering som nu finns i läroplanen har därmed, enligt Chevallards teori, delvis förändrats i relation till det kunskapsinnehåll som utgör programmering för en verksam programmerare.

Nästa steg i transpositionsprocessen sker när läroplanen tolkas av lärare, som genom sina beslut bidrar till att forma det som faktiskt görs i klassrummet, och som kommer att utgöra så kallad *undervisad kunskap* (*taught knowledge*). Den slutliga instansen i transpositionsprocessen utgörs av den kunskap som elever får möjlighet att faktiskt lära sig (*learned available knowledge*). I viss mån sker en återkoppling till tidigare nivåer så det som sker i klassrummet kan komma att påverka och förändra det läraren gör osv. För att beskriva kunskap på de olika platserna i utbildningssystemet har Chevallard utvecklat en praxeologi som beskriver dels val av uppgifter och tekniker; kunskapens praxis, dels underliggande argument och teorier som rättfärdigar dessa val; kunskapens logos.



Figur 1. Chevallards teori om hur kunskap förändras mellan olika instanser, från Bosch & Gascon (2006).

Den studie som presenteras här zoomar in på den didaktiska transpositionen från *avsedd kunskap* till *undervisad kunskap* (markerat med rött i figur 1) i det skede av historien där programmering är ett helt nytt inslag i matematikundervisningen.

2.2 Svenska lärares syn på programmering i skolmatematiken

Tidigare studier om programmering i grundskolans matematik fokuserar ofta elevernas lärande snarare än lärares undervisning. Hittills har endast några få studier genomförts kring svenska lärares initiala reaktioner på införandet av programmering i skolan. Resultatet av dessa visar att lärare i allmänhet har svårt att se ett tydligt samband mellan matematik och programmering men har samtidigt en positiv inställning till inslaget av programmering i matematiken (Kilhamn m fl., 2021; Misfeldt m fl., 2019; Mozelius m fl., 2019). Pörn m fl. (2021) har genom web-baserade enkäter undersökt finlandssvenska grundskollärares spontana syn på kopplingen mellan matematik och programmering. De fann att lärarna i första hand relaterade programmering till att skriva, ge och följa instruktioner samt till olika aspekter av logiskt tänkande. Däremot var det få lärare som kopplade samman programmering med något specifikt område inom matematiken, som exempelvis algebra eller geometri. Vidare visar resultatet av olika intervjustudier att lärare i stor utsträckning relaterar programmering till allmänna förmågor rörande samarbete och kommunikation (Nouri m fl., 2020), samt ser programmering som ett pedagogiskt verktyg som kan öka elevers engagemang på matematiklektionen (Kilhamn m fl., 2021). Utöver frågan kring vad programmering är och vilken koppling den har till matematiken påpekas att det snabba införandet av programmering har skapat stress, framförallt hos lärare som saknar erfarenhet av programmering (Mozelius m fl., 2019). I den här artikeln vill vi bidra med ytterligare kunskap om lärares syn på programmering i matematik genom att analysera hur lärare som planerat och genomfört programmeringslektioner i matematikämnet beskriver lektionernas syfte och lärandemål.

3 Programmering i svensk skolmatematik

För att förstå den transposition av kunskap som äger rum när programmering inkluderas i matematikämnet ges först en historisk bakgrund och en redogörelse för den avsedda kunskapen så som den är formulerad i nuvarande svenska läroplan.

3.1 Historisk bakgrund till programmering i svensk skolmatematik

Första gången datorer dyker upp i den svenska grundskolans styrdokument är 1969 där ”Orientering om datamaskiner” skrevs in under matematikämnets huvudmoment för högstadiet (Skolöverstyrelsen, 1967, s. 137). Undervisningen handlade då mer om datorer än om programmering, men på olika håll, främst på gymnasiet, gjordes innovativa försök att undervisa programmering. Det var en tid då programmerare använde högnivåspråk som t.ex. Fortran, Basic och Pascal, vilka krävde dyra och kraftfulla datorer. Därför var det vanligare att tillämpa lågnivåspråk som Assembly i skolan och lärare kunde använda bordskalkylatorer som räknehjälpmedel (Rolandsson, 2011). I den typen av datorhybrid matades koden in utan spegling (bildskärmar var ovanliga) och utskriften fanns att hämta på speciella skrivmaskiner (teletypers). Ibland användes modem för att ringa upp en stordator på kvällstid och körresultatet hämtades dagen efter. För en lärare som önskade undervisa programmering fanns det därför flera faktorer som måste hanteras: ekonomi, tid till förberedelser, hårdvarans komplexitet samt programmeringsspråkets syntax och långsamma återkoppling. Idag är förutsättningarna för programmering i skolan betydligt bättre, med snabba datorer och programmeringsspråk som är enklare att hantera. En elev av idag kan relativt snabbt skapa, köra och korrigera sin kod tack vare visuell återkoppling från en bildskärm.

En ny läroplan för grundskolan kom 1980, i vilken datalära som ett nytt innehållsområde skulle undervisas inom ramen för ämnena samhällskunskap och matematik (Skolöverstyrelsen, 1980). Alla skolor hade inte tillgång till datorer, och bland de grundskolor som hade datorer fanns i snitt endast fem datorer per skola (Söderlund, 2000). Staten gick därför in med ett stimulanspaket, och en detaljerad studieplan för datalära (Skolöverstyrelsen, 1984) där det tydligt framgick att programmering inte skulle ta för mycket tid i anspråk och att fokus skulle vara matematik och inte programmering för att inte riskera att matematikproblem övergick till programmeringsproblem:

> Ytterligare färdighet i datoranvändning får eleverna i ämnet matematik, där problemlösning med datorer ingår som ett moment i undervisningen. I dessa övningar bör färdiga program användas där så är möjligt, och först som en påbyggnad i undervisningen kan problem lösas med programmering (Skolöverstyrelsen, 1984, s. 10).

Datalära fick en trög start, med mycket fokus på hårdvara och lite fokus på undervisning och lärares fortbildning. I väntan på datorer bedrevs undervisningen mestadels med penna och papper, vilket kan verka kontraproduktivt då programmering inte skulle ta för mycket tid i anspråk. Riis (1987) beskrev i en rapport att lärare med egen erfarenhet av programmering i undervisningen också var kritiska till dess innehåll. I Sverige utvecklades under denna tid en egen skoldator, Compis, med grafiska möjligheter och det egna programmeringsspråket Comal (en version av Basic) för problemlösning i matematik.

I samband med implementeringen av 1994 års läroplan försvann datalära och programmering ur kursplanen för grundskolan (Utbildningsdepartementet, 1994). Datorer omnämndes istället i olika ämnen med fokus på datorns möjligheter för beräkningar, datahantering, ordbehandling och informationssökning. År 2017 återinfördes programmering i grundskolan, nu som en aspekt av vad som kallas digital kompetens (Skolverket, 2017).

I slutet av 1960-talet utvecklades programmeringsspråket Logo (Papert, 1980), som fick stor internationell spridning i skolundervisning, men som inte på allvar slog igenom i Sverige (Hedrén, 1990). Programmeringsspråket innehöll bland annat en grafik där en sköldpadda styrdes på skärmen med enkla kommandon. En grundläggande filosofi i Logo var att användaren skulle uppleva matematik inom ramarna för programmets ”mikrovärldar”, och lära sig matematik genom att reflektera över det som skapas i programmet.

De höga förväntningar som fanns på Logo som matematiskt problemlösningsverktyg och medel för att utforska matematik kom delvis på skam i den internationella forskning som bedrevs under 1980-talet (Noss & Hoyles, 1996).

Kritiken handlade ofta om att programmeringen ansågs tidskrävande och svår, och att lärare behövde mer didaktisk kunskap om programmering för att undervisningen skulle utveckla elevers matematiska förmågor. Enligt Hedrén (1990) framhöll vissa forskare att matematikens semantiska innehåll, speciellt i geometri, blev tydligare genom programmeringen, medan andra kritiserade den bakomliggande filosofin som ett önsketänkande och nyttan med Logo som minimal, eftersom språket adresserade ett smalt område i matematik.

Papert såg tidigt att elever kunde skapa mikrovärldar med grafiska figurer, så kallade ”sprites”. Denna idé plockades upp i Scratch, en programmeringsmiljö med grafiska block med en mängd olika funktioner för att hantera färg, form, musik, bild och animeringar som utvecklats under 2000-talet (Resnick m fl., 2009). Scratch har på senare tid fått stor spridning både internationellt och inom den svenska skolan. Programmet skiljer sig från Logo genom att det inte i första hand är utvecklat för att stödja matematiken, utan för att stimulera unga att programmera interaktiva berättelser, animeringar och spel samt att kunna dela dessa med varandra över internet (Resnick m fl., 2009). Ett engelskt forskningsprojekt har visat att det går att undervisa om programmering i Scratch och samtidigt lära sig en del matematik, men att det kräver mycket arbete för att designa aktiviteter som innehåller viktiga matematiska idéer och som bryggar över mellan matematik och programmering (Benton m fl., 2017).

3.2 Programmering i nuvarande svenska läroplan

Den senaste revideringen av svensk läroplan trädde i kraft 2018, då 2011 års läroplan kompletterades med skrivningar om elevers utveckling av digital kompetens (Skolverket, 2017), varvid programmering inkluderades i ämnena matematik och teknik. I kursplanen för matematik står programmering omnämnt i ämnets syftesbeskrivning:

> Genom undervisningen ska eleverna ges förutsättningar att utveckla förtrogenhet med grundläggande matematiska begrepp och metoder och deras användbarhet. *Vidare ska eleverna genom undervisningen ges möjligheter att utveckla kunskaper i att använda digitala verktyg och programmering för att kunna undersöka problemställningar och matematiska begrepp, göra beräkningar och för att presentera och tolka data.*
>
> (Skolverket, 2017, s. 56, vår kursivering)

Vidare finns programmering inskrivet som en del av det centrala innehållet i *algebra* uppdelat efter stadierna. För årskurs 1-3:

> Hur entydiga stegvisa instruktioner kan konstrueras, beskrivas och följas som grund för programmering. Symbolers användning vid stegvisa instruktioner.

För årskurs 4-6 och 7-9:

> Hur algoritmer kan skapas och användas vid programmering.

I årskurs 4-6 ska programmering ske i *visuella miljöer* och i årskurs 7-9 i *olika programmeringsmiljöer*. För årskurs 7-9 finns programmering även omnämnt i det centrala innehållsområdet *problemlösning*.

4 Metod

För att besvara forskningsfrågorna har vi samlat skriftligt material från lärare som planerat och genomfört programmeringslektioner i matematikämnet. En systematisk sammanställning och kvalitativ innehållsanalys (Bryman, 2012) av datamaterialet har genomförts. Vi börjar med att redogöra för datamaterialet och beskriver därefter analysmetoden med hjälp av exempel från materialet.

4.1 Empiriskt material

Studien har utförts med empiri från ett nationellt utvecklings- och forskningsprojekt för grundskollärare som undervisar programmering. Skolhuvudmän från fem olika kommuner utspridda i landet anslöt sig till projektet, som kom att involvera ca 135 lärare från 15 olika skolor under tre läsår 2017–2020 (se närmare Jahnke, 2020). Lärarna genomförde så kallade lesson studies (LS) där de i grupper om två till fem lärare tillsammans planerade, genomförde, utvärderade och reviderade en lektion (Takahashi & Yoshida, 2004). De formulerade själva lektionens syfte och lärandemål och hade fria händer att välja programmeringsmiljö. All empiri bygger på den dokumentation som respektive lärargrupp skapat i projektet. I den här studien fokuserar vi på de LS som genomfördes inom ramen för matematikämnet, vilket uppgick till 32 stycken. Urvalet av lärare representerar väl grundskolans samtliga årskurser med 12 LS i årskurs F-3, 9 LS i årskurs 4-6 och 11 LS i årskurs 7-9.

Varje lärargrupp genomförde en LS, där de i två till tre cykler arbetade sig igenom olika steg på vägen mot en gemensam lektionsplanering. Innan lärarna påbörjade arbetet med att utveckla lektionsplaneringar deltog de i en mindre utbildning om LS-metodens olika steg och vikten av dokumentation. Här följer en komprimerad version av den information som lärarna fick i början av projektet.

- *Inventering*: Lärargruppen avgör elevernas förkunskaper och avgränsar ett eller flera lärandemål och kunskapsinnehåll lämpligt för elevgruppens ålder.
- *Planering*: Lärargruppen planerar gemensamt en lektion om programmering.
- *Första lektionen*: En lärare genomför undervisningen i en elevgrupp, medan övriga lärare i gruppen observerar och samlar information.
- *Första reflektionen*: Gjorda observationer delas i gruppen för diskussion om resultat av lektionen. Lektionsplanen revideras.
- De två sista stegen upprepas med nya elevgrupper (och eventuellt en annan undervisande lärare) tills lärarna känner sig nöjda med lektionsplanen.

Lärargruppen fick även en mall för dokumentation av sin arbetsprocess där de på egen hand skulle formulera lektionens syfte och lärandemål. I mallen förväntades lärarna även dokumentera didaktiska metoder och på vilket sätt lektionens innehåll relaterar till kursplanen samt utvärdera lektionens resultat. Omfattningen och detaljnivån på dokumentationen varierade mellan lärargrupperna, där somliga endast bidrog med en sammanfattande powerpoint-presentation enligt mallen, medan andra även lämnade planeringsunderlag och exempel på programkod.

De samlade skriftliga dokumentationerna av planering, genomförande och utvärdering av lektioner från dessa 32 LS utgör således det empiriska materialet för den innevarande studien, där varje LS genererat en lektion som under processens gång utprövats i flera klasser. Dokumentationen kring varje lektion har givits ett unikt namn bestående av två bokstäver följt av en siffra som anger årskursen där lektionen utprövades, exempelvis SJ_1.

4.2 Analysmetod

En sammanställning av de 32 dokumenterade lektionerna gjordes med fokus på det innehåll som på olika sätt behandlade matematiklärande. I den här studien definierar vi syfte och lärandemål relaterat till programmering och datalogiskt tänkande och särskiljer det från matematik, eftersom vi är intresserade av relationen mellan dessa två kunskapsfält. Vi använder således termen *matematik* i meningen centralt innehåll i skolmatematiken så som det beskrivits i den svenska läroplanen före införandet av programmering (Skolverket, 2011). Lärandemål relaterat till sociala och praktiska dimensioner av lektionen har analyserats separat (Jahnke, 2020; Sjöberg m fl., 2019; Zhang m fl., 2020). Följande delar av dokumentationen utgjorde underlaget för analysen i den här artikeln:

- matematikinnehållet i det syfte och lärandemål som lärarna formulerat
- det explicita matematikinnehållet i respektive aktivitet
- lärarnas utvärdering av lektionen avseende matematiklärande
- den programmeringsmiljö som användes.

Analysen företogs i flera steg. I första steget identifierandes citat ur materialet där matematikinnehållet tydligt framgick. I en iterativ process reviderades dessa så att de citat som föreföll innehållsligt lika kom att formuleras på samma sätt i en komprimerad sammanställning. Varje gång en formulering ändrades validerades den mot dokumentationen för att säkerställa att ändringen bibehållit innebörden i originaldokumentet. Resultatet av denna process gav en översikt över de 32 lektionerna där olika trender kunde utläsas och analyskategorier formuleras.

Matematikinnehållet i lektionerna (forskningsfråga 1) kategoriserades enligt läroplanens beskrivning av det centrala innehållet i kursplanen innan programmering skrevs in (Skolverket, 2011). Inledningsvis testade vi att analysera innehållet efter kategorier som växte fram ur materialet, men fick problem med att kategorierna blev tvetydiga och beroende av definitionen av matematik. Exempelvis var beskrivningar av lärandemål avseende resonemang, kommunikation och problemlösning svåra att kategorisera. När vi utgick från läroplanens indelning av det centrala innehållet försvann tvetydigheten och en deskriptiv sammanställning kunde genomföras med få inslag av tolkning. Resonemang och kommunikation betraktades därvidlag som förmågor relaterade till det matematiska innehållet som angivits. I de fall där flera matematiska områden aktualiserades i lektionen valdes det som var mest explicit uttalat i lärarnas formuleringar. Exempelvis klassades ett innehåll som geometri om syfte och lärandemål relaterar till geometri trots att algebraiska formler för beräkning av area och omkrets är en bärande del av innehållet. I många lektioner förekommer variabler men utan explicita formuleringar kring variabelbegreppet i relation till algebraiska uttryck. När dokumentationen endast innehöll skrivningar kring hantering av variabler i programmeringsspråket snarare än hantering av variabler i algebra klassades det inte som en lektion i algebra. Denna aspekt återkommer vi till i diskussionen. När det gäller problemlösning, som i läroplanen anges både som förmåga och centralt innehåll, har vi endast klassificerat lektionen som problemlösning om den explicit handlat om matematiska problem eller matematiska lösningsstrategier.

Vad gäller *relationen mellan matematik och programmering* (forskningsfråga 2) var processen att finna kategorierna längre och mer omfattande. Analysen genomfördes först i flera iterationer av författare 1. Därefter validerades tolkningen av övriga författare, och justerades tills vi tillsammans enades om formuleringen av fyra kategorier som kunde återfinnas i materialet. Därefter återvände vi till materialet och kategoriserade ånyo samtliga lektioner för att säkerställa en enhetlig tolkning av kategorierna. Vi fann ett visst överlapp mellan kategorierna på så sätt att kategori 2 även ingår i kategori 3 och 4 eftersom matematiken utgör en kontext för programmeringen även i båda dessa kategorier. De fyra kategorierna beskrivs i följande stycke, med exempel på hur klassificeringen gått till.

1. *Enbart programmering*. Dessa lektioner handlar enbart om programmering.
2. *Matematik som kontext för programmering*. I dessa lektioner utgör matematiken huvudsakligen en kontext för att lära sig programmering. Inget nytt matematiklärande åsyftas, men programmering kan utnyttjas för att repetera eller befästa matematikkunskaper.
3. *Programmering som ett verktyg för att effektivisera beräkningar inom matematiken*. I dessa lektioner finns ett tydligt matematikinnehåll. En dator programmeras och används som ett effektivt sätt att utföra beräkningar.
4. *Programmering som ett verktyg för att utforska matematik*. I dessa lektioner används programmering för att utforska matematiska begrepp eller samband. Programmeringen tillför nya sätt att närma sig matematiken och uppges på så sätt kunna fördjupa den matematiska förståelsen på en nivå som är relevant för den angivna årskursen.

Här följer två exempel från årskurs 1 som visar hur innehållet har klassificerats. SJ_1 beskriver syfte och lärandemål som fokuserar på att styra en BeeBot på en matta med hjälp av pilar. Lärarna skriver:

> Hur entydiga stegvisa instruktioner kan konstrueras, beskrivas och följas som grund för ***programmering***. Symbolers användning vid stegvisa instruktioner. Målet är att eleverna ska förstå hur stegvisa instruktioner kan konstrueras och läsas av genom att lägga egen kod till en BeeBot samt tolka andras kod. [SJ_1]

Lektionen klassificerades i enlighet med kategori 1 ovan, det vill säga som en lektion som innehöll enbart programmering eftersom ingen matematik utöver programmering explicit omnämndes. Inledningen av citatet är direkt taget ur läroplanens skrivningar om programmering, något som förekom i så gott som alla dokumentationer. Citat ur läroplanen gällande övrigt matematikinnehåll fanns inte i den här lektionen. Även när sådana fanns var de inte alltid uppenbart relaterade till lektionens faktiska innehåll, varför störst vikt lades vid lärarnas egenformulerade syften och lärandemål, samt de lärandemål ur kursplanen som även kommenterades i utvärderingen av lektionen.

AS_1 beskriver en lektion i analog programmering som går ut på att beskriva ett händelseförlopp med hjälp av numrerade instruktioner. Lärarna skriver:

> Målet med matematiklektionen är att eleverna ska förstå hur entydiga stegvisa instruktioner kan konstrueras, beskrivas och följas som grund för programmering. Symbolers användning vid stegvisa instruktioner. Naturliga tal och hur de kan användas för att ange ordning. / … / Förutsättningar och förkunskapskrav: Känna till siffrorna 1-8 och kunna ange ordningsföljden. [AS_1]

Lektionen skiljer sig från SJ_1 genom att naturliga tal som ordningstal anges som en viktig del av lektionen, samtidigt som ordningstalen 1-8 betraktades som en förkunskap. Aktiviteten går ut på att eleverna ska ”rita sin morgon” i 8 rutor numrerade från 1 till 8. Lärarna menar att det är en lektion i analog programmering. Lektionen bedömdes innehålla matematik tillhörande det centrala innehållet taluppfattning och tals användning, och klassificerades i enlighet med kategori 2 ovan, där matematiken har rollen av en kontext i vilken (analog) programmering äger rum men utan att det finns ambitioner för något nytt matematiklärande.

5 Resultat

Först sammanfattas resultatet av analysen i tabell 1, där lektionernas matematikinnehåll (forskningsfråga 1) visas på den vertikala axeln och relationen mellan matematik och programmering enligt våra fyra kategorier (forskningsfråga 2) på den horisontella axeln. Tabellen visar också hur lektionerna fördelar sig över olika programmeringsmiljöer enligt följande kategorier: analog programmering helt utan dator eller digitala verktyg (n=4); robotprogrammering med exempelvis BeeBot (n=5); blockprogrammering som vanligtvis innebar Scratch eller Microbit (n=14); textprogrammering som främst innebar programmering i Python (n=8); samt Excel (n=1).

Table 1. Matematikinnehållet relaterat till relationen mellan matematik och programmering.

	Relationen mellan matematik och programmering			
Matematik-innehåll	1.Enbart programmering	2. Matematik som kontext för programmering	3. Programmering som verktyg för att effektivisera beräkningar	4. Programmering som verktyg för att utforska matematik
Enbart programmering n=10	**Analog**: HH_2, TO_4a **Robot**: SJ_1, TB_2, SE_3, TO_4b **Block**: AS_2, AS_9 MA_2, VT_3			
Taluppfattning och tals användning n=8		**Analog**: AS_1 **Block**: TO_3a, ST_5, HG_6 **Text**: SH_7, NB_7, ST_7	**Text**: BB_7	
Geometri n=8		**Analog**: TO_3b **Robot**: TB_1 **Block**: BF_3, MA_5 **Text**: NB_8	**Text**: KV_7	**Block**: AS_4, TO_6
Statistik och sannolikhet n=3			**Block**: TO_9 **Text**: ST_9 **Excel**: AS_7	
Algebra n=0				
Problemlösning n=1			**Text**: BB_8	
Samband och förändring n=2				**Block**: SJ_5, ST_6
n=32	n=10 (31%)	n=12 (38%)	n=6 (18%)	n=4 (13%)

5.1 Matematikinnehållet

Vi ser i tabell 1 att ganska olika matematik utgör innehållet när lärarna ska planera programmeringslektioner i matematik, men att 31% saknar matematikinnehåll. Taluppfattning och geometri är ofta förekommande. Algebra, som är det centrala innehåll läroplanen har placerat programmering inom, saknas helt som explicit matematikinnehåll i dessa lektioner. I följande avsnitt presenterar vi beskrivningar av och exempel på de lektioner som klassificerats i de fyra analyskategorierna med utgångspunkt i de kluster som framträder i tabellen relativt de olika matematiska innehållen.

5.2 Enbart programmering

Nästan en tredjedel (31%) av lektionerna handlar enbart om programmering. Lärandemål gällande programmering är klart och tydligt beskrivna men inga övriga lärandemål finns för matematik. Aktiviteterna går ofta ut på att programmera en robot att röra sig över golvet [SJ_1, TB_2, SE_2], att programmera en person (analogt) eller en virtuell robot i exempelvis ett dansprogram, eller att skapa en presentation i Scratch [AS_2, AS_9]. Det finns ingenting i dessa lektioner som markerar att de är matematiklektioner. Flera av lektionerna i den här gruppen är planerade som ämnesövergripande samarbeten där programmeringen utgör matematikdelen. Ett exempel är en lektion som genomfördes i årskurs 4 som ett samarbete mellan matematik och geografi. Lärarna skriver i sin planering av lektionen:

> Syftet med lektionen är att eleverna får öva på stegvisa instruktioner. Även möta nya begrepp som om (if), villkorssats. Testa felsöka. /…/ Introducera Sverigekartan med några stora städer utmärkt samt "hinder" som finns på ett rutsystem. /…/ De ska lära sig formuleringen: om hinder "hoppa över". Förstå vilka instruktioner de ska använda. [TO_4b]

I lärarnas utvärdering ser vi att det är geografin som utgör kontext för programmering och att någon annan matematik än programmeringsinnehållet inte aktualiseras:

> Eleverna diskuterade engagerat geografi och hittade flera lösningar hur de kunde gå. Arbetet skapade kreativitet och lust hos de flesta eleverna.
> Många frågade om de fick gå nordöst eller om de fick simma hela vägen. Flera kom också på att man kunde skriva tre steg norr istället för att upprepa ett steg norr tre gånger. [TO_4b]

I den här kategorin återfinns exempel på lektioner där lärare och elever arbetar med stegvisa instruktioner och upprepning, två vanliga konstruktioner i programmering. I övriga kategorier kommer vi att se hur dessa och liknande konstruktioner används i mer matematiska sammanhang.

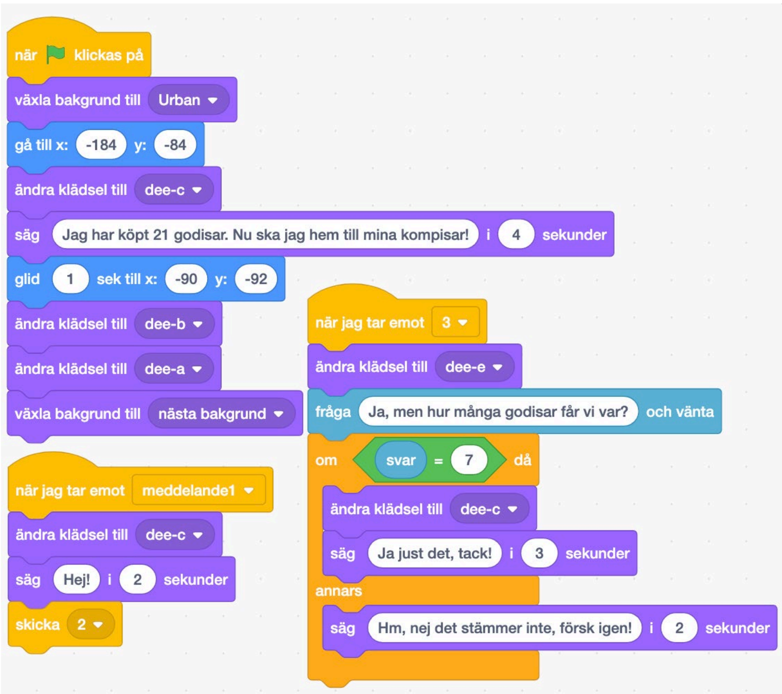
5.3 Matematik som kontext för programmering

Den största gruppen (38%) utgörs av lektioner där matematiken fungerar som en kontext för programmering. Utöver mål relaterade till programmering finns i dessa lektioner ett tydligt matematiskt innehåll relaterat till taluppfattning eller till geometri, med lärandemål som handlar om att repetera och befästa kunskaper, eller om att tillämpa gamla kunskaper i nya sammanhang. Exempelvis beskriver lärarna i ST_5 sitt lärandemål på följande sätt: ”Kunna använda tidigare kunskaper i matematik och programmering för att skapa ett program för att öva en huvudräkningsstrategi”, men konstaterar i utvärderingen att det blev ”Lite prat om matematik och mycket om kod”.

I vissa lektioner försvinner det matematiska innehållet nästan helt genom att andra aspekter av programmeringen hamnar i fokus. Ett sådant exempel är lektion TO_3a där följande aktivitet introduceras:

> Du ska göra en räknesaga i Scratch. Sagan ska innehålla följande:
> ★ Två eller flera sprajter som omväxlande gör saker, t.ex. rör sig eller pratar.
> ★ En mattefråga som den som kör ditt program kan svara på. Programmet ska berätta om man svarat rätt eller fel. [TO_3a]

Eleverna får en exempelkod att utgå från, se figur 2, och arbetar sedan med att först rita sin ”storyboard” och därefter programmera i Scratch. I utvärderingen skriver lärarna ingenting om de räknesagor som eleverna hittade på. Istället är fokus helt på elevernas svårigheter att förstå, välja, hämta och använda olika koder i Scratch. Matematiken blir i sammanhanget underordnad programmeringen.



Figur 2. Exempel på räknesaga i Scratch [TO_3a].

Ett annat exempel är lektion ST_7 som handlar om att lära sig skilja mellan de olika inmatningskommandona `input()` för text, `int()` för heltal och `float()` för decimaltal. Övningsuppgifterna innehåller en del algebraiska formler och matematiska beräkningar, men syftet med lektionen är att lära sig programspråkets struktur, inte att träna matematik eller lösa matematiska problem. Variabelbegreppet tas upp som problematiskt i utvärderingen:

> Eleverna blandade ihop två olika sätt att skapa variabler. Många använde "input" när de skulle skapa en variabel av typen `a="jonas"`. Utifrån detta tänker vi förtydliga skillnaden mellan dessa två sätt genom att ge eleverna en uppgift där de ska jämföra: `namn="Robert" , print(namn*5)` med `namn=input("vad heter du?") , print(namn*5)`. [ST_7]

Det är uppenbart att det här handlar om vad en variabel är i koden och hur programmet tolkar det som skrivs, snarare än variabelbegreppet i algebraisk mening.

Hälften av lektionerna i den här kategorin har geometri som matematiskt innehåll. Ett typexempel är lektion BF_3 där eleverna ska programmera en blue-bot med hjälp av pilar så att den tar sig fram i ett rutsystem. Lärarna beskriver aktiviteten:

> Pedagogen som håller i lektionen ger varje grupp en instruktion. T.ex ”placera blue-boten på kvadraten med ögonen mot cirkeln och ta dig sedan till cylindern. /…/ För att öka svårighetsgraden kommer vi även att ge instruktioner som t.ex. ”ställ blue-boten på den figuren som inte har några hörn med ögonen mot pyramiden. Ta dig sedan till den figuren som har 2 korta och 2 långa sidor. [BF_3]

Trots att lärandemålet uppges vara att ”befästa de geometriska formerna och dess egenskaper” nämner lärarna i utvärderingen ingenting om elevernas förståelse av de geometriska begreppen utan fokuserar helt på deras svårigheter att relatera pilarna till roboten. Matematik fungerar således i första hand som en kontext för programmering där programmeringen används för att befästa matematikkunskaper.

En av geometrilektionerna, NB_8, arbetar med textprogrammering. Liksom i lektion ST_7 används övningar som går ut på att lära sig programspråket Python. Matematiken i NB_8 utgörs av att skriva ett program som beräknar area och omkrets av rektanglar. Eftersom lektionen hålls i årskurs 8 är sambanden mellan sidorna i en rektangel och dess omkrets respektive area inget nytt. Det eleverna ombeds reflektera över är hur koden kommer att se ut och vilket resultat en viss kod kommer att få, inte vilka generella matematiska slutsatser de kan dra om rektanglar. Det är känd matematik som används som kontext för att lära sig begrepp och syntax i ett programmeringsspråk. Det som kommenteras av lärarna är åter igen begreppet variabel.

En sak som flera lärargrupper återkommer till i sina utvärderingar är att det tar lång tid för eleverna att lära sig programmera. Om matematiken endast utgör en kontext för programmering kan eleverna därför uppleva att de inte lär sig matematik på matematiklektionerna. Lärarna som genomfört lektion NB_7 noterar i sin utvärdering att ”Några elever var oroliga för att programmeringstiden tas från tiden för matematikundervisningen”.

5.4 Programmering som verktyg för att effektivisera beräkningar

I tabell 1 återfinns programmering som ett verktyg för att utföra effektiva beräkningar i matematik i sex lektioner med stor variation av matematiskt innehåll. KV_7 är en lektion i geometri, inte helt olik NB_8, men med programmeringsspråket Javascript. I KV_7 beskriver lärarna de matematiska målen för elevernas lärande i termer av att ”Stärka sin förmåga att använda formler som en metod att lösa enkla geometriuppgifter” samt ”stärka sin förståelse för begrepp kopplade till geometrin”. Eleverna får följande instruktioner:

> Nu ska ni få träna er i att använda datorerna som hjälpmedel när ni gör beräkningar med formler. Som vi har gått igenom tidigare är ju formler ett verktyg för att underlätta vid komplicerade beräkningar. Ibland är formlerna ganska enkla: Rektangelns area = basen · höjden (A=b·h), men ofta är de mer komplicerade: Klotets volym = 4 · 3,14 · radien · radien · radien/3 ($V=4\cdot\pi\cdot r^3/3$). [KV_7]

Skillnaden mot NB_8 är att dessa lärare på ett explicit sätt talar om för eleverna vad det är datorn tillför, att det är när beräkningarna är komplicerade som datorn är ett användbart verktyg.

Hälften av lektionerna i denna kategori handlar om statistik och sannolikhet. Två av dem är lektioner i årskurs 9 där datorn användes för att simulera tärningskast vid sannolikhetsexperiment [ST_9; TO_9]. Den tredje är AS_7, som handlar om att föra in värden i ett kalkylblad och sedan beräkna medelvärde, median och typvärde. Den klassificeras därför som en lektion i statistik. Lärarna citerar elevernas utvärdering:

> “Jag har lärt mig hur Variabel, Funktion och om vilkorssats. Jag har även lärt mig hur kalkylark fungerar”. “Jag har lärt hur kan man räkna medelvärde och median och typvärde utan ansträngning”. [AS_7]

Begreppen variabel och funktion relaterar i detta sammanhang mer till hur begreppen används i kalkylark än till deras innebörd inom områdena algebra respektive samband och förändring. Lärarna reflekterar kring just begreppet funktion:

> Vi upptäckte att lärare la olika vikt vid de olika begreppen, vilket senare ledde till en diskussion om begreppens innebörd. Ex: Funktion kan dels beskrivas som “två variabler som är beroende av varandra” och dels som ett sätt att förklara för datorn “vad den ska göra”. Under LS förklarades detta begrepp på olika grunder vilket kan ha ursprung i olika erfarenhet och skolning i excel och matematik. [AS_7]

Vi noterar i den här lektionen dels att begreppet programmering sträcker sig utanför block- och textprogrammering till att även omfatta arbete med kalkylblad, dels att ett matematiskt begrepp som funktion kan ha en annan innebörd i programmering.

5.5 Programmering som verktyg för att utforska matematik

I två av geometrilektionerna ska eleverna använda programmering i syfte att utforska geometriska begrepp eller samband. I AS_4 får eleverna utforska begreppen vinkel och rotation samt fundera på sambandet mellan vinkelns gradtal och andel av cirkel som hel, halv, fjärdedel och tredjedel. Lektionen utförs i både årskurs 4 och årskurs 6. Lärarna beskriver syftet med lektionen på följande sätt:

> Vi har tagit fram den här lektionen för att tydliggöra kopplingen mellan programmering och matematik. Vi vill se om eleverna kan konstruera geometriska objekt med hjälp av sina kunskaper i programmering. Vi vill se om eleverna får mer fördjupade kunskaper i matematiska begrepp med hjälp av programmering t.ex. se sambandet mellan gradantalet i en cirkel och gradantalet i en halv cirkel. Vi vill att de ska visa detta genom att programmera dessa objekt. [AS_4]

Eleverna får en färdig kod som ritar en cirkel, se figur 3. Läraren frågar:

> Var tror ni att jag ska ändra någonstans för att det ska bli en halvcirkel, fjärdedelar och tredjedelar? Testa själv! / … / Hur gör ni för att få halva cirkeln röd och den andra halva blå? [AS_4]

Figur 3. Kod som ritar en cirkel.

I TO_6 får eleverna utforska sambandet mellan arean på en rektangel och en triangel med samma bas och höjd. Exempelkod i Scratch ges för rektangel och ska modifieras för triangel. Lektionen utmärker sig av att lärarna uppger ett utforskande syfte, eleverna utmanas matematisk. De skriver:

> Vi valde området geometri och area då vi kunde se att det skulle vara en lagom utmaning att förstå sambandet mellan arean av en rektangel och arean av en triangel. [TO_6]

Eftersom programmeringsövningen bygger på att eleverna redan känner till sambandet är aktiviteten endast svagt utforskande. Den klassificeras som utforskande eftersom lärarna förväntar sig att programmeringsaktiviteten ska bidra till att skapa förståelse för sambandet. Lärarna skriver i utvärderingen:

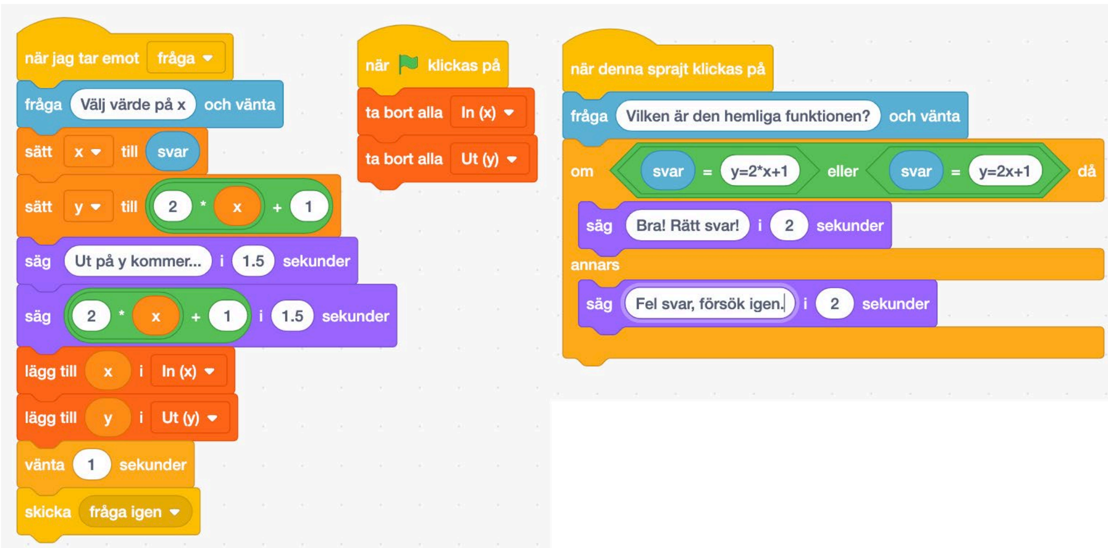
> Eleverna befäste och fick förståelse för areabegreppet. En del kunde sedan innan men en hel del var tvungna att be om hjälp för att förstå. De kunde inte gå vidare med utmaningen att modifiera koden om de inte förstod vad de skulle

> göra. /…/ Lektionens syfte och mål nåddes i hög grad, eleverna förstod area begreppet och kunde modifiera en befintlig kod. [TO_6]

Vidare uppmärksammar lärarna att eleverna ”inte förstod att de kunde stoppa in bas*höjden som en variabel”.

En del av det centrala innehållet som i kursplanen kallas samband och förändring utgörs i årskurs 4-6 av koordinatsystemet, något som utforskas i SJ_5. Lektionen handlar om stegvisa instruktioner där olika figurer ritas i ett koordinatsystem med hjälp av kommandon i Scratch. Det som skiljer den här aktiviteten från lektionerna om stegvisa instruktioner i analyskategori 1 är att instruktionerna ges med hjälp av ett matematiskt representationssystem istället för med ord och ikoniska bilder. I den här aktiviteten används programmering som ett sätt att utforska begreppen koordinater, *x*-led, *y*-led och origo.

Även funktioner och räta linjens ekvation återfinns i kursplanens samband och förändring (dock endast för årskurs 7-9), något som här utgör innehållet i ST_6. Lärarna uppger att lektionen bygger på en tidigare lektion där en analog ”funktionsmaskin” skulle programmeras. Här görs det i Scratch med hjälp av en delvis färdig kod som eleverna ska remixa, se figur 4.



Figur 4. Exempelkod med frågan: På vilket sätt kan koden ändras?

Syftet med lektionen uppges vara att eleverna ska ”fördjupa sig i funktionsbegreppet genom att använda tidigare kunskaper i en ny situation”. De beskriver lärandemålet på följande sätt:

> Målet är att alla ska kunna remixa en exempelkod och skapa en egen funktion som gör att programmet kan räkna ut y-värdet om användaren anger ett x-värde. Funktionen i exempelkoden är räta linjens ekvation, dvs y = k x + m där både k och m är positiva heltal. Ytterligare ett mål är att elever som vill ha mer utmaning ska kunna göra andra typer av funktioner där t.ex. k<1 och m<0, vilket skulle kunna bli en funktion av typen y = x/2 – 4. [ST_6]

Resultatet av att använda programmering som ett verktyg i syfte att utforska begreppet närmare var enligt lärarna blandat. I utvärderingen skriver de:

> Vår uppfattning är att vissa elever fördjupade sina kunskaper om funktioner, medan andra mest härmade koden och inte riktigt förstod hur det hängde ihop med det vi arbetat med tidigare kring funktioner. Något som indikerar att elever fördjupade sina kunskaper om funktionsbegreppet var att flera valde att göra annat än räta linjens ekvation, vilket de inte arbetat med tidigare. I klass nr 2 har 14% av eleverna gjort ickelinjära funktioner. [ST_6]

6 Diskussion

Studiens resultat kommer nu att diskuteras i relation till tidigare forskning samt olika faktorer som är relevanta för att belysa transpositionen av kunskap från den *avsedda kunskapen* i termer av kursplanens syftesbeskrivning och centrala innehåll till *undervisad kunskap* så som den framstår i det empiriska materialet. Den första forskningsfrågan fokuserar på de innehållsliga val lärarna gör när de ska programmera på matematiklektionen och den andra frågan handlar om den relation mellan matematik och programmering som framträder i dessa lektioner. Det går inte att avgöra om valet av innehåll är ett resultat av lärarens syn på relationen mellan de två kunskapsfälten eller om det förhåller sig tvärtom. Men det är tydligt att *vad och hur* samverkar med *varför*. Nedan diskuterar vi denna samverkan i lektioner i det inledande skedet av implementeringen av programmering som ett nytt kunskapsinnehåll i grundskolans matematikämne.

6.1 Syftet med programmering i matematik

Vi har i analysen av lärarnas dokumentation av de lektioner de planerat och genomfört upptäckt en viss otydlighet gällande relationen mellan programmering och matematik. Drygt 30% av de studerade lektionerna handlade uteslutande om programmering utan annan koppling till matematik. Ytterligare 38% av lektionerna handlade om programmering som är inlagd i en matematisk kontext utan att ha som ambition att tillföra matematiklärandet något nytt. Matematiken utgör i dessa lektioner en ram kring programmeringen för att legitimera dess existens som en del av matematikämnet utan att programmeringen tas i anspråk som en resurs för matematiken.

Det går att tolka situationen som att transpositionen av matematiken medför en epistemologisk förändring av matematikämnet. Om programmering finns med som ett innehåll i matematikämnet kommer lärarna att betrakta den som en del av matematiken. Att många av lektionerna vi studerat fokuserar programmering i sig självt snarare än matematiken är inte förvånande då man ser på tidigare försök med programmering i matematik (se avsnitt 3). Det finns här vissa likheter med ämnet statistik. Ursprungligen var statistik ett samhällsvetenskapligt ämne där matematik användes för att förstå samhälleliga problem. Så småningom utvecklades speciella statistiska metoder och representationer som kom att inlemmas som en del av matematiken. Det vi nu ser är hur programmering går från att vara ett eget fält som utvecklats med hjälp av matematik till att inlemmas som en del av matematiken. Därmed förändras innebörden av skolmatematiken så att ämnet får en ökad inriktning mot tillämpad matematik.

En annan tolkning av resultatet är att programmering i det inledande skedet av implementeringen, när programmering även är nytt för lärarna, av nödvändighet handlar mest om programmeringens vad och hur. Senare, när lärarna utvecklat en säkerhet i programmering och eleverna möter det varje skolår, kommer kanske större fokus att riktas mot programmering som verktyg för problemlösning, beräkningar och utforskande av matematik. En sådan argumentation fanns bland en del lärare som intervjuats i en tidigare studie (Kilhamn m fl., 2021). I sammanhanget kan man fråga sig varför det i så fall är just matematikämnet som ska lägga tid på att lära eleverna programmera och varför det är just matematiklärare som ska undervisa om det.

I motsats till kategori 1 och 2 som främst fokuserar programmering som innehåll, relaterar lektionerna i kategori 3 och 4 tydligare till läroplanens beskrivning av syftet med programmering. I kategori 3 används programmeringen som ett verktyg för att utföra beräkningar. Programmering blir en matematisk metod bland många andra med vissa unika fördelar. Alla sex lektionerna i denna kategori genomfördes i årskurs 7-9, vilket skulle kunna tyda på att det är först då som beräkningarna är så komplicerade eller omfattande att programmering verkligen är en hjälp. Att ta in programmering som beräkningsmetod där det är enklare med huvudräkning eller miniräknare är som att använda grävmaskin där det bara behövs en spade. Detta fenomen såg vi i en del av lektionerna där matematiken utgjorde en kontext för programmering, där övningarna inte gick ut på att så smidigt som möjligt utföra beräkningen, utan att lära sig utföra den med hjälp av programmering.

I lektionerna i kategori 4 används programmering för att utforska matematiska begrepp och samband mellan begrepp. De fyra lektionerna i den här kategorin genomfördes alla i årskurs 4-6, i en blockprogrammeringsmiljö. I en kritisk granskning av dessa fyra lektioner framstår inte programmeringen som det bästa verktyget för det undersökande arbetssättet. Om lärarna inte samtidigt haft som syfte att skapa en lektion i programmering skulle de med stor sannolikhet ha nått bättre resultat med hjälp av andra digitala verktyg, som exempelvis GeoGebra. Det kräver stor kunskap om programmet för att åstadkomma bra matematiska visualiseringar i exempelvis Scratch, eftersom programmet inte i första hand är tänkt som ett matematikverktyg (Resnick m fl., 2009).

Genom att läroplanen tar upp digitala verktyg och programmering i samma syftesformulering blir det upp till läraren att avgöra när programmering eller andra digitala verktyg fungerar bäst i relation till syftet. Frågan är också om programmering är att betrakta som ett bland andra digitala verktyg eller om programmering är något i grunden annorlunda. Att en lektion handlar om programmering i kalkylark antyder att det inte för lärarna är uppenbart var gränsen går mellan programmering och andra digitala verktyg.

6.2 Kursplanens centrala innehåll i matematik

Bortsett från de lektioner som klassificerades som endast programmering var det innehållsområdena taluppfattning och tals användning respektive geometri som var vanligast förekommande i den här studien. I majoriteten av dessa lektioner användes dock matematiken enbart som en kontext för att lära sig programmera, vilket ofta ledde till att det matematiska innehållet hamnade i skymundan av programmeringen eller till och med försvann. Detta resultat, och att en tredjedel av lektionerna inte visar någon koppling till matematik, indikerar att flera av lärarna hade svårigheter att hitta meningsfulla samband mellan programmering och matematik. Denna slutsats ligger i linje med vad som hittills framkommit i studier om lärares initiala respons på programmeringens införande i skolmatematiken (Kilhamn m fl., 2021; Misfeldt m fl., 2019; Pörn m fl., 2021). I vår studie framkom dessutom en oro hos flera lärare och elever över att programmeringen tar lång tid att lära sig och därmed stjäl tid från matematikinnehållet. Detta är en oro som bör tas i beaktande, speciellt med tanke på att det redan påvisats att det snabba införandet av programmering i svensk skolundervisning har skapat stress hos lärare, framförallt hos de lärare som saknar erfarenhet av programmering (se Mozelius m fl., 2019).

Några av de lektioner som ingick i den här studien klassificerades som statistik och sannolikhet respektive samband och förändring. Inte helt oväntat användes programmeringen som ett verktyg för att utföra beräkningar under lektionerna i statistik och sannolikhet. I de lektioner som behandlade samband och förändring användes programmeringen som ett verktyg för att utforska matematik, till exempel fördjupade eleverna sina kunskaper om funktionsbegreppet. I en programmeringskontext gav just detta begrepp anledning till en fördjupad diskussion bland lärare, då de insåg att begreppet tolkades olika: under vissa lektioner definierades funktioner utifrån ett matematiskt perspektiv som ”samband mellan variabler” och under andra lektioner utifrån en programmeringskontext som ”instruktioner för att lösa problem”. I ett lärandeperspektiv är denna diskussion mycket relevant med tanke på att man inom programmering och matematik ofta använder samma begrepp och symboler utan att dessa alltid har samma innebörd (se Bråting & Kilhamn, 2020). Funktionsbegreppet är ett typiskt exempel på detta då en funktion inom matematiken alltid innebär en relation mellan två mängder medan en funktion inom programmering ibland likställs med en procedur. För matematiklärare är dessa begreppsliga skillnader av stor vikt att känna till för att inte skapa begreppsförvirring hos eleverna. Studier om tidigt algebraiskt tänkande visar att funktionsbegreppet kan vara problematiskt för många elever även utan att programmeringen förs in i skolmatematiken (Kieran, 2018). Vi menar därför att det är viktigt att matematiklärare inte bara fortbildas i programmering utan också i hur införandet av programmering i matematiken kan påverka elevers matematiska begreppsbildning.

De innehållsområden som förekom minst i vårt empiriska material var problemlösning respektive algebra. Endast en lektion klassificerades som problemlösning och ingen som algebra. Detta resultat är något överraskande med tanke på att programmering har införts som en del av det centrala innehållet i algebra (Skolverket, 2017). Det är dock viktigt att påpeka att variabler förekom i flera lektioner som inte klassats som algebra, exempelvis när eleverna använde formler för att beräkna areor av geometriska objekt. På motsvarande sätt som funktionsbegreppet diskuterade lärarna att det finns olika slags variabler i matematik jämfört med programmering. Lärarna syftade här på att inom programmering kan en variabel innehålla text, vilket skiljer sig från matematiken där variabler vanligtvis innehåller tal (se Bråting & Kilhamn, 2020). Ur ett lärandeperspektiv hävdar vi att det är viktigt att matematiklärare får tid och möjlighet att fördjupa sig i variabelbegreppets olika innebörder inom såväl programmering som matematik.

I den här studien har vi definierat algebra utifrån det centrala innehållsområdet i läroplanen. Efter revideringen 2017 inkluderar innehållsområdet algebra även stegvisa instruktioner, algoritmer och programmering i visuella och textuella programspråk. Det vi kan konstatera är att de mer traditionella områdena inom skolalgebra som till exempel ekvationslösning och mönster inte finns representerade i vårt empiriska material. Algebra är ett område där svenska elever i internationella tester inte brukar prestera så bra, och också ett område som i våra läroplaner haft en svag inramning och otydlig beskrivning (Prytz, 2020). Tydliga exempel på detta är att stora delar av det som nu klassificeras som samband och förändring i tidigare kursplaner har tillhört innehållsområdet algebra samt att man i revideringen år 2017 förde in programmering som en del av algebrainnehållet. I samband med detta har dessutom begreppet algoritm, som tidigare kopplats till aritmetik, flyttats till algebra. Detta visar att ämnet genomgår en transposition som knappast stärker aspekter som traditionellt kopplats till algebra, såsom fokus på generalisering, samband och struktur, och som utgör fokus för internationell forskning kring algebra i grundskolan (Kieran, 2018). Man kan spekulera kring vad som i framtiden kommer att innefattas av skolalgebran, speciellt med tanke på att många lärare kopplar samman programmering med mer allmänna förmågor som att kunna samarbeta, kommunicera och följa instruktioner (Nouri m. fl., 2020; Pörn m fl., 2021), och utveckling av datalogiskt tänkande (Jahnke, 2020).

6.3 Val av programmeringsmiljö och programmeringsspråk

Bland de lektioner som endast handlar om programmering arbetar elever i årskurs 1-4 antingen analogt, med robotar eller med blockprogrammering. I dessa programmeringsmiljöer kan elever enkelt skapa kod och testa den kod de själva skapar. För eleverna blir det ett arbetssätt med många och snabba körningar, där syntaktiska fel uppenbaras om de har skrivit fel i koden och logiska fel om algoritmen inte är korrekt. Ett bra exempel på detta är analog programmering (dans) där elever måste reflektera över ”robotens” handlingar i relation till det önskade rörelsemönstret. Lektionerna i denna kategori visar att lärare har följt kursplanens centrala innehåll genom att undervisa om stegvisa instruktioner. Innehållet har sitt ursprung i programmering som vetenskap snarare än i matematik som vetenskap.

Lärandemålen i dessa lektioner är väl i linje med mål för datalogiskt tänkande som beskrivs av exempelvis Brennan och Resnick (2012). Några uttalade mål kring datalogiskt tänkande finns dock inte i kursplanen för matematikämnet.

I de lektioner där matematiken görs till kontext för programmering är spridningen stor över årskurser och programmeringsmiljöer. Val av programmeringsmiljö följer den progression som kursplanen beskriver, med blockprogrammering i årskurs 4-6 och textprogrammering i årskurs 7-9. I vissa textbaserade språk, som t.ex. Python, kan datorn inte avgöra om ett inmatat tecken är ett värde eller ett tecken. För att arbeta med heltal behöver man använda en instruktion `int()` som omvandlar tecknet till ett värde, vilket eleverna i lektion ST_7 tränar på. Just denna instruktion kan dock bli obegriplig om läraren inte har kunskaper om programmeringens historia, och matematiken riskerar att antingen blandas ihop med, eller hamna i skymundan av, programmeringen.

I de fyra lektioner där programmering används som verktyg för att utforska matematik, används blockprogrammering. I dessa lektioner beskriver lärarna elevers svårigheter att hantera kod och syntax, vilket begränsar utforskandet av matematiska begrepp. En annan begränsande faktor skulle kunna vara lärares bristande kännedom om vilka möjligheter som erbjuds och vilka svårigheter som finns, exempelvis med distraktorer som för fokus bort från matematiken (Benton m fl., 2017; Bråting m fl., 2021). Att få elever att ta steget från att använda program till att skapa program, vilket beskrivs som en av målsättningarna med Scratch (Resnick m fl., 2009), ställer stora krav även på lärares kunskaper och förmåga att hantera programmeringsmiljön som ett undervisningsverktyg.

I en intervjustudie lyfter lärare fram att blockprogrammering främjar elevers kreativitet och engagemang (Kilhamn m fl., 2021). Att gå vidare från ökat engagemang till ökat lärande kräver dock en viss struktur med tydliga målsättningar så att eleverna arbetar mot avsedda kunskapsmål. I de utforskande lektionerna (kategori 4) ser vi att lärarna försöker skapa en sådan struktur i aktiviteterna. Samtliga har valt att arbeta med färdig kod där eleverna förväntas reflektera över matematiska konsekvenser då koden ändras (remixing), ett arbetssätt som Brennan och Resnick (2012) lyfter fram som en viktig datalogisk praktik.

6.4 Avslutande reflektion

Genom införandet av programmering i matematikämnet har Skolverket gjort ett val som påverkar synen på matematik. Många av lärargrupperna citerar följande ur läroplanens syftesformulering.

> Genom undervisningen ska eleverna ges förutsättningar att utveckla förtrogenhet med grundläggande matematiska begrepp och metoder och deras användbarhet (Skolverket, 2017).

När detta syfte relateras till en lektion som helt ägnas åt programmeringsbegrepp och kodning utan att andra grundläggande matematisk begrepp och metoder tas upp kan vi dra slutsatsen att lärarna anser att programmering utgör en del av matematiken och att programmeringsbegrepp nu anses vara matematiska begrepp. Detta resultat skiljer sig från resultatet av vår tidigare intervjustudie, där lärarna sällan uppfattade programmering i sig som en matematisk aktivitet (Kilhamn m fl., 2021).

När lektioner i programmering saknar koppling till traditionell matematik kan det diskuteras om programmering bör förläggas till just matematikämnet. Med tanke på vilket matematikinnehåll lärarna i den här studien sett som lämpligt för arbete med programmering, där ingen lektion klassificerades som en lektion i algebra, kan det också diskuteras om programmeringen verkligen ska anses utgöra en del av det centrala innehållet i just algebra. Om den svenska modellen med programmering som en del av algebran får internationellt genomslag återstår att se.

I den transpositionsprocess från avsedd kunskap till undervisad kunskap som vi beskrivit här ser vi att större vikt läggs vid det centrala innehållet än vid läroplanens syftesbeskrivning som legitimerar kunskapsstoffet och talar om vad det ska användas till. I Chevallards (2006) teori skulle detta kunna uttryckas i termer av större fokus på praxis (vad och hur) än på logos (varför). Att lära sig vad programmering är och hur det går till är mer framträdande i vårt material än frågan om vad programmering kan användas till och varför.

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Opettajaopiskelijoiden näkemyksiä omista valmiuksistaan matematiikka-ahdistusta kokevan oppilaan kohtaamisessa

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Matematiikka-ahdistuksen kohtaaminen ja siihen vaikuttamisen keinot ovat yhä korostuneemmin osa laadukasta matematiikan opetusta. Tutkimuskirjallisuutta niin matematiikka-ahdistuksen vaikutuksista kuin siihen vaikuttamisen keinoista on runsaasti. Sen sijaan kansallista tietoa siitä, millaisena juuri valmistuvat opettajat kokevat valmiutensa ahdistuksen kohtaamiseen on niukasti. Tässä tutkimuksessa selvitettiin haastattelututkimuksen keinoin, millaista on opintojensa loppuvaiheessa olevien opettajaopiskelijoiden tietämys matematiikka-ahdistuksesta ja siihen vaikuttamisesta, sekä millaisia toiveita heillä olisi teeman käsittelemiseksi opettajaopinnoissa. Analyysin perusteella opiskelijat kokivat olevansa kyvykkäitä kohtamaan matematiikka-ahdistusta ja etsimään siihen erilaisia vaikuttamisen keinoja itsenäisesti, mutta toivoivat, että koulutuksessa näitä keinoja tarkasteltaisiin selkeästi nykyistä enemmän.

Avainsanat: matematiikka-ahdistus, opettajankoulutus, koulutuksen kehittäminen

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Prospective teachers' views their readiness to face math anxiety in the classroom

Reducing math anxiety is an increasingly important part of mathematics teaching. There is a lot of research about the effects of math anxiety and how it can be reduced. However, Finnish prospective teachers' thoughts about their readiness for facing math anxiety in the classroom need to be looked more closely. This study focuses to find out what is the prospective teachers' knowledge about facing up math anxiety in the classroom, and how they see the topic should be taken care during the teacher education. Based on the analysis of eight prospective teachers' interviews, we could conclude that the students expected that they were able to face math anxiety in the classroom with several ways. Nevertheless, they hoped that methods for facing math anxiety should be more thoroughly and clearly looked at in teacher education and thus facilitate their preparedness to face math anxiety in the classroom.

Keywords: math anxiety, teacher education, educational development

1 Johdanto

Matematiikan opintojen korostaminen korkea-asteen opiskelijavalinnoissa on kasvattanut matematiikan opiskelun merkitystä niin lukiossa kuin peruskoulussa, sillä hyvä matemaattinen osaaminen rakentuu jo peruskoulusta lähtien (Metsämuuronen, 2017). Eräs maailmanlaajuisesti merkittävä matematiikan opiskelua haittaava tekijä on matematiikka-ahdistus. Iso-Britanniassa Johnston-Wilderin, Brindleyn ja Dentin (2014) tutkimuksen mukaan noin 30 % oppilaista ilmoitti kärsivänsä korkeasta matematiikka-ahdistuksesta. Ashcraft ja Moore (2009) tekivät varovaisen arvion, jonka mukaan voimakkaasta matematiikka-ahdistuksesta kärsii joka kuudes amerikkalainen. Vastaavasti Pisa 2012 tutkimuksen yhteydessä testin tehneistä 31 % ilmoitti olevansa hyvin hermostuneita ratkaistessaan matemaattisia ongelmia (OECD, 2013). Tässä tutkimuksessa Suomi sijoittui niiden maiden joukkoon, joissa matematiikka-ahdistuksen kokemus oli matala mutta kuitenkin selvästi havaittava (Foley, Herts, Borgonovi, Guerriero, Levine & Beilock, 2017). Matematiikan osaamisen vaihtelusta matematiikka-ahdistus selitti 20 % (Kupari, Välijärvi, Andersson, Arffman, Nissinen, Puhakka & Vettenranta, 2013). Jokaisessa koululuokassa on siis todennäköisesti useita matematiikka-ahdistuksesta kärsiviä oppilaita ja siksi matematiikka-ahdistuksen tunnistaminen ja huomioiminen on hyvin tärkeä osa opettajan pedagogista osaamista.

Opettajaopiskelijoiden ja opettajankoulutuksen suhteesta matematiikka-ahdistukseen tehdyt tutkimukset ovat keskittyneet tutkimaan lähinnä opettajaopiskelijoita ja heidän itsensä kokemaa matematiikka-ahdistusta (Dowker, Sarkar & Looi, 2016; Finlayson, 2014; Looney, Perry & Steck, 2017; Lutovac, 2014; Lutovac & Kaasila, 2014). Sen sijaan tutkimuksia siitä, mitä opettajaopiskelijat tietävät matematiikka-ahdistuksesta ja sen käsittelystä, ei ole juuri tehty, eikä varsinkaan suomalaisessa kontekstissa. Tässä tutkimuksessa tarkastellaan, mitä luokanopettajaopiskelijat osaavat kertoa matematiikka-ahdistuksesta ja sen vaikutuksista matematiikan opiskeluun. Lisäksi tarkastellaan luokanopettaja-opiskelijoiden toiveita siitä, millä tavoin matematiikka-ahdistusta tulisi heidän mielestään käsitellä opettajankoulutuksessa. Tutkimus liittyy Itä-Suomen yliopistossa tehtävään matematiikan pedagogisten opintojen kehittämistyöhön toimien osana sen nykytilan kartoitusta.

2 Matematiikka-ahdistus ja sen synty

Matematiikka-ahdistus määritellään yleisesti jännityksen ja ahdistuksen tunteiksi, jotka häiritsevät lukujen kanssa toimimista ja matemaattisten ongelmien ratkaisemista erilaisissa tavallisen elämän ja akateemisen suoriutumisen tilanteissa (Richardson & Suinn, 1972). Matematiikka-ahdistus ei liity oppilaan yleiseen älykkyyteen (Ashcraft, 2002) ja se on lisäksi erillään yleisestä akateemisesta ahdistuneisuudesta (Ramirez, Gunderson, Levine & Beilock, 2012), koetilanteen aiheuttamasta ahdistuksesta (Paechter, Macher, Martskvishvili, Wimmer & Papousek, 2017) ja yleisestä ahdistuneisuudesta (Papousek ym., 2012).

Vaikka matematiikka-ahdistus on tunne, se näyttäytyy myös fysiologisena oireiluna kuten kohonneena sykkeenä, hikoavina käsinä, vatsavaivoina ja huimauksena (Blazerin, 2011). Korkea matematiikka-ahdistus myös rasittaa elimistöä huomattavasti (Faust, 1992).

Matematiikan opiskelussa matematiikka-ahdistuneet oppilaat ovat usein muita oppilaita hitaampia suorittamaan matematiikan tehtäviä. Vaikka heidän vastaustensa tarkkuudessa ei olekaan eroja, ahdistus tulee näkyväksi, kun tehtävän suorittaminen vaatii useampia vaiheita tai sisältää useampinumeroisia lukuja (Cates & Rhymer, 2003; Vukovic, Kieffer, Bailey & Harari, 2012). Matematiikka-ahdistuksen onkin tutkittu kuormittavan työmuistia ja matematiikka-ahdistuneen oppilaan työskentely on hyvin altis kaikenlaisille ulkopuolisille häiriötekijöille (Ashcraft & Kirk, 2002; vrt. Holm, Björn, Laine, Korhonen & Hannula, 2020). Ramirez ym. (2012) esittävät, että työmuisti kuormittuu pelkästä ahdistuksesta niin paljon, ettei matemaattisten ongelmien prosessoimiselle jää juurikaan tilaa. Matematiikka-ahdistus heikentää kaikkia kognitiivisia prosesseja, esimerkiksi lukutaitoa tilanteissa, joissa tekstin konteksti liittyy matematiikkaan. Viimeaikaiset tutkimukset viittaavat siihen, että matematiikan sisällön unohtamisen kognitiiviset prosessit liittyvät myös matematiikka-ahdistukseen (McDonough & Ramirez, 2018).

Vaikka matematiikka-ahdistus vaikuttaa opiskelijan suoriutumiseen matematiikassa Wang, Lukowski, Hart, Lyons, Thompson, Kovas, Mazzocco, Plomin ja Petrill (2015) esittävät, että matematiikka-ahdistuksen ja suoriutumisen yhteys ei ole suoraviivaista, vaan siihen vaikuttaa opiskelijan luontainen motivaatio. Jos oppilaalla on korkea sisäinen motivaatio, voi kohtalaisella matematiikka-ahdistuksella olla suoriutumista parantava vaikutus. Sitä vastoin alhaisen motivaatiotason oppilailla ahdistuksen kasvaminen heikentää suoriutumista. Tämä yhteys näyttää toimivan myös päinvastaiseen suuntaan. Madjar, Zalsman, Weizman, Lev-Ran ja Shoval (2018) raportoivat, että keskiarvoltaan huonommin menestyvät oppilaat raportoivat korkeampaa ja pysyvämpää matematiikka-ahdistusta kuin paremmin suoriutuvat oppilaat. Ehkä taustalla on, että matematiikka-ahdistus heikentää oppilaan minäpystyvyyden tunnetta matematiikan taitajana (Luttenberger, Wimmer, & Paechter, 2018). Mitä voimakkaampi matematiikka-ahdistus, sitä alhaisempi matematiikan minäpystyvyyden kokemus (Gonzalez-DeHass, Furner, Vásquez-Colina & Morris, 2017).

Matematiikka-ahdistus syntyy yleensä opiskelijoiden aikaisemmista huonoista koulukokemuksista (Bekdemir, 2010; Malinsky, Ross, Pannells & McJunkin, 2006). Finlayson (2014) tarkasteli tutkimuksessaan matematiikka-ahdistuneita opettajaopiskelijoita ja hänen havaintonsa ahdistuksen synnystä voidaan luokitella kolmeen luokkaan: 1) matematiikka-ahdistuksen taustalla olevat opiskelijan negatiiviset tunteet kuten itsevarmuuden puute ja epäonnistumisen pelko, 2) opettajan persoonallisuudesta aiheutuva matematiikka-ahdistus kuten oppikirjoihin perustuva opetustyyli, opettajan auktoritatiivinen rooli, huono lähestyttävyys ja opettajan vaade, että matematiikkaa opiskellaan vain hänen tyylillään, ja 3) matematiikka-ahdistusta lisäävät opetusmenetelmälliset ratkaisut kuten joustamaton opetussuunnitelman noudattaminen, kilpailu matematiikan tunneilla ja kokeet. Kaikki tutkittavat kokivat matematiikka-ahdistusta, jos opettajaa oli vaikea lähestyä ja hän sai oppilaat tuntemaan, ettei tuntia saanut häiritä kysymyksillä.

Opettajan vaikutus oppilaan kokemaan matematiikka-ahdistukseen Mizalan, Martínezin ja Martínezin (2015) voi johtua myös opettajan itse kokemasta matematiikka-ahdistuksesta ja sen välittymisestä oppilaisiin. Opettajan matematiikka-ahdistus näkyy oppitunneilla esimerkiksi siinä, että opettaja pyrkii kiirehtimään opetuksessaan päästäkseen eroon tunnin aiheesta tai hän ollessaan epävarma opetusmenetelmistään keskittyy ainoastaan oikean vastauksen saamiseen, eikä luo luokkaan ilmapiiriä, jossa kysymykset ja pohdinnat ovat hyväksyttyjä. Lisäksi matematiikka-ahdistunut opettaja luovuttaa itsekin helposti ja hänen on haastava tukea apua tarvitsevaa oppilastaan (Dowker ym., 2016). Tällainen toiminta opettaa oppilaalle, että matematiikkaa kuuluukin pelätä ja saattaa aiheuttaa pelon siitä, että huonosti pärjäävälle oppilaalle suututaan (Furner, 2017).

Matematiikka-ahdistus vaikuttaa siitä kärsivän elämään niin koulussa kuin arkielämässäkin ja voi johtaa matematiikan välttelyyn (Casad, Hale ja Wachs, 2015). Välttelyn seurauksena ovat heikot matemaattiset taidot ja huonot arvosanat, ja nämä estävät oppilasta läpäisemästä perustavanlaatuisia matematiikan kursseja (Akinsola, Tella & Tella, 2007) tai rajoittavat vaativampien kurssien valitsemista (Ramirez ym., 2012). Lopulta kaikki vaikuttavat matematiikan osaamisen lisäksi myös oppimiskäytänteisiin ja opiskeluvalintoihin (Luttenberger ym., 2018). Matematiikan välttely näyttäytyy matematiikka-ahdistuneiden opiskelijoilla kaikenlaisissa tilanteissa, joissa joudutaan käyttämään matematiikkaa. Ashcraftin (2002) mukaan matematiikka-ahdistusta kokevien oppilaiden matematiikan opiskelua leimaakin negatiivinen kehä (Tuohilampi & Hannula, 2013). Aikaisemmat negatiiviset kokemukset matematiikasta aiheuttavat matematiikan välttelyä. Tämä johtaa huonompaan valmistautumiseen matematiikan opiskelussa ja tuottaa heikompaa osaamista, josta seuraa taas uusia negatiivisia kokemuksia. Negatiivisen kierteen seurauksena matematiikka-ahdistuneet opiskelijat ovat lukiovaiheessa opiskelleet vähemmän matematiikkaa oppilastovereihinsa verrattuna, ja tämä vaikuttaa niin heidän myöhempiin koulutuspaikkojen valintaansa kuin niiden opintoihinkin. Foleyn ym. (2017) mukaan matematiikka-ahdistuneet opiskelijat välttelevät hakeutumista matemaattisille aloille kuten luonnontieteisiin, tai teknologia- ja insinööritieteisiin.

3 Matematiikka-ahdistuksen tunnistaminen ja siihen vaikuttaminen

Voidakseen tukea matematiikka-ahdistusta kokevaa oppilastaan, opettajan on ensin tunnistettava oppilaan matematiikka-ahdistus. Matematiikka-ahdistuksen merkkejä ovat esimerkiksi tyhjän kokeen palauttaminen tai sellainen oppilaan ajattelumalli, jossa oikeat vastaukset ovat hyviä ja väärät huonoja (Finlayson, 2014). Myös laskutoimitusten sujuvuuden ja tarkkuuden arviointi voivat auttaa ahdistuksen tunnistamisessa ja sen tason määrittelemisessä (Cates & Rhymer, 2003). Joskus luova kirjoittaminenkin voi paljastaa oppilaan matematiikka-ahdistuksen. Furner (2017) ehdottaakin tunnistamisen avuksi oppilaille teetettävää kirjoitelmaa, jossa oppilaat jatkavat seuraavia virkkeitä: (1) Kun kuulen sanan matematiikka, minä … (2) Lempiaiheeni matematiikassa on …, (3) Pidän matematiikassa vähiten …, (4) Jos voisin kysyä yhdestä asiasta matematiikassa, se olisi … ja (5) Suosikkini matematiikan opettajista on …, koska ….

Kun opettaja on tunnistanut oppilaassa matematiikka-ahdistusta, hän voi tarttua siihen eri tavoin. Ensinnäkin hän voi tarjota oppilaalle emotionaalista tukea (Beilock & Willingham, 2014; Rodrigues, 2012), toiseksi hän voi tarjota oppilaalle kognitiivista tukea (Federic & Skaalvik, 2013), kolmanneksi opettaja voi tehdä opetusmenetelmällisiä ratkaisuja (Dowker ym., 2016; Finlayson, 2014; Mattarella-

Micke ym., 2011) ja neljänneksi puuttua ahdistukseen yksilöllisin tukijärjestelyin (Brooks, 2014; Finlayson, 2014; Ramirez & Beilock, 2011).

3.1 Opettajan emotionaalinen tuki

Opettajan emotionaalinen tuki voi kohdentua itsesäätelytaitojen ja metakognitiivisten taitojen kehittämiseen esimerkiksi itsearvioinnin avulla. Itsesäätelytaitojen parantaminen kohottaa itsevarmuutta ja vähentää näin matematiikka-ahdistusta (Jain & Dowson, 2009). Kasvava metakognitiivinen tietoisuus taas voi osaltaan vähentää matematiikka-ahdistusta kuten Sarıcam ja Ogurlu (2015) havaitsivat tutkimuksessaan: lahjakkailla oppilailla myös metakognitiivinen tietoisuus on korkeampaa ja näin ollen matematiikka-ahdistus matalampaa. Metakognitiivisen tietouden kehittämisessä itsearvioinnit ovat keskeisessä roolissa (Donham, 2010).

Finlaysonin (2014) mukaan opetuksen yksilöllistäminen ja oppilaiden oppimistyylien huomioon ottaminen opetuksen suunnittelussa ovat keskeisiä työkaluja matematiikka-ahdistuksen vähentämisessä. Positiivinen palaute ja vahvistaminen rakentavat oppilaan itsetuntoa ja itsevarmuutta (Rodrigues, 2012). Palautteella ei ole vaikutusta ainoastaan oppimiseen, vaan myös henkilön kokemiin tunteisiin ja näkemykseen omista heikkouksistaan ja vahvuuksistaan. Tehostaakseen oppimista palautteen tulee kuitenkin olla tarkkaa ja tavoitteeseen liittyvää. Positiiviset tunteet vaikuttavat oppimiseen positiivisesti, kun taas negatiiviset tunteet negatiivisesti. Tätä näkemystä tukevat positiivisen psykologian lisäksi neurotieteet ja motivaatiopsykologia (Voerman, Korthagen, Meijer & Simons, 2014). Positiivinen palaute työskentelystä (Greene & Todd, 2015) alentaa oppilaan syketasoa ja parantaa oppimistuloksia. Positiivista palautetta ja itsevarmuuden tukemista tarvitsevat ennen kaikkea oppilaat, joita matematiikka ahdistaa, sillä matematiikka-ahdistuksen taso on korkeinta heillä, jotka uskovat olevansa huonoja matematiikassa (Al Mutawah, 2015; Jain & Dowson, 2009). Opettajan omiin sanavalintoihin tulisi kuulua kannustaminen ja oppilaan rohkaiseminen niin, että vaikka tehtävä onkin haastava, oppilas varmasti selviää siitä. (Beilock & Willingham, 2014).

3.2 Opettajan kognitiivinen tuki

Emotionaalinen tuki ei kuitenkaan yksinään riitä, vaan sen lisäksi tulisi tukea oppilaan matemaattisten taitojen kehittymistä. Konkreettinen matematiikan taitojen tukeminen on Federicin ja Skaalvikin (2013) tutkimuksessa emotionaalista tukea enemmän yhteydessä matematiikka-ahdistuksen vähenemiseen ja yrittämisen lisääntymiseen. Oppilaan matematiikan perustaitojen vahvistaminen auttaa oppilasta luottamaan omiin taitoihinsa, vähentää matematiikka-ahdistusta (Jain & Dowson, 2009), ja voi jopa suojata nuoria oppilaita matematiikka-ahdistuksen kehittymiseltä (Beilock & Willingham, 2014). Jotkut oppilaat tarvitsevat enemmän ohjausta ja apua taitojensa harjoittamiseen kuin toiset. Oppilaat, joilla on sekä matala työmuistikapasiteetti että korkea matematiikka-ahdistus, tarvitsevat enemmän opettajan tukea ja selkeää ohjeistusta oppiakseen ratkaisemaan soveltavia tehtäviä kuin muut oppilaat (Vukovic ym., 2012).

Opettaja voi myös keskittyä matemaattisen ymmärryksen lisäämiseen opetuksessaan esimerkiksi konkretian keinoin (Looney ym., 2017; Vinson, 2001). Se vähentää oppilaiden kokemaa ahdistusta ja opettaja pystyy opettamaan tehokkaammin (Brady & Bowd, 2005).

Matematiikka-ahdistusta saattavat lieventää myös järjestelmällinen opiskelu ja ulkoa opettelu. Kun matematiikan perusasiat yliopitaan ja esimerkiksi laskutoimituksista tulee sujuvia tai automatisoituja, se auttaa selviämään ahdistuksesta ja antaa itsevarmuutta uuden opiskeluun. Tässä tärkeää on oppilaan oma aktiivisuus; kotitehtävien tekeminen ja muistiinpanojen tai muutoin opiskeltujen asioiden kertaaminen (Cates & Rhymer, 2003).

3.3 Matematiikka-ahdistusta vähentävät opetusjärjestelyt

Matematiikka-ahdistuksen välttämiseksi olisi opetuksen aikajärjestelyt mietittävä tarkasti. Kaikenlainen kelloa vastaan työskenteleminen ja nopeutta mittaava tekeminen ovat ahdistusta tuottavia elementtejä (Boaler, 2009). Sitä vastoin ongelmanratkaisuprosessin kärsivällisyyttä ja sitkeyttä vaativien taitojen tunnistaminen, niistä keskusteleminen ja harjoittaminen tuottavat hyvät mahdollisuudet matemaattisen ajattelun kehittymiselle ja vähentävät matematiikka-ahdistusta. Myös arvioinnin muuttaminen niin, että matematiikan tehtävien ja kokeiden suorittamisesta poistetaan aikarajoitteet, voi vähentää oppilaan kokemaa stressiä, vähentää matematiikka-ahdistusta ja parantaa näin hänen suoriutumistaan (Mattarella-Micke, Mateo, Kozak, Foster & Beilock, 2011).

Toiseksi työskentely osaamistasoltaan heterogeenisessa ryhmässä, auttaa matematiikka-ahdistuneita oppilaita, sillä se mahdollistaa vertaistutoroinnin ryhmän sisällä (Supekar, Iuvulana, Chen & Menon, 2015). Matematiikan opiskelun yksi keskeinen tavoite on oppia kohtaamaan ongelmatilanteita ja harjoitella niistä ulospääsyä. Tätä tietoisuutta voidaan lisätä esimerkiksi ongelmanratkaisuprosessia sanoittamalla ja erilaisia ratkaisuvaihtoehtoja tarjoamalla. Virheiden hyväksyminen sekä niistä oppiminen parantavat oppilaiden mahdollisuuksia selviytyä itsenäisesti ongelmanratkaisutilanteissa (ks. Hannula, 2019).

Opetusjärjestelyissään opettaja voi keskittyä myös siihen, että hän tarjoaa matematiikasta hyödyllisen kuvan arkielämän kannalta ja auttaa oppilaita näkemään yhteyksiä matematiikan ja heidän omien mielenkiinnon kohteidensa välillä (Rodrigues, 2012). Schaeffer, Berkowitz, Levine ja Beilock (2018) havaitsivat pienten lasten ja heidän vanhempiensa iltasatutyyppisten matematiikkatarinoiden lukemisinterventiosta ja sen positiivisista vaikutuksista lasten akateemiseen menestymiseen, että matemaattisten tarinoiden lukemista kannattaisi ottaa myös osaksi opetusta.

3.4 Yksilölliset tukijärjestelyt ja harjoitteet

Finlaysonin (2014) tutkimuksessa matematiikasta ahdistuneet opettajaopiskelijat käyttivät selviytymiskeinoinaan rentoutumista, itsevarmuuden parantamista esimerkiksi helpoista ongelmista vaikeisiin siirtymällä ja onnistumisista iloitsemalla. He pyrkivät tekemään opiskelustaan järjestelmällistä, tekemään kotitehtäviä huolellisesti, pyytämään apua ja tukeutumaan perheeseen, sekä käyttämään internetiä tiedonhaussa.

Brunyén, Mahoneyn, Gilesin, Rappin, Taylorin, ja Kanarekin (2013) mukaan Mindfulnessin harjoitteleminen voi auttaa matematiikka-ahdistukseen, koska sen avulla voidaan vähentää opiskelun häiriötekijöiden vaikutusta. Tutkimuksessaan he havaitsivat, että kun voimakkaasti matemaattiset ahdistuneet opiskelijat harjaannuttivat tietoista hengitystekniikkaa, he kertoivat tuntevansa olonsa paljon rauhallisemmaksi ja suoriutuvansa testeistä paremmin.

Useammassa tutkimuksessa matematiikka-ahdistusta lieventäväksi keinoksi on todettu luova kirjoittaminen, eli matematiikan opiskeluun liittyvien ja sen aiheuttamien tunteiden sanallistaminen kirjoittamisen avulla. Kirjoittaminen vähentää negatiivisten ajatusten viemää tilaa työmuistissa ja antaa näin mahdollisuuden arvioida uudelleen stressaavaa tilannetta (Beilock & Willingham, 2014). Klein ja Boals (2001) huomasivat tutkimuksessaan, että luovasta kirjoittamisesta vaikutti olevan hyötyä erityisesti, jos kirjoitetaan negatiivisista kokemuksista. Heidän havaintojensa perusteella positiivisista tai neutraaleista, jokapäiväisistä asioista kirjoittamisesta ei ollut hyötyä työmuistin paranemisessa, kun taas negatiivisista kokemuksista kirjoittaneiden opiskelijoiden työmuistikapasiteetti parani huomattavasti ja pitkäkestoisesti. Huolista kirjoittaminen voi parantaa myös oppilaiden koesuoriutumista silloin, kun ahdistusta kokeva oppilas kirjoittaa huolistaan juuri ennen koetta (Ramirez & Beilock, 2011). Lutovac ja Kaasila (2011; 2019) sen sijaan ovat havainneet tutkimuksissaan, että toisen henkilön matematiikka-ahdistuksesta selviämisen tarinaan paneutuminen voi toimia oman matematiikkakuvan muokkaajana ja matematiikka-ahdistuksen hälventäjänä.

Matematiikka-ahdistuksen vaikutusta voidaan myös vähentää ajattelumenetelmällä, jossa ahdistus sanoitetaan positiiviseksi (Brooks, 2014) tai esimerkiksi kirjoittamalla matematiikan tunneilla oppimispäiväkirjaa. Päiväkirjassa oppilas voi ilmaista, miten hän ymmärtää erilaiset matematiikan aihealueet ja kertoa tuntemuksistaan matematiikan tunnin aikana. Tämä voi auttaa myös opettajaa ymmärtämään paremmin oppilaan erilaisia ahdistuksen ja turhautumisen tunteita (Furner, 2017).

4 Tutkimuksen toteutus

4.1 Aineisto ja tutkimuskysymykset

Tämän haastattelututkimuksen tavoitteena on selvittää opettajaopiskelijoiden näkemyksiä ja tietämystä siitä, kuinka matematiikka-ahdistusta kokevaa oppilasta voidaan tukea. Lisäksi tuodaan näkyviin opiskelijoiden ajatuksia siitä, miten matematiikka-ahdistusta käsitellään opettajankoulutuksessa. Tutkimuskysymyksinä palvelevat:

1. Millaisilla keinoilla opettajaopiskelijat tukisivat oppilasta, jolla on matematiikka-ahdistusta?
2. Miten opettajaopiskelijat haluaisivat käsitellä matematiikka-ahdistusta opettajankoulutuksessa?

Tutkimusaineisto on hankittu keväällä 2019 puolistrukturoiduilla teemahaastatteluilla (DiCicco-Bloom & Crabtree, 2006; Kelly, 2010). Haastattelujen kesto vaihteli 10-20 minuuttiin ja muodosti litteroituna 8 550 sanan aineiston.

Tutkimukseen osallistui 8 opettajankoulutukseen osallistunutta opiskelijaa. Haastateltavasta yksi oli neljännen vuoden ja muut viidennen vuoden opiskelijoita. Haastateltavat opiskelivat joko luokanopettajiksi (5), erityisopettajiksi (2) tai aineenopettajiksi (1) ja heitä yhdisti se, että kaikilla on valmistumisensa jälkeen pätevyys toimia matematiikan opettajana alakoulussa. Haastateltavat olivat suorittaneet opiskelujensa aikana luokanopettajan monialaiset opinnot. Näihin opintoihin sisältyy pakollisena kurssina matematiikan pedagogiikan 8 opintopisteen kurssi, jonka tenttimateriaalissa käsitellään matematiikka-ahdistusta.

Haastattelun aluksi opiskelijoiden kanssa keskusteltiin matematiikka-ahdistuksesta sekä heille kerrottiin matematiikka-ahdistuksen lyhyt määritelmä:

”Matematiikka-ahdistus määritellään usein jännityksen ja ahdistuksen tunteiksi, jotka häiritsevät numeroiden kanssa toimimista ja matemaattisten ongelmien ratkaisemista erilaisissa tavallisen elämän ja akateemisen suoriutumisen tilanteissa. Se voi estää oppilasta läpäisemästä perustavanlaatuisia matematiikan kursseja tai estää vaativampien kurssien valitsemista” (Richardson & Suin, 1972). Tämän jälkeen tutkimuksen ensimmäisellä haastatteluteemalla selvitettiin, miten opiskelijat tukisivat matematiikka-ahdistunutta oppilasta. Teemaa lähestyttiin kolmella kysymyksellä (a) Kuvittele tilanne, jossa opetat matematiikkaa sellaiselle luokalle, jolla yhdellä tai useammalla oppilaalla on matematiikka-ahdistusta. Miten voisit tunnistaa tällaisen oppilaan? (b) Miten voisit itse opettajana tukea oppilasta, jolla on matematiikka-ahdistusta? (c) Jos pyytäisit apua oppilaan tukemiseen, millaisiin asioihin sitä pyytäisit ja keneltä? Seuraavassa haastatteluteemassa käsiteltiin sitä, millaisia kokemuksia haastateltaville on kertynyt matematiikka-ahdistuksen käsittelemisestä opettajaopintojensa aikana kahden kysymyksen avulla (a) Käsitelläänkö matematiikka-ahdistusta mielestäsi tarpeeksi opettajan opinnoissa? Perustele vastauksesi. (b) Ajatteletko, että sinulla on valmistuttuasi riittävästi työkaluja tukea oppilasta, jolla on matematiikka-ahdistusta? Perustele vastauksesi.

4.2 Analyysin kuvaus

Aineisto analysoitiin teoriaohjaavan sisällönanalyysin avulla. Teoriaohjaava sisällönanalyysi, jota Burnard (1991) kutsuu temaattiseksi sisällönanalyysiksi, mahdollistaa analyysin aineiston ehdoilla, mutta silti aiempaa tutkimusta hyödyntäen (Tuomi & Sarajärvi, 2018). Analyysissä on Eskolan (2018) mukaisesti teoreettisia kytkentöjä, mutta analyysi itsessään ei pohjaudu tai nouse suoraan teoriasta. Teoriaohjaavan sisällönanalyysin kolmen vaiheen avulla muodostettiin käsitteellinen malli matematiikka-ahdistustietouden kokemuksesta (Elo & Kyngäs, 2008). Peruslitteroinnin tarkkuudella tuotettuun aineistoon perehdyttiin ensin huolellisesti kokonaiskuvan saamiseksi, jonka jälkeen aineistosta tuotettiin kaksi erillistä tiedostoa, joista ensimmäiseen tallennettiin haastattelut kokonaisina haastateltava kerrallaan ja toiseen haastateltavien vastaukset kysymyksittäin koottuina.

Seuraavaksi aineistosta etsittiin tutkimuskysymys kerrallaan kaikki ne ilmaukset, jotka vastasivat kuhunkin tutkimuskysymykseen. Nämä alkuperäiset ilmaukset koottiin pelkistettyinä ilmauksina erilliseen tiedostoon sen tutkimuskysymyksen alle, johon ilmaus vastaa, pitäen kuitenkin jatkuvasti analyysin rinnalla alkuperäisiä kokonaisia haastatteluja, jotta ilmausten konteksti ei katoaisi analyysin edetessä (Burnard, 1991).

Analyysiprosessin eteneminen tapahtui esimerkiksi seuraavasti. Lainauksessa on haastateltavan H2 vastaus kysymykseen ”Miten voisit itse opettajana tukea oppilasta, jolla on matematiikka-ahdistusta?”.

> Voisko mahollisesti toiminnalliset työtavat auttaa? Uskosin ainaki, että niinku tämmöset oikeen nuoret ykkös-kakkos-alkuopetusikäset lapset, nii ne vois hyötyä tämmösestä toiminnallisista harjotuksista (H2)

Alkuperäinen ilmaus on pelkistetty ilmaukseksi Toiminnalliset työtavat, joka sijoittuu alaluokkaan 2 Toiminnallisuus. Tämä sijoittuu alaluokkaan 1 Konkreettiset opetusmenetelmät ja lopulta teorialähtöiseen yläluokkaan Opetusmenetelmälliset ratkaisut.

Pelkistetyistä ilmauksista koottiin siis ensiksi toisen tason alaluokkia ryhmittelemällä pelkistettyjä ilmauksia ja nimeämällä ne alaluokkaa yhdistävällä nimellä. Esimerkiksi Alaluokka 2 Toiminnallisuus koostui pelkistetyistä ilmauksista kuten toiminnalliset työtavat, enemmän leikillisyyttä ja pelillisyyttä luokassa, toiminnallista ja pelien kautta (ks. taulukko 1). Sama menettely toistettiin kummankin tutkimuskysymyksen kohdalla niin, että tässä vaiheessa olivat vielä näkyvissä ne haastattelukysymykset, joihin pelkistetty ilmaus oli vastaus. Näin varmistettiin, että pelkistetyt ilmaukset todella vastasivat tutkimuskysymyksiin (Burnard, 1991). Seuraavaksi syntyneet toisen tason alaluokat koottiin pelkistettyine ilmauksineen selvyyden vuoksi omaan tiedostoonsa ja alaluokkien tiivistämistä jatkettiin ryhmitellen ne edelleen ensimmäisen tason alaluokiksi. Esimerkiksi Alaluokka 1 Konkreettiset opetusmenetelmät koostui toisen tason alaluokista: Toiminnallisuus, Apuvälineet ja Matematiikan konkreettiseksi tekeminen.

Ensimmäisen tason alaluokista yhdisteltiin yläluokkia teorian avulla. Esimerkiksi ensimmäisen tason alaluokat Konkreettiset opetusmenetelmät, Eriyttävät opetusjärjestelyt yhdistyivät yläluokan Opetusmenetelmälliset ratkaisut alle. (ks. taulukko 1). Tässä vaiheessa analyysia aiempi tutkimustieto tuli siis mukaan ohjaamaan yläluokkien muodostamista ja nimeämistä. Yläluokat toimivat varsinaisina tutkimustuloksina. Analyysin jatkaminen näitä yläluokkia yhdistäviksi luokiksi, osoitti, että analyysin jatkaminen olisi yhdistänyt liian erilaisia aiheita liian suuren luokan alle (Burnard, 1991). Lopuksi analyysin vaiheet toistettiin ja varmistettiin, ettei mitään jäänyt huomaamatta. Tarkastelussa pidettiin jatkuvasti huolta siitä, että pelkistettyjä ilmauksia käsiteltiin niiden alkuperäisessä kontekstissa, oikeiden asiayhteyksien säilyttämiseksi (Burnard, 1991; Mayring, 2014).

5 Tulokset

Seuraavassa tarkastelemme tuloksia tutkimuskysymys kerrallaan.

5.1 Millaisilla keinoilla opettajaopiskelijat tukisivat oppilasta, jolla on matematiikka-ahdistusta?

Pohtiessaan keinoja, joilla tukea matematiikka-ahdistunutta oppilasta, opiskelijat päätyivät ensin selvittämään ahdistuksen alkuperää ahdistuksen kierteen katkaisemiseksi.

> Ehkä myös sitte selvittäisin, että mistä se ehkä kenties kumpuaa se matematiikka-ahdistus ja voisko sitä kautta niinku puuttua siihen, että se niinku katkeis se kierre. (H1)

Olennaisena mainittiin mahdollisimman varhainen tunnistaminen ja puuttuminen esimerkiksi koulukuraattorin avustamana, jotta ongelma ei pääsisi kasvamaan vuosien myötä. Tässä työssä korostettiin niin kotien kuin moniammatillisen yhteistyön merkitystä.

> Mä ehkä pyytäisin ekana niinku muilta opettajilta, ett miten ne on ratkonu ja onks he tätä samaa oppilasta opettanu aikasemmin, ett miten se näkyy ja missä se ehkä se ongelma on tullu, ett minkä takia se matikka on alkanu ahdistaa. (H4)

Aineistosta nousi teorian ohjaamana kolme yläluokkaa matematiikka-ahdistuneen oppilaan tukemiseksi: (1) opettajan tarjoama emotionaalinen tuki, (2) opettajan tarjoama kognitiivinen tuki, ja (3) opetusmenetelmälliset ratkaisut. Tulokset esitellään tiivistetyssä muodossa taulukossa 1.

Taulukko 1. Matematiikka-ahdistuksen tukemisen keinot

Yläluokka	**Alaluokka 1**	**Alaluokka 2**
Opettajan tarjoama emotionaalinen tuki (Al Mutawah, 2015; Beilock & Willingham, 2014; Jain & Dowson, 2009; Rodrigues, 2012)	Tunteiden käsittely	Keskusteluapu Asiaan liittyvien tunnetilojen käsittely Luottamussuhteen rakentaminen Koulukuraattorin tai -psykologin apu
	Itsetunnon tukeminen	Yrittämään kannustaminen Positiivinen palaute Vahvuuksien löytäminen ja rohkaiseminen Onnistumisen kokemuksien löytäminen
Opettajan tarjoama kognitiivinen tuki (Beilock & Willingham, 2014; Federic & Skaalvik, 2013; Jain & Dowson, 2009)	Matemaattisen osaamisen kehittäminen	Tukiopetus Keskittyminen pelkästään tähän oppilaaseen Taitojen yksilöllinen harjoittaminen Perustehtävien yhdessä katsominen ja taitojen kartuttaminen
Opetusmenetelmälliset ratkaisut (Dowker ym., 2016; Finlayson, 2014; Mattarella-Micke ym., 2011)	Konkreettiset opetusmenetelmät	Toiminnallisuus Apuvälineet Matematiikan konkreettiseksi tekeminen
	Eriyttävät opetusjärjestelyt	Enemmän aikaa (omatahtisuus) Aiheiden ja tehtävien valikointi Erilaiset työtavat Koejärjestelyt

Opettajan tarjoama emotionaalinen tuki jakautui haastatteluissa kahteen alaluokkaan: tunteiden käsittely ja itsetunnon tukeminen (vrt. Al Mutawah, 2015; Beilock & Willingham, 2014; Jain & Dowson, 2009; Rodrigues, 2012). Tunteiden käsittelyä pidettiin tärkeänä tapana lievittää ahdistusta oppilaan kanssa keskustelemalla joko itse, tai ohjaamalla hänet keskustelemaan koulukuraattorin, koulupsykologin tai erityisopettajan kanssa.

> Vois periaatteessa ite kysyä opettajana neuvoo niinku joltain koulukuraattorilta tai sitt kysyy niinku näitä sopivia reittejä pitkin sitte aikanaan, että oisko sen oppilaan mahdollista mennä käsittelemään sitä mahdollista ahdistustansa niinku koulukuraattorin kanssa. (H5)

Opiskelijat näkivät tässä keskeisenä luottamussuhteen rakentamisen, joka mahdollistaa asiaan liittyvien tunnetilojen käsittelyn. Itsetunnon tukeminen puolestaan nähtiin tapana saada oppilas yrittämään matematiikan tehtäviä itsenäisesti. Tätä tavoiteltiin haastatteluissa yrittämään kannustamisen avulla.

Pyrkimyksenä oli saada oppilas haastamaan itseään matematiikassa ja tuntemaan onnistumisen kokemuksia. Opettajaopiskelijat kertoivat pyrkivänsä tähän runsaan ja välittömän positiivisen palautteen avulla ja etsimällä oppilaan matemaattisia vahvuuksia.

> No siis ehkä opettajana sitte semmonen niinku kannustaisin ja semmonen tosi pienillä askelilla, että aina ku se oppilas niinku antaaki merkkejä, että hän yrittää, niin hirveesti kehua ja että hienosti meni ja että nyt niinku etteenpäin ja sitten niinku yrittää vähän sitä oppilasta niinku pittää pinnalla siinä. (H1)

Opettajan tarjoama kognitiivinen tuki keskittyi oppilaan matemaattisen osaamisen kehittymisen tukemiseen (vrt. Beilock & Willingham, 2014; Federic & Skaalvik, 2013; Jain & Dowson, 2009). Opiskelijat kuvasivat oppilaan yksilöllistä kohtaamista esimerkiksi kahden kesken tapahtuvilla tukiopetusjärjestelyillä. Tavoitteena on taitojen kartuttaminen perustehtävistä lähtien ja siirtyen yksilöllisen harjoittelun kautta uuden matemaattisen identiteetin rakentamiseen.

> Jääpä tukiopetukseen, ett käyttää sen hetken, ett tekee sen kaa vaan kahestaan, ett se saa sen oman ajan sille ja niinku sä pystyt keskittyy pelkästään siihen oppilaaseen. (H8)

Kaksi haastateltavaa mainitsi oppilaan perustaitotason parantamisen ja mahdollisten puutteiden korjaamisen ennen kuin matematiikassa edettäisiin pidemmälle. Kaikkien asiasta maininneiden ajatus oli, että taitojen harjoittaminen parantaa oppilaan itsevarmuutta matematiikassa ja helpottaa näin ollen hänen matematiikka-ahdistustaan.

> Koittaa silleen pikkuhiljaa sitä perustaitotasoa niinkun kartuttaa, että se pohja tulee kuntoon ja sitte hän itekki huomaa varmaan pikkuhiljaa, että hän niinku pystyy tähän ja hän niinku selviytyy siitä. (H6)

Eniten mainintoja kertyi yläluokkaan Opetusmenetelmälliset ratkaisut. Kaikki haastateltavat mainitsivat jonkin opetusmenetelmällisen ratkaisun jossakin vaiheessa haastatteluaan. Opetusmenetelmällisiin ratkaisuihin sisältyivät konkreettiset opetusmenetelmät ja eriyttävät järjestelyt (vrt. Dowker ym., 2016; Finlayson, 2014; Mattarella-Micke ym., 2011). Konkreettisista opetusmenetelmistä haastateltavat mainitsivat toiminnalliset, leikilliset ja pelilliset työtavat.

> Voisko mahollisesti toiminnalliset työtavat auttaa? Uskosin ainaki, että niinku tämmöset oikeen nuoret ykkös-kakkos-alkuopetusikäset lapset, nii ne vois hyötyä tämmösestä toiminnallisista harjotuksista. (H2)

Myös konkreettisten apuvälineiden, kuten kymppisauvojen, käyttö esiintyi keinojen joukossa.

> Mä varmaan yrittäsin helpottaa sitä, että käyttäsin enemmän leikillisyyttä ja pelillisyyttä ja sit konkreettisia esimerkkejä, apuvälineitä, ett jos se helpottais, ett ei ois pelkästään ne numerot, vaan vaikka ne kymppisauvat siinä apuna. (H7)

Opettajaopiskelijat mainitsivat myös matematiikan konkreettiseksi tekemisen tuomalla vaikkapa matematiikassa käytetyt esimerkit arkipäivään ja korostamalla, että matematiikka on paljon muutakin kuin laskemista.

> Sais irrotettuu sen oppilaan siitä, ett matikka, niinku ett, ja koko luokalleki sais läpi sen, ett matikka on paljo muutaki, ku sitä pelkkää laskemista. (H8)

Samansuuntaisia ajatuksia esitettiin myös yleisiksi didaktisiksi ratkaisuiksi ja koko luokan kanssa käytettäviksi menetelmiksi. Ne hyödyttäisivät sekä matematiikka-ahdistusta kokevaa oppilasta että koko ryhmää.

> Mä väitän, että myös koko luokkaa auttaa siinä niinku näkemään matematiikan erilaisesta näkökulmasta. (H6)

Eriyttävät opetuksen järjestelyt olivat myös opettajaopiskelijoiden mahdollisia tukikeinoja matematiikka-ahdistuneen oppilaan tukemiseksi. Opiskelijat antaisivat oppilaalle helpompia tehtäviä, enemmän aikaa suorittaa häneltä pyydettyjä asioita sekä pohtisivat vaihtoehtoisia tapoja toteuttaa kokeita.

> Miettisin erilaisia työtapoja ehkä sitte sitä kautta, että pystyiskö sitä matematiikkaa opiskelemaan öö, silleen että sitä tulee vähän niinkun huomaamatta opiskeltuaki. Sitt, jos se liittyy koetilanteeseen niin, tuota, sitt vois miettiä erilaisia tapoja toteuttaa sitä koetta. (H2)

Haastateltavista yksi mainitsi opetettavien aihealueiden karsimisen niin, että oppikirjaa käytettäessä opiskelusta jätettäisiin pois sellaisia aiheita, jotka eivät sisälly opetussuunnitelmaan. Oppilaalle annettaisiin myös mahdollisuus edetä omaan tahtiinsa, jopa niin, että aluksi tavoitteena olisi vain se, että oppilas kykenisi itsenäisesti aloittamaan jonkin tehtävän oppitunnilla.

5.2 Miten opettajaopiskelijat haluaisivat käsitellä matematiikka-ahdistusta opettajankoulutuksessa?

Kaikki haastateltavat olivat sitä mieltä, että matematiikka-ahdistusta ei ole opinnoissa käsitelty tarpeeksi. Tämä muodostikin analyysiin ensimmäisen yläluokan, jonka alle alaluokat Nykytilan riittämättömyyden kokemus, Koulutuksen sisällöt ja Itse hankittu tieto asettautuivat (taulukko 2).

Taulukko 2. Matematiikka-ahdistuksen käsitteleminen opettajankoulutuksessa.

Yläluokka	**Alaluokka 1**	**Alaluokka 2**
Tietoa ei tarjota tarpeeksi (Brady & Bowd, 2005)	Nykytilan riittämättömyyden kokemus	Aiheen merkittävyys Tukemiseen ei ole työkaluja Opiskelija ei osaisi tunnistaa ja tukea
	Koulutuksen sisällöt	Keskittyvät enemmän oppimisvaikeuksiin Mahdollisia vaikeuksia ei käsitellä
	Itse hankittu tieto	Muualta saadun tiedon ja maalaisjärjen soveltaminen Itsenäinen perehtyminen
Koulutuksen kehittäminen (Lutovac & Kaasila 2011; 2019)	Koulutuksen kehittäminen	Käsitteleminen yleisellä tasolla Käsitteleminen teoreettisella tasolla Tukitoimien käsitteleminen

Opettajankoulutus ei haastateltavien mukaan tarjoa tarpeeksi tietoa matematiikka-ahdistuksesta (vrt. Brady & Bowd, 2005). Koulutuksen nykytilan perusteluina opiskelijat kertoivat, että he eivät osaisi nykyisen koulutuksensa pohjalta tunnistaa ja tukea oppilasta, jolla on matematiikka-ahdistusta tai että heillä ei ole riittäviä työkaluja joko kyseiseen aiheeseen tai yleensä matematiikan opettamiseen liittyen.

> Apua. Varmaan siis joku erityisope tai joku, jolla on, on saattanu nähä asiaa ehkä enemmän ja tietäis siitä jottain. Että varsinkaan, kun minä en hirveästi asiasta tiiä, niin tuota niin ehkä sitten jos jottain ulkopuolista joka vois ehkä jotain suunnitelmaa lähetään tekemään, että miten sitä vois sitte lähtee purkamaan. (H1)

Näihin asioihin he kaipasivat koulutuksen tukea. Opiskelijat kertoivat myös, että matematiikka-ahdistusta käsitellään liian vähän suhteessa siihen, kuinka merkittäväksi he sen kokivat. Kritiikkiä saivat myös opettajankoulutuksen sisällöt, jotka opettajaopiskelijoiden mukaan joko keskittyvät enemmän oppimisvaikeuksiin tai eivät käsittele vaikeuksia lainkaan:

> Aika vähän, että enemmän se ehkä keskittyy siihen oppimisvaikeuspuoleen. (H6)

Muualta saatu tieto viittaa tässä opettajaopiskelijoiden näkemykseen siitä, että voidakseen tukea oppilasta, jolla on matematiikka-ahdistusta, heidän täytyy joko hankkia tietoa itsenäisesti tai soveltaa muuta jo opittua tietoa. Opiskelijat kertoivat, että heidän täytyy perehtyä matematiikka-ahdistukseen oman mielenkiintonsa mukaan, koska heidän oma koulutuspolkunsa ei ole tarjonnut mahdollisuutta aiheeseen tutustumiseen. Jotkut opiskelijoista ajattelivat, että heillä on tarpeeksi työkaluja tukea matematiikka-ahdistusta kokevaa oppilasta hyödyntämällä muita opettajankoulutuksessa oppimiaan asioita sekä omaa kokemustaan erilaisista oppijoista.

Haastatteluissa tuli esiin myös opettajankoulutuksen kehittämisideoita matematiikka-ahdistuksen suhteen (vrt. Lutovac & Kaasila, 2011; 2019). Opiskelijoiden esittämät kehittämistoiveet myös vaihtelivat. Joidenkin mielestä matematiikka-ahdistuksen käsittelyssä olisi riittävää, että sitä käsiteltäisiin koulutuksessa yleisellä tasolla, ja opinnoissa olisi esimerkiksi yksi luento, joka käsittelisi mahdollisia vaikeuksia matematiikassa. Kolme haastateltavista kaipasi kuitenkin matematiikka-ahdistuksen käsittelyä teoreettisella tasolla niin, että koulutuksessa tarjottaisiin enemmän tutkimustietoa matematiikka-ahdistuksesta ja sen tukemisesta sekä opettajan roolista vaikeuksien tukemisessa.

> Mun mielestä sitä vois olla enemmän ja ylipäätään yks luento vois käsitellä ett miten, mitkä haittaa ehkä sitä matikan opiskelua [...] ett teoreettinen tieto tuolta alalta ainaki yhen luennon verran vois olla paikallaan. (H7)

Osa haastateltavista esitti konkreettisten tukitoimien käsittelemistä koulutuksessa niin, että opinnoissa paneuduttaisiin tapausesimerkkeihin.

> Ett, semmosia ihan niinkun käytännön työkaluja, ett mitä mää teen, ku mää havaitsen, että jollain oppilaalla on matematiikka-ahistusta. Nii, ni ei mulla oo muuta ku semmosia mitä mä nyt oon niinku tän luokanopettajakoulutuksen aikana oppinu, tämmösiä yleisiä niinku esimerkiks vaikka toi toiminnallinen oppiminen ja eriyttäminen ja näin eespäin. Mutta tota, mut onko sitte matematiikka-ahistukseen, sen tukemiseen niinku olemassa jotain ihan tiettyjä tapoja, nii niistä mä en oo tietonen. (H2)

Toiset opiskelijat taas toivoivat, että opiskelijoita autettaisiin huomaamaan matematiikka-ahdistus oppilaissa ja pyrittäisiin positiivisen matematiikkakuvan rakentamiseen.

6 Pohdinta

Kokonaisuutena opettajaopiskelijat mainitsivat haastatteluissa monia tutkimusten valossa tehokkaita tukikeinoja matematiikka-ahdistukseen. Kuitenkin yksilötasolla tukikeinojen määrässä oli huomattavaa vaihtelua ja osa haastateltavista esitti tukikeinoksi ainoastaan opettajan tarjoamaa emotionaalista tukea jossakin muodossaan. Tämä voinee selittyä osaltaan sillä, että haastateltavilla oli hyvin erilaisia käsityksiä siitä, miten matematiikka-ahdistus määritellään. Tämän vuoksi kaikkien haastateltavien osalta matematiikka-ahdistuksen määritelmän käyminen haastattelun alussa oli perusteltua. Toisaalta määritelmän perusteella luotu käsitys matematiikka-ahdistuksesta ja sen tukemisesta saattaa osaltaan selittää sitä, että tukemisen keinot jäivät nyt varsin yleiselle tasolle.

Matematiikka-ahdistuksen kohtaamisessa emotionaalisen tuen keinoin (Federick & Skaalvik, 2013) opiskelijoiden kuvaukset jäivät varsin yleisiksi. Opiskelijat nostivat esiin positiivisen palautteen ja oppilaan itsetunnon vahvistamisen, mutta haastatteluista ei käynyt ilmi se, että palaute tulisi kohdistaa tavoitteeseen (Al Mutawah, 2015; Jain & Dowson, 2009; Rodrigues, 2012). Opiskelijat tukisivat oppilasta emotionaalisesti kannustamalla ja nostamalla esiin jo hankittua osaamista. Sen sijaan itsesäätelytaitojen kehittämisestä, tunteiden merkityksestä ja metakognitiivisen tietouden lisäämisestä ei tullut mainintoja.

Esitetyt kognitiivisen tuen muodot keskittyivät matemaattisen osaamisen kehittämiseen. Kokonaisuutena tämä yläluokka oli kaikkein kattavin verrattuna teoriassa esitettyihin keinoihin. Haastatteluista jäi uupumaan matematiikka-ahdistusta lieventävinä keinoina kirjallisuudessa esitetyt selkeä ohjeistus (Vukovic ym., 2012), järjestelmällinen eteneminen ja ylioppiminen eli automatisoituneet rutiinit (Cates & Rhymer, 2003). Opetusmenetelmällisten ratkaisujen osalta opettajaopiskelijoiden osaamista tulisi kehittää myös ryhmän sisäisen vertaistutoroinnin (Supekar ym., 2015), virheiden hyväksymisen ja niistä oppimisen (Hannula, 2019), kuin matematiikan tarinallistamisen (Schaeffer ym., 2018) osalta. Lisäksi opettajaopiskelijoiden esittämistä tukitoimista puuttui kokonaan kirjallisuudessa esitetty oppilaan henkilökohtainen tukeminen, kuten esimerkiksi ilmaiseva tai luova kirjoittaminen (esim. Beilock & Willingham, 2014; Furner, 2017;

Klein & Boals, 2001; Ramirez & Beilock, 2011), keskittymisharjoitukset (Brunyé ym., 2013), ja positiivisen ajattelun menetelmä (Brooks, 2014).

Haastateltavat olivat yksimielisiä siitä, että matematiikka-ahdistusta ja oppilaiden mahdollisia vaikeuksia ja niiden tukemista pitäisi käsitellä enemmän opettajaopinnoissa. Tämä vahvistaisi osaltaan itsevarmuutta matematiikan opettamista kohtaan (vrt. Looney ym., 2017). Opettajaopiskelijat toivoivat aiheen käsittelyn olevan konkreettista ja sisältävän tapausesimerkkejä ja esittivät, että koulutuksessa korostettaisiin opettajan asennoitumisen tärkeyttä (vrt. Brady & Bowd, 2005). Opiskelijoiden näkemyksiin ei kuitenkaan sisältynyt opettajaopiskelijoiden oman matematiikka-ahdistuksen käsittelyä, jota esimerkiksi Lutovac ja Kaasila (2019) pitävät keskeisenä opettajankoulutuksen kehittämissuuntana. On tietysti, mahdollista, etteivät tähän haastatteluun osallistuneet opiskelijat olleet kokeneet matematiikka-ahdistusta henkilökohtaisella tasolla, vaikka se tutkimusten mukaan onkin varsin yleistä myös opettajaopiskelijoilla (Lutovac, 2014).

Haastateltavien asenne matematiikka-ahdistuksen tukemista kohtaan vaikutti kuitenkin olevan positiivinen (Dowker ym., 2016; Finlayson, 2014). He uskoivat selviävänsä eteen tulevista haasteista hankkimalla tarpeen mukaan lisää tietoa ja pyytämällä apua moniammatilliselta tukiverkostolta, kuten erityisopettajilta, koulukuraattoreilta ja -psykologeilta, sekä muilta opettajilta ja oppilaiden huoltajilta. Haastateltavien koulutustausta tuli esille erityisesti siinä, miten he näkivät moniammatillisen tuen matematiikka-ahdistuksen kohtaamisessa. Siinä missä erityisopettajaopiskelija kääntyi koulukuraattorin tai psykologin puoleen, luokan- ja aineenopettajaopiskelijalle erityisopettaja oli pääasiallinen kollegiaalisen tuen tarjoaja. Aineiston perusteella matematiikka-ahdistuksen käsittelyssä tulisikin tulevaisuudessa painottaa emotionaalisen ja kognitiivisen sekä opetusmenetelmällisten seikkojen lisäksi oppijan henkilökohtaisen tukemisen muotoja. Tässä myös opettajaopiskelijoiden oman matematiikka-ahdistuksen käsittely ja henkilökohtainen tukeminen nousevat tärkeään rooliin.

7 Johtopäätelmät

Matematiikka-ahdistuksen tunnistaminen ja huomioiminen ovat tärkeä osa opettajan pedagogista osaamista. Opettajaopinnoissa matematiikka-ahdistuksen käsittelyssä olisikin hyvä painottaa matematiikka-ahdistuksen tunnistamisen ja siihen vaikuttamisen keinoja ja tukea niitä konkreetein esimerkein. Oppilaan matematiikka-ahdistusta voidaan helpottaa emotionaalisilla ja kognitiivisilla tukijärjestelyillä sekä opetusmenetelmällisillä ratkaisuilla. Näiden keinojen läpikäyminen toisi opettajaopiskelijoille itsevarmuutta kohdata matematiikka-ahdistusta oppilaissaan. Lisäksi tulisi korostaa matematiikka-ahdistuksen henkilökohtaista aspektia. Opettajaopiskelijoiden omien kokemusten ja mahdollisen matematiikka-ahdistuksen tutkiskelu ja siihen vaikuttaminen esimerkiksi keskittymisharjoitusten, ilmaisevan kirjoittamisen tai vertaisten selviämistarinoiden avulla olisi kannattavaa ei pelkästään matematiikka-ahdistuneiden opiskelijoiden oman ahdistuksen hallinnan, mutta myös pedagogisten valmiuksien kehittymisen kannalta.

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A study of future physics teachers' knowledge for teaching: A case of a decibel sound level scale

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This study examines future secondary physics teachers' knowledge related to the teaching of sound waves, and specifically the topics of sound level and sound intensity. The data is comprised of future teachers' responses to a task in which they had to compose a script for an imaginary dialogue between a teacher and a group of students and to provide a commentary elaborating on their instructional choices. The topics selected for the task were chosen intentionally as they provide authentic and rich opportunities to bridge mathematics and science concepts, while challenging future teachers to consider the logarithmic measurement scale and its role in science. The task provided the participants with the beginning of a dialogue that featured student confusion about the measurement of sound level using a decibel scale. Future physics teachers were asked to extend this dialogue through describing envisioned instructional interactions that could have ensued. The instructional interchange related to the relationship between sound intensity and sound level, and particular teachers' responses to the student ideas related to the meaning of a decibel sound level scale were analysed. These responses were categorized as featuring superficial or deep, and conceptual or procedural knowledge for teaching. We describe each category using illustrative excerpts from the participants' scripts. We conclude with highlighting the affordances of scriptwriting for teachers, teacher educators, and researchers.

Keywords: knowledge for teaching, lesson play, logarithms, scriptwriting, sound level, physics teacher education

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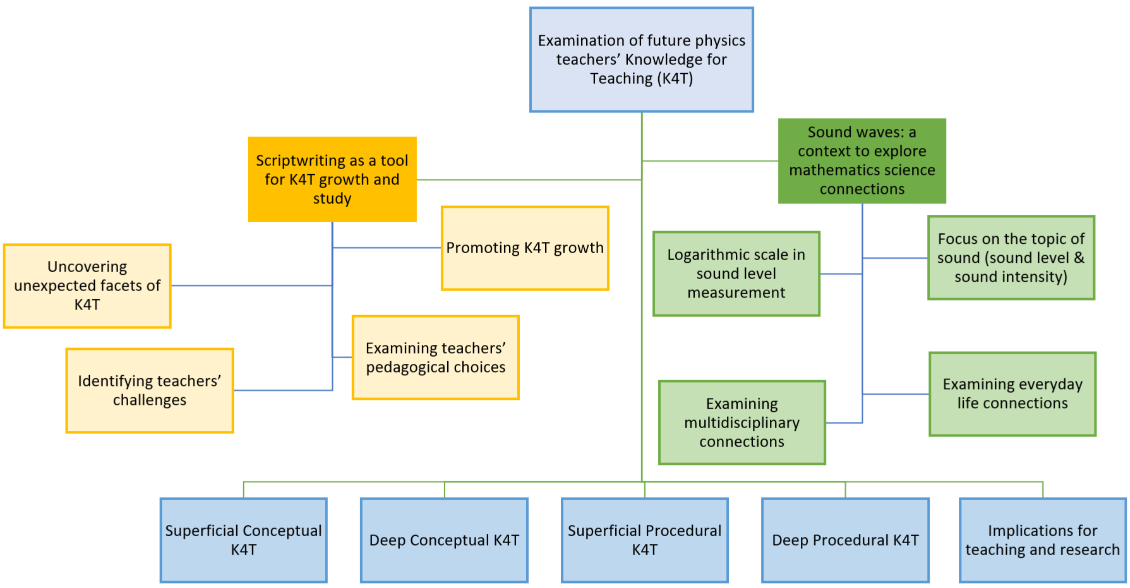


1 Introduction

Exploring ways to describe and strengthen teachers' knowledge has long been of interest to science educators and researchers. Ample attention has been paid to the concepts that pose difficulties for learners in order to propose pedagogical approaches to address them, such as the concepts of electric current, forces, etc. (McDermott, 2001; McDermott et al., 2006). Within this general avenue of science education research, we focus on the topic of sound, specifically, sound level and sound intensity.

We engaged a group of future physics teachers in extending an imaginary dialogue between a teacher and several students, in which a student exhibited confusion about the measurement of sound level and the meaning of the differences in measures expressed in decibels. Our analysis of the scripted dialogues examined the teachers' knowledge for teaching utilized in the envisioned instructional interactions.

2 Background

In this section, we first describe different aspects of teachers' knowledge, focusing on the knowledge needed for teaching. Then, we introduce scripting tasks, a particular kind of task related to approximation of practice, used both to reveal and strengthen teachers' knowledge. We proceed with a brief overview of the concepts related to sound level and sound intensity and conclude with describing prior education research on sound and logarithms.

2.1 Knowledge for Teaching (K4T)

Teachers' knowledge and practice have received substantial attention in education research. Following Shulman's (1986, 1987) classical studies, and his distinction between Content Knowledge (CK or Subject Matter Knowledge SMK) and Pedagogical Content Knowledge (PCK), researchers proposed various elaborations on the components of knowledge needed for teaching. Contemporary frameworks focusing on knowledge for teaching (K4T) have expounded on a variety of additional facets. These include the familiarity with the subject and its overall structure, understanding how the subject matter relates to other subjects and everyday life, awareness of potential student difficulties and willingness to use modern technologies to support student learning (Milner-Bolotin, 2019, 2020). Moreover, descriptions of K4T often highlight topics related to the current state of affairs in the field of education, such as contemporary advances in educational technologies, ethical challenges, student engagement, and unsolved problems. Researchers appear to agree that subject-specific K4T is paramount for effective teaching and this pertains to teaching all subjects, including elementary or secondary mathematics (e.g., Zazkis & Zazkis, 2011) and science (Campbell et al., 2014; Depaepe et al., 2013).

Our research stems from a position that deep and comprehensive understanding of subject matter is a cornerstone of effective pedagogy (Biggs, 1987). Yet, it is well known that an undergraduate science degree hardly guarantees the depth and breadth of knowledge necessary for successful secondary school teaching. For example, researchers have measured the gain in conceptual knowledge acquired by students in introductory science courses. These studies documented that a significant number of undergraduates fail to transcend factual memorization (Hake, 1998).

The realization that subject-matter knowledge is essential for successful teaching and not all teachers might possess it is not new (Shulman, 1986). It has been a key concern of previous education reforms, such as the science and mathematics education reforms of the 1960's, 1980's, and 2000's (Center for Education Reform, 2018). Many notable scientists and mathematicians have collaborated with educators to produce textbooks and resources for elementary, secondary, and even post-secondary teachers (Arons, 1997; Feynman, 1994; Klein, 2004).

Advancing teachers' knowledge is the main goal of teacher education and professional development. Therefore, we should move beyond the general claims of labelling teachers' K4T as "lacking" and to understand the specific details pertaining to particular topics relevant to teaching. This leads to a question: How is it possible to gain access to K4T of a group of future physics teachers in order to design and adjust subsequent instruction? One possibility is to engage teachers in writing a script for a lesson, a method that we elaborate next.

2.2 Lesson play and scripting approaches

Scriptwriting, a valuable pedagogical strategy and an innovative research tool, was originally developed in the context of mathematics teacher education (Zazkis et al., 2013). While scriptwriting was novel in mathematics education research, its roots trace to the Socratic dialog and to the style of Lakatos' (1976) evocative 'Proofs and Refutations' in which a fictional interaction between a teacher and students interrogates mathematical claims.

Initially, scriptwriting was introduced in mathematics teacher education as a *lesson play*, where participants script interactions between an imaginary teacher-character and student-character(s) (Zazkis et al., 2009). Juxtaposed with a classical lesson plan describing merely content and activities, the lesson play aims at revealing how teaching-learning interactions might unfold. In later research, the lesson play approach was extended to an activity of writing an imaginary dialogue that is not necessarily restricted to a lesson, referred to as scriptwriting. In teacher education, scriptwriting opens doors for "approximations of practice", which "include opportunities to rehearse and enact discrete components of complex practice in settings of reduced complexity" (Grossman et al., 2009, p. 283), thus becoming especially valuable for teacher preparation.

Scriptwriting is both an instructional tool and a research data collection tool. It has been implemented in recent research (Kontorovich & Zazkis, 2016; Zazkis & Kontorovich, 2016; Zazkis & Marmur, 2018; Zazkis et al., 2013; Zazkis & Zazkis, 2011), where participants had to identify problematic issues in the presented topics, and subsequently clarify these by designing a scripted dialog. The affordances of scriptwriting for future teachers, teacher educators, and researchers were detailed in the aforementioned studies.

For teachers, writing a script is an opportunity to examine a personal response to a situation, explore erroneous or incomplete approaches of students, revisit and possibly enhance personal understandings of the concepts involved, and enrich the repertoire of potential responses to be used in future teaching. Challenging future teachers to think about the science content from this teaching perspective could encourage them to consult more advanced science resources. Moreover, while the teachers' ability to anticipate and address potential student difficulties depends on the teachers' experience, novice teachers can learn from other educators' experiences. This could motivate future teachers to consult science education literature. Thus, the scriptwriting activity helps uncover and draw on students' prior knowledge, while imagining a lesson as an interactive "living process", as opposed to following a previously designed rigid lesson plan.

For researchers, the scripts result in a rich data source that can be scrutinized from various perspectives, providing a lens for examining images of teaching and insights into the scriptwriter's understanding of the subject matter (Zazkis & Kontorovich, 2016). Scripts composed by teachers may reveal not only their K4T relevant to a particular concept, but also their own difficulties and misunderstandings.

For teacher educators, the scripts provide insights into planned pedagogical approaches that can be consequently highlighted and discussed in working with teachers. Furthermore, scripts composed by future teachers provide teacher educators with a view on the relevant knowledge of their students, which can subsequently be incorporated into instructional activities aimed at extending and strengthening this knowledge (Zazkis & Marmur, 2018).

The chosen science content for our investigation is sound and sound level, which is ultimately related to a logarithmic scale. A brief overview of this topic is provided in the next section.

2.3 A brief physics overview: Sound level and a logarithmic scale

Sound is a mechanical wave (Hawkes et al., 2018). In a fluid, sound consists of compressional longitudinal waves (rarefactions and compressions), while in a solid, sound propagates either as a longitudinal or a transverse wave. Sound waves transfer energy through the vibrations of the medium generated by the sound source such as an oscillating object.

Humans can perceive an extremely wide range of frequencies. The threshold of human hearing for a normal adult is about 20 Hz, while the upper limit is 20 kHz. This is three orders of magnitude difference! The range of frequencies perceivable by different species is shown in Table 1 (Ahlborn, 2004).

Table 1. Ranges of frequencies perceivable by different species

Species	Lower frequency of sound (Hz)	Upper frequency of sound (Hz)
Humans	20.00	20,000
Dogs	50.00	45,000
Cats	45.00	85,000
Bats	20.00	120,000
Dolphins	0.25	200,000
Elephants	5.00	10,000
Birds	1,000.00	4,000

The physical properties of the sound waves influence the physiological features of sound: its loudness, pitch, and timbre. The energy transmitted by a sound wave (E) per unit of time (t) is called the power of the sound wave (P):

$$P = \frac{E}{t} \Rightarrow [P] = \frac{\text{joules}}{\text{second}} = \text{watts (W)}$$

Since sound waves spread isotropically, one can calculate how much energy is transmitted per unit area *S*, located a distance *r* from the source. This quantity is called the sound intensity (*I*) and can be described as the power transmitted by the wave per unit area:

$$I = \frac{E}{t \times S} = \frac{P}{S} = \frac{P}{4\pi r^2}$$

The sound intensity is measured in watts per square meter: $[I] = \frac{\mathrm{W}}{\mathrm{m}^2}$.

So far, we have focused on the physical description of sound waves. In addition, we have to consider the range of sound intensities perceptible by humans, as well as how our ears perceive sound. Examining sound perception helps justify why the concepts of logarithms and a logarithmic scale are used to describe sound intensity.

There is almost 16 orders of magnitude difference between the softest sound an average human can hear ($I_0 = 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}$) and the loudest sound that will completely destroy our hearing by bursting our eardrums ($I_{\max} = 10^4\ \frac{\mathrm{W}}{\mathrm{m}^2}$) (Table 2). To describe a physical quantity with such a vast range of values, it is common to utilize a logarithmic scale. This scale is especially convenient for achieving a manageable range of numbers. Thus, the sound level, β, was originally defined as the logarithm of the ratio of the given sound intensity and the softest sound perceivable by humans, I_0:

$$\beta\ (\text{in B}) = log\frac{I}{I_0},\ \ where\ \ I_0 = 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}$$

Sound level is measured in bel (1 B), in honour of Alexander Graham Bell (1847-1922), a Scottish-born inventor, scientist, and engineer who patented the first telephone, the phonograph (gramophone), and a few other devices. However, this unit is too large (imprecise) to be useful, thus it was suggested that one tenth of the unit, the decibel or dB where 1 B = 10 dB be used. Today, the sound level is most often measured in dB:

$$\beta\ (\text{in dB}) = 10\, log\frac{I}{I_0},\quad \left(I_0 = 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}\right) \qquad (1)$$

There is another reason why the dB sound level scale is useful. Our ears respond logarithmically to changes in intensity (Gray, 2000) and this is what a logarithmic scale is constructed to display. How our ears perceive the change in loudness depends on the ratio between the original and current sound intensities. Let us illustrate what this means with the following example.

Let us assume that the sound level increases from 10 dB to 20 dB. What is the perceived increase in the loudness of sound and what is the increase in sound intensity? We have:

$$\beta_1 = 10 \text{ dB}; \beta_2 = 20 \text{ dB, therefore } \Delta\beta = 10 \text{ dB}$$

We need to find the ratios between the initial and final sound intensities, that is, $\frac{I_2}{I_1}$. We will show that the change of sound level of 10 dB means that the intensity of the sound, I, increased tenfold.

$$\beta_1 = 10 \text{ dB} \Rightarrow 10\, log\frac{I_1}{I_0} = 10 \Rightarrow log\frac{I_1}{I_0} = 1 \Rightarrow \frac{I_1}{I_0} = 10$$

$$I_1 = 10 I_0 \Rightarrow I_1 = 10^{-11}\,\frac{\text{W}}{\text{m}^2}$$

$$\beta_2 = 20 \text{ dB} \Rightarrow 10\, log\frac{I_2}{I_0} = 20 \Rightarrow log\frac{I_2}{I_0} = 2 \Rightarrow \frac{I_2}{I_0} = 100$$

$$I_2 = 100 I_0 \Rightarrow I_2 = 10^{-10}\,\frac{\text{W}}{\text{m}^2} = 10 I_{\text{A1}} \qquad (2)$$

Thus, a sound level change from 10 to 20 dB means that the final intensity is tenfold the original one (Eq. 2). Similarly, a change from 20 to 30 dB, or any other 10 dB increase, will result in a tenfold increase of the original intensity. This derivation follows from the sound level definition (Eq. 1). It relies on a direct application of the rules of operations of logarithms. While a student might think that a 20-dB sound is twice more intense than a 10-dB one, this is not the case. The power delivered to the ear in the second case is 10 times the original power (Eq. 2).

The difference between the perception of sound and the actual amount of energy reaching our ears is the key to understanding why the logarithmic scale is used in describing the sound intensity. This difference follows from our physiology. Our ears function according to a logarithmic and not to a linear scale. This is the topic of psychoacoustics - the study of how humans perceive sound.

To develop intuition regarding the sound level measurement, one can consider how we perceive different sounds and how much power is delivered in each case (Table 2).

Table 2. Sound intensity levels perceptible by humans and corresponding phenomena (The Physics Hypertextbook https://physics.info/intensity) and relevant physical phenomena

Sound intensity level β (dB)	Intensity, *I*, (W/m^2)	Example/effect
$-\infty$	0	Absolute silence
−24	$1 \times 10^{-14.4}$	sounds quieter than this are not possible due to the random motion of air molecules at room temperature (ΔP = 1.27 μPa)
−20.6	$1 \times 10^{-14.06}$	Current world's quietest room (Microsoft Building, Redmond, WA, USA)
−9.4	$1 \times 10^{-12.94}$	former world's quietest room (Orfield Lab, Minneapolis, MN, USA)
0	1×10^{-12}	Threshold of hearing at 1000 Hz, reference value for sound pressure of 20 mPa
10	1×10^{-11}	Normal breathing, rustling of leaves
20	1×10^{-10}	Whisper at 1 m distance
30	1×10^{-9}	Quiet home
40	1×10^{-8}	Average home
50	1×10^{-7}	Average office, soft music, quiet residential area
55	$1 \times 10^{-7.5}$	Dishwasher, electric shaver, electric toothbrush, large office, rainfall
60	1×10^{-6}	Normal conversation, quiet TV
70	1×10^{-5}	Noisy office, busy traffic, air conditioner, automobile interior, alarm clock, background music, loud television, vacuum cleaner, washing machine, hair dryer, flush toilet
80	1×10^{-4}	Loud radio, coffee grinder, noisy restaurant, ringing telephone, whistling kettle, blender, doorbell, food processor
86	$1 \times 10^{-3.4}$	Inside a small single engine plane, such as Cessna, or twin engine such as Piper Seminole
90	1×10^{-3}	Inside a heavy truck or a tractor, very heavy traffic, hand saw, lawn mower, machine tools. Sound generated by Niagara Falls Prolonged exposure is dangerous
100	1×10^{-2}	Noisy factory, siren at 30 m, electric drill, shouted conversation, tractor, truck. Exposure of 8+ hours a day is dangerous
110	1×10^{-1}	Shouting or barking in the ear, boom box, factory machinery, motorcycle, school dance, snow blower, snowmobile, squeaky toy held close to the ear, subway train, and woodworking class. Serious damage from 30 min per day exposure.
113	1×10^{-1}	Loudest clap (Alastair Galpin, New Zeeland, 2008)
120	1	Loud rock concert, pneumatic chipper at 2 m, a clap of thunder, threshold of pain
130	1×10^{1}	Baby cry, peak stadium crowd noise
140	1×10^{2}	Jet airplane at 30 m. Severe pain, damage in seconds
150	1×10^{3}	Jet aircraft at a few meters, during take-off; explosive blast. Severe pain, instantaneous damage
160	1×10^{4}	Bursting of eardrums

It takes effort to gain an intuitive understanding of how sound intensities can be described using a logarithmic scale. The lack of exposure, as well as an inherent complexity associated with logarithms, contribute to many difficulties that students experience when learning these concepts. We attend to research that explored some of these difficulties in the next section.

2.4 Brief overview of prior research on sound and logarithms

Research has shown that the concepts of logarithms and a logarithmic scale pose substantial challenges for students. Even after learning the procedures of operating with logarithms, student conceptual understanding is often lacking (Berezovski & Zazkis, 2006; Berger et al., 1987; Liang & Wood, 2005; Weber, 2016).

For example, Liang and Wood (2005) analysed secondary students' misconceptions related to logarithms. They classified these misconceptions into three categories: (1) knowledge or computational errors, (2) understanding errors, and (3) application errors. The first category comprised routine questions that required direct recall or application of the definition and laws of logarithms, or simple manipulations requiring a minimal number of steps. Not surprisingly, 86% of the students were able to answer these questions correctly. Yet, when the questions became more complex, where students had to decide how to apply their understanding to a slightly unfamiliar situation, only 66% of them were able to provide a correct answer. Finally, only 39% of the students were able to solve questions that required applying their understanding to a novel situation.

Several studies focused on students' misconceptions related to the basic properties of mechanical waves, such as sound (Periago et al., 2009). For example, many students found it difficult to visualize how waves can transfer energy without transferring matter. In the case of longitudinal waves such as sound, this becomes even more complicated. The properties of waves and their interrelationships are also often confused, such as loudness of sound and its pitch. Yet, the investigation of sound level provides an additional complication where the human perception of sound and the immense range of sound intensities provide an obstacle for the students who are used to dealing with linear scales and with small ranges of quantities. The mathematical description of sound level using a logarithmic scale creates an additional obstacle.

3 Theoretical framing

The key premise of this study is that effective teaching is based on the teachers' deep and extensive content understanding, at the level exceeding the one required from students. Paraphrasing a famous mathematician and educator, Felix Klein (2004), one might say that teaching requires educators to acquire an advanced standpoint on the subject. Thus, in order for teachers to become effective, they have to understand the concepts they ought to teach in great depth. But what does it mean "to understand in-depth"? Science and mathematics educators have developed different conceptualizations of this notion.

Richard Skemp's original conceptualization of understanding in mathematics distinguished between *instrumental* and *relational understanding* (Skemp, 1976). *Instrumental understanding* refers to the ability to perform procedures, without necessarily being aware of the reasons behind them. This kind of understanding relies heavily on rote memorization and is driven by the question of *how* and not *why*. A classic example of instrumental understanding, and how deeply it is ingrained in the teaching of mathematics, is a well-known phrase used to "help" students learn to divide fractions: "You don't need to know why, just invert and multiply" (Ma, 1999; Milner-Bolotin, 2018a, 2018b). Note that the emphasis here is not on the *why*, but on the *how*.

Instrumental understanding is not unique to mathematics. For example, in science students recognize patterns and match problems with equations, often without gaining a deeper conceptual understanding of the process or even recognizing various ways of representing the same concepts. In science, instrumental understanding is often expressed by a student ability to solve problems when recognizing patterns and matching givens with formulas, while stumbling with conceptual problems requiring the making of predictions by applying science laws to everyday life (Hake, 1998).

Skemp also described *relational understanding,* which focuses on the reasons and justifications on connections between different concepts and on applications. In science education, it is often referred to as *conceptual understanding* (Milner-Bolotin, 2014). This is in accordance with the distinctions made by Hiebert (1986) between *procedural* and *conceptual knowledge*. *Conceptual knowledge* has been defined as understanding of the principles and relationships that underlie a domain (Hiebert & Lefevre, 1986) or *knowing why,* while the core of *procedural knowledge* is in knowing algorithms and rules for completing tasks and procedures.

Similar descriptions were provided by other researchers. For example, Rittle-Johnson and Alibali's (1999) empirical study about conceptual and procedural knowledge in mathematics defined them as follows:

> We define conceptual knowledge as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. We define procedural knowledge as action sequences for solving problems. (p. 175)

Educators argued for the facilitation of students' conceptual development, noting that a large part of mathematics instruction focusses on procedures and skill development (Hiebert & Grouws, 2007). However, distinguishing "knowledge of procedures" and "knowledge of concepts and relationships" did not provide information about the depths or quality of knowledge. While the conceptual–procedural distinction was adopted by many researchers, Star (2005, 2007) criticized the implicit preference of educators toward conceptual knowledge, arguing that many scholars who are interested in mathematics teaching and learning, tend to conceive of conceptual and procedural knowledge as types of knowledge rather than in terms of qualities of knowledge. Star (2005) argued:

> The term conceptual knowledge has come to encompass not only what is known (knowledge of concepts) but also one way that concepts can be known (e.g., deeply and with rich connections). Similarly, the term procedural knowledge indicates not only what is known (knowledge of procedures) but also one way that procedures (algorithms) can be known (e.g., superficially and without rich connections). (p. 408)

In order to disentangle knowledge type and knowledge quality, Star (2005) introduced a further refinement by making a distinction between *deep procedural knowledge* and *superficial conceptual knowledge*. According to Star, deep procedural knowledge is rich in relationships and may entail the flexible application of procedures. Deep procedural knowledge also entails the ability to choose an appropriate and efficient procedure that justifies its use, while being linked with understanding and critical judgement. Superficial conceptual knowledge, as the term implies, refers to surface level knowledge and the reproduction of known facts. The investigation of this refined distinction is our main focus.

In this study, we are interested in the quality of a teachers' knowledge related to sound level and sound intensity. Our access to teachers' knowledge was through the imagined instructional interactions that they described (see the description of the task in Section 4). We used thematic content analysis (Creswell, 2008) of these imagined instructional interactions to uncover and categorize the participants' knowledge for teaching (K4T). In particular, we focused on the idea that sound level is related to logarithms and the logarithmic scale. Acknowledging this relationship demonstrates the connection between concepts in mathematics and science, and can be considered as conceptual knowledge. But simply declaring that there is a connection, points to a *superficial conceptual K4T*. *Deep conceptual K4T* can be evident, for example, when using multiple representations, drawing connections with daily experiences, and explaining (rather than citing) underlying relationships.

Guiding students into substituting numbers into formulas correctly, or into memorizing the rules of operations with logarithms, is an indication of *procedural K4T*. However, *deep procedural K4T* may be exhibited through the teacher's ability when explaining resulting conclusions, such as "every increase in 3 dB doubles the intensity", or "every increase in 10 dB multiplies the intensity by 10". The teacher's *deep procedural K4T* may also be exhibited when guiding students to extrapolate from these conclusions when considering other changes in sound level. Our operational definitions of the categories as related to K4T are summarized in Table 3.

Table 3. Description of four categories of teachers' K4T that guided this study

Category of teacher's K4T	Description of the category
Superficial conceptual	Declaring connections between concepts without elaboration
Deep conceptual	Explaining underlying relationships between concepts
Superficial procedural	Focusing on the use of procedures without elaborating on the underlying principles
Deep procedural	Connecting procedures to underlying principles

In light of the discussion above, this study aims to address the following research question: What do participants' scripts reveal about the scriptwriters' K4T for teaching the topic of sound, in particular the concepts of sound intensity and sound level?

4 Methodology

4.1 Participants and context

Twelve future physics teachers participated in the study. They all had an extensive physics background at the tertiary level. Eight of the ten participants had a B.Sc. in Physics, two had an M.Sc. in Physics, one had an engineering degree, and one had a Ph.D. in Astronomy. At the time of the study, the participants were enrolled in a methods course for secondary physics teachers in their teacher education program. This required course was delivered in 39 hours over 13 consecutive weeks. It took place in a physics laboratory, and therefore included both theoretical and hands-on activities related to secondary physics teaching.

4.2 Scripting Task

During the course, the participants completed the task, Exploration of Sound, which included composing a script for a play that was referred to as a lesson play (Figure 1). As a preparation for this task, the concept of a lesson play (Section 2.2) was presented and discussed in class. The participants were asked to review the science related to sound waves relevant to the secondary physics curriculum in advance. During class, they discussed the task in small groups. This was followed by a whole class discussion of the underlying concepts, pedagogical approaches, and potential student difficulties. The future physics teachers were then asked to complete the task individually over a two-week period.

The task presents the beginning of a dialogue that features student confusion related to sound measurement. In addition to composing a lesson play, in which the teacher and student characters continue the dialogue and discuss sound levels, the participants were asked to address the following questions:

1. How do you understand the concept of sound level? The way you understand the idea yourself could be different from the way you explain it to students. If this is the case, please indicate how you could clarify the issue to yourself, or to another physics teacher.
2. What are key physics concepts students need to acquire to understand the concept of sound level and how it is measured?
3. What are potential student difficulties, misconceptions or alternative conceptions?

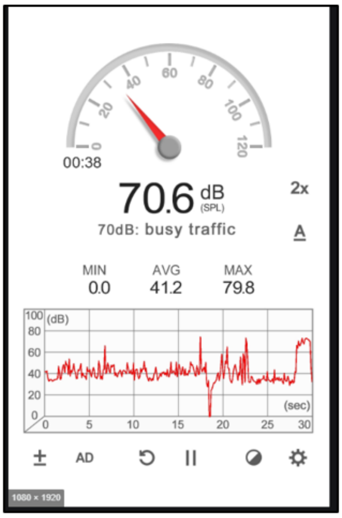
4. How might a teacher help students understand these concepts? What pedagogical approaches would you recommend and why?
5. What resources did you use to write your Lesson Play that helped you figure it out? (Research papers, pedagogical forums, websites, etc.)

Physics 11: Exploration of sound and its properties

https://curriculum.gov.bc.ca/curriculum/science/11/physics

https://www.healthlinkbc.ca/health-topics/tf4173

Play Setting: *The play is set in a Physics 11 classroom. The students have just finished the intro unit on waves and already had two introductory lessons on sound. They discussed the properties and behaviour of waves, sound characteristics, the phenomenon of resonance, how waves are generated and how they propagate. When the teacher entered the classroom the following week, she noticed a group of students arguing excitedly in the corner about the sound level at the concert they just attended over the weekend:*



Student 1: I had a huge headache after the last Saturday's concert. I had my smartphone app and it measured the sound level there to be 91 dB. This was hurting my ears and my head for the entire Sunday. I still feel it.

Student 2: I have read that sound levels that are above 85 dB are harmful to humans, so this is understandable, but that harmful? 91 dB is less than 10% higher than the threshold. So 91 dB should not be such a big deal.

Student 3: It doesn't make sense! I am very confused with this dB thing. What is it? I have a Sound level app on my smartphone. But what do these measurements mean? This is so confusing.

Teacher: This is an interesting conversation. This is a great opportunity to discuss the concept of sound level and what it means…

Roles:

Teacher: A very thoughtful and inspiring new physics teacher

Student 1: ______________________________

Student 2: ______________________________

Student 3: ______________________________

Additional characters: ______________________________

Figure 1. A scripting task on the topic of sound implemented in the study

4.3 Data analysis procedures

The scripts composed by the participants, along with their responses to the five questions, comprised the data for our study. We have read and analysed all the scripts independently, identified major themes, checked for the accuracy of the explanations and the examples, and considered not only what was present in the scripts, but also what the scriptwriters chose to omit. We then compared our analyses and observations and discussed our interpretations of the data.

5 Results and Analysis

In this section, we describe four examples illustrating different categories of K4T in the context of teaching the topic of sound. While we realize that these categories are not disjoint representations of teachers' K4T and can be better represented by a spectrum, we use representative examples from the participants' lesson plays as illustrations. We then discuss how this pedagogical tool can be used in science teacher education to support future teachers in developing their K4T. All the future teachers' names below are pseudonyms.

5.1 Case 1: Superficial Conceptual K4T

5.1.1 Jamie's lesson play

The teacher-character in Jamie's script began the discussion of sound and its properties by connecting sound phenomena to students' prior experiences. While this was a natural start, most of the interactions during the lesson were factual statements either from the teacher or from the students. It appeared as if Jamie tried to include everything that he could remember about sound and sound waves. Jamie's lesson play included the discussion of many sound properties, such as its sources, frequency, pitch, and loudness. Additionally, the use of PhET simulations, while relevant to the general exploration of sound waves or their interference, was irrelevant to the lesson (Wieman et al., 2010). In Jamie's script, the discussion initiated by the teacher left no sufficient time to focus on the physics and mathematics concepts. The following excerpt illustrates this:

Denny: Oh yeah, what is a dB?

Teacher: Well a dB, or a decibel, is the intensity level you hear the sound at. Notice the Deci prefix in the word decibel is Latin for tenth. The intensity level is relative to something we call the Threshold of Hearing, which is 0 dB. And its logarithmic after that. The difference between 10 dB and 20 dB is 10 times greater.

Sanders: Okay, what? So, decibels are on a logarithmic scale?

Wendy: So, if I heard something at 30 dB, does that mean it is a thousand times greater than 1 dB? Or if a sound was 70 dB, and I heard something at 90 dB, it is 100 times louder?

Teacher: Correct. [...]

Wendy: What I thought that was a small 10% difference in threshold is a lot. As the scale gets higher, the magnitude is much greater, it's not a linear relationship with the threshold of hearing.

Teacher: So now you know! We must treasure our hearing. Let's set this on a scale now. Using Health-BC, sounds that are harmful are above 85 dB, and the concert at 91 dB is like a subway, or a shouting conversation. That is loud! Especially over a long period of time, you are putting your ears against very different frequencies, to overly large amplitudes...

5.1.2 Analysis of Jamie's lesson play

This excerpt illustrates why we categorized this script as an example of Superficial Conceptual K4T, as it is rife with lost opportunities for supporting student conceptual understanding. Jamie's script also shows how teacher's Superficial Conceptual K4T could reinforce students' misunderstanding and confusion. For instance, there was no explanation of why the 20-decibel difference meant a hundred-fold increase in sound intensity. It is unclear why a logarithmic scale was chosen to describe the sound level. On one hand, a lot of complex information was given to the students without any explanation, on the other hand, the teacher discussed the meaning of the prefix *Deci* used in the name of the sound level unit (*deci*bel). This was a striking example of a "pedagogical shield" (Koichu & Zazkis, 2013; Marmur & Zazkis, 2018; Zazkis & Leikin, 2008), that is, certain pedagogical choices made by teachers to protect them from exposing their inability to attend to the core of the task.

In the reflection, Jamie acknowledged having a rather shallow understanding of the content, yet he did not try to delve deeper into it. The topics of logarithms and the logarithmic scale were hardly mentioned in the script and were not elaborated upon. This is how Jamie described it: "*I was confused about the approach I would take... I really wasn't sure what sound level meant.*"

5.2 Case 2: Deep Conceptual K4T

5.2.1 Alex's lesson play

We illustrate Deep Conceptual K4T with the case of Alex, who approached the discussion of sound level and sound intensity without requiring students to delve into any formal calculations. Alex split the script into three scenes. **Scene 1** introduced students to the concept of sound level by combining their perception of sound loudness and the measurement of the sound level. This part had an experiential focus and culminated with identifying the discrepancies between students' prior knowledge and the results of their experiences of measuring sound levels with their smartphones. In **Scene 2**, a more formal definition of sound intensity was introduced and the students were given a version of Table 2. In **Scene 3,** the students returned to the initial problem to reconcile their original measurement of a 3-decibel increase in sound level when doubling the number of sound sources.

Alex realized that while a notion of loudness of sound was familiar to the students from everyday life, the concepts of sound level and intensity were not. Moreover, the students did not possess the necessary mathematical background of logarithms to approach this subject computationally. Consequently, the teacher-character in Alex' script supported students in acquiring conceptual understanding without reverting to calculations. To do that, she used a smartphone application that could measure the ambient sound level, so the students experienced the differences in sound levels first hand. This was a deliberate pedagogical choice related to technology use, as students today have access to smartphones, but few use this powerful tool for work in mathematics or science (Maciel, 2015; Milner-Bolotin, 2016).

Alex intended to help her students understand what the concepts of sound intensity and sound level represented through the physical experiences of sound and its measurement. To achieve this, the teacher-character in Alex's lesson play spread the students around the classroom and asked them to measure the sound levels produced by one and then by two identical xylophones:

> Teacher: That's interesting. So, if that's true, that would mean that the decibel scale doesn't measure how loud the original sound was, it measures how loud it is when it gets to you. Now, the other interesting thing is how the number changed. I want to do another demo to make that clearer. Everyone measure and write down how loud this one xylophone is in dB. [The teacher plays one key of one xylophone.] Now, if I played two xylophones and hit just as hard, how loud do you expect it to be compared to just one xylophone?

While the teacher in this script never discussed the logarithmic scale, she challenged the students to think about what would happen if instead of a single sound source (e.g., a xylophone), there were two identical sources. This easy to perform experiment has a significant pedagogical value, and reveals Alex's Deep Conceptual K4T, as illustrated by the following dialogue:

Jiyun: Well, it should be twice as loud because there's two.

Teacher: Great, our first suggestion is twice as loud. Any other suggestions? [The class shakes their heads.] Okay, let's measure it. Don't say your number right away. [The Teacher plays both xylophones at once.] Okay everyone, write down what your phone measured. Now, I want to ask everyone, did the xylophones sound twice as loud as one xylophone? Did you feel like your ears were dealing with twice as much sound?

Julia: I think so.

Austin: Not really! It only sounded a little louder than one xylophone. Like, I could tell there were two, but it didn't hurt my ears or anything.

Teacher: Okay, we have two different opinions! Let's see which one matches how the decibel scale measures. Everyone, please summarize your two measurements.

Julia: I got 75 and 78.

Austin: I got 65 and 69.

Jiyun: I got 70 and 72.

This dialogue illustrates how comparing the measurements of the sound levels of one versus two identical sources can help students develop a conceptual understanding of the key property of logarithms, $\log(ab)=\log(a)+\log(b)$, and the correspondence between linear and logarithmic scales. The teacher-led the students through having an experience of the logarithmic scale, used to measure sound level, without actually realizing that they were using such a mathematical construct. Having two sound sources instead of one does not double the dB value, but adds 3 dB to the original one. In this script, the students experienced how the measurement of the sound level created by two xylophones added about 3 dB to the original sound level. In the follow-up lesson, the teacher introduced the table demonstrating the sound level and sound intensity connection (Table 2), and asked the students to discuss the patterns they might have noticed, thereby connecting what the students observed with their earlier experiences. Thus, the lesson came to a full circle.

5.2.2 Analysis of Alex's lesson play

Alex's choices in the lesson play pointed to her Deep Conceptual K4T. Alex identified key physical, physiological, and mathematical concepts involved in teaching this topic. Through considering potential student difficulties, by evoking powerful examples, and asking questions that encouraged students to confront their own understandings, Alex built on students' prior knowledge yet expanded it while bridging multiple representations of the phenomenon. Her thoughtful approach was based on juxtaposing intuitive student understanding of sound level with its direct measurement using a smartphone application. While Alex avoided introducing a rigorous mathematical description and computational approaches in the script, she helped students build some intuitive conceptual understanding. The deliberate and pedagogically rich choices made in this script were informed by Alex's own understanding of the concept and her awareness of student conceptual difficulties. As Alex wrote in her reflection:

> Talking about sound is difficult because there are a lot of concepts. When you decide to shift the focus of your lesson on the spot you may not have a lesson properly scaffolded for the concept. I am okay at checking for understanding, but my imaginary teacher was not always checking whether people were silent because they did or did not understand.

Alex recalled from her undergraduate physics courses that describing sound intensity required some "advanced mathematics". Yet, Alex decided to help students build conceptual, intuitive understanding, which in turn required the teacher to have a deep understanding of the subject matter. This is how Alex described it:

> I hadn't studied sound of decibels for a long time, and all I remembered was the core math. I had to do a lot of research about air pressure, sound waves, intensity and sound level, and even though air pressure wasn't explained in depth in the lesson play, having extra understanding was really important for knowing how to best help the students. You need to understand a subject much deeper if you want to teach the basics.

This lesson play demonstrates Alex's Deep Conceptual K4T. While she refrained from using procedural knowledge when calculating sound level, her approach could be considered as an important first step in introducing students to the concepts of logarithms and the logarithmic scale.

5.3. Case 3: Superficial Procedural K4T

5.3.1 Valery's lesson play

Valery's lesson play exemplifies Superficial Procedural K4T. Valery had a solid understanding of the relevant concepts and was aware of related student difficulties. In the reflection, he reminisced about an undergraduate physics course where these topics were discussed in detail, as they were still fresh in his memory. For example, Valery clearly distinguished the concepts of loudness and sound intensity. In the words of his teacher-character, Valery emphasized that human perception of sound and the vast range of sound intensities were the reason for choosing a logarithmic scale for describing this phenomenon. He also stressed the difference between the linear and logarithmic scales and how they are used to describe sound properties:

> Teacher: ... I know that all of you are very comfortable with linear scales. If I double a certain quantity, then the number with its associated unit is doubled as well. This does not work when talking about decibels. If I speak at 50 dB, then double the sound intensity, I do not speak at 100 dB. I will now give you a short introduction to logarithms, and in pre-calculus 12 next year, you will go more in depth learning about them.
>
> Teacher writes the following equation on the board: $\beta = 10\log\frac{I}{I_0}$.
>
> This is equation we use when we talk about sound intensity measured in decibels. The *log(x)* says to take the logarithm of the quantity in the parentheses, using base 10. This can be done by using the log function on your calculator. As an example, I want you all to calculate what the logarithm of 10 is in base 10.

At this moment in the script, the teacher-character realized that the students might not have studied logarithms in their mathematics classes, so the focus was turned to the procedure for calculating logarithms using calculators. The teacher-character even told the students that while they might not know what these calculations meant, they would later learn it in their mathematics class. Then, the teacher-character mentioned to the students that "In math, the logarithm is an inverse of the exponential". The teacher focussed on the steps of *how* to calculate logarithms and interpret a logarithmic scale. The teacher also tried to help students gain some intuitive understanding of a logarithmic scale by showing an example of 10^0 and 10^2, emphasizing that the second number is 100 times larger than the first one, while the second exponent is only two steps removed from the first one. The teacher in Valery's script stated "*... so even though the two values are 2 units apart, they actually stand for a factor of 100 between them!*" From this moment on, the lesson focussed on calculating different sound levels and comparing the sound of 86 dB with 91 dB. The teacher led the students in deriving the expression which showed that the intensity of the 91-decibel sound is almost 4 times larger than the intensity of the 86-decibel sound. In this script, the students followed the teacher's derivation written on the board, confirming that it all made sense to them now:

> Teacher: Hopefully you feel better, Jerry! Back to your initial guess, Leah. You were on the right track. The only thing you have to keep in mind about the decibel scale is the (10 dB) term in front. You need to combine that with your exponents to properly compare the two decibel values, like such.
>
> Teacher writes on the board: $10^{\frac{91-85}{10}} \sim 3.98$.

This was a surprising culmination of the lesson, as it is unclear if the students would be able to justify or even follow these steps. To understand the approach to solving this problem, one needs to be familiar with logarithms and the definition of sound level. For some reason, the teacher in Valery's play skipped these steps and only showed the students the final solution. Thus, the entire lesson culminated in a mathematical procedure while ignoring the reasons for it.

5.3.2 Analysis of Valery's lesson play

This lesson play presents a teacher who has deep knowledge of the underlying concepts from both a mathematics and a physics perspective. Yet, Valery chose to focus on the procedures or the steps the students have to go through in order to calculate and compare different sound levels and the sound intensities associated with them. Moreover, while focussing on the procedures, a lot of teachable moments were lost, that could have been used to help students make sense of the fascinating properties of logarithms. As such, we consider Valery's pedagogical choices as an example of Superficial Procedural K4T.

In his reflection, Valery wrote the following:

> I learned this topic in first-year university and so I was well equipped mathematically compared to grade 11 students. Introducing new math functions in a physics class before a math class is not anything new, but I think it is more impactful at the secondary school level.

Surprisingly, Valery did not use a table of various sound intensities (Table 2) to illustrate an exponential growth, and did not compare linear and logarithmic scales using graphical representations. Instead, Valery chose to spend most of the time on the procedural steps of *how* to calculate the sound level and not *why* sound level is calculated in this way. We show how both the *why* and the *how* could be incorporated into the lesson play in the final example.

5.4 Case 4: Deep Procedural K4T

5.4.1 Chris's lesson play

Chris's lesson play illustrates Deep Procedural K4T. While reflecting on the connection between mathematics and science, she wrote:

> I would want to use the example they were talking about. As physicists, this is part of what we do: poke at problems with math. During the lesson I want to give them an opportunity to make noise and measure the resulting sound level in decibels. I would also want to give them at least one other example of a logarithmic scale.

This reveals that Chris is ready to help students bridge mathematical and physical representations with experimental evidence. Chris realized that the students might find it difficult to connect linear and logarithmic scales and decided to expand on it in her lesson play. As such, the teacher-character in Chris's script began by helping students generate experimental evidence through using a smartphone application that measured the sound level of a thud created by a falling textbook.

Sam: Maybe sound adds up. Like, if we are talking and we drop a textbook it will measure the combined sound.

Teacher: Yes, we're aiming to measure the sound of one thing at a time, so we don't want the app to pick up other sounds. Ok, who wants to go first?

Pippin: Me! Ready with your app, Merry? 3 … 2 … 1 … [drops textbook on to desk]

Merry: That was [some number] dB. Teacher: Alright. I would like to go next. [Rustles paper.]

Merry: That was [some other number] dB. Sam: Can I go next?

Teacher: Go ahead. Sam taps a ruler on a desk. Merry: That was [yet another number] dB.

Teacher: The loudness seems to change a lot, are the number of decibels changing a lot too?

Merry: Only from the super quiet sounds to the louder sounds.

In this dialogue, the teacher-character supported students in making connections between linear and logarithmic scales. The students heard a significant increase in the loudness and juxtaposed it with the "small change" in sound level as expressed on a decibel scale. This is important, as up to this point the teacher had not introduced the mathematical description of the sound level. Only at this point, the teacher introduced the formula connecting the sound level with the sound intensity. The teacher prefaced the sound level formula by saying, "*Because we are physicists, we can write down how sound level depends on intensity in a formula*." The teacher then helped the students connect the mathematical description of sound level to the students' physical experiences of loudness:

Teacher writes β(in dB)=10log I/I_o on the board.

Teacher: Where beta is the sound level, I is the intensity of the sound wave we are interested in, and Io is the intensity of the quietest sound that a good ear can hear. The logarithm is base 10. What happens if I is equal to I_0?

Frodo: Then you have log of one.

Teacher: Does beta equal zero?

Merry: No, because there's still sound.

Teacher draws a logarithmic curve on the board.

Sam: Wait, I remember this from math. $\log(1) = 0$. So, if $I = I_0$, then $\beta = 0$ dB.

Teacher: So, if I measure 0 dB, does that mean there is no sound?

Frodo: No, because to get 0 dB I has to be the same as I_0.

Teacher: That's right, and I_0 is the quietest sound a human can hear…

It is noteworthy how connections between mathematical and physical representations of the phenomenon were made. For example, instead of introducing the concept of the threshold of hearing, the teacher invited the students to see for themselves the meaning of I_0 – the quietest sound they could hear. Then the teacher-led the students through the steps of the mathematical description:

Teacher: Let's return to the physics and start thinking about louder sounds now, $I_0 = 1.0 \times 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}$ [writes Io value on the board, is a given, it's]. Sam, you were right that sound levels above 85 dB can do damage. Can we use this formula [points at formula on the board] to find the intensity of the sound wave for that sound level?

Pippin: I can't. I don't like logs.

Teacher: From general properties of logarithms, if log base b of n equals a, then n is b to the exponent a. [Writes equations as she speaks: $\log_b(n) = a \rightarrow b^a = n$.

Sam: I don't like logs either, but with this information I think I can rearrange the formula.

Teacher: Go ahead and give it a try. We are looking for the intensity given that the sound level is 85 dB and $I_0 = 1.0 \times 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}$.

Students attempt calculation.

Teacher: When I rearrange the formula, I get $I = I_0 10^{\beta/10}$. [writes formula on board]. Then, given β =85 dB and $I_0 = 1.0 \times 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}$, $I = 3.2 \times 10^{-4}\ \frac{\mathrm{W}}{\mathrm{m}^2}$. Let's do the same for 91 dB. What is *I* for 85 dB compared to *I* for 91 dB?

Merry: I get 1.3×10^{-3}.

Teacher: Always remember to state your units. You got 1.3×10^{-3} what?

Merry: Right, $\frac{\mathrm{W}}{\mathrm{m}^2}$.

Teacher: So, what is *I* for 91 dB over *I* for 85 dB?

Sam: Just about 4!

Pippin: Always state your units!

Teacher: Well, in this case we've taken something in Watts per meter squared and divided by something also in Watts per meter squared, so we get a unit-less number.

Merry: So, 91 dB is like 4 times more intense than 85 dB, even though the difference in the number isn't that big. That's so weird!

The students in the script discussed ratios of sound intensities and how these ratios could be expressed using a logarithmic scale and decibels. They connected different mathematical representations of the same phenomenon.

The teacher-character led the students through the mathematical procedure while realizing that some students might be apprehensive of it. Yet, the teacher decided not to avoid the mathematical procedure but instead helped the students develop the mathematical skills necessary when dealing with logarithms. The teacher also described how the logarithmic scale is used to describe earthquakes.

The script culminated with the teacher asking the students to calculate the ratio of intensities for 85-decibel and 91-decibel sounds. The teacher modelled the derivation for the calculation on the board by starting with the log rule that students should be already familiar with, $\log(x) - \log(y) = log\left(\frac{x}{y}\right)$. Finally, the teacher used this rule to help students derive the difference in sound levels between two sounds, $\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$, and then asked the final question:

Teacher: Here is a question to try: what $\beta_2 - \beta_1$ do we need to get $\frac{I_2}{I_1} = 2$?

Sam: I get 3.01 dB.

Merry: Wow, only 3 dB.

Teacher: That's right, only 3 dB to double the intensity. So Merry, does it make sense that your head hurts when you experienced 91 dB and 85 dB is where damage will start to occur?

Merry: Yup!

5.4.2 Analysis of Chris's lesson play

This lesson play illustrates the merging of the teacher's Deep Procedural K4T. Chris's teacher-character not only led the students through the mathematical representation of the physical phenomenon, but she also anticipated when the students might have potential pitfalls and misconceptions. The teacher seamlessly moved between different representations and made a deliberate effort to connect these representations to the students' everyday life experiences. The chosen juxtaposition of the difference in decibels with the ratios of intensities is a valuable pedagogical approach, in which we also recognise Deep Conceptual K4T.

Chris' reflection also reveals that although she did not have the physics content knowledge prior to designing this lesson play, she was fully capable of acquiring it and making connections to students' lives while helping them with concrete examples of abstract concepts. This is how Chris described it:

> I think this is a pretty traditional lesson.... I didn't know much about the topic, so I found the relevant sections in Giancoli [a physics textbook] and introduced concepts as they did. There was some discussion, a bit of

> demonstration in which the students were involved, some exploration of the math, and then a tie-back to the opening question.

6 Discussion

This study aimed at addressing the following research question: What do participants' scripts reveal about the scriptwriters' knowledge for teaching the topic of sound, in particular the concepts of sound intensity and sound level? To address this question, we implemented a scriptwriting task (Figure 1) in a methods course for future secondary physics teachers. We found that all of the scripts submitted by the future teachers could fit into one of four broad categories of describing their K4T (Table 3). From analysing future teachers' reflections and juxtaposing them with the scripts, it became clear that future teachers' knowledge of the relevant content was a significant factor in their lesson plays.

When future teachers (e.g., Jamie) felt insecure about their content knowledge, they avoided dealing with the topic directly and digressed into discussing superficial or less relevant issues and avoided any mathematical representations (Superficial Conceptual K4T). This was an example of the pedagogical shield described in the literature (Kontorovich & Zazkis, 2016). On the other hand, some teachers (e.g., Alex) avoided using complex mathematical representations, but were able to focus on the concepts and helped students connect these concepts to their everyday lives. Those future teachers demonstrated Deep Conceptual K4T. We also found that even if future teachers were confident in their mathematical and science content knowledge (e.g., Valery), this did not guarantee that they would demonstrate Deep Procedural K4T. Few future (e.g., Chris) teachers demonstrated Deep Procedural K4T in their lesson plays.

In their reflections, all of the future physics teachers emphasized the value of the scriptwriting activity. It helped them identify their own pedagogical challenges, while encouraging them to imagine the interactions that might happen in a real secondary physics classroom. The focus on the learning as an interactive dialogical process was something that future physics teachers did not encounter during traditional lesson planning activities. Importantly, all of the future physics teachers emphasized the value of the scriptwriting process in helping spur their pedagogical growth.

Moreover, as the scriptwriting activity occurred over an extended period of time, the future teachers were not limited by the knowledge they already possessed but were encouraged to expand their content and pedagogical horizons. The leading questions in the first part of the scripting task (Figure 1) modelled pedagogical experiences of practising teachers preparing for lessons that might be outside of their direct area of expertise. Thus, unlike a traditional lesson plan, the scriptwriting activity provides a fertile ground for challenging future teachers to explore pedagogical approaches when teaching more advanced topics. Furthermore, the scriptwriting activity provides an opportunity for educational researchers to peek into future teachers' K4T and consider how teacher educators can help future teachers expand upon it.

7 Conclusions

Our study adds to the growing body of research that investigates future science teachers' knowledge for teaching (K4T). The study's particular contributions can be grouped into two major categories. First, while scriptwriting was used in a variety of studies in mathematics education, our contribution is in extending the applicability of scriptwriting to physics education. Scriptwriting invites future physics teachers to imagine possible instructional interactions, student questions, and pedagogical approaches. This may help future physics teachers begin to break down the existing rigid subject matter barriers and to consider how future physics teachers can make their physics lessons more engaging and meaningful for their students.

Second, we extend research on teachers' knowledge related to sound intensity, focusing on how such knowledge plays out in an imaginary teaching scenario. While the measurement of sound intensity is based on logarithms, some of the scriptwriters demonstrated how the mathematics could be highlighted in a manner accessible to students, while others illustrated how mathematics could be avoided without hindering the integrity of the instruction. Our work builds upon the studies of researchers who questioned the traditional facets of knowledge, as conceptual and procedural, and who provided refinement of these notions (Star, 2005). We operationalized the facets of K4T (deep/superficial and conceptual/procedural) in the context of sound intensity and sound level and illustrated them using the participants' scripts.

In light of our findings, we believe that incorporating scriptwriting tasks in mathematics and science teacher education has a number of potential advantages. For teacher educators, this process can reveal both the content and pedagogical knowledge of future teachers, as well as their own misconceptions and challenges. This information is valuable for designing effective methods courses. For researchers, analysing the scripts can guide them towards the design of effective prompts that may help generate meaningful scriptwriting tasks for pedagogically powerful experiences for the next group of future teachers. Finally, for future teachers, participating in the scriptwriting activity can help them practise designing and implementing meaningful lessons in a non-threatening and reflective environment while gaining confidence and expanding their K4T.

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Inspiring or confusing – a study of Finnish 1–6 teachers' relation to teaching programming

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There is limited research on teaching and learning of programming in primary school and even less about aspects concerning teaching programming from teachers' viewpoint. In this study, we explore how Finnish 1-6 primary school teachers (N=91), teaching at schools with Swedish as the language of instruction, relate to programming and teaching of programming, one year after the introduction of the new national curriculum that included programming. The teachers' relation to programming is studied by analyzing their view on programming, perceived preparedness to teach programming and their attitudes towards teaching programming. The main results of the present study are that the responding teachers approach programming in school with mixed emotions, but the majority claim to have sufficient preparedness to teach programming, and many of them have a positive attitude towards the subject. The findings indicate that the most important factor for high perceived preparedness and positive attitude is sufficient domain knowledge. The teachers' views on programming are very diverse, ranging from focusing only on the connection to elementary step-by-step thinking to more sophisticated reasoning connecting to central aspects of computational thinking and other educational outcomes. The findings suggest that there is a need for educational efforts to make the connection between mathematical content and programming more visible for primary school teachers.

Keywords: elementary education, mathematics education, programming, teacher professional development, 21st century abilities

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1 Introduction

Digital technology affects our daily lives, and programming and coding are at the very heart of this technology. Teaching computer science and programming, or in a broader sense computational thinking (Papert, 1996; Wing, 2006), has been a much-discussed subject in education during the last decade. There has been an increasing emphasis on integrating computer science into the school curriculum from lower grades in several countries (e.g. Schulte, Hornung, Sentence, Dagiene, Jevsikova, Thota, et al., 2012; Brown, Sentence, Crick & Humphreys, 2014; Duncan & Bell, 2015). This trend is to some extent driven by economic and technological demands for a future workforce (Chen, Shen, Barth-Cohen, Jiang, Huang, & Eltoukhy, 2017), but it has also been stressed that computational thinking and code literacy are important skills for full participation in modern society (Dufva & Dufva, 2016). The notion of code literacy relates to the understanding of code and to the intentions and context of the code. “In the same way that not all literate individuals become authors, not all code-literate individuals become developers. Still, literate people have the necessary skills and the apprehension of reading and writing.” (Dufva & Dufva, 2016, p. 2). In addition, authors point out that computer science education in early grades influences students' persistence in the domain and therefore, also their future career choices (Margolis, Estrella, Goode, Holme, & Nao, 2010; Yardi & Bruckman, 2007).

A practical way to enhance children's code literacy is to incorporate programming at an early stage in the educational system. Several countries, for example, Finland, Sweden, Estonia, the United Kingdom and the United States have included programming in the national core curriculum. This is accomplished in different ways (Hubwieser, Armoni, Giannakos, & Mittermeir, 2014; Hubwieser, Armoni, & Giannakos, 2015). Some countries have introduced computer science as a subject of its own, Computing in England (Department for Education, 2013), while others have decided to integrate programming into other subjects, by, for instance, making programming an interdisciplinary element throughout the curriculum. This is the case in Finland, where programming is included in the generic competencies to be developed in all subjects and explicitly integrated in mathematics and handicraft (FNBE, 2016).

The path from the inclusion of programming in the national core curriculum to enacting lessons targeting it in a relevant manner is complex (Mannila, Dagiene, Demo, Grgurina, Mirolo, Rolandsson, & Settle, 2014). As Mannila et al. point out there are several issues to be discussed and defined to succeed in the implementation process. For example, it is not clear what exactly should be taught at different grade levels, neither what materials should be used. Primary school teachers as generalists need widespread professional development concerning technical skills and understanding of suitable pedagogies to successfully implement new curriculum ideas (Benton, Hoyles, Kalas, & Noss, 2017).

While some studies describe the use and impact of specific programming tools and classroom activities (Falloon, 2016; Sáez-López, Román-González, & Vázquez-Cano, 2016), only a few explore programming from the perspective of primary school teachers. Yet, teachers' knowledge, beliefs and motivation are important to consider if we are to succeed in laying a solid ground in all students' computational thinking and awakening their interest towards programming and technology (e.g. Ertmer, 2005; Ball, Thames, & Phelps, 2008; Sentance, Sinclair, Simmons, & Csizmadia, 2018; Hubwieser et al., 2015). The overall aim of this study is to investigate Finnish primary school teachers' relation to programming and to teaching of programming. The following research questions (RQ) guide the study:

RQ1: What are the studied primary school teachers' views on programming?
RQ2: What is the studied primary school teachers´ perceived preparedness to teach programming?
RQ3: What are the studied primary school teachers´ attitudes towards teaching programming?

2 Literature review

2.1. Programming and computational thinking in school

Programming for K-12 students was first introduced in the 1960s when Logo programming was presented as a potential framework for teaching mathematics (Feurzeig & Papert, 2011; Lye & Koh 2014 p. 52). During programming activities, students are engaged in computational thinking that involves general central concepts from computer science. The origins of computational thinking in mathematics education can be traced back more than thirty years to the work of Papert who developed computer software to facilitate children to engage and explore computer programming as a natural problem-solving tool in their mathematics studies (Papert, 1980, 1996). The term re-entered the pedagogical research community in 2006 when Wing pronounced that computational thinking represents a universally applicable attitude and skill set everyone, not just computer scientists, would be eager to learn and use. She suggested the definition "computational thinking involves solving problems, designing systems and understanding human behavior, by drawing on the concepts fundamental to computer science" (Wing, 2006 p. 33). After that, several organizations and authors have presented different definitions of computational thinking. For example, the International Society for Technology in Education (ISTE, 2020) views computational thinking and its application as a cross-curricular skill. The core components of computational thinking according to ISTE: decomposition; gathering and analyzing data; abstraction; algorithm design; and how computing impacts people and society. These definitions and views are quite general and may indeed involve activities not necessarily directly connected to programming and coding.

Brennan and Resnick (2012) proposed a programming-based view on computational thinking, focusing especially on visual programming using Scratch for K-12 students. They introduced a framework with three dimensions of computational thinking: computational concepts, computational practices and computational perspectives. The first dimension includes concepts that programmers commonly use as they develop programs, such as variable, iteration and function. Computational practices reflect different problem-solving practices that occur in the programming process, such as testing, debugging and reusing. The third dimension, perspectives, involves the programmer's connection and relationship to other members of the programming community and to the surrounding technological world.

The definitions and interpretations of computational thinking are diverse, but the essence in computational thinking comprises at least thinking in a way that can be represented and processed by machines to facilitate a solution. A model is needed for representation and a set of structured computational steps (algorithm) is required for its solution.

In a recent study, Popat and Starkey (2019) reviewed research identifying educational outcomes, other than computer science and computational thinking, of programming in school. Their results concluded that when students are learning to code, a range of other educational outcomes could be learnt or practiced through the process of learning coding. These included mathematical problem-solving, critical thinking, social skills, self-management and more general academic skills. Students learning to code are coding to learn, according to Popat and Starkey (2019).

Criticism against the term computational thinking in an educational context is often based on the lack of consensus of the exact meaning of the term and its multiple interpretations. In addition, the relevance and importance of computational thinking as a general skill in everyday life has been questioned (Grover & Pea, 2013, p. 40). Some authors question the claim that computational thinking is an important skill for all students to learn and others react to why computational thinking should be superior to other types of thinking processes (e.g. Denning, 2017).

2.2. Teachers' relationship to programming

There are many studies about the important role of a teacher when implementing new ideas in school curriculum (e.g. Guskey, 2002; Hijón-Neira, Santacruz-Valencia, Pérez-Marín, & Gómez-Gómez, 2017). It has also been pointed out that teachers' content knowledge and pedagogical knowledge are important and affect student progress and achievement in mathematics (Ball, Thames & Phelps, 2008; Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, U., et al., 2010). Thorough content knowledge is important also in teaching of programming as several studies witness students' difficulties in learning concepts related to programming (e.g. Cetin, 2013; Denner, Werner & Ortiz, 2012). Moreover, the way in which teachers relate to new ideas is crucial for successful curriculum reform. Still, while there is a growing body of studies focusing on students' learning of programming at different school levels (e.g. Sáez-López et al. 2016; Chen et al., 2017; Duncan & Bell, 2015), only a few studies focusing on primary school teachers' and teaching of programming can be found.

Misfeldt, Szabo and Helenius (2019) investigated mathematics teachers' conception of the relationship between mathematics and programming. The results suggest that the teachers, on average, feel that there is a relationship between the two subjects and that mathematics teachers are interested in working with programming but that they do not feel well prepared for taking on that task. Funke, Geldreich and Hubwieser (2016) interviewed six primary school teachers about their views of computer science, and the findings pointed out that the teachers had no clear image of what computer science in school is, but they highlighted the importance of implementing computer science in an early educational stage. Govender and Grayson (2008) studied pre-service teachers' experiences when learning object-oriented programming. These pre-service teachers mostly talked about programming as finding a niche to design or upgrade a program that is needed somewhere. When probing the connections between programming and problem solving, they agreed that there must first be a problem, and then a program is constructed to solve it (Govender & Grayson, 2008). Recently Nouri, Zhang, Mannila & Norén (2019) investigated which skills 19 teachers interested in programming themselves aimed to develop among pupils. Apart from Brennan and Resnick's (2012) dimensions, they found some general skills related to digital competency and 21st century skills.

Several researchers highlight the critical role of the teachers in making explicit and systematic links between programming and students' existing and developing mathematical knowledge (e.g. Benton et al., 2017; Hickmott, Prieto-Rodriguez & Holmes, 2018). Hickmott et al. (2018) state that there is a lack of empirical studies that include concrete ideas or practices for K–12 educators that explicitly link the learning of mathematics and computational thinking. Kilhamn and Bråting (2019) investigate the relationship between programming and algebra in school. They emphasize the awareness of possible pitfalls concerning syntax and semantics in these two areas to avoid confusion among students when working with, for example, algorithms and variables.

Mannila et al. (2014) surveyed teachers' experiences about and perceptions of computational thinking in five European countries, Finland, Italy, Lithuania, Netherlands, and Sweden. An important contribution of the study was the revealing of different aspects of computational thinking that already has become a part of teachers' classroom practices and how this is done. The survey data suggest that some teachers are already involved in activities that have strong potential for introducing some aspects of computational thinking. One of the concluding questions for further research in that study was students' and teachers' attitudes and experiences of introducing computational thinking in the classroom.

There are some studies investigating teachers' attitudes towards programming (Cetin & Ozden, 2015; Cetin, 2016; Hijón-Neira et al., 2017). Cetin (2016) shows that the pre-service teachers who learned programming through Scratch mastered central concepts better than the pre-service teachers in the control group did. They also experienced learning programming more meaningful. Hijón-Neira et al. (2017) investigated primary school teachers' views on programming in schools in one region in Spain through a questionnaire, and they analyzed the responses of 46 teachers. The teachers agreed on the benefits that programming provides in several areas, for example the development of thinking skills, the organization of ideas, the ability of abstraction and problem solving, motivational aspects, and the opportunities offered by teaching through games.

3 Context and educational setting of the study

We commence by briefly describing the Finnish school context and the role of programming in the national core curriculum (FNBE, 2016).

The comprehensive school (grades 1-9, ages 7-16) in Finland is the same for all students as there is no tracking. Hence, the national core curriculum is the same for all students. A primary school teacher teaches almost all subjects in grades 1-6, including mathematics. Primary school teachers in Finland are highly educated since they have a master's degree in education. Finland has two official languages, Finnish (88.7 %) and Swedish (5.3 %). According to existing legislation, education is organized separately for both language groups in parallel monolingual schools that follow the same national core curricula. Approximately 5 % of students in compulsory education attend a school where Swedish is the language of instruction. The target group of this study is primary school teachers that work in schools where the instructional language is Swedish.

The Finnish national core curriculum describes generic competencies as a way to meet the challenges of the future world related to the 21st century skills and integrative instruction across school subjects. Programming is included in the generic competence of information and communication technology. The general task of mathematics education is to develop students' logical, accurate and creative thinking. Programming is included in the content of mathematical thinking skills and applies to all students from grade 1 up to the end of grade 9 (Hemmi, Krzywacki, & Partanen, 2017). Learning programming in mathematics starts in grades 1-2 with constructing simple algorithmic instructions by using symbols in written or oral form and testing them. During grades 3-6, the emphasis is on formulating instructions in a graphical programming environment. Programming is also included in the subject of handicraft from grade 3. In handicraft, students should practice programming through activities in, for example, robotics and automation. In grades 7-9, students develop and deepen their algorithmic thinking and their skills in applying programming in the mathematical problem-solving process.

In Finland, the national core curriculum only offers a general frame, and the municipalities, schools and teachers are to concretize the curriculum intentions (Hemmi, Lepik, & Viholainen, 2013). Teachers can freely choose their curriculum resources, and the mathematics textbooks are commercially produced without any national control (Hemmi, Krzywacki, & Koljonen, 2017).

4 Materials and methods

In this section, we present the data material used, the methods of data collection and how the data is analyzed.

4.1 The questionnaire

The empirical data for this study was obtained using a web-based survey that was sent to Swedish primary schools (grade 1-6) in Finland. The survey contained 39 questions and was divided into four different sections. Some questions were obligatory, some were optional, and some questions were branched. The minimal number of answered questions for each respondent was 28, and the maximal number was 39, depending on the number of optional and branched answers. Section 1 asked for background information on the respondents. Questions on programming were posed in section 2, and section 3 contained questions related to teaching programming. Finally, in Section 4 there were questions on participation in in-service training and the possibility to give general comments. In the present study, answers from 18 questions (obligatory or optional) were collected and analyzed. The connection between the research questions and these survey questions are presented in section 4.3.

4.2. Data collection and informants

The data collection was carried out from the 3rd of April 2017 to the 10th of May 2017, and some preliminary results were reported in an unpublished thesis in educational sciences (Kallio-Kujala, 2017). In all, 110 teachers answered the questionnaire, but 19 answers were removed from the data material due to incompleteness. The final group of respondents consisted of N=91 teachers, 70 female and 21 men. The regional distribution of the respondents between the three major Swedish-speaking regions of Finland where 37/29/19 and six respondents came from other parts of Finland.

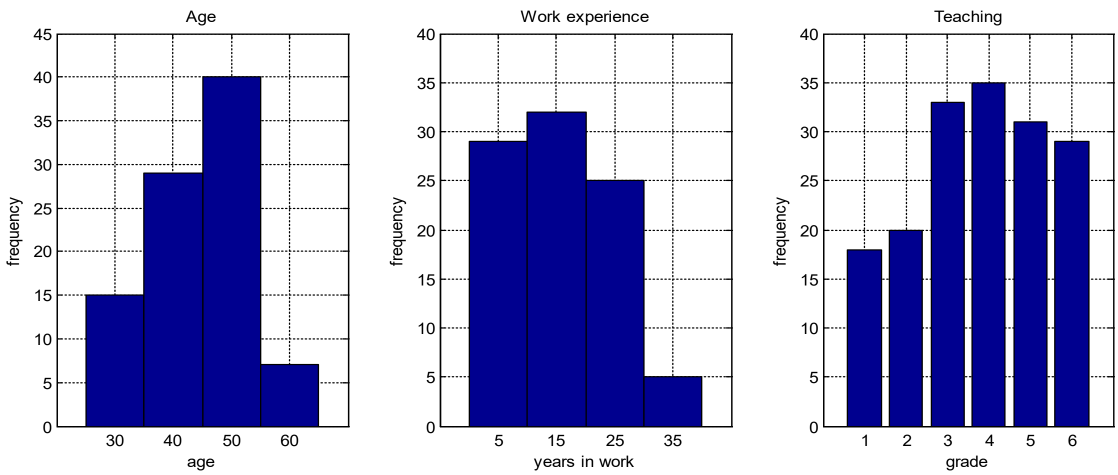

Figure 1. Background overview on the respondents' age, teaching experience and which grade they teach during the on-going year.

Most teachers had an age from 35 to 55 years. The distribution of teachers' work experience was evenly distributed between 0-30 years with a few teachers exceeding 30 years. The extended number of responses in the rightmost chart in Figure 1 is due to that several teachers teach multiple grades. Out of the 91 responding teachers, 84 was certified primary school teachers with a master's degree in education. Four respondents had a bachelor's degree, and three lacked a university degree in education. Programming is explicitly mentioned in the curriculum in mathematics (grade 1-6) and handicraft (grade 3-6) and 86 of the respondents had taught mathematics and 35 had handicraft during the ongoing school year.

4.3. Connection between the survey and research questions

Below, we explain how the research questions are connected to different survey questions.

RQ1: What are the studied primary school teachers' views on programming in school?

This research question was answered by analyzing the teachers' answers of the open question: "What is programming? Please, focus on programming in primary school, but you can also discuss programming in general." From the context of the questionnaire, it is evident that this question is directly related to how the current change of the national curriculum (inclusion of programming) affected the mathematics content. The analysis of teachers' answers, following Bryman (2001), uses an iterative data-driven approach, in our case, going through several cycles of analysis. First, the teachers' written responses (in Swedish) were read and summarily analyzed. During this step, we identified certain similarities and generalities among the answers, which lead to the identification of six different categories. Thus, the identified six categories were a result of the analysis. Then the teachers' responses were read again, interpreted, and assigned to categories, in an iterative approach, by the authors.

RQ2: What is the studied primary school teachers´ perceived preparedness to teach programming?

The term perceived preparedness is associated with teachers' preparation before the curriculum reform. This preparation includes familiarization with the new curriculum, preempting relevant teaching material, collaborative preparation with colleagues, support from school management and self-perceived preparation level. Seven questions in the survey correspond to this topic. The teachers responded to the questions on a 6-point scale ranging from "no … at all" to "very much … ", with alternatives 1-3 on the negative side and 4-6 on the positive side. The translated questions can be found in Appendix A.

The teachers also responded (yes/no) to whether or not they had participated in in-service training. A two-sided unpaired *t*-test with α-level 0.05 was conducted to compare if there were a significant difference in the perceived preparedness for those that had participated in in-service training compared to those that had not participated. Both samples are approximately normally distributed, and the standard deviations are approximately equal. To complement the quantitative items, the teachers had the possibility to respond to two open questions that relate to their perceived preparedness to teach programming:

- What type of support and help have you obtained from your colleagues or school on how to teach programming?
- What type of material do you have and how have you obtained it?

The teacher responses to the last question were categorized into six different categories using an open data-driven approach similar to the one used in RQ1.

RQ3: What are the studied primary school teachers´ attitudes towards teaching programming?

Teachers responded to statements related to their attitude to programming and to teach programming in primary school. Five statements correspond to this topic. The teachers responded to the statements on a 6-point Likert scale (e.g. Oppenheim, 2000) with alternatives 1-3 on the negative side and 4-6 on the positive side. The translated statements can be found in Appendix A. These five statements are similar with the computer programming attitude scale developed by Cetin and Ozden (2015). Their scale included affection, cognition and behavior as three dimensions of attitude. The cognitive dimension consists of beliefs about the attitude object, the affective dimension includes feelings towards the object, and the behavioral dimension refers to action tendencies towards the object. The five scale items in this study relate to affection and cognition. A two-sided unpaired *t*-test with α-level 0.05 was conducted to compare if there were a significant difference in attitude for those that had participated in in-service training compared to those that had not participated. Both samples are approximately normally distributed, and the standard deviations are approximately equal. To complement the quantitative scale items, the teachers also answered one multiple-choice question and two open questions related to attitude.

- Which of the following words describes your emotions to teach programming? (See Table 3)
- What do you think has influenced your attitude to teach programming in primary school?
- State your arguments why students should learn programming in primary school.

The teacher responses to the second question were categorized into five different categories using an open data-driven approach similar to the one used in RQ1.

5 Results

Next, we report the findings and results of our analysis of the teachers' relation to teaching programming. We follow the order of the RQs.

5.1. Analysis and result of RQ1: views on programming

The teachers' views on programming and teaching programming were categorized as 1) sequential, 2) logical, 3) algorithmic, 4) problem-solving, 5) technological and 6) progressional (see Table 1). Due to the openness of the question, one answer could be assigned to several categories. Below in this section, we explain and describe the categories in more detail and exemplify them with teachers' responses translated to English. The number of words in the different teacher answers (in Swedish) varied from one word to 108 words and the mean number of words in the answers were 25. The distribution of the teachers' views on programming in relation to the six analytical categories can be seen in Table 1. One answer can be assignment to several categories. The number of answers assigned to different number of categories are: 0 (6), 1 (48), 2 (24), 3 (10), 4 (2), 5 (1). That is, six answers were uncategorized and 24 answers belonged to two different categories. No answer was assigned to all six categories. The total number of assigned answers are 139.

Below, we describe and exemplify the categories identified for teachers' views on programming in school. A single excerpt often below to several categories. Underlining has been used to indicate belonging to a certain category. Some of the results related to RQ1 has recently been published in a proceeding paper (Pörn, Hemmi, & Kallio-Kujala, 2021).

Table 1. Distribution of teachers' views on programming with respect to the six categories

Category (view on programming)	***n* (% of 91 teachers)**
1. Sequential	59 (65)
2. Logical	29 (32)
3. Algorithmic	10 (11)
4. Problem-solving	17 (19)
5. Technological	9 (10)
6. Progressional	15 (16)
Total	**139**

Sequential view

The sequential view connects programming with the explicit action of giving (or writing or following) step-by-step instructions to a computer, robot or fellow student. This category is the most common among the answers as 65 % of the teachers' response could be connected to this. Teachers connect these kinds of actions to activities associated with spatial thinking and step-by-step procedures. This is exemplified in the following answers:

> In primary school education, it is important to let students test to program a computer, give instructions to another person or to a robot and try to make it complete the desired task. (Teacher 10)
> A simple way is to say; Go two steps to the right, one backwards and then five steps forward. Then you have come to the finish. (Teacher 33)
> Programming is to give detailed step-by-step instructions that do not offer space for misinterpretations or ambiguity. (Teacher 72).

Several teachers pointed out that the instructions need not to be given to a computer or robot, but equally well to a fellow student.

Logical view

This category was the second most common as 32 % of the teacher responses point out that programming is connected to logical thinking or the identification of patterns. Most responses in this category state that programming promotes the development of logical thinking, as shown in the following extracts:

> Programming is, for example, to split a problem into smaller parts, to see relations, to learn to think logically, to create something new. (Teacher 64)
> I think programming is very much about logical thinking and recognizing patterns. (Teacher 45).

Many teachers connect programming to a combination of handling instructions and applying logical thinking.

Algorithmic view

The algorithmic view is connected to the central concepts of computer science and development of programs such as algorithm, abstraction, modularization, planning and testing. Eleven percent of teachers' responses are categorized as algorithmic. A typical example is the following excerpt:

> It is about coding, solving complex problems by splitting them into smaller pieces, identifying patterns, creating abstractions and writing algorithms. (Teacher 16)

Teachers that connect to these concepts may have more in depth knowledge of programming and to the process of applying programming to solve problems.

Problem-solving view

In this view programming is connected to the usage as a mathematical problem-solving tool. This aspect of programming is highlighted in 19 % of the answers.

> Programming is a really good activity that trains the ability to solve problems. (Teacher 43)
> Programming is about logical thinking, ability to solve problems, systematics, and creativity . . . Programming is mathematics. (Teacher 73)

Despite the close and important connection between mathematical problem solving and programming, no teacher answer is giving any explicit example of such a problem-solving activity.

Technological view

A few teacher descriptions (10 %) consider programming from a more general perspective that involves the connection to modern technology and digitalization of society. Some responses address directly the importance of understanding the relation between human and modern technology:

> To realize that everything a machine can do is due to a human that has programmed it. (Teacher 10)

> Several things in our close environment work with aid of programming, e.g. machines, computer games and telephones. Industry uses robots that have been programmed. (Teacher 75)

This category captures more general aspects of programming, pointing out the human-machine relationship and connection to modern technology.

Progressional view

The aspects of curriculum and progression concerning programming in primary school and comments on the importance of knowledge for the future work-life are present in 16 % of the answers. Several teachers saw programming as a positive element in mathematics lessons and important for all students to learn, for example to prepare for future work life.

> We have to prepare them for the working life after school when they must be prepared to think creatively. (Teacher 58)

On the other hand, there were teachers who were not convinced about the importance of learning programming for all students and those that lack clear information on the progress throughout the grades 1-6.

> I think programming is fun, but I do not see it as a useful subject. That type of thinking can be acquired in many other ways. (Teacher 22)
> Interesting, but I would like to have a clearer plan about what to do each school year. (Teacher 69)

Variation in teachers' descriptions

Due to the openness of the question, the range and the depth in teachers' responses varied a lot. Some of the teachers touched several categories while others only responded with short sentences categorized into one category. The following extract is an example of the former and was coded into categories 1, 2, 3 and 6.

> Programming is a working process were you construct an algorithm, a hypothesis or a plan of how something should be executed or work. This plan is then tested and updated in order to work correctly. On a basic level, it can be as easy as working with numbered instructions. For older students it proceeds to the creation of block-based events using apps and computer programs and then finally in the highest grades by coding using a text-based language. (Teacher 62)

This teacher captures several important concepts and practices in computational thinking, such as instructions, events, algorithm, planning and testing as well as the progression of the topic. The next example reflects a logical, algorithmic and problem-solving view on programming.

> Programming is all about logical thinking and problem solving. It is about coding, solving complex problems by splitting them into smaller pieces, identifying patterns, creating abstractions and writing algorithms. You can practice programming using different programs, games and languages. Programming is a new way of thinking. (Teacher 16)

The focus in this answer is on problem-solving, the thinking aspect and the creation of algorithms and abstractions. The last example is coded into categories 2, 4 and 5.

> Programming is a way to teach students logical thinking, understanding of relations and problem solving. They develop both cognitively and linguistically. In time, they will understand that all new technology they use is based on programming. (Teacher 40)

This teacher specifically lifts logical thinking and problem solving as important learning outcomes and the technological view is also present. The answer also highlights the communicative (social) aspect of programming as being important. This social aspect of programming was mentioned in two answers.

Connections to mathematical content

There were few answers that explicitly connected programming to other mathematical content. The reason for this could indeed be the openness of the survey question. Still, no teacher answer spontaneously relates programming activities to measurement, arithmetic expressions, equation solving, nor probability. Some answers made connections to the broad area of problem-solving but provided no explicit example of what type of problem-solving was actually involved. The few examples with mathematical content can be connected to elementary spatial thinking (how to move along a pre-defined path) and simple geometrical shapes (how to form a square).

5.2. Analysis and result of RQ2: perceived preparedness to teach programming

The seven items connected to preparedness were summed together to get a measure of the teachers' overall perceived preparedness to teach programming. The internal consistency reliability (Cronbach's alpha) was 0.88 for this composite variable. The correlation (Pearson's r) between the seven items ranged from 0.26 to 0.81. Figure 2 shows the distribution of the variable "Perceived preparedness" on the scale 1-6 and the mean values of the seven individual items. Fifty-six (62 %) of the teachers scored higher than 3.5 (positive side responses) on the perceived preparedness scale, and 35 (38 %) scored lower than 3.5 (negative side responses). The perceived preparedness of the responding teachers is on the positive side, but several teachers also express a clear lack of knowledge about programming and an explicit need for more education on the subject. The following teacher comments address this need:

> First, I need to know what programming actually means! (Teacher 10)
> I wish for more clarity in what to exactly teach and on which grade. Education is needed. (Teacher 37)
> Much more education and support is needed, since this topic is completely new to me. (Teacher 29)

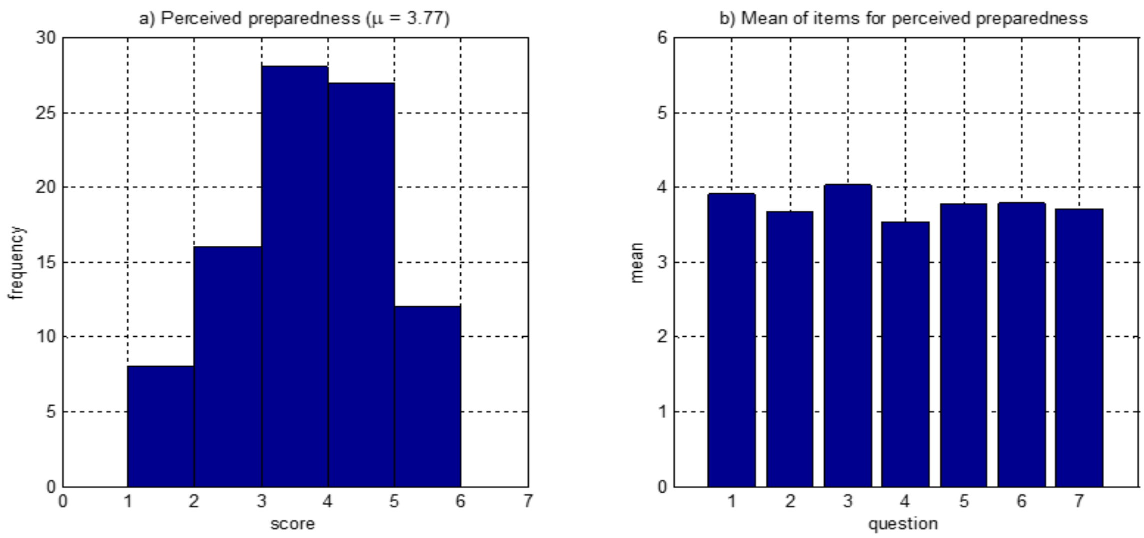

Figure 2. a) Distribution of teachers' perceived preparedness to teach programming, b) mean of individual items for perceived preparedness.

Seventy-one (78 %) of the teachers had participated in at least one in-service training. There was a significant difference in the means for perceived preparedness between group 1 (in-service training; n=71, m=3.972, sd=1.036) and group 2 (no in-service training; n=20, m=3.064, sd=1.130) with t(89)=3.392, p=0.001. The result indicates that teachers who have participated in in-service training had higher perceived preparedness than those that have not.

In addition, the teachers responded to the open question: What type of support and help have you obtained on how to teach programming? Sixty-two (68 %) of 91 teachers claimed that they had obtained sufficient support from their school and colleagues in their preparation to teach programming in primary school. Common examples of supporting factors were the possibility to participate in many in-service training events, discussions with colleagues, explicit interest from and engagement by the principal and systematic visits at school by local ICT-tutors. On the negative side, most comments reflected on inadequate equipment and material at school and some pointed out the lack of a broader discussion regarding a more holistic perspective on the implementation of programming in primary school.

> More practical examples is needed on how to embed programming into a primary school context so that we don't have to do programming just for programming itself. (Teacher 75)

The teachers also responded to the open question: What type of material do you have and how have you obtained it? Fifty-three teachers answered this optional open question. The answers were categorized based on how they had obtained their material. In Table 2, one answer can belong to several categories.

Table 2. Categorization of how the teachers had obtained their teaching material and artifacts.

Self-made	Purchased	Borrowed	Web-based	From in-service training	From colleagues
9	15	6	24	14	10

Self-made material includes, for example, different types of programming cards and games for unplugged activities. A typical example of purchased material was educational robots (e.g. BeeBots, Sphero, Lego Mindstorms). It is also common for schools in the same municipality to have shared material pools with more expensive educational material that can be borrowed. It is most common to obtain programming material from the internet and many teachers use web-based tools like Scratch and code.org as well as applets like ScratchJr and Lightbot. Since many of the responding teachers had participated in in-service training courses, some material is directly obtained through those events. Ten respondents mention explicitly that a colleague provides the material.

5.3. Result of RQ3: attitudes towards teaching programming

The five items connected to attitude were summed to get a measure of the teachers' overall attitude to teach programming. Cronbach's alpha was 0.79. The correlation between different items ranged from 0.13 to 0.73. Figure 3 shows the distribution of the variable "Attitude" on the scale 1-6 and the mean values of the five individual items. Eighty (88 %) of the teachers scored higher than 3.5 on the attitude scale and 11 (12 %) scored lower than 3.5. The responding teachers' attitude to teach programming in primary school is clearly on the positive side. There was a significant difference in the means for attitude between group 1 (in-service training; n=71, m=4.64, sd=0.779) and group 2 (no in-service training; n=20, m=3.888, sd=0.872) with t(89)=2.381, p=0.02. The result indicates that teachers who have participated in in-service training scored slightly higher on attitude towards teaching programming than those that had not.

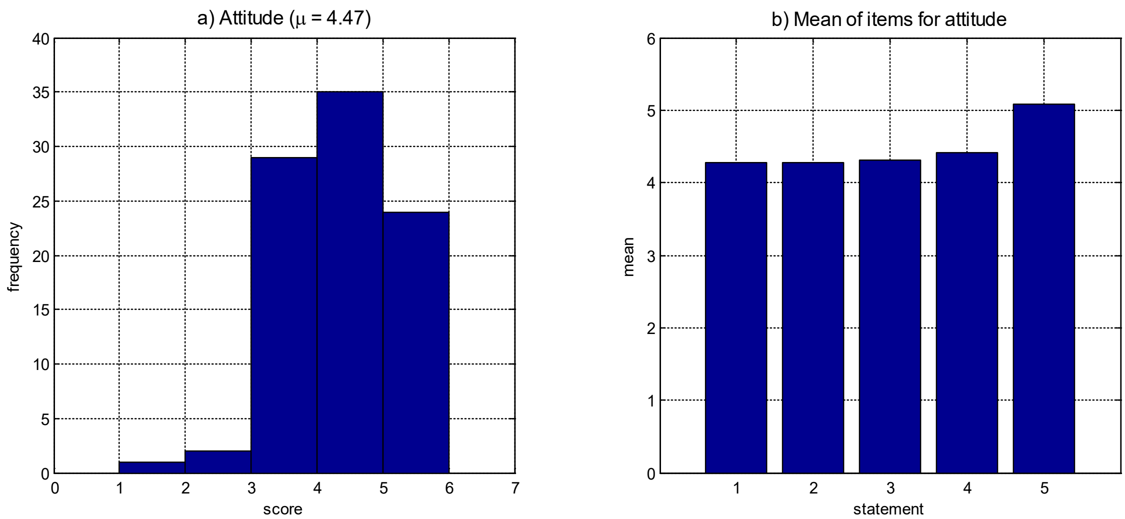

Figure 3. a) Distribution of teachers' attitudes to teach programming, b) mean of individual items for attitude.

In addition, teachers responded to the multiple-choice question: Which of the following words describes your emotions to teach programming? One respondent could choose several words. Table 3 shows that the majority of the answers relate to positive emotions, like inspiration, motivation, joy, enthusiasm and involvement. Many of the teachers also felt insecurity, confusion and even annoyance. Fifty-eight (64 %) of the teachers reported that they felt emotions associated with negative words, 70 (77 %) had positive emotions and 40 (44 %) had mixed (both positive and negative) emotions towards teaching programming. Three respondents did not select a single word. Nine of the respondents stated that they felt both inspired and confused when it comes to teaching programming in primary school.

Table 3. Words that primary school teachers chose to describe their emotions to teach programming.

Negative	***n***	**Positive**	***n***
Annoyance	12	Enthusiasm	33
Fright	0	Inspiration	41
Confusion	23	Involvement	27
Desperation	1	Joy	36
Dread	4	Motivation	41
Insecurity	47	Optimism	25
Indifference	7	Passion	3
Total	**94**	**Total**	**206**

Fifty-eight teachers responded to the question "What do you think has influenced your attitude towards teaching programming in primary school?" The open answers were categorized into the following six categories displayed in Table 4. One answer can belong to several categories.

Table 4. Categorization of factors that influenced teachers' attitude to teach programming.

Own interest	**Participation in in-service training**	**External factors**	**Digitalization and technical development**	**Interested students and colleagues**	**Other**
12	17	12	7	9	8

Participation in in-service training seems to be an important factor for attitude in teachers' experience, as well as own interest in the subject together with external factors. Some teachers also mention students' interest and eagerness to program as a factor that explicitly influenced their own attitude towards teaching programming. Some teacher excerpts that express factors with positive influence on attitude are:

> My interest in mathematics and logic and the knowledge that this is something that is here to stay. (Teacher 1)
> My curiosity and interest for technical development. (Teacher 9)
> The fact that I have seen how interested students are of programming. (Teacher 20)
> It was, without a doubt, the in-service training that gave me courage to try. (Teacher 32)

Several teacher answers highlighted to positive impact of participating in in-service training. The responses that were categorized into the five categories where all positive. Six negative and two positive teacher responses to this question were labelled

"Other". Some of these critical teacher comments reflected insecurity, annoyance, confusion and opposition to teach programming:

> I feel that I am the only one in this world that do not know what this is all about. Neither has it come to my attention why this should be taught in school nor why it is so important. (Teacher 10)
> The management in our school believes that if something is technical or digital it is a good thing. The management does not care about pedagogical content. (Teacher 21)
> I try to avoid everything about programming and give that responsibility to those that are interested. (Teacher 50)

Finally, teachers responded to the open question: State your arguments why students should learn programming in primary school. Some teacher comments were:

> Everybody has the right to learn the basics of programming. There is a future demand for programmers. (Teacher 42)
> Everyone will clearly not need programming, nevertheless it is beneficial to know something about programming. (Teacher 29)
> It is not programming itself that is important, it is the additional value it gives to the student's mathematical thinking and reasoning skills. (Teacher 74)
> There are much more important things to learn. Let the professionals do the programming. (Teacher 50)

The first comment considers programming as an important skill that is certainly useful for everybody, while the second comment tones down the usefulness but agrees on that it is valuable to have some basic knowledge. The third comment focus on the possibility of a supportive effect of programming activities to mathematical thinking and reasoning skills in general. The last comment is clearly negative to the teaching of programming in school due to other more important content. These four examples are representative for the 24 responses to this optional question.

5.4. Correlation between the results concerning different RQs

Finally, we analyzed possible connection and correlation between the different research questions. The composite variables perceived preparedness and attitude are dependent. The correlation (Pearson's r) between perceived preparedness and attitude is 0.58.

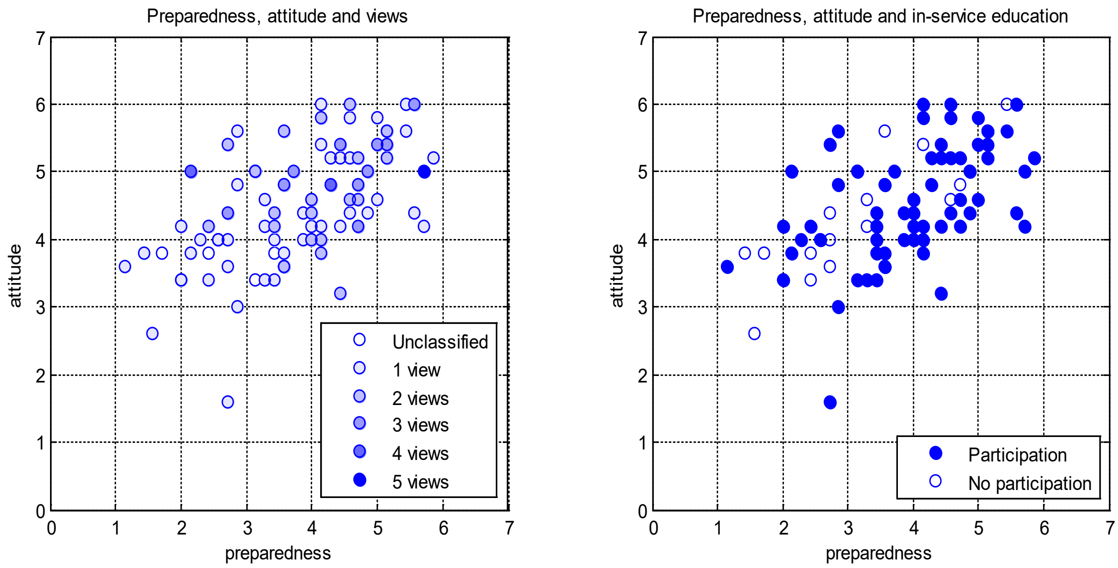

Figure 4. a) Perceived preparedness versus attitude with variety of view as shades of blue. b) Perceived preparedness versus attitude and participation in in-service education.

The variety of the teachers' views on programming increases slightly with preparedness and attitude. Many teachers with high preparedness and positive attitude express a broader and deeper view of programming. The teachers that participated in in-service education had higher perceived preparedness and attitude than those that had not participated. Note that 13 points overlap in Figure 4a and Figure 4b (78 distinct points, 91 data points).

5.5. Summary of results

The teachers' views on programming are very diverse. Most of the teachers in the study had a sequential view on programming that mainly connected programming to writing, giving and following of instructions. Programming was also considered to contribute to the development of logical thinking, serve as a valuable tool in problem-solving and be a useful skill in future work life (technological and progressional view).

Factors that contributed to a high perceived preparedness level were attendance in in-service training courses, supporting discussions with colleagues and existence of relevant teaching material at their school. On the other hand, several teachers had an unclear view on what programming in primary school actually is and some teachers expressed a clear lack of knowledge regarding programming and highlighted an explicit need for more education and support on the subject. The results also showed that teachers that had participated in in-service training courses had higher perceived preparedness than those that had not.

Several teachers saw programming as a positive element in the new national curriculum and important for students to learn. Many of the teachers (44 %) in the study approach programming with mixed emotions. For example, they feel inspired and confused or enthusiastic and insecure at the same time. Inspired and enthusiastic, because programming was considered a modern, relevant and useful topic and many teachers stressed that students have the right to learn programming in school to prepare for future work life. Some teachers felt that they were doing important and valuable work, and programming was also considered a source for inspiration and creativity in the mathematics classroom. In addition, participation in in-service courses gave the confidence to connect to the subject, and the interest and engagement by fellow colleagues and pupils were considered important factors that influenced their own attitude to the subject in a positive way. Some teachers also felt confused and insecure since programming is a new topic for almost all primary school teachers, and many of them found it challenging to position this new topic within the mathematics curriculum.

6 Discussion

The present study contributes with some knowledge regarding teachers' views, perceived preparedness and attitudes of introducing programming in the primary school classroom.

The study reveals that, in all the Swedish speaking regions in Finland, there are teachers that are interested and deeply involved in the development of teaching programming in primary school (late Spring 2017). Although many of the teachers have mixed feelings towards teaching programming, a majority of the respondents consider themselves to have a sufficient level of perceived preparedness and a positive attitude. It is not possible to measure views, attitudes nor beliefs in an absolute sense (Reid, 2006). The reported perceived preparedness to teach programming does not necessarily correspond to actual preparedness. It might be that a teacher with a high perceived preparedness to teach programming has a limited and somewhat narrow view on programming. For example, it can be the case that a teacher has in-depth knowledge of the Scratch program and experiences a high level of preparedness, but if another tool or environment is encountered the knowledge cannot be transferred to the new situation. Heintz and Mannila (2018) also noted and reflected on this when they summarized experiences from a large-scale computational thinking course in Sweden. Teaching programming has often been technology-driven and enthusiastic teachers and other actors have considered what they can do with a particular tool. Therefore, there might be a danger that a holistic picture of the learning path of children is not so clear for primary school teachers (Hemmi, Krzywacki, & Partanen, 2017).

The six identified categories of teacher views have connections to different frameworks for computational thinking developed in the literature. Some of the categories are clearly visible in the model of possible educational outcomes of programming in school (Popat & Starkey, 2019 p. 370) in the form of higher-order thinking skills (logical, algorithmic and problem-solving view) and curriculum and pedagogical design (technological and progressional view). The teachers' answers and the six identified categories also have a connection to the assessment framework by Brennan and Resnick. Many Finnish primary school teachers' use Scratch as a programming tool, and many have attended in-service training courses addressing Scratch. When they are to describe what they consider as programming, it might be that they, to some extent, view programming through the lens of Scratch.

Some of the teachers in this study had a broader and deeper view on programming in school, reflecting their knowledge, enthusiasm and engagement. A majority of the teachers had a positive attitude towards teaching programming, and they felt well prepared for this task. The findings suggest that participation in in-service training courses and education could have a positive impact on preparedness as well as on attitude and it may enrich the teachers' views on programming. Several teachers mentioned this as an important aspect also in their open responses.

On the other hand, some of the participating teachers expressed their lack of resources, content knowledge and a lack of a clear view of programming in school similar to the results of the study by Hijón-Neira et al. (2017) where the authors conclude, "However, many schools face serious teaching difficulties derived from the lack of adequate resources or properly trained teachers". Some teachers with lower perceived preparedness also had a more narrow, or negative, view on programming in school. Several of them also questioned the purpose and potential benefit of the inclusion of programming in the national core curriculum.

There were only a few explicit connections to specific mathematical content among the views. The reason for this could indeed be the openness of the survey question. However, along the lines with the concerns mentioned by Benton et al. (2017), it might be that the primary school teachers do not fully apprehend the interplay between mathematical and programming content and learning. As several researchers point out, there is a need to make explicit the links between mathematics and programming for teachers (Benton et al., 2017; Hickmott et al., 2018; Kilhamn & Bråting, 2019).

This also relates to several teachers' concerns about lack of knowledge, information and relevant materials to be able to concretize the general goals of the national core curriculum. At the time of the study (Spring 2017), there was a lack of educational material for programming, especially material with a relevant and explicit connection to mathematics.

7 Limitations of the study

The teachers who completed the survey may not need to be representative for the whole population of primary school teachers in Finland teaching at schools with Swedish as the language of instruction. The survey was sent out to the principals in primary schools in Finland with Swedish as an instructional language. Participation in the survey was nonobligatory, and it is unclear if the principals enabled all teachers at their school to participate in the study or if the survey was directed only to a few active teachers at the school. Therefore, it is not possible to give any response rate for the survey.

It can also be the case that the sample of teachers in this study has a bias towards higher perceived preparedness and attitude than the "average" primary school teacher. It is likely that some of the respondents were "early adopters" that included programming in their mathematics classroom even before the implementation of the new national curriculum. It may also be the case that the principal directed the survey only to selected active teachers at his/her school.

Unpaired t-tests were conducted. The group sizes in the unpaired t-tests were different (n=71, n=20), but the sample variances were approximately equal. According to Rusticus and Lovato (2014), there is only a modest risk for errors when testing the difference in means between groups with unequal sizes and approximately equal variance.

It is also important to note that the measures of teachers' attitudes and preparedness were all based upon self-reported data.

8 Conclusions

In this paper, we studied 91 Finnish primary school teachers' relation to programming and to teaching programming by analyzing their views on the subject, perceived preparedness to teach the subject and their attitudes towards teaching the subject. Although our study concerns a specific context, the results are important for the international research field as it sheds light on a current issue, the teaching of programming in primary school mathematics from the teachers' perspectives. It is also valuable to have studied teachers' relation to programming directly after the curriculum implementation 2016. The results of our study are relevant to the international research field, as several countries are attempting to implement programming in primary school curriculum from lower grades. The case of Finland can reveal general aspects important to consider also in other countries and in further research targeting the inclusion of programming in primary school. The study also contributes to our knowledge about how primary school teachers relate to teaching programming.

Today primary school teachers have access to a variety of different tools and material when teaching programming. Although some of the responding teachers claimed to have a lack of or insufficient material, the crucial aspect is to use the material properly. To do so, sufficient domain knowledge of programming (and mathematics) is necessary. The findings in this study indicate that teachers' views on programming are very diverse, and this may lead to inequality in education. The findings suggest that participation in in-service training courses and education could have a positive impact on preparedness as well as on attitude, and it may enrich the teachers' views on programming. The findings also suggest that there is a potential need for educational efforts to make the connection between mathematical content and programming more visible for primary school teachers, for example, in the form of well-designed concrete exercises and pedagogical practices. Those working with teachers, teacher education and the production of study materials have an important role in this continuous endeavor.

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Appendix A

Questions related to perceived preparedness

1. To what extent are you familiar with the parts of curriculum where programming is mentioned? I am
   1: not familiar at all 6: very much familiar
2. To what extent are you familiar with what the students are supposed to learn about programming? I am
   1: not familiar at all 6: very much familiar
3. How well do you consider that your school has relevant and useful material for teaching programming? Our school has
   1: no material at all 6: very much material
4. How well do you consider that you have relevant and useful material for teaching programming? I have
   1: no material at all 6: very much material
5. How much support have you obtained from your school in your preparation to teach programming? I have obtained
   1: no support at all 6: very much support
6. How much support have you obtained from your colleagues in your preparation to teach programming? I have obtained
   1: no support at all 6: very much support
7. How well prepared (knowledge, skills, material) do you consider yourself to be to teach programming in primary school? I feel
   1: not prepared at all 6: very well prepared

Appendix B

Statements related to attitude (1: I strongly disagree 6: I totally agree)

1. Programming is an important skill.
2. Programming is interesting.
3. It is important to teach programming in primary school.
4. I relate positively to teach programming in primary school.
5. I feel very insecure with new technology.

The responses to statement 5 were reversed.

Teachers' science camp experiences in southern Chile: Strengthening teacher identity and continuing education

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Science camps for teachers have been held in Chile for several years and are recognized as important opportunities for continuing education, but they have been largely ignored as instances for impactful reflection on teacher self-assessment and identity. This article presents the analysis of teacher experiences at a science camp held in southern Chile called "Explora Va!", which was designed not just as an instance for continuing education for teachers in scientific contents but also for individual and collaborative reflection aimed at strengthening educators' skills as agents of change in their institutions. The question of this study was: how do the participants represent their teacher identities based on their experiences at the "Explora Va!" Camp? Using a qualitative approach, the results of the analysis of narratives from teachers' camp journals are reported here. The narratives from these journals provide an account of two dimensions, personal and contextual, where the importance of the teaching profession and science teaching were explored beyond conventional disciplinary limits. Collaboration, innovation, and personal and professional growth at the camp served to signify and resignify professional identities based on common elements but attending to the particular circumstances and unique backgrounds of each teacher.

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1 Introduction

In the initial and continuing education of teachers in Chile, little focus has been placed on activities that encourage processes of self-criticism and reflection on one's own teaching work. However, in recent years the Ministry of Education (MINEDUC) has provided guidelines aimed at strengthening teacher identity and professional trajectories, generating spaces for teachers to exercise the reflective process regarding their own educational practices.

In a recent evaluation of educational gaps and teaching skills in Chile, teachers obtained a very unsatisfactory assessment in the dimension of reflection on pedagogical practices (MINEDUC, 2020). In the Southern region of Los Ríos, it was reported that only 19% of teachers undertake analysis of the pedagogical strategies that they use, 27% propose adequate strategies, criteria, and assessment instruments, and only 18% formulate reflections that allow them to determine which learning outcomes have been achieved by students (MINEDUC, 2017). Meanwhile, data on teachers' perceptions of their working conditions and professional development indicate that a low percentage consider analytical reflection to be important in their work; however, this factor emerges as quite deficient in teacher evaluations and has been shown to strongly impact teacher identity (MINEDUC, 2017).

In this context, spaces for intensive continuing education for teachers (workshops, courses, skills training) have included educational experiences centered mainly on delivering curricular contents and pedagogical innovations to improve teaching-learning processes. However, these spaces could, in addition, provide a key opportunity to address fundamental dimensions of the teacher's role, such as strengthening professional identity and self-assessment, if they were also designed as instances of collaborative reflection.

In Chile, science camps for teachers have been held for several years as instances of informal continuing education; little attention has been given to how this format addresses reflection and the impact that it can have on factors such as the self-assessment and identity of educational professionals. Likewise, there is little understanding of how, in said spaces, the collaborative model of professional research can be developed, through the collaborative identification of problems, in order for teachers to question and reflect on their own practices and those of other participants with the aim of strengthening teacher identity (Kennedy, 2014).

The research question is: How do the participants represent their teacher identities based on their experiences at the "Explora Va!" Camp? The objective of this research is to describe and analyze the experience of a five-day science camp for science educators held in southern Chile, part of a program called "Explora Va!" Science Camps for Teachers. This camp was designed not just as an instance of teacher training in scientific contents but also as an opportunity for strengthening the competencies of educators as agents of change in their schools and generating personal and collective reflection. Employing a qualitative approach, the results reported here are based on the analysis of narratives from teacher journals, where participants recorded their experiences and reflections during the camp.

2 Theoretical framework

2.1 Continuing education for teachers

The challenges that teachers face throughout their careers are many. In this context, continuing education for teachers represents a possibility to "develop individual abilities, knowledge, experience, and other characteristics as a teacher" (OECD, 2009).

The discourse encouraged in this area, especially at the government level and in public policy in education, holds that the improvement of teacher quality will positively impact the improvement of students (Kennedy, 2005). The continuing education of teachers is crucial to reach this objective (Kennedy, 2014).

In Latin America, the continuing education of teachers tends to be framed in a context of educational reforms and unfavorable work conditions for teachers, including low salaries and excessive workload. But teachers meet professional demands based on the concept of accountability to existing protocols and structures: "programs oriented at teacher development are inserted into antiquated systems in which norms and regulations tend to restrict the possibility of innovation" (Ávalos, 2004, p. 140). Thus, the results of continuing education programs for teachers are lessened when school realities are ignored and the knowledge, skills, and beliefs of teachers are not considered (Qablan et al., 2015).

Diverse studies (Fraser et al., 2007; Kennedy, 2005; Kennedy, 2014) point to distinct theoretical frameworks for analyzing models of continuing education for teachers. There are those related to teachers' personal situations—beliefs, interests, and motivations—and those related to social situations—the context in which the educational experience takes place, including feelings of support and favorable relationships between individuals and groups. Ultimately, a strong link should exist between theory and practice: intellectual stimulation and professional relevance.

The importance of professional reflection is well recognized, as it provides opportunities for the development of teachers' professional agency and control over their learning processes in order to achieve transformational learning (Fraser et al., 2007). Transformational models, as collaborative models of professional research, call for the collaborative identification of problems in order for teachers to inquire and reflect on their practice and that of others. These models have shown effectiveness in enacting educational change at individual, school, and system-wide levels (Kennedy, 2014). As the capacity for teachers' agency increases, so do their achievements in the development of professional autonomy. This stems from a democratic perspective that seeks for teacher motivation to emerge based on their interiority and identification with their role and for learning objectives to be articulated through their beliefs and values alongside the acquisition and application of new knowledge and abilities (Kennedy, 2014).

2.2 Science camps as spaces of continuing education for teachers

While the literature widely addresses camps as an informal learning strategy in the sciences, it tends to center on students' learning contents, with their teachers, in some cases, as participants alongside them (Logerwell, 2009; Luehmann & Markowitz, 2007; Antink-Meyer et al., 2016; Barab & Hay, 2001).

The majority of research on the effectiveness of science camps, offered for different purposes, has been carried out with participants from primary or secondary school (Karaman, 2016). In these studies, experiences influenced participants' attitudes, interests, and learning in science, both short- and long-term, yet while positive results have been demonstrated in terms of affective aspects, there is little evidence available on the impact of camps on cognitive aspects related to the learning of contents (Antink-Meyer et al., 2016).

Meanwhile, studies on camps for science teachers are scarce and focus on content. Jacoby (2013) describes a camp for science teachers as "training" where the teachers learn by "doing" in order to motivate their students to learn physics. A similar approach is offered by Post-Zwicker and Guilbert (1998), who describe a "Plasma Camp" designed to help secondary teachers carry out research and projects in the classroom.

Given the shortcomings of education in the sciences and the limited offering of associated programs (Aflalo, 2014; Backhus & Thompson, 2006), professional development activities for working teachers could compensate for these deficiencies. Informal educational scenarios, such as science camps, constitute an ideal learning environment that permits participants to compensate for their lack of knowledge in the sciences (Fields, 2009; Foster & Shiel-Rolle, 2011; Leblebicioglu et al., 2011; Spector et al., 2012), which is generally a product of their weak initial education in the area (Karaman et al., 2016).

The literature regarding the effectiveness of science camps for teachers is limited (Naizer et al., 2003; Wallace & Brooks, 2014; Karaman et al., 2016), though it has been demonstrated that participants show significant progress in learning, independent of the discipline that they teach (Karaman et al., 2016). More research is required to examine the positive experiences of teachers in science camps as an important contribution to knowledge in this area of education (Karaman et al., 2016).

2.3 Continuing education and teacher identity

Ruohotie-Lyhty (2018) states that teachers' agency is linked to their identity narrative. This is true, firstly, because their professional path is underpinned by self-perceptions of their role and labor based on what they expect, believe, and value in their work. Secondly, narrative activities allow them to organize and give meaning to their experiences, constituting a process of agency using their individual experiences and the interrelationships of educational communities.

In Chile, research related to the concept of teacher identity has awoken a growing interest in the topic since the 1990s. This construct refers to how teachers experience their work subjectively, as well as the perception of the profession held by their colleagues and the rest of society.

While the conceptual notion of professional identity has generated strong and growing interest within academia (De Tezanos, 2012), a shared definition does not yet exist for this idea, with distinct definitions and understandings present in the literature and utilized in different manners in the educational sphere.

A bibliographic review of the construct carried out by Beijaard et al. (2004) pioneered the systematization of research principals in the area of teachers' professional identity, reviewing 22 articles published between 1988 and 2000 in order to determine which characteristics are essential to research on teachers' professional identity, how current research can be characterized, and what problems need to be raised when studying this topic. Among the most important characteristics, the authors highlight that it is a continuous process of interpretation and reinterpretation of experiences; it entails the person and their context; it contains sub-identities; and it points to the centrality of agency, with reference to the need for teachers to be active in the process of professional development. Ultimately, the review concludes that in the majority of studies, the concept is defined differently, or that a clear definition of teachers' professional identity is not present.

Beijaard et al. (2004) also contend that in the definition of teachers' professional identity, the majority of studies emphasize the personal sphere, with an underestimation of the contextual, which also plays a role in identity formation. Thus, in the formation of teacher identity there exists a constant tension between agency (the personal dimension of teaching) and structure (what is socially rendered), translating as a complex equilibrium and dynamic in which self-image is balanced with a variety of roles that teachers feel they must play (Volkmann & Anderson, 1998).

Teachers' professional identity forms part of the social identity of an individual as well as the self-definition that each teacher creates of their person. Beyond the multiple considerations involved in attempting to characterize teacher identity, consensus exists on the fact that it is not a stable entity but rather a process subject to continuous change and construction of meaning and interpretation. Likewise, there is general agreement that teacher identity is influenced by previous experiences, the personal characteristics of the individual, and professional contexts (Beauchamp & Thomas, 2009; Beijaard et al., 2004; Flores & Day, 2006; Hong, 2010; Pillen et al., 2013; Schepens, et al., 2009).

Pillen et al. (2003) offer two arguments that attest to the utility of the construct of teachers' professional identity. On the one hand, it can be used by teachers as a resource that provides meaning, and on the other, it can function as an analytical lens for teacher learning and development. Additionally, Beijaard et al. (2004) state that teachers with a positive self-perception of their identity generate mechanisms that may even eliminate discontent in the face of precarious work conditions.

Some authors hold that the notion of teachers' professional identity is closely related to self-image, which is not separate from the life story or narrative of the individual (Kerby, 1991). In this sense, some studies emphasize the relationship of teachers' professional identity to concepts such as reflection or self-assessment (Cooper & Olson, 1996). It has been posited that it is not possible to speak about oneself without reflection, so to develop a self-image as a teacher is it necessary to develop auto-reflective tools (Antonek et al., 1997). Nonetheless, it is important to consider that the construction of teachers' professional identity begins with their initial education and extends throughout their full professional career, with the potential to be continually reconstructed in spaces of continuing education.

2.4 Science teacher identity

Science teacher identity is an area closely linked to this study since the camp sought to reinforce the identity of science teachers using the ontological coaching approach. Regarding science teacher identity, Avraamidou (2014) carried out a metastudy to examine how its construction has been conceptualized and researched, synthesizing 29 empirical studies that give an account of the utilization of this concept. Science teacher identity offers a powerful and multidimensional lens to study teacher learning and development. It highlights the role of context for the teacher, casts light on teachers' personal stories in relation to science, and permits the examination of the impact of personal characteristics on teachers' learning and development (such as age, gender, emotions, and ethnicity).

A literature review centered on science teacher identity reveals characteristics that can build greater understanding of the particularities of teacher identity as a whole. Some research addresses the disciplinary characteristics of science and science teaching (Badia & Iglesias, 2019; Avraamidou, 2016), while other studies consider community aspects. In this sense, Badia and Iglesias (2019) highlight that teacher identity is composed of experiences with science when teachers were in school and their adoption of the concept of effective science teaching. Likewise, Avraamidou (2016) describes three types of teacher identities, including those centered on the conception of what it means to be a teacher, feelings regarding the scientific discipline, and specific ways of teaching science. Meanwhile, Kier and Lee (2017) propose two key characteristics in science teacher identity: the conception of teaching and learning and the conception of the nature of science that teachers possess.

However, not all of the characteristics identified by these studies reveal the social aspect of science teacher identity, as posed in Avraamidou (2016), Beijaard, Meijer, and Verloops (2004), and Rushton et al. (2020). For Avraamidou (2016), teacher identity consists of interaction between educational biographies and institutional discourses. Meanwhile, Beijaard, Meijer, and Verloops (2004) indicate characteristics that involve the individual with their context, in which teacher identity consists of sub-identities and agency is critical; this points to the necessity of teachers being active in the process of professional development. Rushton et al. (2020) emphasize the importance of shared identity and group membership as catalysts for the development of positive teacher identities, suggesting that being part of a group and socially constructing identities are crucial factors for strengthening teacher identity in general. In this context, teacher identities appear to be reinforced in communities in which their stories are heard in an active and critical manner in order to empower them as agents of change. Correspondingly, the camp experience examined in this study will be understood as a space in which teachers resignified their identities through reflective spaces, strengthening those aspects related to the meaning of teacher identity through shared experiences.

3 Methodology

3.1 The “Explora Va!” Science Camp for Teachers

Motivated by teachers’ interest in participating in educational experiences similar to the summer science camps for youth held in countries such as the USA, Canada, and Finland, the Explora Program of Chile’s Ministry of Science, Technology, Knowledge, and Innovation has implemented “Explora Va!” Science Camps for Teachers since 2017. These camps are utilized by teachers as a strategy to partake in continuing education in the transversal teaching of science that is in tune with the guidelines of the Ministry of Education. Teachers and administrators from all K-12 levels apply to these camps.

This study stems from the camp held in January 2018 in the town of Llifén, located in the Region of Los Ríos in southern Chile and surrounded by an environment of native forests, rivers, and lakes characteristic of the zone. The camp was implemented over the course of five days with a residential arrangement (i.e., “sleepaway camp”). One-hundred-and-four teachers, administrators, and educational assistants from around the country participated, coming from distinct disciplines and establishments both public and private, rural and urban, and with varying levels of social vulnerability.

The objective of the camp was to strengthen teachers’ competencies as agents of change in their schools through the presentation and development of tools that allow value to be added to the educational community based on different dimensions of science and technology. The Explora Program understands the teacher as an agent of change with the capacity to design, manage, and implement activities inside and outside the classroom in order for students to develop thinking skills and individual attitudes in the sphere of science and technology: critical and reflective thinking, the development of curiosity, the ability to plan, and rigor and honesty in the collection of data and evidence.

During the camp, as secondary objectives, the teachers reflected on their own abilities and attitudes in the area of science and technology, the meaning of their teaching, and their links to other disciplines, as well as identifying the main nodes that hinder the development and appropriation of the skills and attitudes inherent in science and technology on the part of students. They also visualized alternative methodologies for science teaching based on reflection on their practice in collaboration with other teachers and school professionals, and generated learning communities for the collaborative construction of knowledge about educational improvement.

From the methodological point of view, the camp was designed using ontological coaching (Medina et al., 2019; Ortega, 2012) and the concepts of the reflective practical identity model (Galaz, 2011; Schön, 1983). In the case of the former, it is effective in the sphere of teacher professional development “because it contributes to the elevation of the consciousness of the teacher in diverse dimensions (including emotional and motivational) that condition ways of acting in educational contexts” (Medina et al., 2019, p. 21). Therefore, it leads participant teachers to recognize and seek out solutions to problems (Ortega, 2012), stressing their conceptions and knowledges in the areas of teaching, science, and their own pedagogical practices.

The reflective practical identity model is present in the leadership of teachers in the training program, who interact with their peers through dialog, daily coexistence, collaboration in activities, and moments of personal introspection. The activities promote processes of reflection and introspection on teachers’ own identity constructions. The teachers recognize personal elements of what is learned socially on the nature and epistemology of science, which influences their agency as educational practitioners, and they resignify and reaffirm different components of their professional identity.

Based on both of these theoretical contributions, this instance proved to be meaningful for the teachers because they were able to reflect on themselves, their professional activities, and their educational environments and contexts. These spaces strengthened their competencies in terms of scientific research and the teaching of it, as well as those abilities linked to attitudes and values, including active listening, teamwork, and assertive communication. In all of the activities at the camp, including the journal-writing, there were moments aimed at generating a “bridge” between what was learned and experienced at the camp and teachers’ agency in their own educational communities.

The contents of the camp included, in a progressive manner, the characteristics of scientific knowledge and their pedagogical transposition; the principles of science education based on inquiry; the links and overlaps between the sciences and other curricular subjects; reading comprehension; the error as a source of learning; stereotypes and learning in the sciences; appropriation and curricular development; the importance of questions in the development of scientific inquiry; the organization of learning environments; and lastly, the creation and monitoring of communities.

3.2 Design of activities

Activities were coordinated through a set of modules that provided for the integration of the Participatory Action Research methodology together with ontological coaching. In total, 11 pedagogical and ontological activities were included, complemented by recreational activities (see Table 1).

Table 1. Activity programs implemented at the camp, in chronological order.

Title	Dimension	Description
1. I am part of a "rope team"	Ontological	The "rope team" (referring to the concept from climbing/mountaineering) as a metaphor allowed for the strengthening of personal and professional development collaboratively and synergistically, enhancing camaraderie, unity, cohesion, and teamwork, and utilizing them as a learning process. Established at the start of activities, each member of the "rope team" of 15 teachers, diverse in gender and teaching level, together with a facilitating teacher, made and shared a postcard with their expectations and wishes when arriving at the camp.
2. Pedagogical paths	Pedagogical	These were used to carry out a collective diagnostic on teachers' pedagogical practices. On a path with three "stations" passing along a scenic natural route (lake, forest, mountain), each teacher reflected on their teaching career, mediated by lead questions: What has my relationship been with nature and how do I feel today in connection with nature; what has the nature of my work as a teacher been; what experiences have distinguished my career trajectory; how do I feel today in relation to the profession? A pedagogical assessor, as facilitator, offered context to each of these questions, making a connection between the path through nature and its changes, and the changes and experiences in their career paths in terms of identity and teaching practices. Teachers shared their reflections in small groups, and the facilitator summarized them in a plenary session afterwards.
3. The day's harvest and journal writing	Ontological	At the end of each day, the teachers did their journaling, a reflective activity oriented toward ontological coaching, which consisted of recording their daily reflections on a form with open-ended questions, "harvesting" the essential elements gathered during the day in the context of activities, science teaching, and learning at the camp. The last day, the question focused on the process of personal self-assessment in terms of the full experience of the camp. These narratives were utilized to identify and analyze aspects related to teacher identity in this continuing education experience.
4. Delving into our pedagogical practices	Pedagogical	On the second day, through small group conversations, group inquiry was carried out into pedagogical practices and conceptions, considering the nature of pedagogical knowledge in the sciences. The teachers were shown the elements of a learning community in order to gain a collaborative vision of their work across different levels. The questions for this exercise included the following: What factors explain collaborative work, and what advantages and disadvantages does it have; what causes and consequences can impinge on collaboration; what elements of the camp experience mirror the experience of the classroom, and how? The group conclusions were shared in a plenary session.
5. Identifying pedagogical nodes	Pedagogical	The objective here was for teachers to identify the critical nodes that impact their pedagogical work, considering the change of paradigm in science teaching from instructional to constructivist. After carrying out a recreational activity of moderate difficulty (kayaking, trekking, climbing), they discussed as a full group the negative and positive emotional states experienced, and how these reflected daily pedagogical practices. Group discussions addressed the value of nature as a classroom, its potential as a complement to the traditional classroom, and the challenges that its utilization entails. Then, the teachers looked for solutions to these problems and challenges, using the elements of the previous recreational activity, that is, collaborative and reflective work and the relevance of personal pedagogical knowledge as a source of improvement for science teaching. The results were shared in a plenary session.

6. Project creation	Pedagogical	Each "rope team" designed two plans of action to implement as solutions to the critical nodes identified in the previous activity. Each project contained the elements identified in the construction of learning experiences that promote inquiry in any scientific discipline, using the individual and collective pedagogical knowledge of the participants as a theoretical framework. The activity was divided into four stages (one per day). During brainstorming (stage 1), they agreed on two plans of action to work with. In project formulation (stage 2), they developed the theoretical framework, hypothesis and objectives, methodological design, expected results, and projections. During poster creation (stage 3), they synthesized the main elements of the project for their exhibition, feedback, and exchange at the "Teachers' Convention" (stage 4).
7. Inquiry and experimentati on in the natural sciences	Pedagogical	Visiting the region's coastal area, the teachers carried out an inquiry activity on the identification of marine fauna in the intertidal zone based on the Inquiry Cycle (Feinsinger, 2014). Utilizing this natural laboratory, and based on guided questions on zone formation and diversity, they carried out observation, information gathering, data analysis, discussion, synthesis, reflection, and generation of new questions. With their collected information and the instrumental information available, the teachers were challenged to construct an inquiry activity for their students incorporating the principles of the Inquiry Cycle and those of a learning community (i.e., construction of collective, conversational, reflective knowledge, centered on the error as a source of learning) with a focus on the process of participatory action research by students. The question was constructed based on the identification and classification of samples with the parameters they defined considering the level of their students. The results were presented in a plenary session using scientific posters.
8. Inquiry and experimentati on in the social sciences	Pedagogical	Similar to the previous dynamic, this activity involved the construction of social scientific knowledge in an on-site museum, where the teachers had to observe and characterize the visitors, recording patterns of dress, age, gender, and behavior associated with specific sections and most-visited attractions in the museum. This characterization permitted the exploration of a social laboratory as a source of scientific inquiry to be extrapolated to nearer social contexts (to the teachers) such as the school yard, the classroom, the neighborhood, or the community.
9. Agents of change and the creation of learning communities	Ontological	The objective was for the teachers to adopt significant changes in terms of their vocation and their empowerment as agents of change in their educational communities. To establish this dynamic, the world café was utilized, a methodology that creates networks of collaborative dialog utilizing powerful conversational processes. Complex questions were explored, connecting the different conversations in order to generate new discoveries. The findings, learning, and opportunities for action were systematized and shared in a final plenary session.
10. Dialogs with leading ambassadors in education	Ontological	This was an instance of conversation moderated between a teacher/principal of an educational establishment promoting notable STEAM initiatives, and small groups of teachers. The ambassadors related their experiences in educational improvement, achievements, difficulties, and solutions. They reflected on their skills and attitudes toward the sciences, the meaning of their teaching, and their links with other disciplines. Additionally, a dialog with students was carried out with noteworthy participants in STEAM activities and/or students who had experienced significant motivation in the area thanks to their teachers. They related their school experiences and how this influenced the quality of their learning and their valuation of science and technology.
11. Community celebration ceremony	Ontological	This ceremony encouraged the participation of each attendee in order to recall the experiences of the camp, considering the networks built, the reflections developed regarding their teaching practices, and the changes in attitude that arose within them. The ceremony contained symbolic elements that appealed to the emotions and community spirit, with a recap of everyone's experiences via audiovisual captures in an inspiring, reflective, and festive atmosphere. Participants' sense of belonging and loyalty was encouraged and strengthened.

3.3 Objectives and study design

The objective of this study was to analyze the continuing education experience of the teachers participating in the Explora Va! Camp and the effects of this experience on teachers' professional identity, from the perspective of the teachers themselves.

The general methodology utilized for the gathering and analysis of information followed the protocols of a qualitative study from a comprehensive theoretical perspective. The journal analysis was carried out using directed qualitative content analysis centered on the contextual meaning of the text (Tesch, 1990). Hsieh & Shannon (2005), analyzing three approaches to content analysis, recommend this method when key concepts have been identified that guide the coding of initial categories. In the case of this study, current theoretical developments regarding the construction of teacher identity served as an analytical lens for understanding teachers' reflections on the processes experienced at the camp.

The technique utilized for the collection of information was the daily recording of statements by each participant, which were written in a reflection journal designed using the Google Forms application. With the objective of evaluating the impact of daily programmed activities, participants answered questions in the "daily harvest" that invited reflection on the camp in a different manner each day. The daily reflection questions were related to the objectives of the day's activities, which were designed by the camp's pedagogical team (see Table 2).

The aim for was the teachers attending the camp, who formed the unit of analysis of this study, to carry out an open, evaluative reflection on the dimensions of the experience that affected them most. These narratives would later by analyzed. The discourses registered in the journals were examined through qualitative content analysis (Denzin & Lincoln, 2005), and the answers to the questions were entered as text documents into the qualitative analysis software MaxQda (version 11 for Windows). Coding was initially carried out by seeking textual fragments that indicated reflections associated with teacher identity. In the second level of analysis, these textual fragments were placed in pre-determined categories of teacher identity, either personal or contextual. Once the texts were grouped into these two category types, the next step was to select and interpret those that indicated self-perceived changes that teachers believed were a result of the camp.

Table 2. Reflection questions in the journals.

Day	Activity/Reflection questions
Journal day 1	Hello and welcome! Today was a day for us to get to know each other and talk about our expectations. Write about your sensations, experiences, and impressions from today and what you hope for during these five days of Camp. We'll also reflect on who we are, and on our path as teachers. What have you harvested from this pedagogical path?
Journal day 2	Hello! Today we carried out challenging activities in the fresh air, reflecting on teamwork and learning communities. How do you think that you could promote collaborative work in your pedagogical role? What advantages and/or disadvantages do you see in teamwork? What factors could impinge on collaboration? Tell us your reflections. During the course of the afternoon we also talked about the difficulties that we confronted as professionals (pedagogical nodes). What sticks with you from this conversation?
Journal day 3	Today we visited various places where we could sense and experience the construction of scientific knowledge in distinct disciplines. What are your reflections on the visit and inquiry carried out? Why do you think the research question is important, and in what way do you think that today's experience could be useful to you in your pedagogical practice?
Journal day 4	Today we created an inquiry activity for our students as a group. What did you take from this experience of collective construction? We also had the opportunity to converse with educational leaders. What did you harvest from this exchange with your colleagues? We are one day away from the end of Camp. How have you felt during the past several days? (Consider both the positive and negative aspects of this experience).
Journal day 5	On this final day of camp, compare the agent of change that you were before and the one you are today. Reflect on this.

It should be noted that in order to grant validity and reliability to this study, triangulation was carried out (Martínez, 2006). Thus, during the camp and research process, professionals participated from the areas of anthropology, sociology, the natural sciences, and education, offering different disciplinary viewpoints. These materialized through discussions and dialogs during in-person meetings as well as via digital communication.

4 Results

4.1 General findings

The narratives in the journals give an account of the heterogeneity of teachers' experiences at the camp and relate to the dimensions of their professional identity.

The most abundant categories represent aspects that are mostly confined to the personal dimension of teachers' professional identity. Among these, reflections that describe self-perception stand out, as teachers valued positive personality traits that were not fully recognized before. Narratives of change in their self-definition as teachers also emerge, as well as a resignification of their role in the educational community.

In terms of the categories of the contextual dimension, those related to professional performance and the difficulties of working in adverse political/institutional settings emerge. This class of category appears in much lesser magnitude than those of the personal dimension. Figure 1 illustrates the main emerging categories associated with the journal texts.

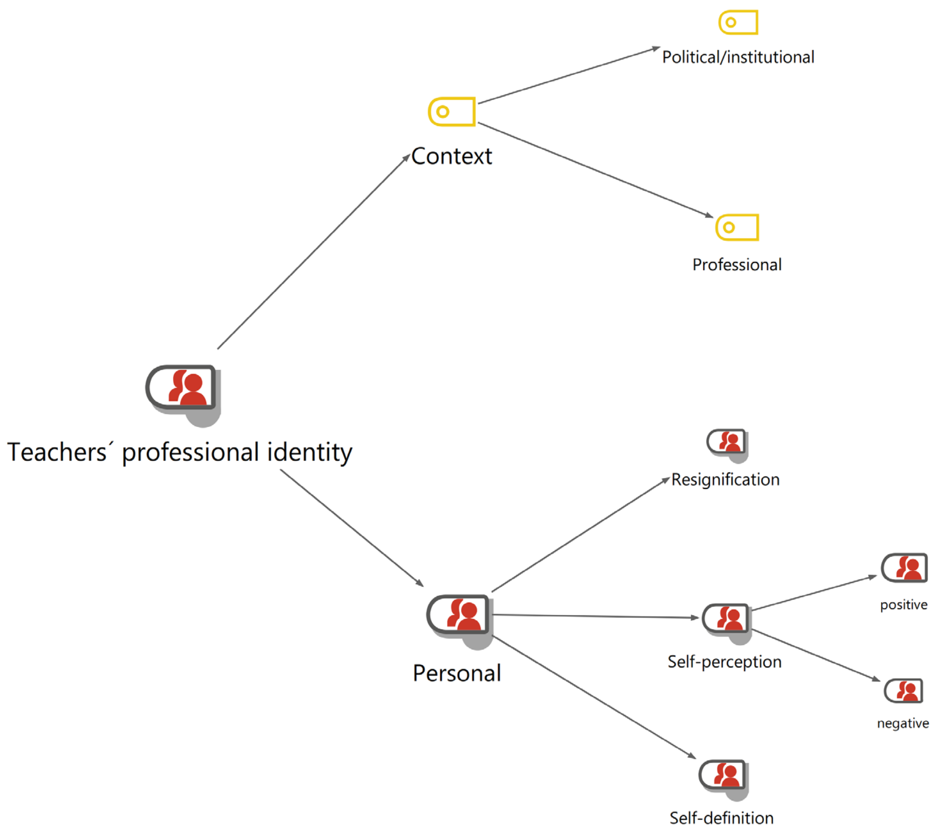

Figure 1. Main categories associated with the journal texts, related to dimensions of teachers' professional identity.

4.2 Personal dimension

In their narratives, the teachers mention expectations that are strongly linked to the emotions and feelings that emerge before an unfamiliar situation. Thus, in their first entries they speak of anxiety alongside the desire to enrich their professional and pedagogical practices through exchanges with their peers and the tools provided in the activities.

However, gradually they begin to indicate that the natural and social environment developed at the camp has offered them a distinct personal and professional learning opportunity centered on themselves. Thus, they signify it as a unique time and space to rethink both satisfying and challenging experiences. In this sense, they emphasize common elements in the definitions of their identities, mainly associated with professional and affective factors that they share, such as passion for their work and love for their students, along with recognizing shared hopes and issues.

In this regard, some teachers express that they have experienced changes in terms of their behavior in the presence of others. They self-define as more sociable, which is expressed in the desire to get to know others, have fun, and feel comfortable with their peers. This attitude invites them to innovate as well as to confront failure or error in the presence of others.

> "Before the Explora Va! Camp I felt like I had fewer social skills, less ability to advance on projects, was less decisive, with little capacity to innovate, to take the first step, with fear of errors or failure. After the Camp I see myself looking hugely forward to working, creating spaces to improve students' learning, with more strategies to motivate them" (A.S. Primary School Teacher)

Self-confidence is another category that emerges from the narratives. Somewhat related to the sensation of greater sociability mentioned previously, some teachers self-defined as having fewer apprehensions about relationships and interactions with others.

> "Before arriving to the camp I had many fears, apprehensions, I didn't want them to ask me about some specific issue because of fear of talking, I didn't know how I was going to relate with the other people and much less what was coming. However, from the first day that I arrived, those fears left me" (P.A. Primary School Teacher)

A resignification of cognitive capacities also seems to have occurred. Some teachers describe leaving the camp with more skills and capacities for learning and knowing. These changes are expressed in the narratives as feeling more intelligent, understanding things, and feeling more intellectually capable.

> "The agent of change that leaves this Camp is one who carries a backpack full of new experiences and a great enthusiasm for innovating in my pedagogical work. I think that the one who arrived was a person with expectations to learn only knowledge perhaps, but the one who leaves today is a person that knows that the most important thing is developing abilities to be able to understand that knowledge and capture it in a good way for my students" (R.I. Primary School Teacher)

Some teachers highlight the resignification of their teaching role during the camp experience following a loss of motivation in their professional work influenced by colleagues at their school. The experience of the camp is underscored as an opening-up and change of internal locus from worrying about problems to worrying about solutions.

> As a teacher, I only have three years of work experience, so arriving at my first job, which is where I still am today, I came with my backpack full of ideas and projects to do. Thankfully, I found myself with an administrative team that supported each of my ideas, so we were able to do many new things. However, as time passed, I began to notice the malaise of my colleagues, which greatly discouraged me, causing me to limit my creativity and desire to generate changes in the establishment. During this camp, I've regained my desire to contribute to change, empowering myself based on my abilities and changing focus, because I can't limit the well-being of my students for the comfort of my colleagues" (Pre-school Educator)

In addition, they resignify their work based on the use of their surroundings as a space of learning in the sciences and as an alternative to the classroom. Referring to specific activities from the camp, they describe feeling like students again and experiencing newfound curiosity. The notion of "meaningful learning" repeatedly appears as a justification for this type of educational experience. They also value being introduced to inquiry-based learning and its transversality in distinct subjects, but they identify it mainly as a resource applicable in natural environments and to a lesser degree in their social environments. One example of this distinction came about when certain teachers recognized the difficulties that they had experienced in observing and forming questions in the case of social inquiry. In general, they internalized the importance of the question within the systematic process that the inquiry method addresses, justifying its relevance as a guiding element for learning amongst their students as well as in the process of scientific research.

It follows that some narratives give an account of reflective processes that place value on practical knowledge not centered exclusively on curricular knowledge; the latter often generated fears of exposing educational weaknesses among the other teachers.

> "From that point I was no longer afraid of giving my opinion, presenting my lived experiences, giving my ideas to execute some project, or I even thought of the possibility of stopping on the stage to share one of our big projects. I feel that I'm leaving as an agent of change with the desire to implement a new type of education for my students and also share my experiences with my colleagues and motivate them to participate in the next Camps so we can all be agents of change and we can connect and establish a community with innovative agents" (P.A. Primary School Teacher)

Meanwhile, some teachers exhibit a greater acceptance of the uncertainty and risk involved when desiring to innovate in pedagogical practice. They also indicate that the experience helped them as a new motivational boost to continue on the same path, but now with a wider range of possibilities in their work.

Other aspects that emerge from the narratives center around a very positive evaluation of the camp, of the organization's personnel and the design of the activities. One aspect that stands out is the frequent mention of the experience as a space of emotional support, which could point to the scant concern for mental health encountered by teachers today in the exercise of their profession. This corresponds to the links forged in the social dimension of professional identity, specifically in terms of the recognition of teachers' work, which they consistently find to be less than deserved. Participants also recognize the prevailing educational accountability systems in Chilean education, in which standardization and measurements play an overbearing role.

4.3 Contextual dimension

Although present to a lesser degree, the narratives that point to changes in the contextual dimension of teachers' professional identity illustrate a sensation of greater professional competence and greater ability to achieve meaningful learning with their students. This latter point, for many teachers, constitutes the ultimate objective of their professional identity and duty throughout their career paths. It provides motivation that helps them withstand the various difficulties that arise at personal, institutional, political, or cultural levels.

Amongst these contextual factors, the teachers' entries emphasize the scant recognition of their role and the conditions in which they carry out their teaching. Both situations are linked to the characterizations of their work that they believe to be present in Chilean society, as well as to the discrepancies between what they desire and what they experience on a daily basis in their schools.

> "Oftentimes we feel insignificant in this big world, and as teachers: an undervalued job. However, today I felt proud to be one. Listening to my other colleagues talk about their work and their labor with so much love and devotion, made me feel truly important as a teacher… Today my harvest is that, feeling proud to be a teacher." (P.H. Secondary School Teacher)

Some professors describe feeling more productive and creative, and their discourses reveal a greater valuation of collaborative work. In this sense, they express that the camp helped them realize that they feel alone in their profession at times. However, this new experience and the work with their peers through dialog and reflection allowed them to visualize problems shared in common and feel like part of a professional community. Indeed, in some cases, a heightened opinion of collaborative work is communicated, and there seems to be a change of attitude toward innovation in educational work based on viewing knowledge as a process of inquiry more than a transmission of information. It follows that certain attitudes toward teaching through cooperative strategies also shifted.

> "I timidly began to assume certain roles that generated some different and positive dynamic in my establishment, but now I have realized all the dimensions in which one can develop that attitude of change. The camp opened my mind to take on that challenge. The experience has given me the capacity to analyze my work and activate my creativity to incorporate the meaning of collaborative and personal work within my practice, as well as to realize that being an agent of change is much more than a mere activity, it is a professional lifestyle. Before I simply thought that one different activity was enough, but today by contrast I know that this is not so, rather, I must have a sustained, patient, responsible, and challenging attitude, day-to-day, in my future pedagogical work" (T.D. Secondary School Teacher)

5 Discussion

The journal narratives provide an account of key moments and insights experienced during five intense days of camp. The teachers make explicit, among other things, the definitions that each one has of themselves as professionals, the place that they occupy in their work context, and their motivations regarding the work of educating.

In this sense, the experience of the camp acted as a mechanism that facilitated collective reflection with the goal of learning from the experience and from the others present, in order to generate knowledge from and for the practice of teaching (Schon, 1998). This type of experience could have a strong effect on positive self-perception, and therefore on changes in conduct in which self-perceptions organize experiences and guide behaviors (Swann et al., 2007).

The participation of teachers in the camp allowed them to question their personal impressions, beliefs, and expectations about traditional forms of capacitation and improvement that they had experienced on other occasions. These experiences have been described previously as institutional actions guided by experts that deliver disciplinary contents, procedures, or pedagogical methods that aspire toward improved professional performance, aimed at resolving the shortcomings or deficits in teachers' work (Vezub, 2013). Differences have been reported in this context between the theoretical aspects of teacher education and their pedagogical practice in situ at schools, claiming that the former are removed from the necessities and interests of educators in the daily realities of their work and professional roles (Qablan et al., 2015; Busquets et al., 2016; Marcelo & Vaillant, 2013; Vaillant, 2019). Theory often points to a rationalist, technocratic, and utilitarian vision steered toward individual performance, based on quantifiable elements of effectiveness-efficiency with pre-established models or navigation charts approved for achieving expected learning outcomes in students. However, such approaches do not consider the voice of the educators—their cognitive, emotional, and valuation processes (Brito, 2018; Casassus, 2006; Ferrada, 2017; Vezub, 2013).

This conservative approach to education is not absent from the area of the sciences. Several teachers participating in the camp declared an initial expectation of gaining tools that would simply enable them to teach the sciences (mainly natural). Although based in an epistemology of moderate rationalism, they ultimately voiced the need and interest to obtain knowledge and teaching strategies to promote more meaningful learning in their studnts. This is important in terms of how they address the world stage and contemporary global issues, which due to their complexity must be analyzed and resolved from different perspectives based in the natural sciences as well as the social sciences (Busquets et al., 2016; González-Weil et al., 2014; Fernández et al., 2002; Quintanilla et al., 2020; Triviño, 2018).

Fernández et al. (2002) argue that these ways of thinking about scientific activity are based on epistemologies that have produced distorted visions of science teaching at different educational levels. Such visions tend to hold science as a solitary, individual activity centered on observation; experimentation without hypotheses or guiding theories; rigid and timeless work sequences disconnected from the social issues of the moment; and an analytical and cumulative version of knowledge.

Likewise, these conceptions related to the radical rationalist or positivist view (Quintanilla et al., 2020) define teacher education based on a version of science teaching that purports to disseminate scientific knowledge to students in an abstract, standardized, and homogeneous manner that is detached from the present times. This vision, reflected in the writing of some teachers participating in the camp, could be linked to their own school trajectories, their undergraduate education, and other instances of continuing education in which they have participated, which together have instilled a certain way of conceiving of and teaching science (González-Weil, et al., 2014; Quintanilla et al., 2014).

Based on a moderate rationalist epistemology, many participants demonstrated interest in and need for increased knowledge and teaching strategies to promote more meaningful learning for their students in terms of how they can confront the world stage, which today includes environmental problems, food shortages, mass migration, pandemics, large-scale violence, and discrimination, among other issues. Due to their complexity, such issues must be analyzed and resolved from different perspectives, based on both the natural and social sciences (Busquets et al., 2016; González-Weil, et al., 2014; Fernández, et al., 2002; Quintanilla, et al., 2020; Triviño, 2018).

Accordingly, discrepancies can be observed in the camp participants in terms of differing perceptions, conceptualizations, objectives in science teaching, and teachers' genuine interest in helping their students achieve scientific capabilities and skills, which may be linked to the personal experiences of each educator with science, and which in their professional work moves between positivist and moderate rationalism (Quintanilla et al., 2020). Lapasta (2018) states that it is necessary to understand professional learning from a constructivist viewpoint, incorporating learners' previous, current, and future experiences: "…if epistemological and psycho-pedagogical reflection is not promoted at the same time in relation to the area of disciplinary knowledge, it is unlikely that they will manage to make a change in terms of the practice of science teaching" (p. 113). These elements are important for achieving innovations on the current world stage, where science resonates energetically as a means to find certainties and answers. Likewise, to achieve changes in teachers' practices, instances of continuing education should incorporate specific, concrete activities where innovations and new theoretical advancements in the disciplines taught are undertaken in classroom contexts. This implies teachers experiencing, in a situated manner, the knowledge that has emerged from new research (Solbes et al., 2018). Camp activities involving natural and social inquiry allow the experience of constructing and teaching scientific knowledge based on visions of education removed from disciplinary limits and conventional teaching strategies aimed at simply reproducing knowledge. They are situated in scenarios that go beyond personal professional identity toward collective identities, related to collaborative learning and teaching of science in meaningful settings, which can be carried over to the classroom with the adaptations and contextualizations that teachers deem necessary.

In line with these arguments, González-Weil et al. (2014) state that educators should create a shared view on the meaning of science teaching, inquiring and reflecting personally and collectively on their practices and recognizing the value of the experience as knowledge that allows for learning in situated settings that are diverse and particular to each school, grade level, and classroom (Quintanilla, et al., 2020).

In this sense, our study demonstrates teachers' positive valuation of participating in the camp modality, an opportunity for continuing education to recall and reflect on personal and professional experiences and inquire further based on educators' narratives and personal stories (Pirrone, 2006). However, Alsup (2006) indicates that these narratives are influenced by both personal and external truths about the concept of being a good teacher, held by colleagues and the wider society, that exist in a process of negotiation in the construction and resignification of teacher identity between their agency and the environment, which is not always harmonious. The new experiences acquired at the camp could either link up or create tension with conceptualizations of the self and of the profession (Beauchamp & Thomas, 2009; Beijaard et al., 2004; Schepens et al., 2009; Ruohotie-Lyhty, 2018). This, despite the fact that the Explora Va! Camp is fully in keeping with the current tenets and principles of continuing education in Chilean public policy (Ruffinelli, 2016), which aim for professionals to research, reflect, and collaborate amongst themselves and other school actors. Teachers' learning, therefore, takes place throughout their personal and professional journeys in both formal and informal activities (Vezub, 2013). The latter are carried out, according to Calvo (2014), "through professional encounters and conversations among peers; reading and inquiry on the diverse factors involved in the work of teaching; the detection of one's own educational needs; and reflective and informed decision-making and returning to decisions to refine and transform them" (p. 113).

In relation to this latter point, teachers underscore the processes of individual and collective reflection that occurred throughout the diverse activities carried out at the camp, rating them as a unique opportunity, a new territory, and a time to contemplate about themselves, their peers, and the different school environments that they have inhabited and co-inhabited during their professional journeys. This produces interesting results since the literature has uncovered little in the way of spaces for critical reflection in initial or continuing education (González-Weil, et al., 2014; Quintanilla, et al., 2020; Vezub, 2013). This represents a barrier to teacher self-definition and to their relationship with their students, considering that their agency and authority could be confronted in the classroom in situations in which their personal and professional identity must be demonstrated when they have not reflected on it previously (Alsup, 2018; Fraser et al., 2007).

In the area of the sciences, the limited space available for reflection and communication also has a great impact, considering that teaching in schools should aim for students to become conscious of their intervention and participation in society based on "(...) making informed decisions, and establishing robust value judgements by activating, in an autonomous and critical manner, those cognitive-linguistic competencies that give coherence to their thinking, their discourse, and their actions regarding the natural world" (Quintanilla et al., 2014, p. 17). Toward this end, the development of critical thinking skills, metacognition, and capacities associated with empathy, solidarity, camaraderie, and curiosity should also be a concern for teachers throughout their professional journeys. Utilizing a reflective perspective in teaching education activities permits going beyond pedagogical aspects to incorporate the interior world of each teacher; their thoughts, principles, emotions, explanations of their undertakings, and their own construction of their professional being (Vanegas & Fuantealba, 2019). Participants valued the camp spaces that allowed for personal-professional dialog and conversation with their colleagues. These served to reaffirm and reconstruct their professional identity based on both divergent and convergent viewpoints of being, doing, and knowing the role of the teacher (Ruohotie-Lyhty, 2018). This occurred both in the sphere of the sciences and in those areas that they identify as strengths, weaknesses, and common needs as educators, independent of the subjects they teach. In fact, common discourses emerged in the camp's reflection activities, despite the diversity of specializations, professional stages, age ranges, geographical locations, and realities of the participants, revealing that the work of educating is linked to a tradition of teaching as a profession already received in its scope of action and domain of performance (De Tezanos, 2012), drawing from common knowledges learned during preparation and teaching practices (Tardif, 2010; Beijaard et al., 2004). This structure can enhance collective identity, but it could also impinge on innovation and the changes demanded by society, generating resistances if teachers perceive that public policy is implementing transformations that do not align with the reality of the school (Hargreaves, 2003; Marcelo & Vaillant, 2013).

To achieve effective transformations, as evinced by the analysis of the journals, educators must first reflect on their own professional identity and the different changes that they will go through across the span of their career path (Day, 2011). This "...allows thinking about the transformation of teachers' practices, since it opens the spectrum of relationships among subjects, knowledges, and cultures involved therein" (Vanegas & Fuantealba, 2019, p. 120).

Likewise, Vaillant (2019) and Calvo (2014) emphasize that the teaching profession should stop being understood as a solitary labor, given that current challenges and daily school dynamics allow lifelong learning by teachers based on their experiences, reflections, decisions, and shared actions with their colleagues. Teaching itself is a supportive profession that incorporates individual but also collective knowledges learned along the way, and the central idea of collaborative professional learning is recognizing that teachers learn from their pedagogical practices (Calvo, 2014) and that this occurs based on their interactions with colleagues in various shared instances, both planned and spontaneous.

For the teachers, the camp represented an accommodating space, as they positively valued the coordination of pedagogical and recreational activities, the organizational routine, and the open-air spaces as scenes for connection with their biographical and professional stories and those of their colleagues, as well as with nature itself, above all feeling recognized in their professional role. This finding is important given the loss of prestige and recognition of teaching in Latin America resulting from distrust of institutions, weaknesses in initial and continuing education, the increasing precariousness of teaching as a profession (Elacqua et al., 2018; Marcelo & Vaillant, 2013; Vaillant, 2019), and the discrepancy between the demands of the profession and the social, economic, political, and cultural transformations that have taken place during the past few decades (Ferrada, 2017; Hargreaves, 2003;

Marcelo & Vaillant, 2013; Vezub, 2013). In this sense, the camp contributes to the development of educators' socio-affective competencies, mainly regarding professional self-esteem—understanding their relationships with satisfaction, identity construction, and the performance and evaluation carried out by other actors in the school in terms of the work of teaching (Peñaherrera et al., 2014).

6 Final reflections

Based on the narratives of teachers from the camp, we can establish that any instance of continuing education should not only be linked to pedagogical knowledge and competencies, but also seek to incorporate information about professional self-recognition and self-worth as important elements of pedagogical labor, which requires commitment, motivation, and a positive perspective on the role that each educator performs in the classroom (Peñaherrera et al., 2014). In this sense, continuing education in the camp modality aspires to highlight the importance of the teaching profession and of science teaching beyond the traditional canons and disciplinary limits conventionally envisioned. Based on shared issues encountered throughout the participants' careers, collaboration, innovation, and change in and with others signifies and resignifies professional identities beginning with common elements, but attending to the particulars and unique stories of each person. Following the camp, the new challenge that arises is the creation of professional networks that endure in time and position participants in their respective educational communities as a cohort of professionals with a vision and scientific tools to support their colleagues in the continuous search for solutions to the quotidian problems that are experienced in schools and society in general. The "Explora Va!" Science Camp for Teachers envisages teacher education as a model that operates in an active manner, generating spaces that permit reflection and increasing teachers' agency and critical thinking about their pedagogical practice.

In this sense, it is interesting to highlight that the experiences of both the camp and of this study took place within the context of Latin America and specifically that of Chile. For this reason, these projects attempt to contribute to the development of situated knowledge both drawn from and meant for communities of teachers. In addition, they seek to identify important elements of the teaching profession based on the voices of its practitioners, which can help public policy designers, researchers, and all those who find themselves working in the area to take the actions most relevant to the reality of schools and teachers.

Lastly, new questions also arise for future research in this area, including, among others: What is the long-term impact of teachers' camps on the process of teacher identity formation? How do the professional capacities developed strengthen the pedagogical practices of teachers in their educational communities? What is the impact of teachers' camps on their epistemological beliefs regarding science and its teaching?

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Co-designing cross-setting activities in a nationwide STEM partnership program – Teachers' and students' experiences

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STEM partnerships are popular initiatives but can be challenging to implement in practice. Accordingly, within the context of a nationwide, cross-setting STEM partnership program in Norway – Lektor2 – a co-design tool was introduced to support teachers to collaborate with STEM professionals in developing curriculum units involving authentic STEM problems and practices. Thus, the purpose of this study was to describe the teachers' and students' experiences from the curriculum units based on the co-design tool and how the tool might help facilitate partnerships in STEM education. Teacher and student data were collected in 2015-2018 (N=2479), and responses to open-ended questions were coded using a grounded theory approach. Findings indicate that the co-design tool, particularly "the commission" – where students are commissioned by STEM professionals to design solutions to authentic problems – enhanced teachers' collaboration with STEM professionals, led to changes in pedagogical approaches, and enabled the teachers to differentiate in their teaching. Student experiences from participating in the co-designed curriculum units are characterised as more expansive views of STEM, STEM learning, and increased STEM engagement. We discuss how the co-design tool enabled teachers to overcome partnership challenges and what aspects of the commission appeared to be important for the students' experiences. This study provides a specific example of a co-design tool that can enhance pedagogical designs developed through STEM partnerships.

Keywords: STEM partnership, authentic STEM education, co-design tool, student outcome

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1 Introduction

Collaborations between classroom teachers and STEM professionals working in the public and private sectors (including non-governmental organisations, businesses, government, and higher education institutions) can potentially provide access to authentic STEM experiences that are connected to young people's communities and everyday life (e.g. Braund & Reiss, 2006; Stromholt & Bell, 2018). Such authentic STEM experiences can enhance understanding of scientific inquiry and practices, influence students' attitudes towards science, and exhibit potential career pathways (Houseal, Abd-El-Khalik, & Destefano, 2014), and prepare students for responsible citizenship (European Commission, 2015; Stromholt & Bell, 2018). However, several challenges with STEM partnerships exist, such as challenges with communication between partners (Frøyland & Langholm, 2009; Moreno, 2005), connecting outdoor experiences to classroom curriculum (Anderson, Kisiel, & Storksdieck, 2006), and lack of teacher ownership (Fallon, 2013). This indicates a need to support teachers to design and implement STEM curriculums that involve collaboration with STEM partners outside school.

Accordingly, the present article addresses this need in the context of a nationwide partnership program in Norway involving hundreds of teachers and their students. More specifically, this study describes a co-design tool that was developed to support teachers to design educational outdoor STEM experiences and facilitate collaboration between teachers and STEM professionals. In brief, the co-design tool involves discussing appropriate topics, alignment with the STEM professional practices and the national STEM curriculum, and authentic, local issues faced by the professional STEM community. Within this context, this study aims at describing teachers' and students' perspectives on the curriculum units co-designed by teachers and STEM professionals using the developed co-design tool, and to discuss how such a co-design tool can help overcome common challenges related to STEM partnerships and cross-setting STEM experiences.

Before presenting further details about the study context, method, and findings, the concepts of authenticity and relevance in STEM education are considered, and literature concerning benefits and challenges with STEM partnerships and how co-design and partnerships can be supported, is reviewed.

2 Literature review

2.1 Authenticity and relevance in STEM education

Authentic science experiences are important as it can support the integration of scientific knowledge, enhance student attitudes and interests toward science learning, promote collaboration between students, and enable students to take responsibility for their own learning (Braund & Reiss, 2006).

From a theoretical perspective, authenticity can be associated with situated learning and cognition, which consider learning as a process of participation in a particular community of practice, in which learners acquire the tools, ways of thinking, and culture of a discipline or community by engaging actively in that practice (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991). Luehmann and Markowitz (2007) build on this view and define authentic practices as engaging students in the complex processes of using scientific tools to, e.g. ask questions, draw conclusions, and report findings. Braund and Reiss (2006) complement this definition, stating that authentic science learning experiences should include activities similar to what professionals do and be student centred and open-ended. Murphy, Lunn, and Jones (2006) distinguish between cultural and personal authenticity, where the former is closely linked to the practices of professionals, and the latter refers to the individual understanding of purpose and relevance while participating in an activity. Thus, personal authenticity can be evident when students follow their personal interests and investigate their own questions, transforming their identity towards or against science culture, or when students actively contribute to out-of-school practices for instance by communicating scientific information to an audience outside school (Anker-Hansen & Andrè, 2019). As an activity can be authentic in terms of science culture, without being personally relevant to a student, it can be useful to define relevance, which is another widely used term in science education (Stuckey, Hofstein, Mamlock-Naaman, & Eilks, 2013). Stuckey et al. (2013) identified three dimensions of relevance: (1) individual, which refers to science activities matching students' interests and providing skills relevant for their everyday life, (2) societal, which can be ascribed to societal participation and the interaction between science and society, and (3) vocational, which refers to awareness and understanding of future professions and careers. Referring to Dewey (1973), Stuckey et al. (2013) argue that relevance is closely connected to meaningfulness, which means that connecting science to students' everyday life makes learning more meaningful. This understanding of meaningful corresponds to Stuckey et al.'s (2013) individual dimension of relevance.

2.2 Benefits and challenges of STEM partnerships

Access to authentic scientific practices and relevant experiences can be achieved by establishing collaborations with settings outside school that provide materials and professional practices that are typically not available in classrooms (Braund & Reiss, 2006; Houseal et al., 2014). Such collaborations between schools and STEM professionals can result in a variety of experiences for students, including cognitive, social and affective learning (Houseal et al., 2014; Shein & Tsai, 2015; Tsybulsky, 2019; Tsybulsky, Dodick, & Camhi, 2018; Tytler, Symington, Williams, & White, 2018). For instance, Houseal et al. (2014), studying teachers' and students' experiences, found that partnerships with scientists enhanced the students' understanding of and about scientific inquiry as well as improved their attitudes towards science and scientists. Such insights can also inform students about possible STEM careers (Archer, DeWitt, & Dillon, 2014; Jensen, 2015), which corresponds to Stuckey et al.'s (2013) vocational dimension of relevance. However, STEM partnerships do not necessarily have an impact on students' career plans. The students in Archer et al.'s (2014) study increased their knowledge and awareness about STEM professions after participating in a STEM partnership program, without changing their career aspirations. Although it may not impact students' career choices, other research indicates that STEM partnerships can promote environmental citizenship (Alkaher & Gan, 2020) and lead to social recognition of young people as contributors to STEM and their community and develop students' personal awareness of their roles as scientifically literate citizens (Stromholt & Bell, 2018). Furthermore, collaborations with partners outside school can give access to STEM experiences across various settings, including, e.g. universities, museums, and industrial premises (Braund & Reiss, 2006; Rennie, Venville, & Wallace, 2018). Such informal environments can be particularly important for developing and validating students' interests, skills, and identities and provide an expansive view of science (Bell, Lewenstein, Shouse, & Feder, 2009). According to Parvin & Stephenson (2004), industrial contexts and contact with industry can also provide students with a reason for doing science and give them a context for learning in the classroom. Accordingly, partnerships offering cross-setting STEM experiences have the potential to enrich and enhance students' appreciation of science by providing them with new connections with science and its applications and relations to society (Braund & Reiss, 2006; Parvin & Stephenson, 2004; Rennie et al., 2018). The potential of societal connections appears to align with Stuckey et al.'s (2013) societal relevance.

Despite the richness in possible outcomes from STEM partnerships, teachers and STEM professionals may experience various challenges in their collaboration. Establishing contact with partners outside school, and lack of support, in terms of dedicated time and financing, from the school are identified as external obstacles (Ng & Ferguson, 2019; Penuel, Lee, & Bevan, 2014; Sagar, Pendrill, & Wallin, 2012; Wormstead, Becker, & Congalton, 2002). Internal obstacles include cultural differences between teachers, students, and STEM professionals, as they differ in terms of knowledge, tools, resources, practices, and attitudes (Falloon & Trewern, 2013; Kisiel, 2014; Penuel et al., 2014). For instance, teachers experience that STEM professionals lack interest in students and have a higher working tempo that does not allow collaboration (Sagar et al., 2012). Such obstacles can impede communication, and consequently, how the partnership is designed and implemented (Falloon, 2013; Moreno, 2005). Studies of collaboration between museum guides and teachers indicate that the communication often focuses on practicalities rather than the content and the pedagogy of the museum visit (Frøyland & Langholm, 2009; Tal, Bamberger, & Morag, 2005), potentially reducing the collaborative activity to a guided tour or a lecture where teachers and students are passive. In a study of teacher-scientist-partnership, Falloon (2013) found that the teachers simply implemented a curriculum unit created by the scientists, resulting in a lack of teacher ownership in the partnership. Dolan and Tanner (2005) describe such partnerships as a "provider-recipient" approach, in which professionals serve as content providers, whereas teachers and students are receivers of their expertise. Possible consequences are that teachers may not perceive the partnership as collaborative, the partnership is treated as "add-ons" by teachers, and the field trip becomes a 'day out' for teachers and students, which is not really connected to the classroom curriculum (Anderson et al., 2006; Falloon, 2013; Moreno, 2005).

2.3 Supporting co-design and STEM partnerships

In spite of various interpretations in the research literature concerning co-design, active participation and involvement from all partners during the design process is commonly emphasised (e.g. Durall, Bauters, Hietala, Leinonen, & Kapros, 2019; Zamenopoulos & Alexiou, 2018). Related to education, Penuel, Roschelle, & Shechtman (2007) define co-design as a highly facilitated process that engages teachers, researchers, and developers in designing, developing and testing educational innovations. This is in agreement with other research, seeing co-design as a facilitated collaboration between researchers and practitioners (Aksela, 2019; Kelly, Wright, Dawes, Kerr, & Robertson, 2019). In the present study, co-design is used to describe the facilitated process where teachers and STEM professionals share their knowledge, skills and resources to collaboratively design a curriculum unit in Lektor2. This understanding is in line with Dolan & Tanner's idea of "true" partnerships, defined as involving 'two or more people, each with expertise or skills to contribute, working towards a common goal' (p. 28).

The literature offers recommendations to support teachers in their collaboration with STEM professionals. Penuel et al. (2014) recommend design principles that engage stakeholders with diverse expertise in a structured, facilitated co-design process, where teachers work with partners to develop, try out, and evaluate an educational innovation. Similarly, Tal, Alon, and Morag's (2014) design principles for field trips recommend teachers and partners plan together and discuss goals, means, and collaboration patterns, as well as connect the field trip to the school curriculum, provide student-centred learning activities, and involve both teachers and partners during the field trip. Shein and Tsai's (2015) model for teacher-scientist partnerships emphasises the collaborative process for planning, implementation and evaluation of partnership activities. During planning, teachers and scientists exchange expertise – teachers provide information about the students, educational and curriculum contexts, whereas scientists provide science content knowledge. Both teachers and scientists bring pedagogical content knowledge into the teaching.

The aforementioned recommendations for STEM partnerships emphasise the collaborative process before, during and after the partnership experience for students. However, there is a need to describe the scaffolds that support teachers in co-designing authentic cross-setting STEM experiences, whether the challenges of partnerships are addressed, and the kinds of experiences students gain from such contexts. The present study was designed to address these gaps by investigating teachers and students' experiences participating in a nationwide partnership program named "Lektor2", where teachers and STEM professionals collaboratively design curriculum units using a developed co-design tool. The study involves 378 teachers, 2101 students and 407 unique STEM partnerships, which contributes to the research literature by including a higher number of participants reporting from a large number of different curriculum units developed through partnerships between teachers and STEM professionals. Accordingly, the following research questions are addressed:

1. How do teachers in Lektor2 describe their experiences with designing curriculum units using the co-design tool?
2. How do students and teachers in Lektor2 describe the students' experiences from participating in the co-designed curriculum units?

3 Study context

Lektor2 is a national STEM partnership program in Norway offering a multi-year professional development for teachers, and financial support for schools, to involve local STEM professionals in their teaching. The term "STEM professional" is used broadly to mean any professionals using STEM in their daily job. Lektor2 aims to promote student learning and engagement in STEM and increase students' awareness of STEM careers. Teachers and STEM professionals collaborate on developing and implementing curriculum units involving cross-setting STEM experiences, such as a field trip to professionals' workplaces, use of professional scientific equipment, or visits from professionals in the classroom. [1]

3.1 The co-design tool

Starting in 2009, Lektor2 has involved STEM professionals in teaching STEM, bringing expert information to teachers and students, and acting as role models. In 2014, an external evaluation of Lektor2 found considerable variation in students' perceptions of quality and learning outcomes (Sjaastad, Carlsten, & Opheim, 2014). While some students found the experience interesting, others described it as "boring" and that they "learned nothing". Many of the partnerships were implemented according to the "provider-recipient" approach (Dolan & Tanner, 2005) and thus remained add-ons for teachers and students, disconnected from the national STEM curriculum. Following Sjaastad et al.'s (2014) recommendations to strengthen the theoretical basis for pedagogical designs, the program staff built on the Extended Classroom model – a design tool for cross-setting learning experiences in science (Remmen & Frøyland, 2017) – to develop the following co-design tool for teachers and STEM-professionals when collaboratively designing curriculum units in Lektor2[2]:

1. *The teacher and STEM professional choose a topic* that is authentic and relevant to the students, the national STEM curriculum, and the work of the STEM professional.

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[1] Lektor2 is funded by the Norwegian Ministry of Education and run by the Norwegian Centre for Science Education. All secondary and upper secondary schools across Norway can apply to participate. Since 2009, 500 schools from all over Norway and 800 different STEM organisations have participated, involving about 15.000-20.000 students each year.

[2] The co-design tool as it is presented to the teachers can be found on www.lektor2.no

2. *The teacher and STEM professional identify a commission for the students* that is derived from the STEM professional's work and engages students in authentic STEM practices.
3. *The teacher identifies the knowledge* (scientific theories, key concepts, etc.) *and practices* (collecting and analysing data, weighing options, communicating results, etc.) *required to complete the commission.*
4. *The teacher and STEM professional discuss how the STEM professional can contribute to authentic experiences for students.* The contributions should reflect the STEM professional's practices and/or workplace practices which students are not exposed to in school.
5. *The teacher designs the activities in the curriculum unit.* Students should participate in activities, both within and outside the classroom, that support them to acquire the STEM knowledge and practices required for completing the commission.

3.2 Commission

Step 2 in the co-design tool asks teachers and STEM professionals to identify and collaboratively design an authentic task for the students – a commission – where the students are commissioned by the STEM professional to do a specific job requiring authentic, complex problem-solving. To clarify what a Lektor2-commission is, five criteria were developed, derived from the literature and analysis of about 200 Lektor2-commissions. A Lektor2 commission is an authentic task that

- replicates authentic situations and problems from the STEM professional's work
- requires students to adopt professional practices requiring application of STEM knowledge and practices – i.e. the solutions cannot be "Googled"
- is sent from the STEM professionals' office, making the STEM professionals "clients" and students "contractors"
- demonstrates the purposes of the STEM professions in society
- engages students in decision-making regarding tools, solutions, approach etc.

The commission is often presented to students in a formal way, such as a commission letter. For example, in one school located in a smaller town in Eastern-Norway, students in year 10 (aged 15) were commissioned by Eidsiva, an energy company, to evaluate new uses for excess heat created at a local heating plant (Figure 1).

Eidsiva

To: Bjørnsveen secondary school
From: Eidsiva Bioenergi AS
Subject: Excess heat from the remote heating plant in Gjøvik
Date: 08.11.2017

Use of excess heat energy

In the near future the remote heating plant in Gjøvik, which burns unused wood chips, will produce an excess of heat energy or 'waste heat' during the warmer months (May through September). Waste heat is thermal power in the form of hot air, water, or steam that is not used and therefore gets lost to the surroundings. The waste heat is a consequence of the demands for heat during the summer being lower than the minimum effect on the chip steam boiler. This means that the boiler is producing more heat than the community is using. Furthermore, Eidsiva plans to start collecting waste heat from additional sources in the city, first by collecting waste heat from the purification plant at Rambekk. This will increase the amount of excess heat in the remote heating plant even further.

The amount of available power and energy is approximately 1500 kW and 5.000.000 kWh/year. The lowest production cost is approximately 0 cent/kWh.

Eidsiva Bioenergi AS requests an evaluation of new possible uses for the waste heat. We would like you to evaluate the following initiatives and propose your recommendations:

- Heating of the outdoor pool *Fastland*
- Heating of a deep pool in Mjøsa
- Extending the season for hothouses or garden-stores
- Heat-driven cooling
- Other possible initiatives

The evaluation and recommendations should include written and visual components, including models, pictures, drawings, etc. The project is to be completed within week 8, and the best solutions will be presented for a committee consisting of employees from Eidsiva Bioenergi in week 10 or 11.

In the case of need for additional information, please contact Eidsiva Bioenergi AS to make an appointment.

Figure 1. The commission letter given to students (15 yrs.) from Eidsiva (translated from Norwegian)

3.3 Facilitation of the co-design process

The implementation of the co-design tool was facilitated by Lektor2 staff (Kostøl & Braathen) and 11 local, trained coordinators, through national conferences, regional meetings, and online resources. Teachers used the co-design tool to develop and evaluate their curriculum units. Figure 2 visualises how the co-design process is facilitated and organised through various activities, resources, staff and participants in the Lektor2.

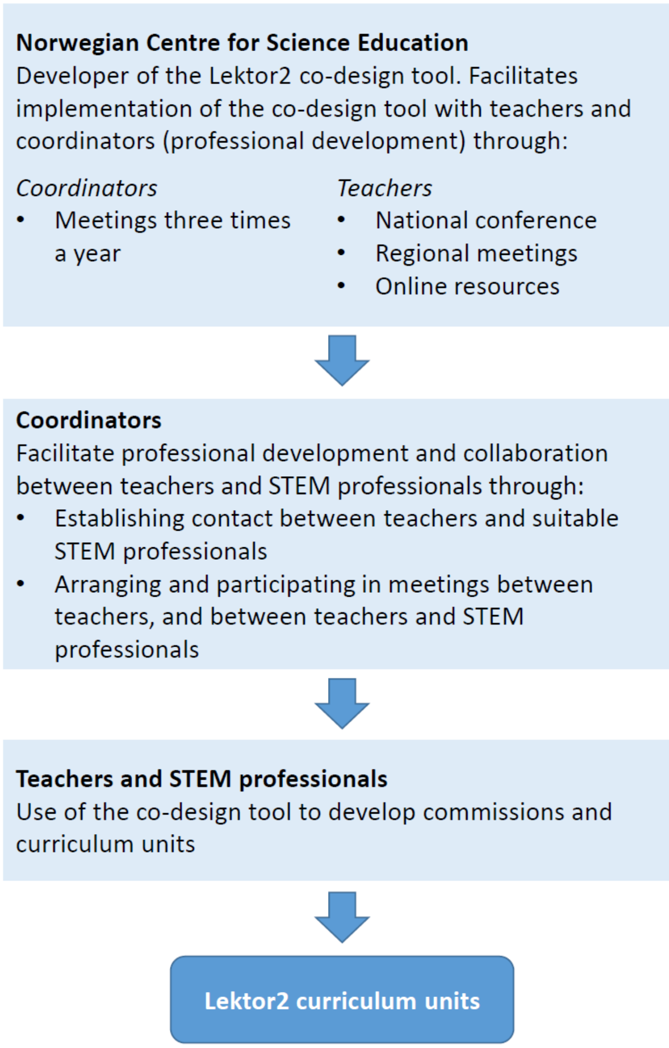

Figure 2. The facilitated co-design process on a national, regional and local level

4 Methodology

4.1 Data collection

To allow all participants in Lektor2 to describe their experiences, electronic surveys were used. This enabled data collection from teachers and students across Norway, which captured a large number of STEM partnerships and hence collaboration with STEM professionals from a variety of workplaces. Although self-reported data have limitations, it was considered appropriate in the present study, as the purpose was to describe the teachers' and students' experiences.

Three surveys (2015, 2017 and 2018) were given to teachers to collect information from both before and after implementation of the co-design tool, and one survey (2017) was given to students in secondary and upper secondary schools (age 13-19) to investigate their experiences from participating in the co-designed curriculum units. The surveys were developed through evaluation and refinement of similar surveys given to previous participating teachers and students in Lektor2. The revised surveys were vetted by the research team to further enhance face validity.

Some teachers designed and implemented two curriculum units with different STEM professionals, resulting in two responses to the survey. In total, responses from 378 teachers and 2101 students were collected, representing 407 unique curriculum units designed by pairs of teachers and STEM professionals using the co-design tool. The surveys and resulting datasets are described below (see Supplemental material for more details).

4.1.1 Datasets addressing RQ1

- *Teacher Survey 2015* collected data about teachers' experiences with Lektor2 *before* the implementation of the co-design tool. The survey was mandatory and was given to all participating teachers. Responses from 205 unique curriculum units were collected.
- *Teacher Survey 2017* collected data about teachers' experiences with Lektor2 *after* the implementation of the co-design tool. The survey was mandatory and was given to all participating teachers. Responses from 202 unique curriculum units were collected.

  In addition to answering closed-questions similar to the questions in Teacher Survey 2015, the teachers responded to the following open-ended question:

    - What do you think of *commission* as a learning approach for the students? (Keywords: different student types, different academic level, work effort etc.)

  Note that keywords were added to guide the teachers' reflections.
- *Teacher Survey 2018* collected data from 71 teachers across Norway who volunteered to respond to open-ended questions about the co-design tool in their collaboration with STEM professionals, and in their teaching practices. Teachers attending mandatory Lektor2 regional meetings were asked to participate.

4.1.2 Datasets addressing RQ2

- *Student Survey 2017* collected data from 2101 students who had participated in a curriculum unit designed by a teacher and STEM professional using the co-design tool. The question that gave qualitative descriptions of student experiences, and hence was analysed, was the open-ended question: "What do you think about participating in a Lektor2 curriculum unit where you have collaborated with STEM professionals?". The completely open question was asked in order to not give the students any direction of what experiences to describe, making it possible for the students to highlight experiences they themselves found to be important, accounting for all student ideas, including unanticipated experiences.
- *Teacher Survey 2017* collected data of the teachers' perceptions on student outcome through the open-ended question: "How would you evaluate the student's outcomes from completing the Lektor2 curriculum unit? (Keywords: learning outcomes, motivation, understanding of how STEM is used in work life etc.)"

4.2 Data analysis

As two of the authors were involved in the program being studied, two independent researchers having extensive knowledge within learning across settings were included in the research team performing the data analysis; one with previous experience with Lektor2 and one external, international researcher with no connection to Lektor2. Open-ended responses in each dataset were coded using a grounded theory approach (Strauss & Corbin, 1998) similar to Luehmann and Markowitz (2007). Codes were not identified beforehand but as emergent in order to privilege teachers' and students' voices and account for unanticipated experiences. The analysis was conducted as follows: 1) All open-ended responses from teachers and students were identified and analysed by a team consisting of the four researchers. 2) Responses that provided a substantial description of experiences or outcomes of the partnership or the co-design tool were subjected to a deeper analysis. 3) Through discussion and collaborative coding of a small set of survey responses, a coding scheme was agreed on, which was applied to the full dataset. 4) The full dataset was coded by the external researcher, resulting in an expanded and revised coding scheme. 5) The revised coding scheme and a subset of data were reviewed and discussed by the research team, revising, elaborating on, and justifying each code. 6) The full dataset was re-coded by the external researcher, attending to disconfirming evidence for each category and revisions when that evidence was found. 7) Finally, the team reviewed the expanded coding scheme and the coded data, analysing a subset for consistency of code application. To identify patterns and better understand the variety of teacher and student experiences, the qualitative data was summarised by calculating frequency distributions and percent frequency of each emerging code. Frequencies are limited in this kind of data as they are difficult to compare across studies and evaluate without further statistics. However, because it can be helpful for, e.g. seeing patterns in the dataset and examine representativeness, it was considered appropriate.

Reliability of the analysis was addressed by including independent researchers and through constant data comparison, comprehensive data use, and inclusion of negative cases. To enhance validity, investigator triangulation and theoretical triangulation (Denzin, 1979) was used, enabling consistency testing and a richer, more careful defining and understanding of the data and codes.

In addition to coding of open-ended responses, responses to closed questions from the teacher surveys in 2015 and 2017, before and after the implementation of the co-design tool, were compared concerning the nature of learning activities provided by the Lektor2 curriculum units. The comparison was made by calculating the percent frequency of reported teaching methods mainly used in the curriculum units in 2015 and 2017, respectively. The research questions with related datasets and analysis are summarised in Table 1.

Table 1. Overview of the data collection and analysis

Research question	Dataset	Total responses	Number of coded responses	Analysis
RQ1: How do teachers in Lektor2 describe their experiences with designing curriculum units using the co-design tool?	Teacher Survey 2015	205	-	Descriptive statistics
	Teacher Survey 2017	202	-	Descriptive statistics
	(closed and open questions)		156	Qualitative coding
	Teacher Survey 2018	71	71	Qualitative coding
RQ2: How do students and teachers in Lektor2 describe the students' experiences from participating in the co-designed curriculum units?	Student Survey 2017	2101	319	Qualitative coding and descriptive statistics
	Teacher Survey 2017	202	174	

5 Findings

In the following, findings from the analyses of teachers' experience with the co-design tool in Lektor2 (RQ1) and teacher and student reflections on student experiences from participating in the co-designed curriculum units (RQ2) are presented.

5.1 RQ1: Teachers' descriptions of their experiences with designing curriculum units using the co-design tool

From the analyses, four types of responses emerged: collaboration with STEM professionals, pedagogical approaches, differentiation, and transfer to teaching in general.

5.1.1 Collaboration with STEM professional

Forty-five of the 71 teachers (63 %) responding to the 2018-survey, expressed that the co-design tool made collaboration with STEM professionals easier and more accessible than before. For instance, one teacher described that it lowered the threshold for initiating contact:

> It [the co-design tool] has made you actually make contact and collaborate. (…)

One-third of these responses mentioned the commission in particular, stating that the commission made the collaboration “more authentic”, and that teachers and STEM professionals were more involved in the entire co-design process of planning, implementing, and evaluating the curriculum unit. The following responses illustrate these points:

> The Lektor2 co-design tool makes us collaborate more closely with the businesses. They are involved in the entire process.
>
> The collaboration becomes more authentic when we have a concrete commission as a starting point.

However, three teachers (4 %) described negative experiences with the co-design tool. This was related to the challenges of having the Lektor2 curriculum unit fit into the traditional curriculum and having time for planning together with the STEM professional:

> I think the commission model itself has worked well, but it has been difficult to make the model a natural part of school life and teaching. Need a lot of time for collaboration between the STEM professional and the school, clarification of expectations etc. (...)

5.1.2 Pedagogical approaches

In the 2015-survey, before the implementation of the co-design tool, 38 % of the teachers reported that the Lektor2 curriculum unit mainly consisted of traditional teaching methods, like lectures and guided tours with the STEM professional. However, 24 % of the teachers reported that their students collected and processed data and 37 % stated conducting experiments as an important student activity.

In the 2017-survey, after the implementation of the co-design tool, 88 % of the teachers reported inquiry-based learning activities in the Lektor2 curriculum unit. Specifically, 57 % of the teachers reported that the students collected and processed data, and 61 % stated practical activities where students had to use the equipment.

In the 2018-survey, changes in the pedagogical approaches used in partnerships were highlighted by the teachers. Of 54 teachers who had implemented the curriculum unit at the time of the survey and thus could compare their collaboration experiences with and without the co-design tool, 46 teachers (85 %) emphasised changes in student activities in their responses. Twenty-eight of these teachers (61 %) included the commission in their evaluation, reporting that it led to more student engagement:

> Students have done more practical work at the business than before (not just a guided tour). They have also had a specific goal/commission to work towards (inquiry-based learning).
>
> (...) The students get tasks/commission instead of lectures. The students no longer remain passive but are engaged in the process.

5.1.3 Differentiation

In the open-ended question in Teacher Survey 2017 about commission as a learning approach for students, 40 % of the teachers expressed that the commission enabled them to differentiate the teaching to students with diverse strengths, preferences, and interests. The teachers reflected on the commission as an opportunity for all students to contribute, regardless of academic level. Therefore, these responses were coded as *differentiation*, illustrated below:

> It has been easier than usual to differentiate the teaching (...)
>
> All students regardless of academic level can contribute, a mastering sensation for those who are not "good at school". (...)

5.1.4 Transfer to teaching in general

In 2018, 43 of 71 teachers (61 %) reported that collaborating with STEM professionals through the co-design tool, and on the commission in particular, influenced their teaching more generally. Of these, 30 teachers felt more capable of designing more "practical" or "open-ended" activities, and/or connecting activities to the local community, as illustrated by the following teacher response:

> [I] try to provide commissions requiring students to do a job for someone outside school (fictive or real). More focus on open-ended tasks that have several possible solutions.

Some teachers also used commissions in their teaching of other curriculum units. One teacher even saw opportunities to transfer the commission to other school subjects:

> The commission model makes me think that students can contribute in every partnership with work life (e.g., investigations, data collection, computation, production etc.). There are partnerships between the school and work life in subjects other than STEM and Lektor2, and I hope that the commission-model can be applied in those settings as well.

5.2 RQ2: Students' and teachers' characterisation of the students' experiences from the co-designed curriculum units

The analysis of open-ended responses about student experiences, collected through Teacher Survey 2017 and Student Survey 2017, revealed a diversity of experiences as summarised in Table 2 (see Supplemental material for a complete overview). Relating experiences were grouped into three main outcomes: Expansive views of STEM, STEM learning and STEM engagement. Details about each outcome are presented subsequently.

Table 2. Findings from the coding of responses describing experiences for students

	CODE	DESCRIPTION	STUDENT		TEACHER	
			Frequency	Percent frequency	Frequency	Percent frequency
OUTCOME 1: EXPANSIVE VIEWS OF STEM	**Connection to real world**	Descriptions of connections/relationship between schoolwork and work life, or seeing things "in practice", applicability of school STEM in the "real world".	76	24 %	96	55 %
	Possibilities	New awareness of future possibilities or realising that STEM is not for them.	44	14 %	14	8 %
	Understanding of work life	Understanding of how a business or the real world works, or an assessment of the job type (boring, interesting, etc.). Not just reference to the link between school and work, but to new understandings of what work life is like.	75	24 %	49	28 %
OUTCOME 2: STEM LEARNING	**Content knowledge**	Specific content knowledge.	5	2 %	26	15 %
	Increased understanding	Benefits in terms of greater outcomes or increased learning/understanding.	17	5 %	28	16 %
	Practices knowledge	Student engagement in practices (putting stuff together, calculations, lab work, writing surveys, etc.).	7	2 %	38	22 %
OUTCOME 3: STEM ENGAGEMENT	**Meaningful**	The relevance of the curriculum unit to everyday life, or as personally important/useful.	14	4 %	20	11 %
	Motivating	Catchall code to note the curriculum unit was engaging or that the students were engaged in deeper/different ways than usual.	18	6 %	87	50 %
	Positioning	Specific descriptions of a student in relation to the work – often a new position/role.	8	3 %	4	2 %
		Total number of coded responses (one response often has several codes)	**319**		**174**	

5.2.1 Outcome 1: Expansive use of STEM

The common characteristic across the three codes *understanding of work life*, *possibilities*, and *connection to real world*, described in Table 2, is that both teachers and students experienced connections between schoolwork and STEM related work outside school and a new awareness of STEM possibilities and work life. Therefore, the three codes were merged into the outcome *Expansive views of STEM*. Specifically, the curriculum unit helped increase students' awareness of jobs in STEM, including a deeper understanding of STEM professionals' tasks and practices, and helped students see the applicability of STEM in contexts outside school. These findings are illustrated by the subsequent student response (coded as *connection to real world*, *understanding of work life*, and *possibilities*) and teacher response (coded as *connection to real world* and *understanding of work life*):

> Student: I thought this was fun, fun to see what we learn about performed in the workplace. That what we are working on now can be our future job.

> Teacher: Several of the students expressed: "Now I understand the point of the mathematics we learn!" (...) The students also gained a very good and tangible insight into how important STEM is in a high-tech business.

Along with a deeper understanding of STEM professions, 44 students (14 %) and 14 teachers (8 %) went further to make connections to possible job opportunities for students in the future. As in the quote below, students especially described the Lektor2 experience as giving them a better overview of possibilities for themselves, a better idea of what they want to do, or informing their decisions about further education or professional pathway.

> Student: (...) I have gained a new view of how many jobs exist within STEM and4 the possibilities around this (...)

Expansive views of STEM also included responses from students, such as the quote below, who spoke to the benefit of Lektor2 even though they were not planning to pursue STEM in the future:

> (...) very good for us to see what opportunities exist and especially opportunities for STEM. This makes it easier to choose both profession and upper secondary school, even though I'm not thinking about going into STEM.

5.2.2 Outcome 2: STEM learning

STEM learning comprises the codes *content knowledge*, *increased understanding*, and *practices knowledge*. Three teachers (2 %) reported that the Lektor2 experience did not contribute to their students' learning, whereas 79 teachers (45 %) described positive outcomes regarding STEM learning such as students gaining increased understanding of key concepts and engaging in STEM practices, exemplified in the quote below:

> Teacher: The curriculum unit provides students with a good starting point for discussion, reflection and writing of argumentative texts (...). Students get positive nature experiences and understanding of predator's biology, role in ecosystems, and society.

A few teachers also reported that students gained increased understanding of the importance of STEM education:

> Teacher: They used their knowledge in a new setting. It became more close to reality, and led to increased understanding of why we learn what we learn at school.

While the teachers were specific about content and practices, the students' were more general in their descriptions of STEM learning outcomes, expressing that they "learned a lot" or "got a better understanding". As indicated by the response below, coded as *increased understanding*, some students experienced that Lektor2 helped them to be more engaged in their own learning:

> Student: I think it was far more educational and easier to pay attention when working with the subject in a more practical context.

5.2.3 Outcome 3: STEM engagement

Another important finding was that the Lektor2 curriculum unit engaged students in STEM education, often in more profound or different ways than usual, as described by the codes *meaningful*, *motivating*, and *positioning*. Eighty-seven teachers (50 %) commented that experiencing, applying and practicing STEM work outside school led to increased student engagement, typically described as an increase in students' effort during the curriculum units. In the following example, the teacher's response refers explicitly to the commission as critical for motivating students.

> Teacher: (…) The fact that they were given an authentic commission was motivating in itself, and they have worked very hard with the commission. They have seen that mathematics and science is important in everyday life and in work life (…)

Many students used terms like “interesting” (17 %), “enjoyable” (29 %), and/or “exciting” (19 %) when characterising the curriculum units. Eighteen of these students went further, describing deeper engagement, as illustrated by the following responses:

> Student: I found it interesting, and it made me want to become more competent in the [STEM] subject.
>
> Student: In short, it was amazing and very inspiring! (…) it made the students in my class participate and be more engaged.

The teachers often explained students’ motivation and deeper engagement by referring to the commission as authentic, meaningful and connected to students’ lives. The following response coded as *motivating* and *meaningful* demonstrates this:

> Teacher: (…) the students were motivated by the fact that this [the commission] is an authentic problem, and easily transferable to their lives.

Fourteen students (4 %) highlighted their experience of working with the commission as authentic, useful and relevant, and not just “doing another task given from the teacher for a grade”. The student responses coded as *meaningful* also emphasised that the commission was connected to their local community or everyday life, inviting them to contribute in decision-making. As with the teachers, the students described how the experience of doing something important led to increased engagement and interest, as exemplified below:

> Student: I think it has been a good unit where you have been given an authentic assignment, instead of something that only is to be graded
>
> Student: This unit has been fun and educational. It is good that we can collaborate with the people in the municipality and be part of making decisions in our local community (…)

The importance of authenticity was further supported by a teacher describing a situation where the commission lost some of its value. In this case, the students were commissioned by a forest owner to plant trees after logging. This required the students to assess which types of trees to plant:

> Teacher: Great motivation related to the commission. (...) one father informed the student that plants to the felling area had already been bought. This spread rapidly among the students and we lost the context and the feeling of working with a real commission. (...) they considered this as just another school assignment and the motivation disappeared.

Eight students described that working with the Lektor2 curriculum unit inspired them to take a new position or reconsider themselves in relation to STEM. Therefore, these responses, such as the following example, were coded as *positioning*:

> Student: It was really exciting and refreshed my interests, and I became happy and would do it again, and I'm considering STEM.

This code also captured student responses showing a shift or confirmation of their decision of not aspiring to STEM careers:

> Student: Thought it was a great experience that showed me how STEM is being used in work life and showed that it wasn't for me.

Positioning also emerged in 13 of the teacher responses, where teachers expressed a shift in recognition of their students and their assets in the classroom. The teachers described students showing new sides of themselves during their work with the commission, for example, students who were quiet or less motivated engaged more deeply than usual. They also described students who were capable to accomplish new and more complex tasks. Examples of responses illustrating these points are:

> Teacher: (...) A student who earlier took a long time to get started, and did not see the benefits of math threw himself into the task and worked very hard (...)

> Teacher: The commission worked very well. Students show new sides of themselves when they are doing commissions, and it's especially nice to see that some really flourish and show you assets that are difficult to see in regular classroom settings.

Twenty students (6 %) found the curriculum unit to be disengaging, expressing that they did not find the commission interesting, or that it was overwhelming or irrelevant to the national curriculum. However, even the students who gave negative comments most often included something positive when describing their experience with Lektor2:

> Student: Fun to collaborate with work life, but the topic and [what] the workplace was doing was boring. It was more fun than regular education (...)

6 Discussion

The purpose of this study was to describe teachers' and students' experiences from curriculum units co-designed by teachers and STEM professionals, using a specific co-design tool developed within a national cross-setting STEM partnership program. Specifically, teachers were asked how they experienced using the co-design tool, and teachers and students described students' experiences after participating in the co-designed curriculum units. Our findings indicated that the co-design tool, particularly the commission, enabled teachers to collaborate with STEM professionals, design learning activities that engaged students in authentic STEM practices, differentiate teaching for diverse students, and transfer new teaching strategies to other areas of their teaching. The coding of teacher and student responses regarding student experiences resulted in three main outcomes: Expansive views of STEM, STEM learning, and STEM engagement. The two latter outcomes align with cognitive and affective outcomes described in the literature on outdoor science activities (Rickinson et al., 2004), including partnerships (e.g., Tysbulski, 2019). However, within each of the three outcomes, several codes that refine the possible outcomes from such partnerships were identified. Our analyses therefore provide a more fine-grained view of the outcomes students can possibly gain from partnerships involving co-design of curriculum units. One reason for this could be that our study included a much larger number of teachers and students, and a range of different co-designed curriculum units, than what has been reported in the research literature earlier.

Some of the reported codes have a low frequency (Table 2). This can be explained by the fact that not all codes were applied to each student response. This is a consequence of our methodological choice, in that students responded to an open question designed not to lead them in any particular direction (see Student Survey 2017). Therefore, whether each particular curriculum unit resulted in all three outcomes for all students cannot be answered. However, the findings from this study describes what the students themselves chose to highlight as important experiences from participating in the co-designed curriculum units.

When reflecting on the co-design tool, the teachers tended to describe the commission in particular, and both teachers and students described the students' work using phrases such as "authentic assignment", "authentic problem" or "real". Therefore, it becomes important to discuss how the commission, as a specific part of the co-design tool and a unique feature of this study, might address partnership challenges and promote different student outcomes.

6.1 Addressing challenges with partnerships: the teachers' perspectives

In contrast to other studies (e.g. Sagar et al., 2012), teachers participating in Lektor2 did not experience the challenge of establishing contact with STEM professionals. This can be partly ascribed to how Lektor2 is organised – coordinators facilitated contact and meetings between teachers and STEM professionals (Figure 2), in line with design principles for collaboration about cross-setting STEM experiences (Penuel et al., 2014; Tal et al., 2014). Further, the co-design tool helped teachers to reduce the challenges related to collaboration. For instance, the tool enabled teachers to initiate collaboration with STEM professionals, as illustrated by responses describing the collaboration as "closer" and the commission as an effective starting point for the co-design process. This contrasts with previous findings cited earlier, indicating challenges with communication in the collaborative process (e.g., Falloon, 2013; Sagar et al., 2012). Furthermore, the teacher describing the cross-setting activity as "students get tasks/commission instead of lectures" from the STEM professional, and the student expressing that "it made the students in my class participate and be more engaged", show that the curriculum units engaged students as active participants in their learning, in contrast to being passive receivers of information from the STEM professional. The notion of active learning is in line with Tal et al.'s (2014) recommendations for productive cross-setting STEM experiences.

A few teachers shared the view of other research findings, indicating that teachers find it difficult to include partnership activities in their regular classroom teaching (Ng & Fergusson, 2019; Sagar et al., 2012). Notably, only a few teachers emphasised the challenges of using the co-design tool, particularly with respect to time and making the commission fit into school traditions. This indicates that the co-design tool, even though the co-design process is facilitated by Lektor2-staff, does not enable all teachers to overcome the external challenges with STEM partnerships. Clearly, the commission requires teachers to adopt innovative teaching strategies that engage students in scientific problems and practices.

6.2 Aspects of the commission and related student outcomes

The number of teachers and students who chose to highlight the commission when describing their Lektor2 experience, indicate that the commission was particularly important for the students' outcomes from the curriculum units. As seen in Table 2, both teachers and students experienced expanded views of STEM and STEM engagement to a greater extent than increased STEM learning. An obvious explanation for these findings could be that the students were not asked to describe what they learned, as the different curriculum units focused on different topics and the students included in the study ranged from secondary to upper secondary school. Therefore, it becomes important to discuss our findings with respect to the potential of using a commission to support student outcomes from cross-setting STEM experiences across topics and school levels.

6.2.1 Commissions provide authentic experiences that are relevant

As indicated by the relatively high frequency of the codes *connection to real world* and *understanding of work life* in the students' responses, and *connection to real world* in the teachers' responses (Table 2), the curriculum units based on a commission appeared to demonstrate how knowledge learned in school was applicable in real-life situations. This corresponds to Murphy et al.'s (2006) cultural authenticity, defined as relevance for professional practices, and Stuckey et al.'s (2013) societal dimension of relevance. This may be related to the commission's goals of presenting authentic problems and requiring students to adopt professional practices and actively apply STEM knowledge and skills. However, that students appreciate the commission as relevant for the society does not necessarily mean that they perceive it as personally relevant, as suggested by the relatively small proportion of the *connection to real world*-responses that were also coded as *meaningful*. None of the criteria for commissions in Lektor2 requires that the commission should be personally relevant – this would be difficult to achieve given the various interests and backgrounds that exist in a student group.

Regardless, it can be concluded that many students in Lektor2 experienced the commission as relevant in one or more dimension (vocational, societal, and/or personal). This may be ascribed to the co-design process, in which teachers and STEM professionals plan the commission together, as described in Study context, confirming the literature emphasising collaboration between the partners in the planning process (e.g., Shein & Tsai, 2015; Tal et al., 2014). Therefore, in order to make sure that students perceive the commission as relevant in a societal, vocational and/or personal way, it seems important that teachers and STEM professionals can share their expertise to design the commission – the teacher's knowledge about the students and the STEM professional's knowledge about authentic problems and practices.

6.2.2 Commissions engage students as contributors to authentic STEM work

According to many teachers in this study, students engaged more deeply when working with the commission, including students who typically struggle with their motivation for STEM in school. This finding confirms that out-of-school experiences can enhance students' interests and motivation (Braund & Reiss, 2006; Tytler et al., 2018). In addition, our findings suggest that the commission in particular inspired some students to work harder than usual (coded as *motivating*). One reason could be that the commission in Lektor2 put students into roles as contributors by communicating their own findings and new ideas to an authentic "client" – who asks for, and often needs, the students' contributions (see criteria under Commission). As exemplified by the student who highlighted the experience of being part of making decisions in the municipality, the students valued the opportunity to contribute to the STEM professionals' work. This finding aligns with Stromholt and Bell's (2018) argument that students should have opportunities to recognise themselves as contributors to STEM and their community. This was also made clear by a contrasting example, in which the teacher's response described a decrease in students' engagement when they learned that the STEM professional did not really need their contribution after all. Thus, authenticity in commissions – from problem to solution – appears to be critical in order to ensure that students experience the partnership as engaging and meaningful. Furthermore, it might be important to emphasise that Lektor2 differs from citizen science projects where student contribution is typically limited to data collection (Wormstead et al., 2002), as the commission also requires deep STEM knowledge, engagement in STEM practices such as designing solutions to real-life problems, analysing data, and arguing based on evidence, and communicating solutions back to STEM professionals (see Commission).

6.2.3 Commissions make students aware of new possible futures

In addition to relevance, the outcome Expansive views of STEM included perspectives on future possibilities, exemplified by a student stating that the current work with the commission could also be their work in the future. Similar to Archer et al. (2014), some of the students in our study recognised a new awareness of job opportunities within STEM when working with the commission. Notably, there was a slight difference in the proportion of student and teacher responses coded as *possibilities* (Table 2), indicating that more students than teachers recognised the professional relevance – i.e., that STEM-related work and jobs were modelled through the curriculum unit, suggesting that these students themselves made connections between STEM in school and professions. This increased awareness of future professions and careers resembles Stuckey et al.'s (2013) vocational dimension of relevance.

However, not all student responses included reflections on whether they considered the job opportunity as relevant for them personally. This dimension was instead captured by the code *positioning*, in which the student responses signalled a shift or confirmation in aspirations through the Lektor2 curriculum unit. This included both student responses reflecting a willingness to consider further STEM education and a recognition that STEM was not for them. Both are important in order to make informed decisions about possible futures, regardless of whether it is going into STEM or not (Jensen, 2015). Of course, there are uncertainties here, as the survey was anonymous and information about the students' background is therefore not accessible. Nonetheless, given that students often base their career aspirations on previous experiences with STEM (Jensen, 2015), the commission in Lektor2 could be one of several authentic STEM experiences in the process of choosing STEM career pathways.

6.2.4 Commissions provide opportunities for students with different academic capabilities

The commission provided opportunities for students to engage more deeply, collaborate with their peers, and discover new sides of their abilities and learning, as evident by responses coded as *motivating* and *differentiation*. Motivation, collaboration and responsibility for learning can be realised by including outdoor science activities in the formal science curriculum (Braund & Reiss, 2006), but it is worth noting that many of the teachers experienced that students with different academic strengths or interests were able to engage in the commission. This is in agreement with Lesseig, Slavit, Nelson, & Seidel (2016), reporting that teachers experienced all their students, including low-achieving students, to be motivated and empowered when faced with complex, open-ended problems. Thus, it seems that the commission can potentially address the broader purposes of cross-setting STEM experiences as expanding access to STEM learning opportunities to promote equity and diversity in STEM education (Penuel et al., 2014). However, this conclusion needs to be addressed in future research on why and how the commission allows students with different interests and capabilities to participate in STEM education.

7 Conclusions and implications

From our study, it can be proposed that the co-design tool enabled teachers in Lektor2 to overcome commonly reported challenges with STEM partnerships and enabled the design of authentic STEM experiences that resulted in a variety of outcomes for students. Figure 3 visualises the co-design tool as essential for creating connections between teachers, STEM professionals, and students, through facilitation of partnership and co-design of curriculum units. It also summarises the teachers' experiences with designing curriculum units in Lektor2 and descriptions of the resulting student experiences. Specifically, teachers experienced that the co-design tool made collaboration with STEM professionals more accessible and authentic, enabled them to focus more on student-centred activities, and facilitated student engagement in authentic STEM work, regardless of academic level. For students, participation in the co-designed curriculum units in Lektor2 resulted in a variety of experiences involving more expansive views of STEM, STEM learning and STEM engagement.

The commission – a part of the co-design tool that was frequently mentioned by the teachers – has been discussed in order to identify its potential for facilitating the outcomes for students. However, this article only provides an overview of the possible student outcomes from curriculum units based on the co-design tool in Lektor2. Further research may investigate how such facilitated STEM partnership programs contribute to individual student's outcomes across all three domains – engagement, learning and expansive views of STEM. Furthermore, as this study was based on teachers' and students' self-reports, observational studies of co-design processes between teachers and STEM professionals are needed to investigate how such processes may influence students' work with authentic tasks across settings.

Nevertheless, based on our findings we argue that the proposed co-design tool – and the commission in particular – helped the teachers in Lektor2 to overcome some of the recurring challenges related to STEM partnerships, and resulted in curriculum units leading to a diversity of student experiences important for authentic STEM education.

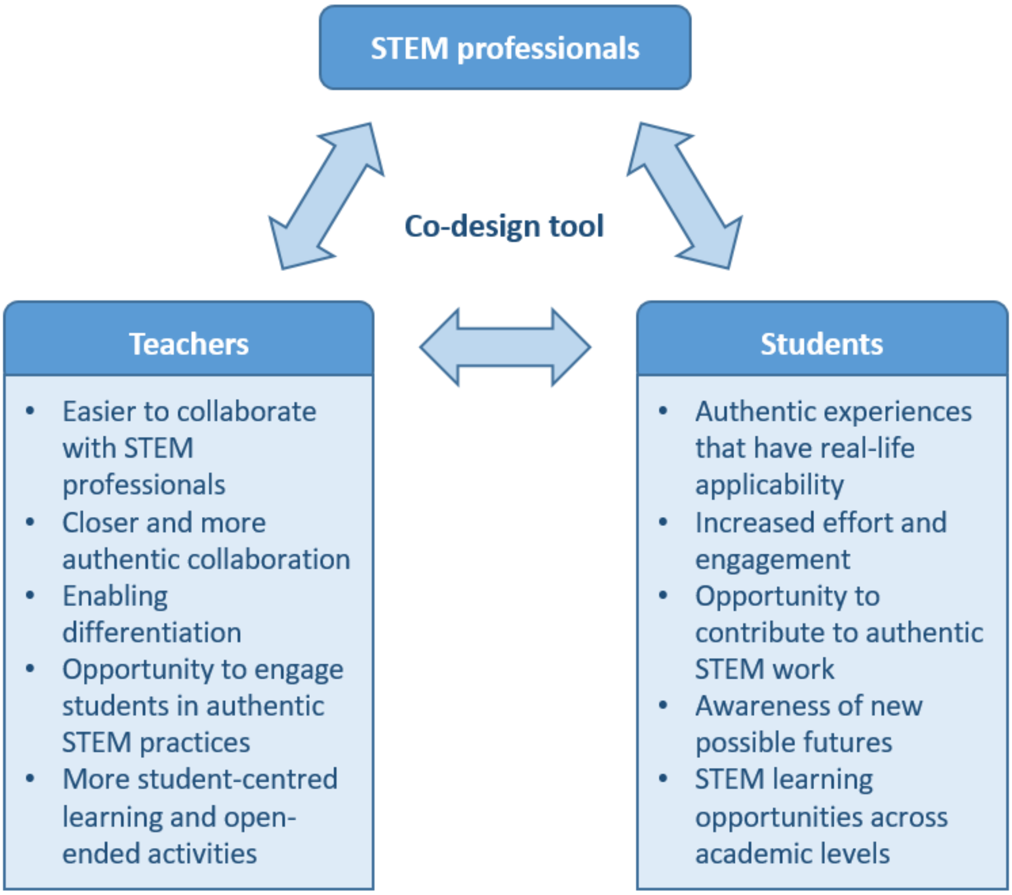

Figure 3. A summary of what the co-design tool can contribute to for teachers and students in Lektor2.

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Matematiikan parhaiden osaajien siirtyminen toiselle asteelle: koulutusvalinnat ja matematiikan osaamisen kehittyminen

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Tutkimus on osa pitkittäistutkimusta, jossa samaan ikäluokkaan kuuluvia oppilaita seurattiin perusopetuksen kolmannelta vuosiluokalta toisen asteen koulutuksen loppuun neljällä eri mittauskerralla. Tutkimuksessa käytetään Opetushallituksen ja Kansallisen koulutuksen arviointikeskuksen vuosien 2005–2015 aikana keräämää kansallisesti edustavaa tutkimusaineistoa. Tutkimusaineisto käsittää kaikkiaan 3896 oppilasta. Tutkimuksessa keskitytään tarkastelemaan matematiikan parhaita osaajia, joita on yhteensä 292 (7,5 %). Poikien osuus on 64,0 % (n = 187) ja tyttöjen 36,0 % (n = 105). Osaaminen määritetään yhdeksännen vuosiluokan kokeessa menestymisen perusteella. Kansallisten matematiikan kokeiden lisäksi oppilaat ovat vastanneet erilaisiin kyselyihin, joissa heiltä on kerätty tietoa yksilöön, kouluun ja kotitaustaan liittyvistä tekijöistä. Tutkimuksessa selvitetään näiden tekijöiden yhteyttä toisen asteen koulutusvalintaan ja osaamisen kehittymiseen toisen asteen opintojen aikana. Tulosten analysoinnissa käytettiin päätöspuuanalyysia (DTA) ja regressioanalyysia. Tutkimuksessa havaittiin, että suurin osa (60,0 %) yhdeksännen vuosiluokan parhaista osaajista oli parhaita osaajia myös toisen asteen päättyessä ja muiden osaaminen laski hyvien tai keskitason osaajien tasolle. Yksilöön liittyvät tekijät selittävät parhaiten matematiikassa menestymistä myös toisella asteella. Myönteiset asenteet matematiikkaa kohtaan ja vahva matematiikan osaamisen pohja perusopetuksessa luovat edellytyksiä menestyä matematiikassa erinomaisesti toisella asteella. Matematiikan parhaiden osaajien osaamisen taso heikkenee todennäköisemmin, jos oppilas ei mene lukioon tai ei suorita lukiossa vähintään 11 matematiikan kurssia. Yhdeksännen vuosiluokan parhaista osaajista lukioon hakeutuivat todennäköisemmin ne, jotka menestyivät arvosanatiedon perusteella erinomaisesti äidinkielessä.

Avainsanat: matematiikan parhaat osaajat, pitkittäistutkimus, kansallinen arviointi, matematiikan oppimistulokset, toinen aste

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1 Johdanto

Useimmat matematiikassa parhaiten menestyneistä oppilaista eriytyvät muista oppilaista jo perusopetuksen varhaisessa vaiheessa. Niemen, Metsämuurosen, Hannulan ja Laineen (2020) mukaan valtaosa yhdeksännen vuosiluokan parhaista osaajista erottui muista jo kolmannen vuosiluokan alussa, ja erot osaamisessa näkyivät selkeästi kuudennella vuosiluokalla. Kansainvälisesti arvioituna erinomaisten matematiikan osaajien osuus suomalaisoppilaista on kuitenkin laskenut huomattavasti. Tämä näkyy muun muassa PISA-tuloksissa vuosien 2003 ja

2015 välisenä aikana (Vettenranta ym., 2016, s. 40). Osaamisen lasku on huolestuttavaa, koska vahvoja matematiikan osaajia tarvitaan eri aloilla. Ongelmana onkin, miten erinomaiset matematiikan osaajat voidaan tunnistaa ja miten heidän osaamistaan voidaan tukea peruskoulun ja toisen asteen opintojen aikana.

Tämä tutkimus on osa pitkittäistutkimusta, jossa samaan ikäluokkaan kuuluvia oppilaita seurattiin vuosina 2005–2015 perusopetuksen kolmannelta vuosiluokalta toisen asteen koulutuksen loppuun. Tutkimusaineisto on Opetushallituksen ja Kansallisen koulutuksen arviointikeskuksen (Karvi) keräämä. Tarkentavassa aineiston analyysissa selvitettiin, miten matematiikassa parhaiten menestyvät yhdeksäsluokkalaiset erottuvat muista oppilaista ja miten heidän osaamisensa on kehittynyt perusopetuksen aikana. Tutkimustulokset osoittivat muun muassa, että oppilaan aikaisempi osaaminen, käsitys omasta osaamisesta ja vanhempien koulutustaso olivat selkeitä parempaa osaamista selittäviä tekijöitä (Niemi ym., 2020).

Useat tutkimukset ovat osoittaneet, että omiin kykyihinsä luottavat ja motivoituneet oppilaat menestyvät matematiikan opinnoissa parhaiten. Esimerkiksi Murayama, Pekrun, Lichtenfeld ja vom Hofe (2012) tutkivat, miten motivaatio, opiskelutyyli ja älykkyys selittävät matematiikassa edistymistä Saksassa viidenneltä luokalta kymmenennelle. Tutkimuksessa seurattiin 3500 koululaisen matematiikan osaamisen kehitystä viiden vuoden aikana. Tulokset osoittivat, että motivaatio ja opiskelutyyli ovat älykkyyttä tärkeämpiä tekijöitä matematiikan osaamisen edistymisessä; älykkyydellä on merkitystä vain opintojen alussa.

Tämän tutkimuksen tarkoituksena on selvittää, mitä muutoksia matematiikassa parhaiten menestyneiden yhdeksäsluokkalaisten osaamisessa tapahtuu toisen asteen opintojen aikana ja kuinka paljon yksilöön liittyvät tekijät ja toisaalta koulu- ja kotiympäristöön liittyvät tekijät ovat yhteydessä matematiikan osaamiseen ja siinä tapahtuviin muutoksiin. Tutkimus tarjoaa koulutuspoliittisesti merkittävää tietoa matematiikan parhaista osaajista ja siitä, miten heidän osaamisensa voidaan tunnistaa opintojen aikana. Tirri ja Kuusisto (2013) ovat esittäneet, että erinomaisten ja lahjakkaiden oppilaiden tunnistaminen ja tukeminen on yksi suomalaisen koulujärjestelmän haasteista. Koulujärjestelmässämme on ollut periaatteena pitää huolta heikoista oppilaista ja tukea niitä, joilla on oppimisvaikeuksia (Tirri & Kuusisto, 2013). Peruskoulu-uudistuksen lähtökohtana 1970-luvulla oli edistää koulutuksen tasa-arvoa ja tasoittaa oppilaiden koulutusmahdollisuuksia. Tällaisen koulutuksellisen tasa-arvon nähdään olevan myös yksi tekijöistä Suomen PISA-menestyksen taustalla (Niemi, 2012). Opetussuunnitelman perusteiden mukaan kaiken opetuksen pedagogisena lähtökohtana tulisi olla eriyttäminen, joka perustuu oppilaiden tarpeille ja mahdollisuuksille muun muassa edetä yksilöllisesti (Opetushallitus, 2014, s. 5). Tällainen opetuksen eriyttäminen koskee kuitenkin useimmiten vain oppilaita, joilla on oppimisvaikeuksia, ja lahjakkaiden oppilaiden tukeminen riippuu yksittäisestä opettajasta (Laine, 2016, s. 13). Tutkimuksessa onkin tarkoitus selvittää, millainen merkitys kouluun liittyvillä tekijöillä on matematiikan parhaiden osaajien osaamisen kehittymisessä verrattuna oppilaan yksilöllisiin tekijöihin.

2 Matematiikan parhaat osaajat

Matematiikan parhaiden osaajien määrittely ei ole yksiselitteistä. Tässä tutkimuksessa tarkastellaan oppilaita, jotka ovat menestyneet erinomaisesti koulumatematiikkaa mittaavissa tehtävissä. Matematiikassa erinomaisesti menestyneitä oppilaita käsittelevien tutkimusten voidaan nähdä tarkastelevan menestymistä seuraavista näkökulmista. Oppilas saa huipputuloksia, koska hän on poikkeuksellisen lahjakas eikä hänen tarvitse tehdä töitä osaamisensa eteen vaan hän omaksuu asioita vain pienellä ohjauksella (mm. Sternberg & Davidson, 2005). Vaihtoehtoisesti oppilaan voidaan sanoa olevan matemaattisesti lupaava, joka omaksuu helposti matemaattisia asioita (Sheffield ym., 1999) tai, että oppilas saa erinomaisia tuloksia, kun tekee paljon töitä oppimisensa eteen (mm. Boaler, 2015).

Matemaattinen lahjakkuus tai lahjakkuus ylipäätänsä on vaikeasti määriteltävä käsite eikä sen määrittämiseksi voida käyttää yhtä ainoaa erityistä kriteeriä (mm. Sternberg, 1993; Gagné, 1995; 2000; Wellisch & Brown, 2012). Muun muassa Sternbergin ja Davidsonin (2005) mukaan matemaattisen lahjakkuuden ajatellaan olevan synnynnäinen persoonan ominaisuus ja matemaattiset taidot ovat seurausta matemaattisesta lahjakkuudesta. Krutetskii (1976) määrittelee matemaattisen lahjakkuuden yksilölliseksi kokoelmaksi matemaattisia taitoja, jotka mahdollistavat menestymisen matematiikassa. Kun yksilöllä on tarvittava määrä matemaattisia kykyjä ja hänen persoonallisuutensa piirteet sekä myös jotkin geneettiset ominaisuudet ovat sopivat, voidaan puhua matemaattisesta lahjakkuudesta. Matemaattinen lahjakkuus voi ilmetä taitona suorittaa menestyksekkäästi kokeita ja testejä, hankkia tietoa, mutta myös kyvykkyyttä kehittää ja tuottaa uutta (Singer ym., 2016). Matemaattisesti lahjakas ei kuitenkaan aina välttämättä saavuta huipputuloksia matematiikassa eikä matematiikassa hyvin menestyvä oppilas ole välttämättä matemaattisesti lahjakas (mm. Brandl & Barthel, 2012; Szabo, 2015). Oppilaan sisäisellä motivaatiolla on keskeinen merkitys matematiikan osaamisessa ja sen kehittymisessä, ja opettajan rooli kiinnostuksen herättäjänä ja ylläpitäjänä on tärkeä (Krutetskii, 1976).

Lahjakkuus ylipäätänsä nähdäänkin nykyään usein mallina, jossa kognitioon, motivaatioon ja ympäristöön liittyvät tekijät ovat toisiinsa yhteydessä ja johtavat lahjakkaaseen käyttäytymiseen (Vlahovic-Stetic ym., 1999). Muun muassa Renzulli (1985; 2002) näkee lahjakkuuden koostuvan kolmesta elementistä, jotka ovat keskitason ylittävä kyvykkyys (above-average ability), opiskelumotivaatio (task commitment) ja luovuus (creativity). Renzullin mallissa keskitason ylittävä kyvykkyys viittaa henkilön kognitiivisiin kykyihin, opiskelumotivaatio henkilön kiinnostukseen tai sitoutumiseen aiheeseen ja korkea luovuus liittyy ajatuksen omaperäisyyteen. Henkilö on lahjakas, kun hänellä on kaikki kolme ominaisuutta. Mönks (1992) sisällyttää jaotteluun myös koulun, perheen ja opiskelutoverit. Mönks ja Mason (2000) kuvaavat, miten yksilölliset tekijät (stressitekijät, sosiaaliset tekijät ja motivaatiotekijät) ja ympäristötekijät (kouluun, ikätasoon ja vanhempiin liittyvät tekijät) vaikuttavat lahjakkaan käyttäytymiseen.

Matemaattisen lahjakkuuden rinnalle on esitetty käsitettä matemaattinen lupaavuus (Sheffield ym., 1999). Käsitteen kuvauksen mukaan matemaattisia kykyjä voidaan kehittää eikä matemaattinen osaaminen ole puhtaasti synnynnäinen ominaisuus, kuten lahjakkuudesta ajatellaan. Tämän mukaan suuremmalla määrällä yksilöitä on mahdollisuus yltää erinomaisiin matematiikan tuloksiin. Matemaattinen lupaavuus määritellään koostuvan neljästä osatekijästä, jotka ovat toisiinsa vastavuoroisessa yhteydessä. Oppilas voi saavuttaa korkean matemaattisen suorituskyvyn, kun hänen potentiaalinsa pääsee toteutumaan koko laajuudessaan. Matemaattinen lupaavuus koostuu Sheffieldin ja kanssakirjoittajien (1999) mukaan kyvykkyydestä (*ability*), motivaatiosta (*motivation*), uskomuksesta (*belief)* ja kokemuksesta (*experience*) tai mahdollisuudesta (*opportunity*). Osatekijät ovat toisiinsa yhteydessä ja kaikkia niitä on kehitettävä, jotta oppilaan potentiaali saadaan maksimoitua. Lupaavuuden käsite korostaa olosuhteiden vaikutusta osaamisen kehittymiseen. Dweckin (2006) mukaan oppilaat, jotka uskovat matemaattisen osaamisensa olevan täysin synnynnäistä (fixed mindset), pärjäävät heikommin kuin ne keskitason oppilaat, jotka tiedostavat, että voivat kehittää osaamistaan (growth mindset). Boalerin (2015) mukaan jokaisella oppilaalla on mahdollisuus menestyä matematiikassa tekemällä töitä oppimisensa eteen.

Myös Leikin (2014) näkee, että erinomaiset taidot matematiikassa ovat kehitettävissä. Älyllisten kykyjen kehittämiseen vaikuttavat kuitenkin yksilön persoonalliset piirteet, kuten motivaatio, stressitekijät, kiinnostuksen kohteet, minäpystyvyys (*self-efficacy*) ja itseluottamus (*self-belief*) (Bandura & Schunk, 1981; Zimmerman, 2000; Pajares, 2003) Myös useat matemaattiseen lahjakkuuteen liittyvät pitkittäistutkimukset osoittavat, että personallisuusominaisuuksilla, kuten motivaatiolla ja tunnetiloilla on tärkeä rooli poikkeuksellisen kyvyn kehittämisessä (Lubinski & Benbow, 2006).

Seuraavaksi tarkastellaan, miten eri tekijät ovat aikaisempien tutkimustulosten mukaan yhteydessä matematiikan osaamiseen ja sen kehittymiseen.

3 Matematiikan osaamiseen yhteydessä olevia tekijöitä

Matematiikan osaamiseen yhteydessä olevia tekijöitä voidaan tarkastella monesta eri näkökulmasta. Metsämuuronen (2009) on luonut mallin, jossa oppimiseen, oppimistuloksiin ja oppimistulosten muutokseen vaikuttavia tekijöitä tarkastellaan kahdeksasta eri näkökulmasta: opiskelijaan liittyvät yksilölliset tekijät, vertaisryhmään liittyvät tekijät, kotiin ja perheeseen liittyvät tekijät, opettajaan ja opettamiseen liittyvät tekijät, koulun johtamiseen liittyvät tekijät, koulun fyysisiin olosuhteisiin liittyvät tekijät, taloustekijät ja demografiset tekijät. Näistä taustatekijöistä tässä tutkimuksessa tarkastellaan kolmea keskeistä näkökulmaa: yksilöön liittyvät tekijät, kouluun liittyvät tekijät ja kotitaustaan liittyvät tekijät. Yksilöön liittyviin tekijöihin luokittuvat esimerkiksi sukupuoli, aikaisempi osaaminen ja asenteet. Kouluun liittyviin tekijöitä ovat esimerkiksi opetukselliset tekijät ja vertaisryhmä koulussa. Kotitaustaan liittyviä tekijöitä ovat esimerkiksi oppilaan kielitausta, sosioekonominen tausta ja kodin antama tuki matematiikan opiskeluun.

3.1. Yksilöön liittyvät tekijät

Oppilaan yksilöllisistä tekijöistä osaamiseen ja sen kehittymiseen vaikuttavat kognitiivisten tekijöiden kuten aikaisemman osaamisen lisäksi muun muassa sukupuoli ja matematiikkaan liittyvät asenteet. Tässä luvussa keskitytään tarkastelemaan sukupuolta ja asenteita, jotka ovat aikaisempien tutkimustulosten valossa keskeisiä matematiikan osaamiseen yhteydessä olevia tekijöitä.

Sukupuolten välisistä eroista osaamisessa raportoidaan jatkuvasti sekä kansallisissa että kansainvälisissä oppimistulosarvioinneissa. Kansallisten oppimistulosarviointia koskevien lähtötasomittaustulosten mukaan matematiikan osaamisessa on havaittu olevan sukupuolten välillä eroa jo koulun aloitusvaiheessa (Ukkola & Metsämuuronen, 2019). Kansainvälisesti tarkasteltuna Suomessa tytöt ja pojat osaavat matematiikkaa keskimäärin yhtä hyvin, mutta erot näkyvät kaikkein korkeimmilla taitotasoilla, joissa pojat ovat yliedustettuja. Esimerkiksi PISA 2018 -tulosten (Leino ym., 2019) mukaan suurin osa suomalaistytöistä ja -pojista menestyy koulussa yhtä hyvin, mutta poikien osaamisessa on enemmän vaihtelua.

Pojat ovat yleisesti tyttöjä parempia matematiikassa ja tytöt lukemisessa ja poikien osaamisen vaihtelu on tyttöjen osaamisen vaihtelua suurempaa maailmanlaajuisesti ja koulutuksellisista ratkaisuista riippumatta (mm. Machin & Pekkarinen, 2008). Meta-analyysin (O'Dea ym., 2018) mukaan poikien suurempi vaihtelu on pysynyt samanlaisena viimeisten 80 vuoden aikana ja se näkyy erityisesti matemaattisluonnontieteellisissä aineissa. Tällaiselle vaihteluhypoteesille (variability hypothesis) on ehdotettu löytyvän osaselitys perinnöllisyydestä (Johnson ym., 2008).

Yksilön matematiikkaan liittyvien asenteiden yhteyttä osaamiseen on tutkittu paljon sekä kansainvälisesti että kansallisesti ja osaamisen ja asenteiden on havaittu välillä olevan selkeä yhteys (Ma & Kishor, 1997). Yhteyttä ei ole kuitenkaan kovin laajasti ja tarkasti analysoitu, ja asenteiden ja osaamisen välinen syy-seuraussuhde on jäänyt epäselväksi (esim. Leder, 2006).

Kansallisten oppimistulosarviointien (mm. Tuohilampi & Hannula, 2013; Metsämuuronen, 2017) mukaan asenteilla ja matematiikan osaamisella on vahva positiivinen yhteys. Sen voimakkuus kuitenkin vaihtelee vuosiluokkakohtaisesti vahvistuen ikävuosien myötä (Tuohilampi & Hannula, 2013). Pääsääntöisesti oppilaiden asenteet koulunkäyntiä ja oppimista kohtaan ovat ensimmäisinä kouluvuosina positiivisia. Oppilailla on silloin yleisesti myönteiset asenteet niin koulunkäyntiä kuin myös eri oppiaineita kohtaan (Harter, 1999; Metsämuuronen ym., 2012; Tuohilampi ym., 2013).

Tuohilammen ja Hannulan (2013) mukaan asenteiden muutos näkyy pitkittäisaineistossa kolmannelta luokalta yhdeksännelle aluksi vain matematiikasta pitämisessä, joka heikentyy olennaisesti kuudennella luokalla. Minäpystyvyys ja hyödyllisyyden kokeminen laskevat selvästi yläkoulun aikana. Tuohilammen ja Hannulan (2013, s. 236) mukaan tällaisella asenteiden muutoksella kielteisemmiksi voi olla seurauksia opiskeluvalintoihin ja tuleviin opintoihin sekä elämässä menestymiseen.

Minäpystyvyyden ja osaamisen välisen suhteen kausaliteetista on tehty erilaisia päätelmiä. Positiivisen minäpystyvyyden on havaittu selittävän hyviä oppimistuloksia ja vastavuoroisesti hyvät oppimistulokset selittävät luottamusta omaan osaamiseen. Muun muassa Williamsin ja Williamsin (2010) PISA-aineistoon pohjautuvan kansainvälisen vertailuanalyysin mukaan matemaattisen minäpystyvyyden vaikutus suoriutumiseen on Suomessa suhteellisen pieni muihin maihin verrattuna, mutta tilastollisesti merkitsevä, ja matematiikan osaamisen vaikutus minäpystyvyyteen yksi suurimmista. Banduran (1986) sekä Pajaresin ja Millerin (1994) mukaan minäpystyvyyden tunne selittää tulevaa osaamistasoa muita matematiikkaan liittyviä asenteita paremmin, kun minäpystyvyys ohjaa yksilön käyttäytymistä ja valintoja. Myös uudemmissa tutkimuksissa vahvan käsityksen omasta osaamisesta on todettu ennustavan parempaa koulumenestystä (mm. Bryan ym., 2011; Jiang ym., 2014; Suárez-Álvarez ym., 2014). Myös PISA2012-tutkimuksessa matematiikan minäkäsityksen ja suoritusluottamuksen on havaittu selittävän vahvimmin matematiikan osaamista kansallisella tasolla (Kupari & Nissinen, 2015).

Hannulan, Bofahin, Tuohilammen ja Metsämuurosen (2014) tekemän pitkittäisanalyysin mukaan osaaminen vaikuttaa minäpystyvyyden kehittymiseen luokilla 3–6, ja luokka-asteilla 6–9 osaaminen ja minäpystyvyys ovat vastavuoroisessa suhteessa. Hannulan ja Laakson (2011) mukaan osaamisen ja asenteiden välinen yhteys voimistuu iän karttuessa, kun käsitykset muuttuvat realistisimmiksi. Matematiikan parhaisiin osaajiin keskittyvän pitkittäisanalyysin mukaan (Niemi ym., 2020) vahva minäpystyvyys selittää asenteista parhaiten oppilaan kuulumista parhaiden osaajien joukkoon yhdeksännellä vuosiluokalla.

3.2. Kouluun liittyvät tekijät

Jotta kyvykkyys, motivaatio ja minäpystyvyys voivat kehittyä, täytyy oppilaalle tarjottujen mahdollisuuksien vastata hänen potentiaaliaan (Leikin, 2014). Winner (2000) sekä Phillips ja Lindsay (2006) näkevät, että sopivilla haasteilla koulussa on suuri vaikutus erityisesti motivaatioon. Korkean motivaation saavuttamiseksi opiskelijat tarvitsevat vakautta, psykososiaalista tukea ja haasteita kognitiivisella tasolla (Ryan & Deci, 2000). Lisäksi mielekkäät oppimistilanteet ovat yhteydessä minäpystyvyyden vaalimiseen ja alisuoriutumisen välttämiseen (Colangelo ym., 1993; McCoach & Siegle, 2003).

Oppimistilanteet liittyvät vallitsevaan ympäristöön, jossa oppiminen tapahtuu. Voidaan puhua esimerkiksi luokkarakenteesta (*classroom structure*) (Ames, 1992) ja luokkailmapiiristä (Chionh & Fraser, 2009). Luokkarakenne liittyy opetukseen ja siihen, miten opettaja suunnittelee tehtävät, ohjaa oppimistapahtumaa ja arvioi oppilaiden edistymistä. Itävaltalaistutkimuksen (Lüftenegger ym., 2015) mukaan koulussa erinomaisesti menestyvät 15-vuotiaat nuoret kokivat keskitasoa heikompia osaajia enemmän autonomiaa ja kokivat saaneensa enemmän taitotasonsa mukaisia oppimistehtäviä ja osallistuneensa aktiivisemmin opiskelua koskevaan päätöksentekoon. Luokan ilmapiirin nähdään liittyvän opiskelijoiden yhteenkuuluvuuden tunteeseen ja sen on havaittu olevan positiivinen ennustaja saavutuksille (Chionh & Fraser, 2009). On kuitenkin hyvä muistaa, että jokaisen oppilaan yksilölliset tekijät vaikuttavat siihen, miten oppilas kokee opetustilanteen ja mitä hän oppii opetustilanteen aikana (Metsämuuronen, 2013).

Kansallisesti tarkasteltuna oppilasta aktivoivien menetelmien nähdään tuottavan parhaita oppimistuloksia. Oppimista ja opetusta ohjaa Perusopetuksen opetussuunnitelman perusteet, joka pohjautuu konstruktivistiseen oppimiskäsitykseen, jossa oppiminen ymmärretään yksilölliseksi ja yhteisölliseksi tietojen ja taitojen rakennusprosessiksi (Opetushallitus, 2004, s. 18). Uusissa opetussuunnitelman perusteissa korostetaan, että oppilas on aktiivinen toimija ja hän oppii asettamaan tavoitteita ja ratkaisemaan ongelmia sekä itsenäisesti että muiden kanssa (Opetushallitus, 2014, s. 17). Yleisesti muun muassa opettajien pitkän koulutuksen ja ammattitaidon on todettu olevan PISA-menestyksen selittäviä tekijöitä (Kansanen, 2003; Niemi, 2011; Niemi & Jakku-Sihvonen, 2011; Sahlberg, 2011).

Hannula ja Oksanen (2013) ovat selvittäneet opetuksellisten tekijöiden yhteyttä matematiikan osaamiseen kansallisessa pitkittäisanalyysissä. Tulosten mukaan tärkeimpiä osaamista kehittäviä opetuksellisia tekijöitä olivat vähäinen oppilaiden osallistaminen tavoitteiden asetteluun ja arviointiin, oppilaiden toistensa neuvominen, mahdollisuus saada luokkaan toinen opettaja ja opettajan kyvykkyys oppilaiden käyttäytymisen hallinnassa (Hannula & Oksanen, 2013). Myös kotitehtävien runsaammalla määrällä on havaittu olevan selkeä positiivinen yhteys matematiikan osaamiseen (Mattila & Rautopuro, 2013). Toisella asteella keskeinen osaamista selittävä tekijä oli se, kuinka usein opiskelijat kokivat opiskeltavien asioiden tulevan selväksi. Parhaita oppimistuloksia saatiin ryhmässä, jossa opettajajohtoisuuteen yhdistyy eriyttäminen taitotason mukaisesti ja saatujen tulosten järkevyyttä pohdittiin (Metsämuuronen, 2017). Saarinen (2020) analysoi PISA-aineistoista vuosina 2003–2015 käytössä olleiden opetusmenetelmien yhteyttä

15-vuotiaiden suomalaisten osaamiseen ja havaitsi, että itseohjautuvuutta edellyttävät menetelmät olivat yhteydessä heikompiin oppimistuloksiin ja opettajalähtöisten menetelmien käyttö korkeampiin oppimistuloksiin. Myös kansallisessa matematiikan oppimistuloksia mittaavassa tutkimuksessa (Metsämuuronen, 2010) havaittiin, että parhaatkin oppilaat hyötyivät opettajajohtoisesta opetuksesta ja konkreettisten havaintovälineiden käytöstä 6. vuosiluokalle tultaessa.

Koulussa viihtyminen, koulukiusaaminen ja työrauhaan liittyvät tekijät vaikuttavat osaltaan oppimiseen koulussa. Metsämuurosen (2017) mukaan koulukiusaaminen, luokan työrauhaongelmat ja heikko viihtyminen koulussa estävät osaamisen kasvua erityisesti yläkoulussa. Se, että oppilaat neuvovat toisiaan, parantaa keskitasoa parempien osaajien oppimistuloksia verrattuna keskitasoa heikompiin osaajiin.

3.3. Kotitaustaan liittyvät tekijät

Yksilöllisten ja kouluun liittyvien tekijöiden lisäksi oppilaan käyttäytymistä ohjaavat yksilön kotiin ja perheeseen liittyvät tekijät, joista tässä luvussa tarkastellaan kielitaustaa, sosioekonomista taustaa ja kodin antamaa tukea matematiikan opiskeluun.

Kansallisissa oppimistulosarvioinneissa erot suomen- ja ruotsinkielisten koulujen oppilaiden välillä kaventuvat ylemmillä luokka-asteilla, kun eroa osaamisessa suomenkielisten oppilaiden hyväksi on ollut havaittavissa jonkin verran alakoulun luokka-asteilla (Metsämuuronen, 2010; 2017). Osalla oppilaista kotikieli on muu kuin suomi tai ruotsi ja osa heistä opiskelee suomea tai ruotsia toisena kielenä -oppimäärän mukaan. Kansallisen matematiikan osaamista selvittävän tutkimuksen mukaan kotikielenään muuta kuin suomea tai ruotsia puhuvien oppilaiden osaaminen oli selvästi heikompaa kuin suomen- tai ruotsinkielisten oppilaiden osaaminen 6. ja 9. luokalla (Räsänen ym., 2010). Yhdeksännellä luokalla oppilaiden joukossa, jossa kotikieli oli muu kuin suomi, oli yli kolminkertainen määrä heikoiksi luokiteltuja oppilaita (Räsänen & Närhi, 2013). Hotulainen ja kanssakirjoittajat (2016) ovat tutkineet metropolialueen nuorten eriytyviä kehityspolkuja yläkoulun aikana ja heidän toisen asteen valintoja. Tutkimustulosten mukaan maahanmuuttajatausta on yhteydessä heikompaan lähtötasoon, mutta ei osaamisen kehitykseen yläkoulun aikana. Lähtötasoerot maahanmuuttajataustaisten ja kantaväestöön kuuluvien oppilaiden välillä selittyvät vanhempien koulutustaustan ja suomen kielen osaamisen eroilla (Hotulainen ym., 2013).

Oppilaan sosioekonomisen taustan on monissa tutkimuksissa todettu olevan yhteydessä osaamiseen ja selittävän matematiikan osaamisen eroja (mm. Suárez-Álvarez ym., 2014; Välijärvi, 2017). Sosioekonominen status määritellään eri tavoin ja määrittelyssä käytetään vanhempien tulo-, koulutus ja ammattitietoja. Ei ole olemassa yhtä yleisesti hyväksyttyä tapaa mitata sosioekonomista statusta (APA, 2007, s. 5; Bradley & Corwyn, 2002). Vanhempien koulutusta pidetään kuitenkin yhtenä keskeisistä sosioekonomisen statuksen osatekijöistä ja sitä on käytetty oppimistulosarvioinneissa yksinkertaisena sosioekonomisen statuksen indikaattorina. Vanhempien ylioppilastutkinto on selittänyt kansallisissa tutkimuksissa selvästi osaamisen eroja eri oppiaineissa (mm. Ouakrim-Soivio & Kuusela, 2012; Hildén & Rautopuro, 2014; Härmälä ym., 2014; Metsämuuronen, 2013; Kuukka & Metsämuuronen, 2016). Vanhempien koulutuksella näyttää olevan keskeinen rooli matematiikan osaamisen kehittymisessä jo 3. luokalta lähtien toisen asteen koulutukseen asti (Metsämuuronen, 2013; 2017).

Hiltunen ja Nissinen (2018) ovat tutkineet PISA 2015 -tutkimuksessa erinomaisesti menestyneitä suomalaisia matematiikan osaajia. Heidän mukaansa perheen korkealla sosioekonomisella statuksella on merkitsevä yhteys erinomaiseen osaamiseen ja erityisesti isän korkealla koulutustasolla on merkitystä. Välijärven (2017) mukaan sosioekonomisella taustalla on vahva yhteys myös oppilaan suoritusmotivaatioon. Oppilailla, joilla on korkeampi sosioekonominen tausta, on parempi suoritusmotivaatio kuin niillä, joilla on matalampi sosioekonominen tausta. Välijärven (2017) mukaan korkean sosioekonomisen taustan nuoret saavat enemmän tukea koulunkäyntiinsä ja tämä on yhteydessä parempiin oppimistuloksiin.

Sosioekonomisen taustan lisäksi tutkimukset ovat osoittaneet, että kodin antaman tuen merkitys oppimiselle on keskeinen. Vanhemman antama tuki, asenteet ja vaikutteet heijastuvat lapseen. Jos vanhemmat antavat lapselle tukea ja ovat kiinnostuneita lapsen koulunkäynnistä, lapsi menestyy paremmin koulussa (Robinson & Harris, 2014; Schneider, 1993). Kodin tuen yhteyttä osaamiseen on tutkittu toisen asteen opiskelijoiden osalta kansallisessa matematiikan arvioinnissa (Metsämuuronen, 2017). Tutkimustulosten mukaan kodin tuki selittää merkitsevästi osaamista sekä lukiossa että ammatillisessa koulutuksessa. Lukiossa yhteys näkyy voimakkaammin: mitä enemmän opiskelija koki saavansa tukea opiskeluun, sitä korkeampaa hänen osaamisensa oli. Ero osaamisessa kodin tukea saaneiden ääriryhmien välillä vastasi kahden vuoden opintoja. Ammatillisessa koulutuksessa vaikutus näkyi niin, että tukea erittäin vähän saaneiden opiskelijoiden osaamisen taso oli merkitsevästi matalampaa kuin muissa ryhmissä. Merkityksellisenä tekijänä kodin tuesta lukiossa osoittautui se, pitävätkö vanhemmat matematiikkaa oppiaineena tärkeänä ja ammatillisessa koulutuksessa se, arvostavatko vanhemmat koulutusta. Toki vanhempien ylioppilastaustalla saattaa olla merkitystä kodin antaman tuen kanssa, koska heillä on paremmat lähtökohdat esimerkiksi tukea lasta koulutehtävissä.

4 Matematiikan osaaminen toisen asteen koulutuksessa

Suomalainen koulujärjestelmä on kansainvälisesti verraten melko yhdenmukainen kaikille yhteisen perusopetuksen ajan. Sen jälkeen koulutus eriytyy lukioon ja ammatilliseen koulutukseen, jotka molemmat mahdollistavat etenemisen korkea-asteen koulutukseen. Käytännössä kuitenkin usein ammatillisesta koulutuksesta siirrytään suoraan työelämään ja lukiokoulutuksesta korkeakouluihin. Kalalahden, Zacheuksen, Laaksosen ja Jahnukaisen (2019) mukaan siirtymä toiselle asteelle sisältää tasa-arvoon liittyviä haasteita. Toisen asteen koulutukseen hakeminen ja siellä opiskelu sekä valmistuminen ovat yhteydessä opiskelijan perhetaustaan, koulumenestykseen ja koulunkäynnin sujuvuuteen.

Peruskoulun päättöarvosanat ovat keskeisessä asemassa toisen asteen koulutuspaikan valinnassa ja sinne pääsemisessä. Keskiarvoissa on huomattavia eroja koulujen ja kuntien välillä. Ouakrim-Soivio (2013) ja Hotulainen ym. (2017) ovat tutkimuksissaan osoittaneet, että opettajat suhteuttavat antamansa arvosanat muiden oppilaidensa osaamistasoon eivätkä valtakunnallisiin kriteereihin. Mahdollisuudet menestyä toisen asteen koulutuksessa eivät ole yhdenvertaisia (Hotulainen ym., 2017). Erot peruskoulun päättöarvosanoissa tyttöjen ja poikien välillä ja eri oppiaineissa uhkaavat oppilaiden yhdenvertaista kohtelua toiselle asteelle siirryttäessä. Tähän on puututtu, ja Opetushallitus on laatinut kriteerit perusopetuksen päättöarviointiin kaikissa oppiaineissa. Kriteerien mukaiset päättöarvosanat annetaan kaikissa oppiaineissa koko ikäluokalle keväällä 2022. On kuitenkin huomioitava, että päättöarvosanat eivät vastaa toisiaan eri lukioiden välillä. Osaamistasoltaan matalampien oppilaitosten parhaita arvosanoja saaneet opiskelijat saattavat olla heikompia kuin korkeimpia tuloksia saaneiden lukioiden heikoimpia arvosanoja saaneet opiskelijat (Metsämuuronen, 2017).

Matematiikan opintojen määrät ja tavoitteet ovat erilaiset lukiossa ja ammatillisessa koulutuksessa. Lukiossa matematiikan pitkän oppimäärän opinnoissa on kymmenen pakollista kurssia ja kolme syventävää kurssia. Lisäksi lukiot voivat tarjota näiden lisäksi muitakin kursseja. Lyhyen oppimäärän opinnoissa on kuusi pakollista ja kaksi syventävää kurssia (Opetushallitus, 2003). Matematiikan osaamista toisen asteen lopussa kuvaavien tutkimustulosten mukaan lyhyen matematiikan vähimmäiskurssimäärän suorittaneet opiskelijat säilyttävät 9. luokan osaamisen tason, ja pitkän oppimäärän suorittaminen kasvattaa osaamista merkittävästi (Metsämuuronen, 2017). Ylioppilastutkintojärjestelmä mahdollistaa, että pitkän matematiikan oppimäärän opiskellut voi kirjoittaa lyhyen matematiikan ylioppilaskokeen.

Ammatillisessa koulutuksessa ei ole lukiokoulutusta vastaavaa kurssijärjestelmää. Metsämuurosen (2017) mukaan ammatillisen koulutuksen matematiikan sisällöt vastaavat lukiokoulutuksen lyhyen matematiikan kursseja *Lausekkeet ja yhtälöt* ja *Geometria* tai pitkän matematiikan kurssia *Funktiot ja yhtälöt* ja toista seuraavista kursseista: *Polynomifunktiot* tai *Geometria*. Ammatillisen koulutuksen tutkintojen perusteissa (Opetushallitus, 2009) korostetaan matematiikan taitojen käyttämistä ja soveltamista ammattiin liittyvissä tilanteissa. Metsämuurosen (2017) mukaan ammatillinen koulutus tarjoaa mahdollisuuden saavuttaa lukion matematiikan lyhyttä oppimäärää vastaavan osaamisen tason, mutta matematiikan osaaminen jää usein vähäiseksi, mikä on keskeinen ongelma jatko-opintojen kannalta.

5 Tutkimuskysymykset

Tutkimuksessa selvitetään yhdeksännen vuosiluokan matematiikan parhaiden osaajien yksilöön liittyvien tekijöiden, kouluun liittyvien tekijöiden ja kotitaustaan liittyvien tekijöiden yhteyttä toisen asteen koulutusvalintaan ja osaamisen kehittymiseen toisen asteen opintojen aikana.

1. Mitkä tekijät selittävät yhdeksännen vuosiluokan matematiikan parhaiden osaajien hakeutumista ammatilliseen koulutukseen ja lukioon?
2. Miten yhdeksännen vuosiluokan matematiikan parhaiden osaajien matematiikan osaamistaso muuttuu toisen asteen opintojen aikana?
   (a) Miten suuri osa yhdeksännen vuosiluokan matematiikan parhaista osaajista on parhaita osaajia myös toisen asteen lopussa?

(b) Mitkä tekijät selittävät joidenkin yhdeksännen vuosiluokan parhaiden osaajien putoamista parhaiden osaajien joukosta toisen asteen lopussa?

6 Tutkimusmenetelmät

6.1. Tutkimuskohde

Tutkimuksessa käytettävä aineisto on Opetushallituksen ja Kansallisen koulutuksen arviointikeskuksen (Karvin) keräämä pitkittäisaineisto, jossa samaan ikäluokkaan kuuluvien oppilaiden matematiikan osaamisesta, asenteista ja taustatekijöistä on kerätty tietoa vuosien 2005-2015 aikana oppilaiden ollessa kolmannella, kuudennella ja yhdeksännellä vuosiluokalla sekä toisen asteen päättyessä. Aineisto on kansallisesti edustava ja siinä on huomioitu muun muassa maantieteellinen jakauma.

Tutkimusaineisto käsittää kaikkiaan 3896 oppilasta, joista erityisen tarkastelun kohteena ovat matematiikan parhaat osaajat. Parhaat osaajat määritetään yhdeksännen vuosiluokan kokeessa menestymisen perusteella niin, että heidän saamansa kokonaispistemäärä on vähintään 1,5 hajontayksikköä kokeen keskiarvoa korkeampi. Vertailuryhmänä tutkimuksessa ovat hyvät osaajat ja keskitason osaajat. Hyvien osaajien kokonaispistemäärä on 1,0–1,5 hajontayksikköä kokeen keskiarvoa korkeampi ja keskitason osaajien kokonaispistemäärä on enintään 1,0 hajontayksikön päässä kokeen keskiarvosta. Heikoimmat osaajat on rajattu tarkastelusta pois tässä tutkimuksessa.

Yhdeksännen vuosiluokan kokeessa parhaiten menestyneitä oppilaita on yhteensä 292 (7,5 % koko otoksesta). Poikia on 187 (64,0 %) ja tyttöjä 105 (36,0 %). Taulukossa 1 on tietoa parhaiden, hyvien ja keskitason osaajien koepisteistä yhdeksännellä vuosiluokalla.

Taulukko 1. Koepisteet yhdeksännellä vuosiluokalla

Koepisteet	Parhaat osaajat (n = 292)	Hyvät osaajat (n = 326)	Keskitason osaajat (n = 2658)	Koko otos (n = 3896)
Keskiarvo	722,7	641,5	506,9	510,5
Min.	673,3	617,1	409,6	131,0
Max.	1029,5	665,1	616,1	1029,5
Keskihajonta	49,8	14,5	57,2	106,3

6.2. Matematiikan koetehtävät tutkimuksessa

Kansallisissa arvioinneissa olevat matematiikan kokeet mittaavat opetussuunnitelmien toteutumista eli kokeiden sisältöalueet on valittu perusopetuksen ja toisen asteen opetussuunnitelmien perusteiden mukaan. Aineisto perustuu perusopetuksen ja lukion opetussuunnitelman perusteisiin (Opetushallitus, 2003; 2004) sekä ammatillisen tutkinnon koulutuksen perusteisiin (Opetushallitus, 2009). Perusteet on päivitetty myöhemmin tämän aineiston keräämisen jälkeen. Kansallisten kokeiden tehtävät, arvosteluperusteet ja pisteytysohjeet laaditaan yleisesti asiantuntijaryhmissä. Lisäksi tehtäväsarjojen laadun arvioinnissa käytetään asiantuntijoita ja esitestausta. Tehtäväsarjoissa on vaikeustasoltaan helppoja, keskivaikeita ja vaikeita osioita (Metsämuuronen, 2009).

Matematiikan arviointi kohdistuu matematiikan kokonaisosaamiseen, joka koostuu eri osa-alueista taulukossa 2 esitetyn jaottelun mukaan.

Taulukko 2. Matematiikan osa-alueet eri vuosiluokkien kokeissa

	Luokka-aste	Osioiden määrä	Maksimi-pistemäärä	Reliabiliteetti (α)
Kokonaisosaaminen	3.lk	38	44	0,86
	6. lk	39	52	0,85
	9. lk	68[1]	84[1]	0,94
	lukio	28	52	0,87
	ammatillinen	30	46	0,84
Luvut, laskutoimitukset ja algebra	3. lk	22	24	0,81
	6. lk	21	28	0,78
	9. lk	36	40	0,88
	lukio	3	3	0,27
	ammatillinen	3	3	0,26
Geometria	3. lk	10	14	0,67
	6. lk	10	14	0,66
	9. lk	16	22	0,83
	lukio	7	14	0,73
	ammatillinen	7	14	0,65
Tietojen käsittely, tilastot ja todennäköisyys	3. lk	6	6	0,55
	6. lk	8	10	0,47
	9. lk	7	9	0,61
	lukio	2	2	0,34
	ammatillinen	5	5	0,56
Algebra	lukio	6	8	0,71
	ammatillinen	6	8	0,71
Funktiot	lukio	11	31	0,82
	ammatillinen	12	22	0,66

1 Sisältää viisi Funktiot-osa-alueen laskua

Toisella asteella toteutetuista matematiikan kokeista tehtiin kaksi versiota, toinen lukioon ja toinen ammatilliseen koulutukseen. Kokeiden pohjana käytettiin yhdeksännen vuosiluokan koetta niin, että 78 prosenttia tehtävistä oli samoja tehtäviä. Joitain tehtäviä otettiin myös kuudennen luokan ja kolmannen luokan kokeista. Lisäksi ammatillisen koulutuksen kokeessa oli kaksi tehtävää vuoden 1998 ammatillisen koulutuksen matematiikan kansallisesta kokeesta ja yksi lyhyen matematiikan ylioppilastehtävä. (Metsämuuronen, 2017).

Jotta eri vuosien kokeiden ja eri koeversioiden tuloksia voidaan vertailla, täytyy pistemäärät vertaistaa eli saattaa yhteismittalisiksi. Vertaistamisessa on käytetty osio-vasteteoriaan (*Item Response Theory*) perustuvaa IRT-mallitusta (Rasch, 1960; Lord & Novick, 1968). Opetushallituksen ja Karvin pitkittäisvertailujen (esim. Metsämuuronen, 2006; 2013) lisäksi vertaistamista on käytetty myös kansainvälisissä PISA- (esim. OECD, 2007; 2010; Hautamäki ym. 2008) ja TIMMS-tutkimuksissa (esim. TIMSS, 2009). IRT-mallituksessa oppilaan osaamisen taso ja tehtävän vaikeustaso saatetaan vastaamaan toisiaan. Eri koeversioissa olevien osioiden vaikeustasoa voidaan arvioida linkkitehtävien avulla. Toisin sanoen eri koeversioihin määritetään linkkitehtäviä eli identtisiä tehtäviä, joiden avulla selvitetään, kuinka paljon osaamista tarvitaan, että osiot voidaan ratkaista ja tämän jälkeen kuinka paljon osaamista tarvitaan kunkin pistemäärän saavuttamiseen koko kokeessa ja osamittareissa. Tässä aineistossa vertaistamiseen on valittu perustasoksi 9. luokan koe ja opiskelijoita verrataan siinä keskitasoisesti menestyneeseen oppilaaseen (ks. tarkemmin mm. Metsämuuronen, 2017, s. 213–214).

Tutkimuksessa osaamista kuvaavat pistemäärät esitetään samalla asteikolla kuin PISA- ja TIMSS-tutkimuksissa. Tässä asteikossa 9. vuosiluokalla osaamiseltaan keskitason oppilas saa 500 pistettä ja keskihajonta on 100 pistettä (ks. Metsämuuronen 2017, s. 214–215). Osaamisessa tapahtunutta muutosta tarkastellaan kokonaispistemäärässä tapahtuneen muutoksen mukaan. Toisen asteen koe on linkkitehtävillä saatu vertailukelpoisiksi ja kokeen vertaistetut pistemäärät ovat suoraan verrattavissa yhdeksännen luokan osaamistasoon.

6.3. Asenteiden kartoittaminen tutkimuksessa

Matematiikan osaamista mittaavien kokeiden lisäksi oppilailta on koottu tietoja asenteista matematiikkaa kohtaan. Asenteiden kartoittamisessa käytetään 15 osion Likert-asteikollista mittaria, joka pohjautuu laajalti käytettyyn Fenneman ja Shermanin (1976) matematiikka-asennemittariin. Kolmannella vuosiluokalla käytettiin lyhennettyä versiota standardimittarista ja sanamuotoja muokattiin konkreettisemmiksi. Mukana ei ollut oppiaineen hyödylliseksi kokemisen osa-aluetta, koska kysymykset liittyivät pitkälti jatko-opintoihin ja työelämään. Lisäksi asteikko oli neliportainen muuten viisiportaisena käytetyn Likert-asteikon sijaan. Kuudennella ja yhdeksännellä luokalla sekä toisella asteella käytetyt mittarit olivat keskenään samanlaiset, mutta yhdeksännellä vuosiluokalla ja toisella asteella kerättiin tietoa myös oppilaiden matematiikka-ahdistuksesta. Lisäksi toisella asteella on kartoitettu oppilaiden tunnetiloja (innostus, kiinnostus, tylsyys, pitäminen, turhautuminen, viha, ahdistus, avuttomuus, tyytyväisyys) matematiikan opiskelussa. Tunnetiloista on muodostettu kaksi faktoria: positiiviset ja negatiiviset tunnetilat. (Metsämuuronen & Tuohilampi, 2017).

Asenteita on kartoitettu eri väittämin taulukossa 3 näkyvien osa-alueiden mukaan.

Taulukko 3. Asennemittareiden osa-alueet eri vuosiluokilla

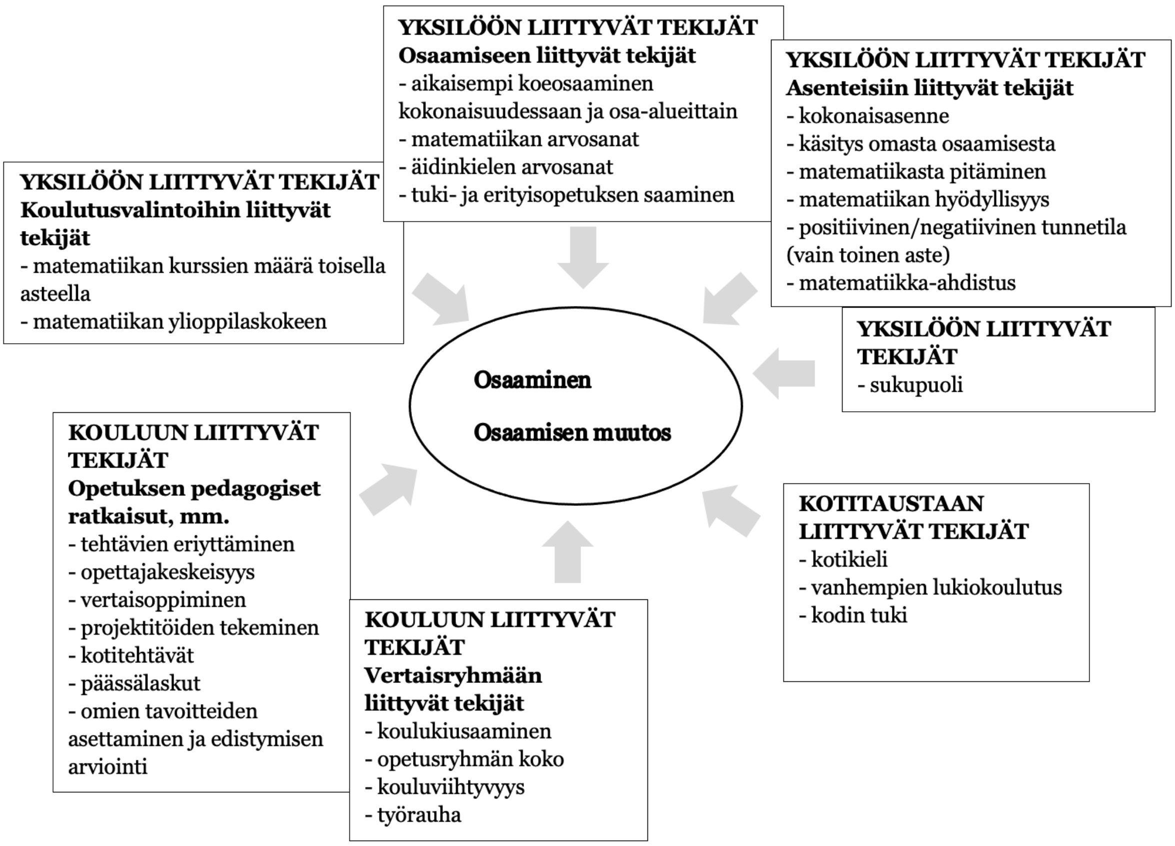
	Luokka-aste	Osioiden määrä	Maksimi-pistemäärä	Reliabiliteetti (α)
Kokonaisasenne[1]	3.lk	8	32	0,86
	6. lk	10	50	0,88
	9. lk	10	50	0,91
	lukio	15	60	0,92
	ammatillinen	15	60	0,91
Käsitys itsestä matematiikan osaajana (OSAA)	3. lk	4	16	0,79
	6. lk	5	25	0,82
	9. lk	5	25	0,88
	lukio	5	20	0,86
	ammatillinen	5	20	0,87
Matematiikasta pitäminen (PITÄÄ)	3. lk	4	16	0,88
	6. lk	5	25	0,89
	9. lk	5	25	0,90
	lukio	5	20	0,92
	ammatillinen	5	20	0,91
Matematiikan koettu hyödyllisyys (HYÖTY)	9. lk	5	25	0,53
	lukio	5	20	0,83
	ammatillinen	5	20	0,83
Matematiikka-ahdistus	9. lk	4	20	0,71
	lukio	3	13	0,76
	ammatillinen	3	13	0,76
Positiiviset tunnetilat matematiikan opiskelussa	lukio	9	36	0,90
	ammatillinen	9	36	0,90

1 Kokonaisasenteessa 3. ja 6. luokalla mukana OSAA- ja PITÄÄ-osa-alueet

6.4. Taustamuuttujat

Matemaattista osaamista selitetään erilaisilla muuttujilla, joita on selvitetty oppilaille suunnattujen taustakyselyiden avulla osaamista kartoittavien kokeiden yhteydessä. Kuviossa 1 on malli, jossa on esitetty taustamuuttujat, joiden yhteyttä osaamiseen ja osaamisen muutokseen selvitetään tässä tutkimuksessa. Malli on luotu mukaillen Metsämuurosen (2009) mallia, jossa oppimiseen, oppimistuloksiin ja oppimistulosten muutokseen vaikuttavia tekijöitä tarkastellaan kaikkiaan kahdeksasta eri näkökulmasta.

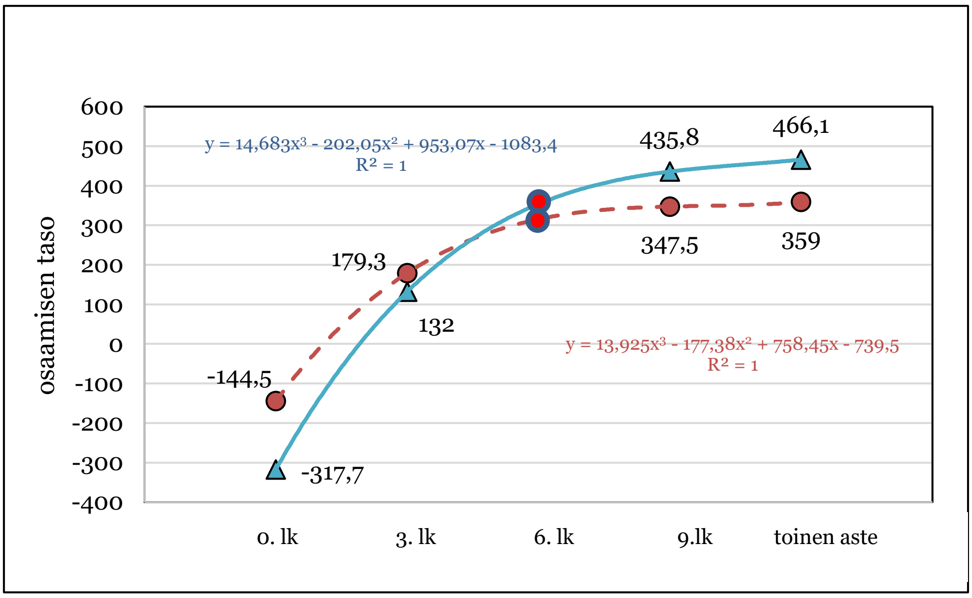
Tässä tutkimuksessa mallissa ovat mukana yksilöön liittyvät tekijät, kouluun liittyvät tekijät ja opiskelijan kotitaustaan liittyvät tekijät. Yksilöön liittyvät tekijät jakautuvat osaamiseen, asenteisiin ja koulutusvalintoihin liittyviin tekijöihin sekä sukupuoleen. Kouluun liittyvät tekijöihin kuuluvat opetuksen pedagogiset ratkaisut ja koulun vertaisryhmään liittyvät tekijät. Kotitaustaan liittyvät tekijät käsittävät kotikielen, vanhempien lukiokoulutuksen ja kodin antaman tuen matematiikan opiskeluun.



Kuvio 1. Taustamuuttujien käsitteellinen malli

6.5. Puuttuvien havaintojen mallittaminen aineistossa

Pitkittäisaineistossa on puuttuvaa tietoa opiskelijoista, joilta ei ole saatu kerättyä tietoa kaikilta neljältä eri mittauskerralta. Tällöin yksittäistä opiskelijaa koskevia puuttuvia havaintoja on mallitettu muiden mittaustulosten avulla hyödyntäen lineaarista ja epälineaarista regressioanalyysiä. Vuoden 2008 eli 6. vuosiluokan aineistossa oli vähiten puuttuvia tapauksia. Kuuden puuttuvan tapauksen arvot interpoloitiin epälineaarisesti käyttämällä 3. ja 9. vuosiluokan sekä toisen asteen koulutuksen arvoja. Mallituksessa hyödynnettiin myös niin sanotun 0. luokan tietoa, joka oli mallinnettu asiantuntijamenettelynä niin, että arvioitiin, millaisista 3. luokan tehtävistä koulun aloittavat oppilaat olisivat todennäköisesti suoriutuneet. Kuviossa 2 on esitetty kahden oppilaan puuttuvan tiedon korvaaminen 6. vuosiluokalla. Vuoden 2012 eli 9. vuosiluokan datasta puuttui 128 tapauksen arvot. Ne interpoloitiin lineaarisesti hyödyntäen 6. vuosiluokan ja toisen asteen koulutuksen arvoja. Vuoden 2005 eli 3. vuosiluokan datasta puuttui 625 tapauksen arvot, jotka mallinnettiin lineaarisesti tietäen 6. ja 9. vuosiluokan mittaustulokset. Ennuste oli kohtalainen, vaikka 3. luokan osaaminen ei ole yhtä vakaata kuin myöhemmillä luokka-asteilla.



Kuvio 2. Kahden oppilaan puuttuvan tiedon korvaaminen 6. luokan aineistossa

Vuoden 2015 puuttuvien arvojen laskeminen oli monimutkaisempaa kuin puuttuvien arvojen laskeminen aikaisempien vuosien osalta. Ensinnäkin puuttuvia tapauksia oli eniten vuoden 2015 osalta. Koko aineistosta puuttuvia tapauksia oli 47,4 % (n = 1846) ja parhaiden osaajien osalta 14,0 % (n = 41). Puuttuvien tapausten suuren määrän vuoksi mallinnuksen tulos ei ole yhtä vakaa kuin vuosien 2008 ja 2012 mallinnus. Viimeisen mittausvuoden mallinnus pystyttiin tekemään hyödyntäen opiskelijoiden rekisteritietoja suoritettujen matematiikan kurssien lukumäärästä, niiden arvosanoista ja matematiikan ylioppilaskirjoituksen tuloksista. Toiseksi mallinnus oli järkevää tehdä erikseen lukion ja ammatillisen koulutuksen opiskelijoille ja kolmanneksi kun lukion ja ammatillisen koulutuksen puolella opiskelijat jakautuivat eri alaryhmiin koulutusvalintojen suhteen, mallinnus oli järkevää tehdä myös eri alaryhmille erikseen.

Lukiossa opiskelijat ovat opiskelleet matematiikan pitkän (vähintään 12 kurssia) tai lyhyen oppimäärän (vähintään 7 kurssia). Lyhyen oppimäärän opiskelleista osa suorittaa matematiikan ylioppilaskokeen ja on opiskellut kursseja enemmän kuin vähimmäismäärän. Osa opiskelijoista ei kirjoita matematiikkaa ylioppilaskokeessa ja on suorittanut matematiikan kursseja vain vähimmäismäärän. Osaamistasojen erot näiden kolmen ryhmän välillä ovat tilastollisesti merkitseviä (Metsämuuronen, 2017; Metsämuuronen & Tuohilampi, 2017). Puuttuvat arvot mallinnettiin näissä kolmessa ryhmässä erikseen. Lukioaineistossa 9. luokan tulos ennusti osaamista yksinään 57 % (R^2 = 0,57). Kun lisäksi tiedettiin lukion aikana suoritettujen matematiikan kurssien määrä, kokonaisselitysaste nousi 72 prosenttiin (R^2_{Adj} = 0,72).

Ammatillisessa koulutuksessa opiskelijat suuntautuivat kahdeksalle opintoalalle (humanistiset tieteet ja koulutus, kulttuuri, yhteiskuntatieteet, liiketalous ja hallinto, luonnontieteet, tekniikka ja liikenne, luonnonvarat ja ympäristö, sosiaalipalvelut, terveys ja liikunta sekä matkailu, ravitsemus ja talous), joissa matematiikan tarve eroaa toisistaan huomattavasti ja joissa opiskelijoiden osaamistasot eroavat tilastollisesti merkitsevästi toisistaan (Metsämuuronen & Salonen, 2017). Puuttuvat arvot mallinnettiin näissä ryhmissä erikseen. Mallinnuksessa käytettävää tietoa oli ammatillisessa koulutuksessa vähemmän, koska matematiikan kurssien lukumäärä, kolme, on kaikille sama. Viisi muuttujaa, osaaminen vuosina 2012, 2008 ja 2005, matematiikan kurssien keskiarvo ja ammatillisen koulutusyksikön keskimääräinen osaaminen, selittivät 63 % ammatillisen oppilaitosten tuloksista (R^2_{Adj} = 0,63). Koska vuoden 2015 aineistossa puuttuvia havaintoja on paljon, on toista astetta koskeviin tuloksiin syytä suhtautua myös kriittisesti. Puuttuvien havaintojen mallituksessa käytettyjen mallien selitysasteet olivat kuitenkin korkeita, joten ennusteet ovat riittävän tarkkoja uskottavien johtopäätösten tekemiseen.

6.6. Tulosten analysointi

Tulosten kuvailussa käytetään perustunnuslukuja kuten frekvenssi- ja prosenttijakaumia sekä keskiarvo- ja keskihajontalukuja. Ryhmien välisiä eroja analysoidaan parametrisin testein kuten t-testillä ja yksisuuntaisella varianssianalyysillä.

Osaamista ja osaamisen muutosta selittäviä tekijöitä analysoidaan monimuuttujamenetelmin. Ensiksi keskeisten ennustekijöiden selvittämisessä käytetään päätöspuu-analyysia (*decision tree analysis*, DTA). Sen avulla voidaan eksploroida laajaa aineistoa ja selvittää muun muassa millaiset tekijät osoittavat oppilaan kuuluvan parhaiden osaajien joukkoon toisen asteen lopussa. Menetelmä löytää oleelliset muuttujat, jotka erottelevat ja luokittelevat selitettävää muuttujaa uskottavasti. Analyysissa käytettiin CHAID-algoritmia, joka etsii tilastollisesti samankaltaisia arvoja selittävän ja selitettävän muuttujan välillä. Algoritmi etsii ja luokittelee ryhmiä, joiden välinen ero on mahdollisimman suuri vertaamalla testien p-arvoja. Jos selittävä muuttuja on jatkuva, p-arvoja etsitään F-testin avulla. Jos muuttuja on järjestysasteikollinen, p-arvoa etsitään likelihood-ratio -testillä. Mikäli muuttuja on luokitteluasteikolliinen, p-arvoa etsitään khiin neliö- testillä tai likelihood-ratio -testillä (Kass, 1980; Metsämuuronen, 2003, 738–739; 2013, 56–57). Tässä tutkimuksessa menetelmää käytetään löytämään keskeisiä erottelevia tekijöitä eri osaajaryhmien välillä.

Toiseksi asiayhteyksien mallintamisessa ja osaamista ennustavien tekijöiden analysoinnissa hyödynnetään lineaarista regressioanalyysiä, jonka avulla saadaan muun muassa muuttujien selitysosuudet paremmin näkyviin. Regressioanalyysissa käytetään askeltavaa menettelyä (stepwise selection). Tabachnick ja Fidell (2007) kutsuvat menettelyä tilastollisiksi menettelyiksi (statistical regression), koska selittävät muuttujat valitaan malliin pelkästään tilastollisin perustein. Askeltavassa menettelyssä yhtälöön lisätään riippumattomia muuttujia yksi kerrallaan ja niitä voidaan myös poistaa, kun uusia paremmin selittäviä muuttujia tulee tilalle. Lopulliseen malliin jää selitysvoimaltaan tilastollisesti merkitsevät muuttujat (Pedhazur, 1982; Metsämuuronen, 2003; Tabachnick & Fidell, 2007). Regressioanalyysin oletuksena on, että selittävät muuttujat eivät korreloi liian voimakkaasti toistensa kanssa. Selittävien muuttujien välistä voimakasta korrelointia, multikollineaarisuutta tutkitaan analyysin tekemisen yhteydessä. Regressioanalyysin tulokset esitetään niin, että muuttujat ovat analyysin esittämässä järjestyksessä. Mallissa ensimmäisenä esitetty muuttuja selittää selitettävän muuttujan vaihtelua parhaiten ja seuraavat muuttujat lisäävät mallin selitysastetta. Lineaarisen regressioanalyysin käytössä tulee ottaa huomioon, että muuttujien yhteydet eivät ole puhtaan lineaarisia. Havaittu vaikutus saattaa syntyä esimerkiksi muuttujan toisen ääripään voimakkaasta vaihtelusta (Metsämuuronen, 2009, s. 49).

7 Tulokset

Tuloksissa tarkastellaan ensin yhdeksännen vuosiluokan matematiikan parhaiden osaajien hakeutumista ammatilliseen koulutukseen ja lukioon ja sitä, mitkä tekijät selittävät valintaa. Sen jälkeen selvitetään, miten yhdeksännen vuosiluokan parhaiden osaajien osaaminen on muuttunut toisen asteen opintojen aikana ja mitkä tekijät selittävät osaamisen vaihtelua toisella asteella.

7.1. Matematiikan parhaiden osaajien hakeutuminen toisen asteen koulutukseen

Koska yhdeksännen vuosiluokan matematiikan parhaista osaajista vain 25 opiskeli ammatillisessa koulutuksessa, pidetään yhdeksännen vuosiluokan matematiikan hyvät osaajat tässä toisen asteen koulutusvalintaa selittävässä osiossa mukana. Näin ammatillisen koulutuksen ja lukiokoulutuksen välisestä erottelusta saadaan selkeämpi.

Selvä enemmistö parhaista sekä hyvistä osaajista opiskeli lukiossa (taulukko 4). Parhaista osaajista ja hyvistä osaajista noin joka kymmenes opiskeli ammatillisessa koulutuksessa.

Taulukko 4. Yhdeksäsluokkalaisten opiskelu ammatillisessa koulutuksessa tai lukiossa

	parhaat osaajat (n = 292)	**hyvät osaajat (n = 326)**	**keskitason osaajat (n = 2658)**	**koko otos (n = 3896)**
ammatillinen koulutus	8,6 %	12,9 %	45,1 %	46,2 %
lukio	91,4 %	86,9 %	54,1 %	53,8 %
kaksoistutkinto	0,0 %	0,3 % (1)	0,8 % (21)	0,8 % (33)

Enemmistö parhaista ja hyvistä osaajista suoritti pitkän matematiikan ylioppilaskokeen toisen asteen lopussa (taulukko 5). Tuloksissa keskitytään jatkossa kuitenkin tarkastelemaan koulutusvalintaa vain ammatillisen koulutuksen ja lukiokoulutuksen välillä eikä huomioida erikseen matematiikan opintojen laajuutta toisella asteella.

Taulukko 5. Yhdeksäsluokkalaisten matematiikan opintojen laajuus ylioppilaskoetiedon mukaan

	parhaat osaajat (n = 292)	hyvät osaajat (n = 326)	keskitason osaajat (n = 2658)	koko otos (n = 3896)
lyhyt matematiikka	10,0 %	16,6 %	24,8 %	20,0 %
pitkä matematiikka	65,8 %	54,0 %	12,3 %	18,0 %
ei tietoa tai ei kirjoita	24,3 %	29,4 %	62,9 %	62,1 %

DTA-analyysin avulla etsittiin tekijöitä, jotka selittävät, hakeutuuko yhdeksännen vuosiluokan hyviin tai parhaisiin osaajiin kuuluva oppilas toisella asteella ammatilliseen koulutukseen vai lukioon. Erottelevia tekijöitä etsittiin ensin osa-alueittain käsitteellisen muuttujamallin osioiden (ks. kuvio 1) mukaisesti ja koottiin sitten osamalleista löytyneistä tekijöistä kokonaismalli.

Koulutusvalintaa erottelevia muuttujia löytyi kaikista osa-alueista. Yksilöllisistä osaamiseen liittyvistä tekijöistä toisen asteen koulutusvalintaa selitti parhaiten äidinkielen arvosana 9. vuosiluokalla. Lukioon hakeutuneista oppilaista 64,2 prosenttia oli saanut äidinkielen arvosanaksi 9 tai 10 ja ammatilliseen koulutukseen hakeutuneista oppilaista 76,5 prosenttia sai arvosanaksi 6, 7 tai 8.

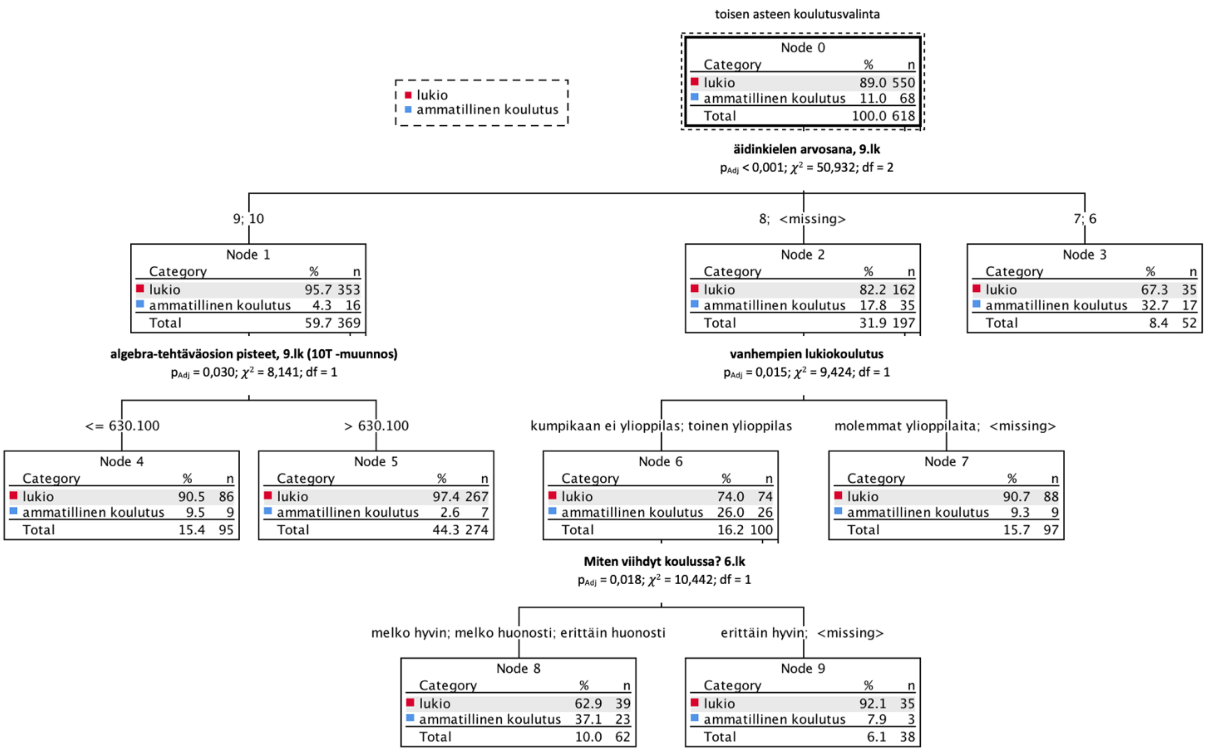
DTA-analyysi löysi vain yhden asenteisiin liittyvän muuttujan, joka erotteli toisen asteen koulutusvalintaa. Oppilaan arvio 3. luokalla siitä, onko matematiikka yksi oppilaan lempiaineista, erotteli parhaiten toisen asteen koulutuksen valintaa ($\chi^2(1)$ = 7,750; p = 0,038). Erottelu ei kuitenkaan ole kovin selkeä, vaikkakin lukioon hakeutuneiden osuus on 56,7 prosenttiyksikköä pienempi, jos oppilaan arvio väitteelle on ollut 0–1 (asteikolla 0–4) kuin jos arvio väitteelle on ollut yli 1. Toisaalta ammatillisen koulutuksen suhteen tällä ei ole juurikaan merkitystä.

Kotitaustaan liittyvistä muuttujista vanhempien lukiokoulutus erotteli parhaiten, hakeutuuko oppilas ammatilliseen koulutukseen vai lukioon ($\chi^2(1)$ = 24,104; p < 0,001). Malli jakoi tiedon vanhempien suorittamasta ylioppilastutkinnosta kahteen ryhmään: 1) molemmat ylioppilaita ja 2) kumpikaan ei ylioppilas tai toinen ylioppilas. Lukioon hakeutuneista oppilaista 47,5 prosentilla molemmat vanhemmat olivat ylioppilaita ja vastaavasti ammatilliseen koulutukseen hakeutuneista oppilaista 16,2 prosentilla molemmat vanhemmat olivat ylioppilaita.

Kouluun ja tarkemmin vertaisryhmään liittyvistä tekijöistä toisen asteen koulutusvalintaa selitti parhaiten koulussa viihtyminen 6. luokalla. Malli jakoi arvion kahteen ryhmään: 1) erittäin hyvin ja 2) melko hyvin, melko huonosti tai erittäin huonosti. Vain arvio erittäin hyvin erosi muista arvioista, jotka malli jakoi samaan ryhmään. Oppilaista, jotka olivat arvioineet viihtyvänsä koulussa erittäin hyvin 6. luokalla, 96,5 prosenttia hakeutui lukioon.

Opetuksen pedagogisista ratkaisuista hakeutumista ammatilliseen koulutukseen tai lukioon erotteli parhaiten 9. luokan väittämä, jonka mukaan tunneilla sovelletaan matematiikan taitoja arkielämän tilanteisiin ($\chi^2(1) = 13{,}606$; $p = 0{,}007$). Oppilaista suurin osa (79,5 %) oli arvioinut, että soveltamista on joskus, harvoin tai ei lainkaan. Malli jakoi nämä arviot samaan ryhmään. Opetuksen pedagogisten ratkaisujen erottelukyky toisen asteen koulutusvalinnan suhteen oli heikko.

Kokonaismallia etsivään analyysiin otettiin mukaan osamalleissa esille tulleet erottelevat tekijät. Kokonaismallissa (kuvio 3) toisen asteen koulutusvalintaa erotteli parhaiten äidinkielen arvosana 9. luokalla ($\chi^2(2) = 50{,}932$; $p < 0{,}001$). Lukioon hakeutumista indikoi erityisesti se, että opiskelija oli saanut äidinkielestä arvosanan 9 tai 10. Näiden oppilaiden valintaa ammatillisen koulutuksen ja lukion välillä erotteli seuraavaksi se, kuinka hyvin oppilas menestyi algebran osaamista mittaavissa tehtäväosioissa 9. luokan kokeessa ($\chi^2(1) = 8{,}141$; $p = 0{,}030$). Oppilaiden, jotka saivat äidinkielen arvosanaksi 8, koulutusvalintaa erotteli vanhempien lukiokoulutus ($\chi^2(1) = 9{,}424$; $p = 0{,}015$).



Kuvio 3. Toisen asteen koulutusvalintaa selittävien tekijöiden kokonaismalli DTA-analyysilla

7.2. Yhdeksännen vuosiluokan matematiikan parhaiden osaajien matematiikan osaaminen toisella asteella

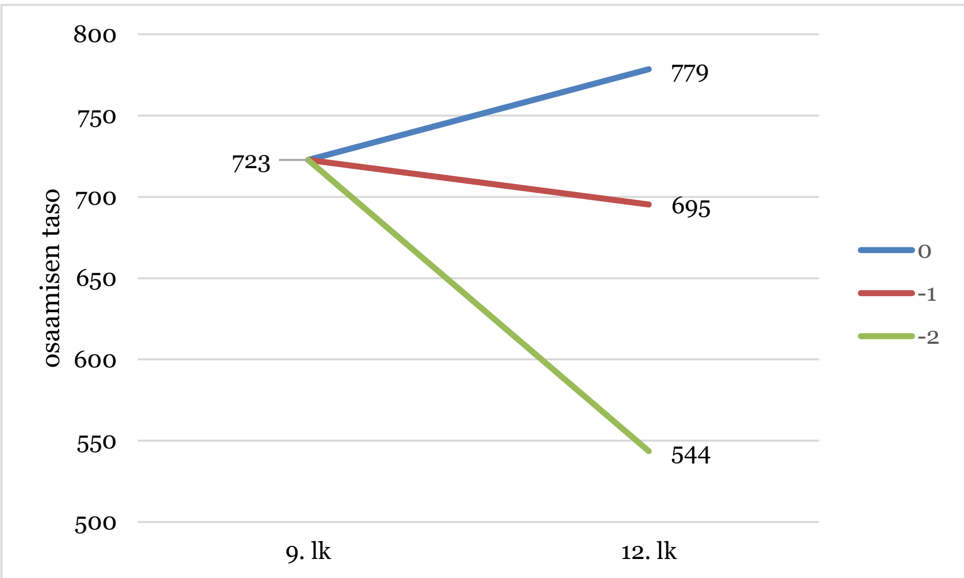
Taulukkoon 6 on koottu tiedot siitä, miten 9. vuosiluokan kokeessa parhaiten menestyneiden oppilaiden matematiikan osaaminen on muuttunut toisen asteen opintojen aikana, kun osaamista on arvioitu uudelleen toisen asteen lopussa. Nähdään, että 9. luokan keskitason osaajissa on tapahtunut eniten muutoksia. Keskitason osaajia löytyy kaikista osaajaryhmistä toisen asteen lopussa. Yhdeksännen vuosiluokan hyvät ja parhaat osaajat sijoittuvat toisen asteen lopussa keskitason, hyvien ja parhaiden osaajien ryhmiin.

Taulukko 6. Osaamisen muutos eri osaajaryhmissä

12.lk / 9.lk	alle keskitason	keskitaso	hyvät	parhaat	yhteensä
keskitaso	183	2275	145	55	2658
hyvät	0	115	126	85	326
parhaat	0	44	78	170	292
yhteensä	183	2434	349	310	3276

Kun katsotaan tarkemmin 9. luokan parhaita osaajia (n = 292), heidän osaamisessaan tapahtuneista muutoksista saadaan muodostettua kolme profiilia, joita merkitään arvoin 0, -1 ja -2 kuvaamaan osaamistason muutosta. Ensimmäisen profiilin (0) mukaisessa osaamisessa ei ole tapahtunut muutosta, vaan 9. luokan parhaat osaajat ovat parhaita osaajia myös toisen asteen lopussa (58,2 %). Toisen profiilin (-1) mukainen osaaminen on laskenut niin, että 9. luokan parhaat osaajat ovat toisen asteen lopussa hyviä osaajia (26,7 %). Kolmannen profiilin (-2) mukainen osaaminen on laskenut eniten niin, että 9. luokan parhaat osaajat ovat keskitason osaajia toisen asteen lopussa (15,1 %).

Kuviossa 4 näkyy, miten parhaiden osaajien osaamisen muutos jakautuu kolmeen suuntaan toisen asteen opintojen aikana.

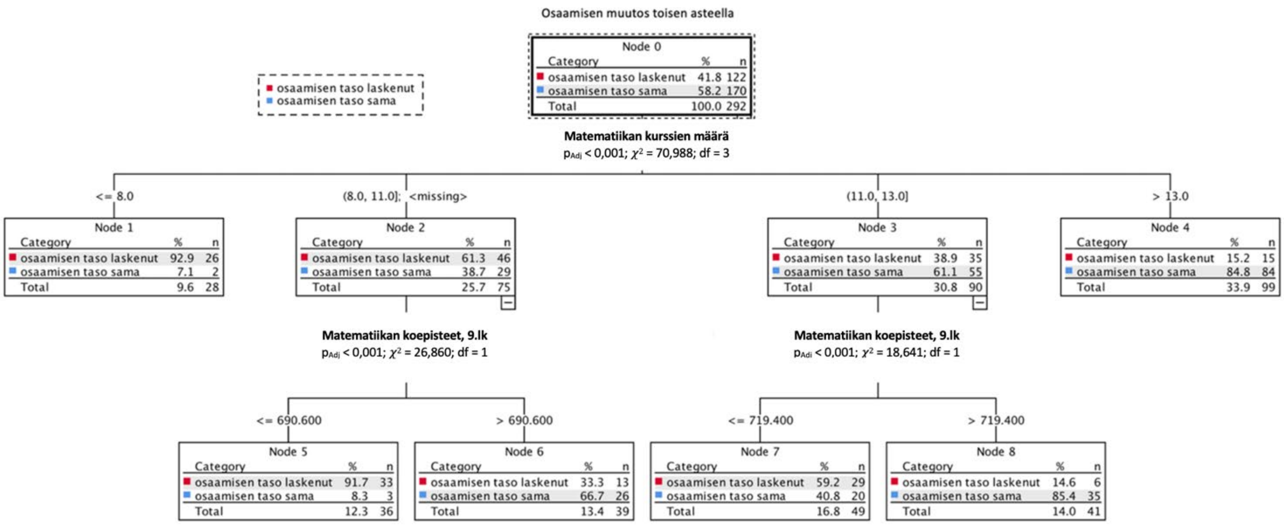


Kuvio 4. Parhaiden osaajien muutosprofiilit osaamisessa yhdeksännen luokan lopusta toisen asteen loppuun

Seuraavaksi on tarkoitus selvittää, mitkä tekijät selittävät muutoksen suuntaa. Etsitään selittäviä muuttujia ensin osa-alueittain, jotka on esitetty kuviossa 1. Asenteista mukana ovat vain perusopetusta koskevat muuttujat. Osamallien perusteella selittävistä muuttujista kootaan lopuksi kokonaismalli.

DTA-analyysien avulla osaamisen muutosta erottelevia muuttujia löytyi yksilöllisistä tekijöistä ja kouluun liittyvistä tekijöistä opetuksen pedagogisten ratkaisujen osalta (ks. kuvio 5). Oppilaan osaamista ja valintoja koskeviin malleihin luotiin ensin jako ammatilliseen koulutukseen ja lukioon, jotta erottelevat tekijät saatiin paremmin näkyviin, koska monet toisen asteen muuttujat liittyvät suoraan joko ammatilliseen koulutukseen tai lukioon. Kokonaismallissa ammatillisen koulutuksen opiskelijat sijoittuivat korkeintaan 8 kurssia suorittaneiden opiskelijoiden joukkoon.

Osamalleissa selittäviksi muuttujiksi jäivät seuraavat muuttujat, jotka otettiin mukaan kokonaismallia etsivään analyysiin: matematiikan kurssimäärä lukiossa, 9. luokan koeosaaminen, käsitys omasta osaamisesta 9. luokalla, matematiikan hyödyllisyyden kokeminen 9. luokalla sekä opiskelijan arvioi siitä, että opiskeltavat asiat tulevat selväksi, päässälaskujen harjoittelu ja oman taitotason mukaiset tehtävät toisella asteella.



Kuvio 5. Osaamisen muutosta selittävien tekijöiden kokonaismalli DTA-analyysilla (ammatillisen koulutuksen opiskelijoiden kurssimäärä <= 8 kurssia)

Kokonaismallissa osaamisen muutosta erotteli parhaiten matematiikan kurssimäärä lukiossa ($\chi^2(3) = 70{,}389$; $p < 0{,}001$). Malli jakoi kurssimäärät neljään ryhmään: korkeintaan 8 kurssia, 9–11 kurssia, 12–13 kurssia tai yli 13 kurssia. Tulosten mukaan raja osaamisen muutokseen parhaista hyviin osaajiin tapahtuu 11 kurssin kohdalla. 9. luokan parhaista osaajista, jotka olivat parhaita myös toisen asteen lopussa, 81,8 prosenttia oli opiskellut matematiikkaa 12–13 kurssia tai enemmän.

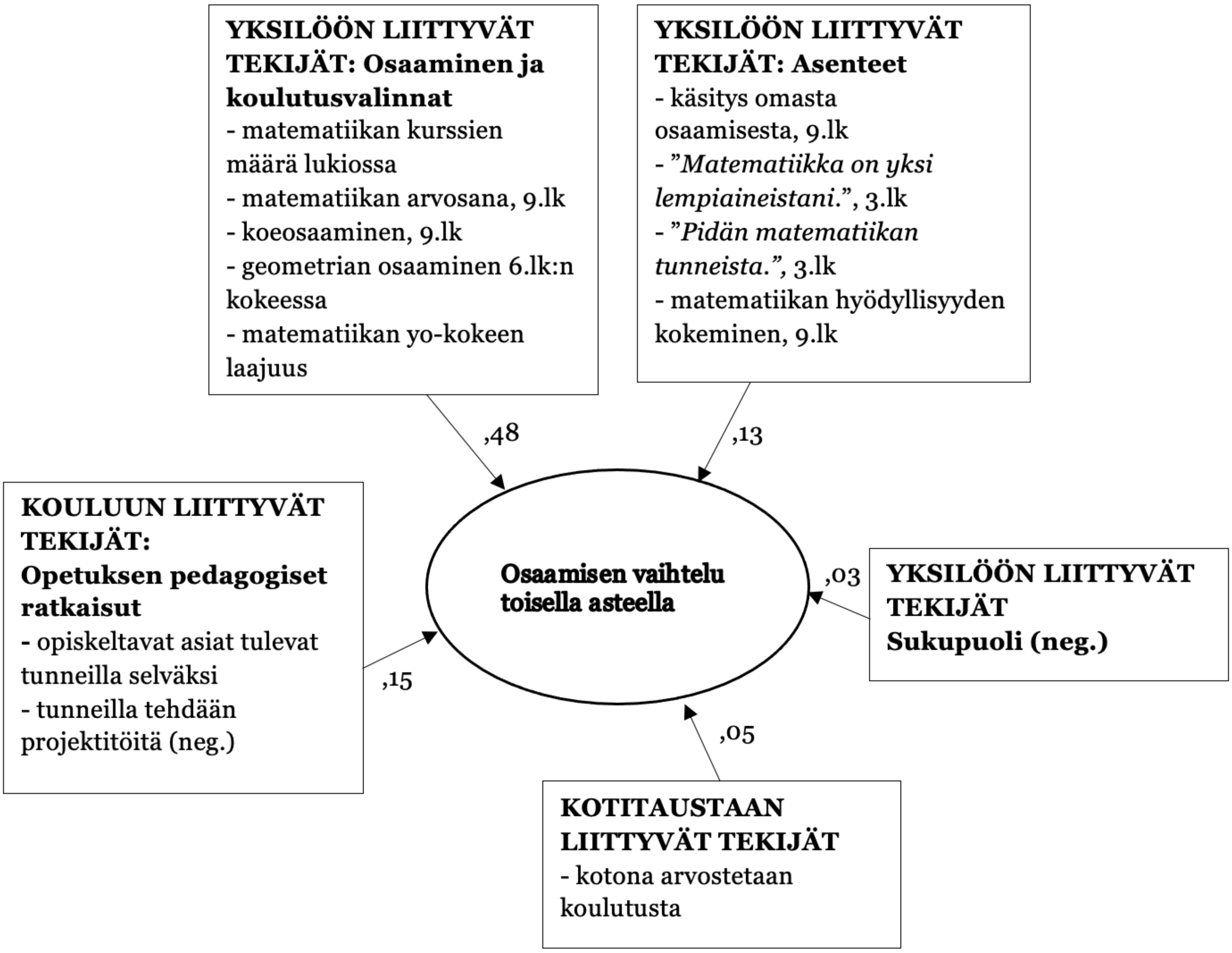
Toinen erotteleva tekijä oli 9. luokan koeosaaminen, kun oppilas oli opiskellut matematiikan kursseja 9–13. Toisin sanoen menestyminen 9. luokan kokeessa ennustaa osaamisen muutosta toisella asteella. Jos yhdeksännen vuosiluokan parhaisiin osaajiin kuuluva oppilas oli saanut 9. luokan kokeesta keskimäärin alle 700 pistettä, hänen osaamisensa taso todennäköisesti laski toisen asteen opintojen aikana.

Lineaarisen regressioanalyysin osamallien tulokset on koottu taulukkoon 7. Vertaisryhmään liittyvistä tekijöistä ei löytynyt selittävää mallia. Askeltavan menetelmän avulla malleihin on jäänyt vain tilastollisesti merkitsevät muuttujat.

Taulukko 7. Osaamisen vaihtelua toisella asteella selittävien osamallien tulokset.

	Parhaiden osaajien osaamisen vaihtelu toisella asteella			
	B	*SE*	*beta*	*p*
Malli 1 – Yksilöön liittyvät tekijät: osaaminen ja koulutusvalinnat				
Vakio	176,584	68,610		0,011
Matematiikan kurssien määrä	9,001	1,785	0,358	< 0,001
Matematiikan arvosana 9. lk	27,146	6,644	0,228	< 0,001
Kokonaisosaaminen 9.lk	0,220	0,074	0,170	0,003
Geometria -osa-alueen osaaminen 6.lk	26,522	9,027	0,160	0,004
Matematiikan yo-kokeen laajuus	16,576	7,093	0,166	0,020
Malli 2 – Yksilöön liittyvät tekijät: asenteet				
Vakio	619,390	32,073		< 0,001
Käsitys omasta osaamisesta, 9. lk	16,919	8,076	0,138	0,037
”*Matematiikka on yksi lempiaineistani.*”, 3.lk	25,459	6,443	0,403	< 0,001
”*Pidän matematiikan tunneista.*”, 3.lk	-21,178	7,313	-0,289	0,004
Matematiikan hyödyllisyyden kokeminen, 9.lk	15,707	6,659	0,151	0,019
Malli 3 – Yksilöön liittyvät tekijät - sukupuoli	-23,533	8,382	-0,163	0,005
Vakio	773,506	12,085		< 0,001
Malli 4 – Kotitaustaan liittyvät tekijät				
Vakio	662,331	29,082		< 0,001
Kotonani arvostetaan koulutusta, toinen aste	19,378	6,588	0,214	0,004
Malli 5 – Kouluun liittyvät tekijät: opetuksen pedagogiset ratkaisut				
Vakio	616,652	26,925		< 0,001
Matematiikan tunneilla opiskeltavat asiat tulevat selväksi (toinen aste)	28,309	6,604	0,309	<0,001
Matematiikan tunneilla tehdään projektitöitä (toinen aste)	-20,658	7,692	-0,194	0,008

Taulukossa 7 on esitetty osamallien tilastollisia tunnuslukuja ja kuviossa 6 näkyy näiden osamallien selitysosuudet osaamisen muutoksen vaihteluun toisella asteella.



Kuvio 6. Osaamisen vaihtelua toisella asteella selittävät osamallit ja niiden selitysosuudet

Kuviosta nähdään, että osaamiseen ja koulutusvalintoihin liittyvän osamallin selitysosuus on malleista korkein (R = 0,700; R^2_{Adj} = 0,477; $F(5, 193)$ = 37.073; p < 0,001). Tulosten mukaan osaamisen muutosta selitti ensisijaisesti matematiikan kurssien määrä. Myös 9. luokan matematiikan arvosana sekä kokonaisosaaminen kansallisessa kokeessa selittivät, miten parhaiden osaajien osaaminen vaihteli toisen asteen lopussa.

Toiseksi eniten osaamisen vaihtelua selitti opetuksen pedagogiset ratkaisut (R = 0,395; R^2_{Adj} = 0,146; $F(2, 168)$ = 15,517; p < 0,001). Malliin jääneet muuttujat koskevat toisen asteen opetusta. Tulosten mukaan osaaminen pysyi vakaampana toisella asteella, jos opiskeltavat asiat tulivat selväksi. Projektitöiden tekeminen näytti heikentävän osaamista toisella asteella.

Kolmanneksi eniten osaamisen vaihtelua selittivät asenteet (R = 0,379; R^2_{Adj} = 0,129; $F(4, 231)$ = 9,668; p < 0,001). Asenteista ensisijaisena selittäjänä oli oppilaan käsitys omasta osaamisesta yhdeksännellä vuosiluokalla. Osaamisen muutosta selittivät tulosten mukaan myös 3. luokan kaksi asenneväittämää. Jos oppilas oli pitänyt matematiikkaa yhtenä lempiaineistaan 3. luokalla, pysyi osaaminen vakaampana toisella asteella. Toisaalta matematiikan tunneista pitäminen sai mallissa negatiivisen arvon, mikä tarkoittaa, että oppilas ei ollut pitänyt matematiikan tunneista 3. luokalla. Näiden kahden asenneväittämän voidaan nähdä arvioivan kahta eri asiaa. Oppilas pitää matematiikkaa todennäköisesti yhtenä lempiaineenaan, kun kokee menestyvänsä siinä, mutta matematiikan tunnit eivät kuitenkaan välttämättä ole oppilaalle mieluisia. Lisäksi osaamisen muutosta toisella asteella selitti se, miten hyödylliseksi oppilas koki matematiikan 9. luokalla.

Kotitaustaan liittyvät tekijät ja oppilaan sukupuoli selittivät osaamisen vaihtelua vähiten. Kotitaustan osalta osaamisen muutosta selitti parhaiten se, kuinka paljon oppilaan kotona arvostetaan koulutusta (R = 0,214; R^2 = 0,046; F(1, 180) = 8,653; p = 0,004). Sukupuolen selitysosuus osaamisen vaihtelusta oli vain 0,2 prosenttia (R = 0,163; R^2 = 0,026; F(1, 290) = 7,883; p = 0,005). Tulosten mukaan parhaisiin osaajiin kuuluvien poikien osaaminen pysyi tyttöjä paremmin saman tasoisena myös toisella asteella.

Osamallien perusteella laadittiin vielä kokonaismalli, joka selitti parhaiden osaajien osaamisen vaihtelusta toisella asteella 41,3 prosenttia (R = 0,660; R^2 = 0,436; R^2_{Adj} = 0,413; F(6, 147) = 18,908; p < 0,001). Malliin vaikutti kuusi muuttujaa tilastollisesti merkitsevästi. Tulokset näkyvät taulukossa 8. Tulosten mukaan osaamisen vaihteluun vaikutti ensisijaisesti, kirjoittiko oppilas matematiikan pitkän oppimäärän ylioppilaskokeen. Tämän selitysosuutta osaamisen vaihtelusta vahvisti matematiikan kokonaisosaaminen 9. luokan kokeessa. Opetuksen pedagogista ratkaisuista malliin vaikutti, miten hyvin opiskelija koki toisella asteella, että opiskeltavat asiat tulevat oppitunneilla selväksi. Sillä, että kotona arvostetaan koulutusta, näytti olevan myös keskeinen merkitys sille, miten parhaiden osaajien osaaminen muuttui toisen asteen opintojen aikana. Malliin vaikutti 6. luokan kokeen osalta menestyminen geometria-tehtäväosiossa. Mallissa viimeisenä selittävänä muuttujana oli matematiikan kurssien määrä toisella asteella.

Taulukko 8. Osaamisen vaihtelua toisella asteella selittävän kokonaismallin tulokset

	Parhaiden osaajien osaamisen vaihtelu toisella asteella			
	B	*SE*	*beta*	*p*
Kokonaismalli				
Vakio	258,606	68,437		< 0,001
Matematiikan yo-kokeen laajuus	25,024	9,095	0,233	0,007
Kokonaisosaaminen, 9.lk	0,280	0,084	0,221	0,001
Opiskeltavat asiat tulevat tunneilla selväksi, toinen aste	20,033	5,556	0,230	< 0,001
Kotonani arvostetaan koulutusta.	17,009	5,388	0,197	0,002
Geometrian osaaminen, 6. lk	29,226	11,872	0,160	0,015
Matematiikan kurssien määrä, toinen aste	6,189	2,601	0,202	0,019

8 Pohdinta

Tutkimuksessa selvitettiin ensin, mitkä tekijät selittävät yhdeksännen vuosiluokan matematiikan parhaiden osaajien hakeutumista ammatilliseen koulutukseen ja lukioon. Ensisijaisesti valintaa erotteli äidinkielen arvosana 9. luokalla. Oppilaista, jotka saivat äidinkielestä arvosanan 9 tai 10, hakeutui lähes kolminkertainen määrä lukioon verrattuna niihin, jotka saivat arvosanaksi 8. Äidinkielen osaaminen on merkityksellistä lukiossa. Äidinkielessä hyvin menestyvillä oppilailla on enemmän valmiuksia ja motivaatiota opiskella lukio-opinnoissa vaadittavia laajempia kokonaisuuksia. Niillä, jotka saivat äidinkielen arvosanaksi 8, valintaan vaikutti se, olivatko molemmat vanhemmat ylioppilaita. Koulutuksen voidaan nähdä periytyvän, kun akateemisten vanhempien lapset valitsevat todennäköisemmin akateemisen uran (mm. Myrskylä, 2009; Suominen, 2013).

Toiseksi tutkimuksessa selvitettiin, miten yhdeksännen vuosiluokan parhaiden osaajien matematiikan osaaminen muuttuu toisen asteen koulutuksen aikana ja erityisesti, mitkä tekijät selittävät muutosta, jossa osa yhdeksännen vuosiluokan parhaista osaajista ei ole parhaita osaajia enää toisen asteen päättyessä.

Yhdeksännen vuosiluokan parhaista osaajista lähes 60 prosenttia oli parhaita osaajia myös toisen asteen päättyessä, ja noin 40 prosentilla osaamisen taso laski toisen asteen hyviin osaajiin tai keskitason osaajiin. Tulosten mukaan osaamisen taso heikkenee todennäköisemmin, jos oppilas ei mene lukioon tai ei suorita lukiossa vähintään 11 matematiikan kurssia. Myös yleisellä tasolla osaamisen eroja lukio-opiskelijoiden joukossa voidaan selittää matematiikan kurssimäärällä ja kurssien arvosanoilla. Metsämuurosen (2017) mukaan lyhyen matematiikan vähimmäiskurssimäärällä saadaan säilytettyä 9. luokan matematiikan osaamisen taso. Opiskelijoilla, jotka ovat suorittaneet yli 13 kurssia, nousee osaamisen taso selvästi, jos he ovat saaneet lukion opinnoista vähintään arvosanan 8. Tutkimustuloksia tarkasteltaessa tulee kuitenkin ottaa huomioon, että puuttuvia havaintoja mallinnettiin osin kurssimäärän perusteella.

Osaamisen vaihtelua osaajaryhmien välillä toisen asteen lopussa selitti vahvasti kokonaisasenne matematiikkaa kohtaan toisella asteella. Tämä oli regressioanalyysin kokonaismallissa ensisijainen selittävä muuttuja yhdeksännen vuosiluokan parhaiden osaajien osaamisen vaihtelulle toisen asteen lopussa. On merkityksellistä, että asenteita matematiikkaa kohtaan ylläpidetään ja vahvistetaan toisen asteen koulutuksen aikana. Hannulan ja Tuohilammen (2017) mukaan korkea vaatimustaso ja vertaisryhmän tasaisuus saattavat alentaa lukio-opiskelijoiden asennoitumisen positiivisuutta.

Vahva matematiikan osaamisen pohja perusopetuksessa luo edellytykset sille, että oppilas menestyy matematiikassa erinomaisesti myös toisella asteella. Tulosten mukaan vahva osaaminen kokonaisuudessaan 9. luokan kansallisessa kokeessa ja geometrian vahva hallinta 6. luokan kokeessa ennustavat erinomaista osaamista myös toisella asteella. Tämä vahvistaa aikaisempia tutkimustuloksia siitä, että aikaisempi osaaminen on merkitsevä ennustaja myöhemmälle osaamiselle (mm. Niemi ym., 2020).

Opiskelijan arvio siitä, kuinka paljon kotona arvostetaan koulutusta, oli myös yksi merkitsevä tekijä yhdeksännen vuosiluokan parhaiden osaajien osaamisen kehittymiselle toisen asteen opintojen aikana. Tiedetään, että yhdeksännen vuosiluokan parhaista osaajista lähes puolella molemmat vanhemmista ovat ylioppilaita (Niemi ym. 2020), ja voidaan ajatella, että koulutuksen arvostaminen on yhteydessä koulutustasoon. Metsämuurosen (2017, s. 107) mukaan akateemiset vanhemmat saattavat herkemmin kannustaa lapsiaan parempiin suorituksiin jo varhaisina vuosina ajatellen tulevia jatko-opiskelumahdollisuuksia.

Tutkimuksessa käsiteltiin matematiikassa parhaiten menestyneitä oppilaita. Heidän osaamisensa määritettiin kansallisessa kokeessa menestymisen perusteella. Matemaattista lahjakkuutta (mm. Sternberg & Davidson, 2005) on vaikea tunnistaa tämän tutkimuksen perusteella, mutta tutkimuksen parhaissa osaajissa voi nähdä tunnuspiirteitä matemaattisesta lupaavuudesta (Sheffield ym., 1999). Tutkimuksessa havaittiin, että keskitasoa korkeampi kyvykkyys, myönteiset asenteet matematiikkaa kohtaan ja kodin ja koulun tarjoamat mahdollisuudet luovat edellytyksiä sille, että oppilas menestyy matematiikassa erinomaisesti toisella asteella.

Tämä tutkimus tuotti jatkokysymyksiä muun muassa siitä, miten matematiikan parhaiden osaajien kykyjä ja myönteistä suhtautumista matematiikkaa kohtaan voidaan tukea ja vahvistaa. Seuraavaksi olisi kiinnostavaa selvittää, miten matematiikan parhaiden osaajien asenteet matematiikkaa kohtaan kehittyvät.

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Innovative collaborative instructional strategies: it's effect on secondary school students' achievement in biology as moderated by verbal ability

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Education is changing rapidly. Schools are gradually shifting away from the traditional mode of instruction and toward a more active model of learning, in which students are collaborating on projects in small groups and then sharing their work with the class. Africa cannot afford to be left behind in this change. Though collaborative teaching and learning are quite popular in Africa, its variants/forms, consensus group and cooperative reflective journal writing are not. The effect of collaborative instructional strategies (consensus group and cooperative reflective journal writing) on students' achievement in biology as moderated by verbal ability was determined in this study. Three hundred five senior secondary school II students from two local governments' area within Ibadan Metropolis participated in the study. The Students' Biology Achievement Test (SBAT) and the Students' Verbal Ability Test (SVAT) were the main data collection tools used for this study. Data generated were analysed using Analysis of Covariance (ANCOVA) and Bonferonni post hoc test. Results show that both forms of collaborative instructional strategies improved students' achievement in biology. Results showed that students exposed to the cooperative reflective journal writing achieved more in biology followed by students in the consensus group strategy. Collaborative strategy can be a feasible alternative approach to teaching biology as it fairly addresses issues of interaction in the classroom. This has helped students develop their communication and also improve their socialisation skills in the classroom and beyond.

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1 Introduction

Teaching and learning is a key aspect of any educational process. It aims is to enrich the learners experience, skills and overall development to function or integrate into the society. Since students' learning is the fulcrum upon which any teaching/learning activity is anchor upon, it behoves that the teaching strategies to be employed should foster students' learning. The strategy employed in the classroom by the teacher can either improve or mar the creative ability of the students. However, in most science classrooms, the strategy employed by teachers only encouraged rote learning and regurgitation of facts, which does not allow for creativity, (Usman, 2008). This, according to Adepitan (2003) and Okoronka (2004), is the way sciences are being taught in Nigeria schools does not allow the students to derive a maximum benefit, because science instructions are mostly teacher-centred. This has led to many students facing learning difficulties and not performing to their optimised level in the classrooms. This strategy termed variously as lecture, expository, traditional or conventional strategy, do not foster interaction between teacher-students and between students-students, (Olatoye, Aderogba, & Aanu, 2011).

Azubuike, (2012), Cepni, Tas and Kose (2006), Okoli and Egbunonu (2012) posited that the use of traditional instructional approach in teaching only allowed students to understand the subject content at the knowledge level as they usually memorised the science phenomena, concepts and theories without understanding the real meanings. Wood and Gentile (2003) opined that these traditional methods of conveying knowledge have been shown to be relatively ineffective on students' ability to master and then retain important concepts. Learning through these methods is passive rather active and does not tend to foster critical and creative thinking and collaborative problem-solving (Olatoye, et al, 2011) in students. As a consequence, students do not have a good comprehension of the science concepts being taught and this leads to poor performance. It is now being acknowledged that there are better ways to learn than through the traditional methods of instruction (Wood & Gentile, 2003). Stakeholders are now coming to the realisation that students needs and characteristics need to be taking into consideration when planning for a lesson.

Students' academic achievement in biology and any other disciplines is a function of the instructional strategy adopted by teacher in the classroom. According to Azubuike (2012) and Salau (2002), researches have attributed poor performance in public examinations to the instructional delivery approaches adopted by many teachers. They noted that most teachers utilise the traditional instructional delivery approaches. These approaches do not take into cognisance each student peculiarities and in effect do not encourage students' active participation in the lesson. Akale (1990) and Azubuike (2012) averred that the most pronounced and important factor that generally influence students' academic achievement in science is the teacher and the teaching methods adopted in the classroom. In other word, the role of the teacher and the instructional strategy adopted in the classroom is a sine-qua-non to students' achievement.

To stem the tide of poor academic achievement among students in the classroom, the idea of a sage on the stage must give way to a guide on the side. As Brown (1997) puts it, effective instruction requires the teacher to step outside the realm of personal experience into the world of the learners. For meaningful learning to occur, the learner must make the commitment to learn and must also be engaged (Akinsola & Animasahun, 2007). According to Iroegbu (1998), learners tend to derive maximum benefits in learning cognitive skills when the teaching strategy adopted involves the use of a mixture of different methods while at the same time creating an opportunity for the learners to practice skills as a meaningful whole. It means for the students to learn meaningfully and be successful academically, the students must be made actively involved in their learning with the guidance of the teacher.

There had been several attempts made to improve the academic achievement of students in biology through the discoveries and application of innovative, effective and students-centred instructional strategies. Some of these strategies include cooperative learning, reflective journal writing, jigsaw, buzz, concept map, peer tutoring, among others. These strategies had all proven to be capable of improving the academic achievement of students, but more needs to be done. These strategies as according to Ukoh and Adejimi (2018), have the potential of enhancing positive interactions and friendship among students. In order to explore further the effects of innovative student-centred teaching approach on students' achievement in biology, this study, therefore, seeks to determine the effect of two innovative, collaborative instructional strategies (consensus group and cooperative reflective journal writing) on students' achievement in biology, as moderated by verbal ability.

1.1 Theoretical framework

The study is premised on the Social Interdependent Theory (SIT) approach to teaching and learning. The SIT was first known and called theory of Cooperation and Competition and was developed by Morton Deutsch (1949a, 1949b, 1973 &1985). It was further elaborated by David W. Johnson (Johnson & Johnson, 1989). The historical roots of social interdependence theory can be traced to a shift from mechanistic to field theories in physics (Deutsch, 1968). This shift especially influenced the emerging school of gestalt psychology at the University of Berlin in the early 1900s (Johnson, 2003). Building on the principles of Gestalt psychology, Kurt Lewin proposed that the essence of a group is the interdependence among members which results in the group being a "dynamic whole" so that a change in the state of any member or subgroup changes the state of any other member or subgroup (Lewin, 1935; 1948).

Lewin (1935) further states, group members are made interdependent through common goals. As members perceive their common goals, a state of tension arises that motivates movement toward the accomplishment of the goals. Morton Deutsch (1949; 1962) extended Lewin's notions by examining how the tension systems of different people may be interrelated. This (aroused tension) forms the basis of the SIT. As Johnson and Johnson (2005) stated, for interdependence to occur, there must be more than one person or entity involved and the people or entities must influence each other, in that a change in the state of one causes a change in the state of the others. This influence reflects in the immediate situation, as each person's behavior is determined by how the situation is perceived, rather than by objective or historical factors (i.e., the principle of contemporaneity). As a person's life space is dynamic (not static), so that, as individuals interact and events occur, each individual's perceptions of the situation change. Within this life space, people's behaviour is motivated by states of tension that arise as they perceive their desired goals. It is this tension that motivates movement toward the accomplishment of the goals. The perception of common goals in conjunction with the joint motivation to achieve them is the source of interdependence among group members.

Social interdependence exists when the accomplishment of each individual's goals is affected by the actions of others (Deutsch 1949a, 1962; Johnson 1970, 2003; Johnson & Johnson 1989; 2005). There are two types of social interdependence; positive (cooperation) and negative (competition) interdependence. Positive interdependence will support the attainment of a group goal, while negative interdependence will hinder it.

Deutsch (2006) stated, positive interdependence occurs when the goals are linked in such a way that amount or probability of an individual attaining his/her goal is positively related with the amount or probability of another reaching his/her goal. Negative interdependence occurs when the goals are linked in such a way that the amount or probability of an individual attaining his/her goal is negatively related with the amount or probability of another reaching his/her goal. Deutsch (1949, 1962) opined that positive interdependence creates the psychological processes of substitutability (i.e., the degree to which actions of one person substitute for the actions of another person), positive cathexis (i.e., the investment of positive psychological energy in objects outside of oneself, such as friends, family, and work), and inducibility (i.e., the openness to being influenced by and to influencing others).

Negative interdependence tends to create non-substitutability, negative cathexis, and resistance to influence.

In essence, positive interdependence will lead to healthy rivalry (promotive interaction) among students which will, in turn, improve students' achievement; negative interdependence will lead to unhealthy rivalry (oppositional or contrient interaction) among students which may leads to poor achievement in trying to better each other. Promotive interaction is the efforts put in by individuals in assisting each other to complete tasks, achieve, or produce in order to reach the group's goals. It consists of number of factors/variables; mutual help and assistance, exchange of needed resources, effective communication, mutual influence, trust and constructive management of conflict. In other word, Oppositional interaction is individuals in discouraging and obstructing each other's efforts to complete tasks, achieve, or produce in order to reach their goals; individuals focus both on increasing their own productivity and on preventing any other person from producing more than they do. It consists of such variables as obstruction of each other's goal achievement efforts, tactics of threat and coercion, ineffective and misleading communication, distrust, and striving to win in conflicts (Johnson, Johnson, & Smith, 2007).

The basic premise of social interdependence theory is that the ways in which participants' goals are structured determine how they interact, and the interaction pattern determines the outcomes of the situation (Deutsch, 1949, 1962). This structure and interaction forms the bases upon which collaborative instructional strategies are derived. A positively structured interdependence leads to promotive interaction which in turn leads to high effort to achieve, positive relationships and psychological health, while a negatively structured interdependence leads to oppositional interaction which leads to low effort to achieve, negative relationships and psychological illness.

1.2 Literature review

As society advances more into the knowledge age, it is becoming increasingly obvious that cognitive/knowledge work is more effective when done in collaboration with other students. Consensus is a form of collaborative, non-coercive decision-making strategy that allows everybody express their opinion and feels they are heard. It is a creative and unique way of reaching agreement between all members of a group. It is neither compromise nor unanimity – but aims to pull together everyone's best ideas and key concerns – a process that often results in surprising and creative solutions, inspiring both the individual and the group as a whole (seedsforchange.org.uk). According to Sartor and Young Brown (2004), consensus is a conscious agreement by everyone. The procedure that leads to consensus-or at least attempts to find consensus is called -the "consensus process". It is characterised by listening, sharing, trusting and respecting the opinions of one another.

Consensus is a decision-making model utilised by prehistoric tribes and adopted by organisations, communities, and groups in coming to a unanimous decision, one that works for everyone (Schutt, 2001). Consensus confers many advantages to those engages in it; promote shared authority and responsibility in making decisions, enhances students' self-expression, encourages full student participation, stimulates creative decision-making, nurtures the development of a conscious community, shows that education can be a practice of freedom, and helps learners to form good self-concepts, heighten their level of engagement, and improve their ability to apply learning in new contexts (Blinne, 2013; Bruffee, 1999; Freire, 1998; Hooks, 1994; MacDougall, 2013; Mitchell, Foulger, Wetzel & Rathkey 2009; Sartor & Young Brown, 2004; Wolk, 1998).

There is not much study with which to support the result from the study with, since little study has been carried out on the effect of consensus group instructional strategy on students' achievement in biology. According to Sartor and Young Brown (2004), the major factor militating against the use of consensus, is it effect on academic performance or scores on standardised tests. This concern was echoed by Fetalvero (2017), when he stated that there was no comparable empirical study with which to compare his result with as at then. Nevertheless, Fetalvero (2017) reported, despite the fact that there was no significant difference in the academic achievement of students exposed to Consensus-Based Education (CBE) and Conventional Education (CE), the consensus-based education showed the prospect of improving students' academic achievement in bioenergetics. He submitted that the effectiveness of CBE over CE became obvious when the students gained scores were categorised into a five-point interval and an item-by-item analysis conducted across the achievement scores grouped by topics and cognitive domain. The students in the CBE group outperform the students in the CE group; this confirms the prospect of CBE over CE.

Cooperative reflective journal writing combines the features of both cooperative learning and reflective journal writing strategies which allows students to reflect together as they learn in a cooperative manner or environment. Ige and Adu (2016) define cooperative reflective journal writing as a strategy that involves students working cooperatively as they reflect on classroom tasks. It, therefore, means that a cooperative reflective journal allows students to reflect cooperatively in the classroom as they learn together in a group. Cooperative learning is arguably one of the most researched strategies of all instructional strategies. It is a form of collaborative work that enables students to work together within a small group to maximise each other potential. It is a pedagogical approach that helps students to gain and create both academic and social relationships as well as to accomplish shared goals, (Johnson, D.W & Johnson, 2002; Lou, Abrami, Spence, Poulsen, Chambers & d'Apollonia, 1996; Slavin, 1996).

The success achieved in the implementation of cooperatively learning and reflective journal writing respectively in promoting students' achievement prompted some researchers to combine cooperative learning and reflective journal writing to seek the effects on students learning. Ige and Adu (2016) reported that cooperative reflective journal writing improves students' achievement in biology more compare to individualised reflective journal writing and conventional instructional strategies. Also, Güvenç (2010) reported that the achievement of students taught with cooperative learning combine with reflective journal writing better those of students taught with cooperative learning alone. These reports bring to the fore that, when strategies are combined they produce a better result than one single strategy.

Asides teaching strategies that have effect on students' academic achievement, students verbal prowess (verbal ability) can also have an effect on students' academic achievement. Given the nature of the two collaborative strategies (consensus group and cooperative reflective journal writing), verbal ability can greatly affect the achievement of students when exposed to both types of strategies. Verbal ability is an important element in human intelligence (Widhiarso & Haryanta, 2016). It is the most compelling feature of school learning and is automatically assessed (Richard & Giovanni, 1990), repeatedly consciously and/or unconsciously. According to Adegbile and Alabi (2007), it may show a significant level of relationship with students' grades. It has a strong connection with many academic disciplines like reading, writing, speaking, mathematics and sciences (Rinderman, Michou & Thompson, 2011; Walker, Green, Hart, & Carta, 1994).

Several studies have shown that there is a connection between students' verbal ability and their academic achievement. Awofala, Balogun and Olagunju (2011), Corengia, Pita, Mesurado and Centeno (2013), Ige and Adu (2016), Vilia, Candeias, Neto, Franco and Melo (2017) and Tzu-Ling Wang (2008) in their respective studies all affirmed that students' verbal ability has an effect on their academic achievement, that is students with high verbal ability achieve higher than students with low verbal ability. On the other hand, results from the studies by Ezenandu (2012), Maduabuchi (2006) and Makinde (2004) all reported that students' verbal ability has no effect on students' academic achievement. In view of divergent reports on the importance of verbal ability on students' achievement, this study will further investigate the variable with respect to consensus group and cooperative reflective journal writing on students' achievement in Biology.

1.3 Aim of the Research

Historically, biologists have identified teaching as cooperative behaviour in which the "teacher" changes his or her actions to aid a naive "student" in acquiring knowledge or skills (Caro & Hauser, 1992). But knowledge acquisition has been reduced to a process by which information is poured from a jug (teacher) into receptacles (students). This situation has made the students passive in their learning and negatively impacted their achievement in biology. Biology is a 'simple' and an important subject in the daily living of man, literature has revealed that students' performance in biology on the Senior Secondary School Certificate Examination (SSSCE) has been consistently poor, and this is mostly attributed to the instructional strategy adopted by the teacher which is teacher centred.

Education is changing rapidly. Schools are shifting away from the traditional mode of instruction and toward a more active model of learning, in which students are collaborating on projects in small groups and then sharing their work with the class, (eSchool News, 2017). This student-centred strategy has engendered the spirit of cooperation among the students, and subsequently leads to an improvement in their academic achievement in biology. This study, therefore, seek to determine the effect of two innovative collaborative instructional strategies (consensus group and cooperative reflective journal writing), with the moderating effect of verbal ability on students' achievement in biology. No known study has been conducted on the effect of consensus on students' achievement in biology or any other discipline in Nigeria, while the only known study on the effect of cooperative reflective journal writing on students' achievement in biology was conducted in 2016 by Ige and Adu.

1.4 Hypotheses

The following three null hypotheses will be tested:

1. There is no significant main effect of treatments on students' achievement in biology
2. There is no significant main effect of verbal ability on students' achievement in biology
3. There is no significant interaction effect of treatment and verbal ability on students' achievement in biology

1.5 Scope

Six co-educational Senior Secondary Schools were purposively selected from Ibadan North and Ibadan North West Local Government areas of Oyo State Nigeria. Intact class of one arm each was used in all selected schools. The schools were randomly assigned to treatment and control groups. The study focused on the effects of consensus group and cooperative reflective journal writing instructional strategies with moderating effect of verbal ability on students' achievement in some concepts in Biology. The topics that were treated during the course of the study were ecological management and nutrient cycling in nature.

2 Materials and methods

2.1 Research Instruments

The following instruments were used to collect data for the study:

1. Students' Biology Achievement Test (SBAT)
2. Students' Verbal Ability Test (SVAT)
3. Teacher's Instructional Guide on Consensus Group Instructional Strategy (TIGCGIS)
4. Teacher's Instructional Guide on Cooperatively Reflective Journal Writing Instructional Strategy (TIGCRJWIS)
5. Teacher's Instructional Guide on Conventional Instructional Strategy (TIGCIS)

Students' Biology Achievement Test

Students' Biology Achievement Test (SBAT) was developed by the researchers to measure the achievement level of the students on the concepts of ecological management and nutrient cycling in nature in biology before and after the implementation of the intervention. The instrument consisted of forty (40) multiple choice questions with options A – D. Each correct answer in SBAT was rewarded one mark; to make a total of 40 marks. The face validity of the instrument was done by science education experts to determine its suitability and the reliability coefficient of 0.74 was obtained using Kuder-Richardson formula- 20 (KR-20). The choice of KR-20 was premised on the inequality in the level of difficulty of the items in the Biology achievement test.

Students' Verbal Ability Test (SVAT)

Students' Verbal Ability Test (SVAT) was adapted from the Australian Council for Educational Research (ACER) to tests students' verbal ability. It has gone through several modifications and revalidation for use by some Nigerian authors (Abimbade, 1987: Aimunmondion, 2008: Awofala, Balogun & Olagunju, 2011: Ezenandu, 2012: Fakeye 2006: Ige & Adu, 2016: Maduabuchi, 2002: Olaboopo, 1999) since its first introduction into the Nigerian education system by Obemeata in 1974. However, the researchers re-validated the test to ascertain its suitability for this study. The SVAT was tested in trials on twenty (20) senior secondary school II students in a school that was not selected for the main study. The data collected were analysed using Kuder-Richardson formula 20 (Kr20), and a reliability of 0.78 was obtained. Kr20 was used because of the inequality of the difficulty level of the items in the SVAT. The SVAT was administered to the participants once before the treatment began.

Teachers' Instructional Guides

Teachers' instructional Guide on Consensus Group Instructional Strategy (TIGCGIS), Teachers' Instructional Guide on Cooperative Reflective Journal Writing Instructional Strategy (TIGCRJWIS) and Teachers' instructional Guide on conventional method (TIGCM) are the lesson notes, which were prepared weekly for the six weeks of the treatment for the study. The duration for each lesson was 80 minutes (double periods). The essence of these instruments was to guide the research assistants (teachers) on the use of steps and procedure followed during the treatment.

Treatments	Procedures
Consensus Group Instructional Strategy	Division: Divide a class into small groups, usually of about 5 learners Provision (Assignment) of task: Assigned a task, usually designed ahead of time, for the small groups to work on Deliberation takes place among group members Group members reach a consensus Teacher reconvenes students into a plenary session to hear the reports from the small groups and negotiate a consensus of the class as a whole Lead students to compare the class's plenary consensus with the current consensus of the knowledge community in order to arrive at a better consensus/decision/judgement Evaluate explicitly the quality of students' work.
Cooperative Reflective Journal Writing Instructional Strategy	Teacher presents the topic Teacher tells the students the task to be done Teacher highlights the major idea within the topic to be taught Teacher groups the students in five-member heterogeneous teams by gender only. Each group appoints a leader and a clerk Teacher gives group some few minutes to review the lesson and share their views The team writes a group journal based on the following guidelines: What question do you have about this lesson? What have you learned in the lesson? What areas did you find difficult? What areas did you find interesting? How do you think this lesson will be useful for you to apply outside the classroom? Teacher collects the group journal for compilation of entries Raised group questions were thrown to the groups for answer in the next lesson Students learning were evaluated based on group entries.
Conventional Strategy	The teacher introduces the lesson by asking questions based on the students' previous knowledge. Teacher presents instructional aid and discusses the contents of the lesson with the students. Teacher directs students to write the chalkboard summary in their notebooks. Teacher evaluates the lesson by asking students some questions in class, later on homework/assignment.

2.2 Research design

This study adopted a quantitative pretest-posttest, control group quasi-experimental design involving a 3X2 factorial matrix. This design was employed because the participants were from intact classes in a natural school setting where random assignment was not possible, and the distraction of class structure was avoided to the minimum. The treatment was the instructional strategy at three levels (consensus group, cooperative reflective journal writing and conventional method). The moderator variable was verbal ability at two levels (low and high). Students' achievement in biology was the dependent variable.

2.3 Sampling and Sampling Technique

The study population was all the senior secondary school two students within Ibadan metropolis Oyo State, Nigeria. A multistage sampling technique was used to pick 305 students for the study. At the first stage of sampling, two local governments were randomly selected out of the five local governments within the metropolis. At the second stage, three coeducational schools that were distantly located were selected purposively in each of the local government areas to make a total of six schools. This was done in order to avoid or minimise experimental contamination. In all the schools intact classes were used.

2.4 Schematic Representation of the Design

Groups	Pre-test	Treatment	Post-test
Consensus Group Instructional Strategy (E_1)	O_1	X_1	O_2
Cooperative Reflective Journal Writing Instructional Strategy (E_2)	O_3	X_2	O_4
Conventional Method Instructional Strategy (C)	O_5	X_3	O_6

Where O_1, O_3 and O_5 represent the pre-test scores of consensus group strategy (E_1), cooperative reflective journal writing strategy (E_2) and conventional method (C), respectively. O_2 and O_4 are the post-test scores of the treatment groups (E_1 and E_2), and O_6 is the post-test of the control group. X_1 represents treatment for experimental group one E_1 (Consensus Group) X_2 represents treatment for experimental group two E_2 (Cooperative Reflective Journal Writing) X_3 represents treatment for control, group C (conventional method)

2.5 Study Procedure

In carrying out the treatment, the following procedure was adopted: During the study, the first week was used to train the research assistants (classroom teachers), the second week used to conduct the pre-tests. Treatment lasted for six weeks utilising the 80 minutes periods. Post-tests were conducted for all groups in the last week of the study.

2.6 Data Analysis

The post-test achievement scores were subjected to a two-way analysis of covariance (ANCOVA) using the pre-test scores as covariates. Analysis of covariance is used to test the main and interaction effects of categorical variables on a continuous dependent variable, controlling for the effects of selected other continuous variables, which co-vary with the dependent. The control variables are called the "covariates". It's also used to control for factors which cannot be randomised but which can be measured on an interval scale in experimental designs. The ANCOVA reduces experiment error by statistical rather than by experimental procedure (Coolican, 1994). The Bonferroni post hoc test was used to determine which of the groups causes the significant main effect, while the interaction effect was explained by the aid of a graph. The Bonferroni post hoc test was employed in order to be certain that the treatments (consensus group and cooperative reflective journal writing) strategies has positive effects on students' academic achievement in comparison to the conventional mode of teaching.

3 Results

3.1 Hypothesis 1

There is no significant main effect of treatment on students' achievement in Biology. The summary of this result is given in table 1.

Table 1. Analysis of Covariance (ANCOVA) of Post-Achievement by Treatment and Verbal ability

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	6933.707	6	1155.618	220.213	0.000	0.816
Intercept	11508.070	1	11508.070	2192.966	0.000	0.880
Pre-Achievement	659.273	1	659.273	125.630	0.000	0.297
Treatment	769.898	2	384.949	73.355	0.000*	0.330
Verbal ability	273.710	1	273.710	52.158	0.000*	0.149
Treatment x Verbal ability	59.905	2	29.953	5.708	0.004*	0.037
Error	1563.820	298	5.248			
Total	278635.000	305				
Corrected Total	8497.528	304				

R Squared = 0.82 (Adjusted R Squared = 0.81) *denotes significant p<0.05

Table 1 reveals that there is a significant main effect of treatment on students' achievement in biology ($F_{(2, 304)} = 73.36$; $p<0.05$, partial $\eta^2 = 0.33$). The effect size, 33.0%, showed a small effect size. Nevertheless, it showed that there was a statistical difference among students in the treatment groups from the conventional group. Therefore, hypothesis 1a was rejected. In order to explore the magnitude of the significant main effect across treatment groups, the estimated marginal means of the treatment groups were carried out and the result is presented in Table 2.

Table 2. Estimated Marginal Means for Post-Achievement by Treatment and Control group

Treatment	**Mean**	**Std. Error**	**95% Confidence Interval**	
			Lower Bound	**Upper Bound**
CGIS	31.85	0.31	31.24	32.45
CRJWIS	32.21	0.59	31.05	33.36
CMIS	27.07	0.29	26.50	27.64

Table 2 reveals that students in the CRJWIS treatment group 2 had the highest adjusted mean score in their post-achievement in biology (32.21), followed by those in the CGIS treatment group 1 (31.85) and their counterparts in the CMIS control group (27.07). To determine which of the groups causes this significant main effect of treatment on students' achievement in biology, the Bonferroni post hoc test is carried out across the groups, while the result is presented in Table 3.

Table 3. Table 3: Bonferroni Post-hoc Analysis of Post-Achievement by Treatment and Control Group

Treatment	Mean	CGIS	CRJWIS	CMIS
CGIS	31.85			*
CRJWIS	32.21			*
CMIS	27.07	*	*	

Table 3 indicates that the post-achievement mean score in biology of students in CRJWIS is not significantly different from those taught with the CGIS but significantly different from those exposed to CMIS. Table 3 also indicates that the difference in the post-achievement mean scores of students exposed to CGIS and their counterparts in the CMIS is significant. This indicates that the significant difference revealed by the ANCOVA result is not due to the difference between the treatment groups (CRJWIS and CGIS) but between the treatment groups and the control group as students' post-achievement scores in biology is concerned.

3.2 Hypothesis 2

There is no significant main effect of verbal ability on students' achievement in Biology.

The result of the analysis of covariance from Table 1 shows that there was a significant main effect of verbal ability on students' post-test achievement scores in biology ($F_{(1, 304)} = 52.16$; $p<0.05$, partial $\eta^2 = 0.15$). The effect size 15.0% showed a small effect size. Hypothesis 2 was therefore rejected. This implies that verbal ability has a main significant effect on students' achievement in Biology irrespective of their treatment status.

Table 4. Estimated Marginal Means for Post-Achievement by Verbal ability

Verbal ability	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Low	28.57	0.44	27.71	29.42
High	32.18	0.22	31.74	32.62

Table 4 reveals that high verbal ability students had higher adjusted mean score in post-achievement in biology (32.18) than their low verbal ability counterparts (28.57). This implies that high verbal ability students have better achievement scores in biology than the low verbal ability students and this difference is significant.

3.3 Hypothesis 3

There is no significant interaction effect of treatment and verbal ability on students' achievement in biology

The result from Table 1 revealed that there was an interaction effect of treatment and verbal ability on students' achievement scores in biology ($F_{(2, 304)} = 5.71$, $p<0.05$; partial $\eta^2 = 0.04$). The effect size of 4.0 revealed a small effect size. Hypothesis 3 was rejected. Treatment and verbal ability had a significant effect on students' achievement in biology. An interaction effect is the simultaneous effect of two or more independent variables on at least one dependent variable in which their joint effect is significantly greater (or significantly less) than the sum of the parts. The inclusion of an interaction term effect in an analytic model provides the researcher with a better representation and understanding of the relationship between the dependent and independent variables. Further, it helps explain more of the variability in the dependent variable, (Encyclopedia of Survey Research Methods, 2008). The significant interaction effect of the treatments and verbal ability showed that the result of the study was not by chance.

The use of a line graph gives a further illustration of the interaction effect.

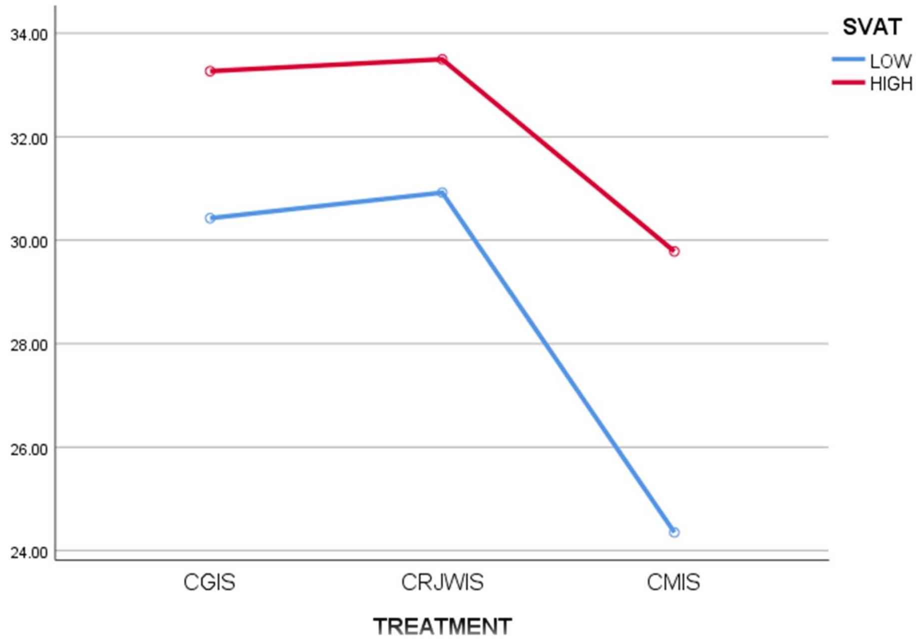

Figure 1. Treatment and verbal ability on students' achievement in biology

Figure 1 revealed that students with high verbal ability achieved better compare to students with low verbal ability regardless of the treatment conditions. Students under the CRJWIS had the highest scores in verbal ability, followed by students under CGIS and students under the conventional method had the least scores in verbal ability. This means that the same set of students achieved more in biology based on their scores in the SVAT.

4 Discussion

Collaborative strategies were more effective at improving students' achievement in biology than the conventional method. The effectiveness of collaborative strategy over the conventional method may be due to the fact that the strategy helped the students to work collectively and actively participate in their learning activities. Furthermore, the effectiveness of the two modes of collaborative strategy (consensus group and cooperative reflective journal writing) may be attributed to the fact that these strategies enhance students' engagement, communication and listening skills, sharing of ideas, and students are able to arrive to joint decisions or knowledge agreeable to all, which is often superior to an individual decision or knowledge. This finding conform to the findings of (Alvarez, Salavati, Nussbaum & Milrad, 2013; Awofala, Fatade, & Ola-Oluwa, 2012; Nneji, 2011; Olabiyi and Awofala, 2019).

Collaborative strategies enhance not only student cognitive skills, which aids long-term retention of learned contents. It also enhances non-cognitive skills like self-confidence, critical thinking, persuasion, problem-solving, work ethics, time management and leadership traits. In the course of collaborating, each student endeavour to articulate his/her thought and strive to persuade one another of the strength of their argument in order to arrive at a solution to a problem. Since they have to work within a time frame and under a peaceful atmosphere, a leader will be appointed to coordinate the affairs of the group. These assertions are supporting by the findings of (Bezerra, 2018; Hartmann, Toksvang & Berg, 2015; Petersen, Toksvang, Plovsing & Berg, 2014). All these skills are required even outside the four walls of the classroom and the school generally.

The studies by Fetalvero (2017) and Ige and Adu (2016) on the effect of Consensus-Based Education and reflectively journal writings on students' achievement in bioenergetics and biology respectively highlighted the effectiveness of these strategies. According to these authors, the incorporation of personal feelings, intuition, experience, wisdom, and insights coupled with reflection helps individual to gain access to multiple sources of information.

Verbal ability had a significant main effect on students' achievement in biology. Students with high verbal ability continuously achieved better than students with low verbal ability. This lend credence to the studies of Adegbile and Alabi (2007), Awofala, Balogun and Olagunju (2011), Ige and Adu (2016) that irrespective of the instructional strategies adopted by the teacher, verbal ability will affect students' achievement. It is against the findings of Ezenandu (2012), Maduabuchi (2006) and Makinde (2004) who all reported that students' verbal ability does not affect their achievement. The findings imply that students should be helped and encouraged to develop their vocabulary, and this can be achieved by exposing them to reading diverse biological and/or science texts. Biology and science in general are both expressive and symbolic, as a result, students need to be acquainted with its language.

Students' academic achievement in any subject, biology inclusive is a function of their verbal prowess in the language of that subject. Verbal ability enables students to be able to interpret and present information, thoughts or ideas in a concise, logical and analytical manner which in turn increases their chances to improve performance. This can only be achieved by students who are proficient in the language of the subject. This is supported by the findings of Adegbile and Alabi (2007) who states that students' verbal ability is associated with their grades and Iyamu (2005), who states that verbal ability is important to effective and successful school learning.

There was a significant interaction effect of treatment and verbal ability on students' achievement in biology. The higher the scores of students in the verbal ability test, the higher their scores in the biology achievement test. This result is supported by the finding of Adegbile and Alabi (2007), Awofala, Balogun and Olagunju (2011) who reported a significant interaction between treatment and verbal ability on students' achievement, but in contrast to the findings of Ezenandu (2012) and Ige and Adu (2016) who reported no interaction effect between treatment and verbal ability on students' achievement.

5 Conclusion and Recommendations

The study confirmed that the application of collaborative instructional strategy has the potential of improving students' achievement in biology. The consensus group and cooperative reflective journal writing instructional strategies were both effective in improving students' achievement in biology over the conventional strategy. The two strategies both enhance students' engagement, communication, listening and interpersonal social skills. Students' verbal ability also play a vital role within the two strategies since students have to communicate with each other in their respective groups. Students with high verbal ability were able to express their thoughts and feeling within the group and were able to seek solutions where they encountered problems, while the low verbal ability students were not able to express their thoughts and feeling and by so doing may not get solution to their problems. The importance of this present study is its addition to the number of innovative teaching approaches available biology teachers in the teaching and learning of biology in Nigeria secondary schools.

Based on the findings from the study, the following recommendations were made,

1. There is need to popularise the use of both form of collaborative instructional strategy among secondary school teachers in the teaching of Biology.
2. Biology teachers should be encouraged to adapt these strategies to their classroom setting in order to improve the achievement of their students in Biology.
3. Government and teachers' professional bodies should endeavour to expose Biology teachers to the use of these strategies through seminars and workshops and in teacher training institutions to facilitate better performance of secondary school Biology students.
4. The medium of instruction used in the classrooms should be geared towards easy assimilation by all students.
5. Adequate caution should be taken during group discussion in order for students not to derail from the objectives of the lesson
6. Also, teacher should endeavour to ensure the participation of all students during groups' discussion.

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Quadratic equations in Swedish textbooks for upper-secondary school

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This paper analyzes the content and tasks involving quadratic equations in eight mathematics textbooks published during the period 2000-2012 at the upper-secondary level in Sweden. The study applies the theoretical *hypothetical learning trajectory* (HLT) framework combining *conceptual* and *procedural* knowledge. The analysis includes horizontal and vertical dimensions within an HLT. The aim is to explore embedded HLTs and learning opportunities from both dimensions in these textbooks. A total of 250 examples and 1,068 tasks have been examined. Results show that all the textbooks contain algebra identities and four different methods for solving quadratic equations as well as their applications as a core hypothetical learning trajectory but differ in how an HLT starts and ends. Geometrical representations for some algebra identities and completing the square are widely used in both theoretical presentations and tasks, which implies that conceptual learning is encouraged among the Swedish textbooks. At the same time, procedural knowledge is still emphasized as a basic but important learning process.

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1 Introduction

In the Swedish national mathematics curriculum (2011) for second-year upper-secondary schools, quadratic equations are among the central content. The same content is taught in many other countries in the world, although in different years of high school. Within the area of quadratic equations, students often study algebra concepts and identities, different solution methods, and applications of quadratic equations in problem-solving. Solving a quadratic equation using the *pq* formula[1] is one of the most common solution methods in Swedish classrooms (Olteanu & Holmqvist, 2012). Demanding prior knowledge of many algebra rules or identities, quadratic equations take up much knowledge space in the curriculum and form a complex teaching area. Therefore, it is important to present this new content comprehensively so that students will have opportunities to understand the abstract

[1] The *pq* formula is a quadratic formula of the type $x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$. It is used for solving quadratic equations of the type $x^2 + px + q = 0$.

content.

Different approaches for students to solve quadratic equations have been studied since 2000; for example, factoring quadratics (e.g., Bossé & Nandakumar, 2005), completing the square with geometry representations (e.g., Allaire & Bradley, 2001; Fachrudin et al., 2014), and using the *pq* formula (e.g., Olteanu & Holmqvist, 2012). Learning and understanding quadratic equations and their solution methods have also been studied; for example, students' understanding of quadratic equations (e.g., Vaiyavutjamai & Clements, 2006), and students' understanding of factoring quadratic equations and their difficulties (e.g., Didis & Erbas, 2015). Sharing the current study's focus, a number of recent studies directly relate to the analysis of different solution methods, for instance, a Polish study on reviewing traditional solution methods in the Polish curriculum and textbooks (Pieronkiewicz & Tanton, 2019). The Polish study demonstrates applications of Viete's formula [2] and the AC method [3], which are methods of factoring quadratic trinomials in solving quadratic equations for two types of quadratic equations: $x^2 + px + q = 0$ and $ax^2 + bx + c = 0$. The study also showed that the application of area models, which is based on geometrically using the combinations of rectangles and squares, to solve quadratic equations through completing the square is found in the textbooks. This approach originates from the history of the stages of algebra (e.g., Katz & Barton, 2007). Unlike using traditional approaches for solving quadratic equations, an American study suggests using different graphs of quadratic functions drawn with the program GeoGebra to help students understand the quadratic formula through working with the symmetry of a graph (Edwards & Chelst, 2019). Some studies are textbook analyses, comparing the content of quadratic equations in textbooks from different countries (Hong & Choi, 2014; Saglam & Alacaci, 2012; Winsløw, 2004). Most of the studies mentioned above focus on one or a few solution methods within areas of learning or teaching quadratic equations or analyzing textbooks' content. However, studying a whole learning process within the content involving both algebra rules as prior knowledge of and solution methods for quadratic equations, there seems to be a lack of attention to

[2] Viete's formula is $r_1 \cdot r_2 = q$; $r_1 + r_2 = -p$ if r_1 and r_2 are the roots to the equation type $x^2 + px + q = 0$. This type of equation can be solved by finding a pair of integers for which the product is *q* and the sum is -*p*. The integers found are the roots to the equation.

[3] The AC method is based on the idea of Viete's formula and is used for equations of the type $ax^2 + bx + c = 0$. It contains four steps: find *ac*; find the factors of *ac* which add to *b*; if these factors are *p* and *q*, replace *bx* with *px*+*qx*; complete the factorization (Pieronkiewicz & Tanton, 2019, p. 109).

quadratic equations in mathematics education.

Regarding how quadratic equations and the related content are presented in detail as a whole learning process, analyzing mathematics textbooks is an efficient way to examine what is offered to help students learn this abstract content. Textbooks contain not only subject-content knowledge but also pedagogical intentions (Pepin et al., 2001). Textbooks are widely used in mathematics classrooms in Sweden (Jablonka & Johansson, 2010; Madej, 2021). Mathematics teaching in Swedish classrooms is often based on textbooks (Madej, 2021). Mathematics teachers plan and prepare their lessons mainly by use of textbooks, and 45% of the teachers use the textbooks as exercises books (Lepik, et al., 2015). Students spend a great amount of time doing exercises from textbooks and the mathematics knowledge teachers present in the classroom mainly stems from the textbooks they use (Bergwall & Hemmi, 2017; Johansson, 2006). As intended, implemented and enacted curriculum material, textbooks play an important role in mathematics education (Valverde, et al., 2002). They contain embedded pedagogy by reflecting content in a certain way to suggest appropriate sequences of the content and pedagogical situations where activities, explanations, examples and exercises are selected to play particular roles; these roles are fixed and unchanging in textbooks (ibid, p. 12). Cognitive requirements on mathematics tasks provide different types of tasks and offer students varied opportunities in learning mathematics (Gracin, 2018). Teaching, learning and using artifacts converge in a textbook and make it multifunction in transferring subject knowledge. Considering embedded pedagogy and a wide range of content on algebra elements and quadratic equations in sequences, the author of this article chooses to analyze mathematics textbooks instead of studying classroom interactions on students learning quadratic equations within a limited area. The study's aim is to explore an embedded hypothetical learning trajectory (HLT) for learning quadratic equations from algebra rules to solving and applying quadratic equations to a wide extent by analyzing the related content, including tasks in Swedish mathematics textbooks. The research questions for this study are:

1. What embedded hypothetical learning trajectories regarding quadratic equations can be identified in Swedish textbooks for helping students understand abstract algebra concepts and procedures?
2. What learning opportunities regarding quadratic equations within an embedded HLT are offered by tasks in Swedish textbooks?

2 Previous studies on mathematics textbooks regarding algebra

The current study analyzes mathematics content in textbooks as subject matter knowledge, which in the textbook analysis is defined as a product-oriented approach (Johnsen, 1993). Some Nordic studies have involved analyzing mathematics textbooks in relation to the curriculum and subject of school algebra. Based on the framework of algebra big ideas[4], Hemmi et al. (2018) exam how algebra is presented in Swedish school curricula and textbooks as well as teachers' discourses for grades 1-9 from the diachronic and synchronic perspectives in order to characterize Swedish school algebra. It is found that EEEI is the most represented category in the textbooks for grades 1-3 and even 4-6; FT and VAR are also well represented, whereas GA is the least represented category in the textbooks. The focus study by Hemmi et al. (2019) has been carried out to analyze introductions of early school algebra particularly related to EEEI and GA in textbooks for grades 1-3 in Estonia, Finland and Sweden (Hemmi et al., 2019). The focus study finds that inverse properties are used in textbooks from the three countries. Creating letter expressions and equations are found in Estonian textbooks. With the same framework, the related study on identifying algebra thinking in Swedish primary textbooks and curriculum (Madej, 2021) shows that EEEI is the main algebra idea in the Swedish context but not GA. Palm Kaplan (2019) analyzes six mathematics textbooks for Swedish lower-secondary schools published in 1995-2015 in order to understand algebra characteristics in terms of school algebra discourses and algebraic activities. The results show that five algebra discourses[5] are identified in the textbooks. An early study identified school algebra according to three periods[6] in Swedish upper-secondary textbooks for the years 1960-2000 by Jakobsson-Åhl (2006) with the approach of phenomenography and hermeneutics.

To sum up, school algebra in analyzed Swedish textbooks for primary and lower-secondary schools mainly involves categories of equivalence, expressions, equations, and inequalities (EEEI); function thinking (FT) and variables (VAR) have also been found. Algebra as generalized arithmetic (GA) is not found in the Swedish textbooks

[4] Algebra big ideas as an analytical framework used by Blanton et al (2015) in Hemmi et al. (2018) for studying students' algebraic thinking. They refer to 1) equivalence, expressions, equations and inequalities (EEEI); 2) generalized arithmetic (GA); 3) functional thinking (FT); 4) variable (VAR).

[5] Five algebra discourses are symbolic, arithmetical, geometrical, (un)realistic and scientific.

[6] Three algebra periods are pre-New Math, New Math and post-New Math.

for years 1-9. School algebra in the analyzed textbooks at Swedish upper-secondary level has been developed from the focus of algebra manipulation to algebra application in real-world problems. Despite the various studies relating to analyzing the contents of algebra in textbooks, none of them directly focuses on quadratic equations.

However, three international studies (Hong & Choi, 2014; Saglam & Alacaci, 2012; Winsløw, 2004) look at quadratic equations in textbook analyses. Among these studies, some textbooks regard polynomials or binomials or geometrical representations of quadratics as prior knowledge before quadratic equations (Saglam & Alacaci, 2012). Introducing quadratic equations with a real-world problem (Hong & Choi, 2014; Winsløw, 2004) or by reviewing linear equations (Hong & Choi, 2014) or directly presenting the quadratic formula after the formal definition of quadratic equations (Winsløw, 2004) are different ways to approach quadratic equations in analyzed textbooks. Solving quadratic equations with factoring and completing the square illustrated with geometrical representations are found as common solution methods (Hong & Choi, 2014; Winsløw, 2004), while some quadratic equations are solved with the square root method or graphical approach (Winsløw, 2004). Different pedagogical intentions have been explored in these studies. For example, encouraging students to reason and explain the concepts, and to use the mathematical thinking process rather than algorithms (Hong & Choi, 2014); or emphasizing the procedural practice of using the formula (Winsløw, 2004). Among these three studies, there is an apparent lack of information in their results as to exactly which topics and what types of tasks have been examined, and where they start and end, as well as how the content is organized in order to develop abstract algebraic thinking.

To benefit research on school algebra in Swedish textbooks at upper-secondary school level, the current study analyzes how various algebra content regarding quadratic equations, including theoretical presentations with given examples and provided tasks, are connected and developed as a whole hypothetical learning trajectory utilizing the theoretical framework of hypothetical learning trajectory (HLT) (Simon, 1995).

3 Analytical approach

3.1 General framework

The general analytical framework used in this study is based on the concept of HLT (Simon, 1995; 2014). An HLT (Simon, 1995) "is made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process – a prediction of how the students thinking and understanding will evolve in the context of the learning activities" (p.136). "The latter two parts are interdependent" (Simon et al., 2018, p. 102). It is a theoretical model for the design of mathematics instruction (Simon, 2014). The central pedagogy of HLT is mathematics teaching for understanding (Simon & Tzur, 2004). Its aim is to develop students' mathematics thinking within a designed conceptual learning progression through different sequences of tasks. HLT "refers to the teacher's prediction as to the path by which learning might proceed. It is hypothetical because the actual learning trajectory is not knowable in advance. It characterizes an expected tendency" (Simon, 1995, p. 135).

Simon and Tzur (2004) emphasize that tasks selected for learning activities play an important role in providing effective mathematics instructions, and cognitively demanding tasks can develop students' cognitive abilities and mathematics thinking. Compared to routine tasks, which are applied as procedural practice, cognitively demanding tasks can offer students opportunities to learn mathematics concepts.

The analyses of mathematics examples and tasks in this study applied the concepts of conceptual and procedural knowledge of mathematics (Hiebert and Lefevre, 1986). Conceptual knowledge is characterized as knowledge that is rich in relationships and can be thought of as a connected web of knowledge (ibid, p. 3). Conceptual knowledge grows through the construction of relationships between existing knowledge and new information. It is labeled as meaningful learning. While "procedural knowledge is made up of two distinct parts. One part is composed of the formal language or symbol representation system of mathematics. The other part consists of algorithms or rules, for completing mathematical tasks" (ibid, p. 6).

Different cognitive demands on mathematics tasks provide opportunities for students to develop procedural and conceptual knowledge. Gracin (2018) interprets cognitive demands on mathematics tasks based on features of rich tasks, low- and high-level tasks, and mathematical competencies. Rich mathematics tasks have high cognitive demands: 1) memorization, 2) procedures without connections, 3) procedures with connections while at the same time developing deeper levels of understanding of mathematical concepts and ideas, and 4) doing mathematics requiring complex and non-algorithmic thinking with great cognitive effort. Low-level tasks contain the first two features, while high-level tasks are based on the last two features and often require students to understand, interpret, apply mathematics knowledge and skills from different sources to accomplish work. A balanced curriculum requires both low- and high-level tasks with a full range of problem types. High cognitive demands in mathematics competencies (Niss, 2015) entail mathematical thinking, problem handling, modeling, reasoning, representation, symbol and formalism, communication, and aids and tools.

In the current study, the HLT is used in such a manner that the author explores an embedded HLT concerning quadratic equations in eight textbooks through identifying three components: the learning sub-goals and final goals; the intended hypothetical learning processes; and the learning activities, as provided learning tasks in the respective related topics. As the overall learning goals are the same in all the textbooks – that is, how to solve and use quadratic equations – this analysis focuses on exploring the embedded learning process and analyzing related learning tasks within and between learning sub-goals. The learning sub-goals are directedly related to the topics before the final goals of solving and applying quadratic equations are addressed. The HLT is applied at two levels: a major level in a horizontal dimension, there theoretical presentations including examples within every topic from basic algebra concepts and rules to quadratic equations, are examined; a minor level in a vertical dimension, there provided tasks under every topic, are analyzed (Figure 1).

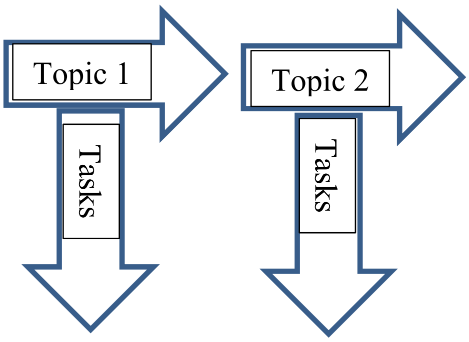


Figure 1. The whole HLT at two levels consists of a number of topics as content in a horizontal dimension. Every topic provides a group of tasks in a vertical dimension.

At the major level, an HLT is interpreted through exploring how every single topic is presented and exemplified so that students can understand an algebra concept or rule, later how the single topics as sub-goals connect each other to reach the final goal of solving and using quadratic equations in a textbook so that intended mathematics thinking develops from basic stages to abstract level as a hierarchical order. The horizontal HLT is examined as potential instructional material provided for teaching.

At the minor level, analyses of learning tasks within a single topic are carried out through applying analytical categories: procedural and conceptual knowledge; applications; and the framework of HLT. Hiebert and Lefevre (1986) expressed that tasks aimed at training students in mathematical procedures are applied to foster their procedural knowledge regarding rules and procedures for solving mathematics problems; tasks focusing on training in mathematics concepts are intended to foster students' conceptual knowledge, which involves a deeper understanding of mathematical relationships. In line with the category of conceptual knowledge, tasks focusing on applying recently learned mathematics concepts, rules, or solution methods for solving real-world mathematics problems are identified as applicational tasks which require comprehension, interpretation, and application of mathematical knowledge and skills (Gracin, 2018). In a sequence of tasks, the embedded HLT is identified by determining how the provided tasks are arranged and developed from basic computation procedures to abstract mathematics generalization in a hierarchical order.

The analyses of tasks intend to explore what learning opportunities are provided for students to learn basic algebra concepts and rules; and solving and using quadratic equations in a textbook from a learning perspective.

3.1.1 Analytical criteria in the vertical dimension

In detail, mathematics tasks are categorized in this study as such:

1. Tasks require students to simplify an algebra expression with algebra identities or rules; to factorize an algebra or quadratic expression or equation; to solve a linear or quadratic equation; to draw the graph of a quadratic function; and to find solutions for quadratic equations using a graphical calculator. These are categorized as *procedural tasks*. The characters of this type of task involve the following: formal mathematical symbols, algorithms and rules (Hiebert and Lefevre, 1986); the solution procedures or algorithms have been presented in previous theoretical presentations; the procedures can be found in textbooks and imitated; and they are routine tasks (Brändström, 2005).

2. Unlike procedural tasks, tasks requiring students to do mathematics through investigating, generalizing, reasoning, assessing, proving, and so on are categorized as *conceptual tasks*. They are carried out by connecting task information to existing knowledge (Hiebert and Lefevre, 1986). The following are different types of requirements for conceptual tasks in the eight textbooks:

- Requiring students to find the missing parameter (an unknown, a coefficient, or a constant) in a given algebra identity or an equation; or find the relationship among a coefficient, a constant, and roots of a quadratic equation.
- Requiring students to prove or explain algebra identities or rules using geometrical representations or in words.
- Requiring students to evaluate or assess whether a mathematics statement or calculation procedures or solutions to quadratic equations are correct, and if not, to reason why it is wrong and give a correct answer.
- Requiring students to set up an algebra expression or quadratic equation according to given statements, a word problem, or a visual representation through interpreting given contexts and applying algebra ideas and symbols to solve it.

3. Tasks asking students to solve a word problem in the application of an algebra expression or a quadratic equation are called *applicational tasks* in this study. Word problems are often constructed by relating to living world subjects: economy, geometry, physics, and arithmetic. In this study, such problems are called real-world problems; while they are not authentic real-world problems, they are intended to be familiar from students' daily life. Most of them can be solved with the help of the given information in a textbook. They require students to set up a mathematical model, such as an algebra expression or a quadratic equation, after interpreting the given text. However, a few of them require students to exercise high mathematical thinking, which means that the information needed to solve a task is not fully provided in the textbook (Palm, Boesen & Lithner, 2011).
4. A sequence of tasks requires students to first calculate a certain type of tasks and discover the patterns of calculation, then generalize or prove them with algebra expressions or rules, finally prove, apply, or develop the rules. The goal is to understand abstract mathematics through a learning progression in a hierarchical order. The sequence of these steps consists of both procedural and

conceptual knowledge and is usually constructed from simple to abstract steps. This category is a level above the first three categories since it covers all of them. This type of tasks is called *HLT-tasks*.

Briefly, the criteria of four categories of tasks are mainly based on procedural and conceptual knowledge (Hiebert & Lefevre, 1986) and the HLT framework (Simon & Tzur, 2004). At the same time, the cognitive requirements for Categories 2-4 are in line with the cognitive requirements for high-level tasks (Gracin, 2018) and mathematics competencies (Niss, 2015).

4 Method

The analyzed textbooks are eight Swedish mathematics textbooks used within the national program of Natural Science (NV) at the upper-secondary level (Appendix A). The textbooks were, and are, frequently used just before and after the establishment of the new national curriculum (Skolverket, 2011), according to information from the library of the Swedish National Resource Center for Mathematics. Half of the books (*Matematik 5000 2c*, *Matematik 2c*, *Exponent 2c*, *Origo 2c*) were published in 2011-2012, and have content on quadratic equations with complex numbers and solving a root equation, as well as analyzing a discriminant of a quadratic equation, which follows the requirement of the new curriculum. The other half (*Delta NT/a+b*, *Nya Delta A och B*, *Exponent B Röd*, *Matematik 4000 Blå*), published before 2011, contain quadratic equations dealing with real numbers only.

The analyses covered all the related topics and their respective tasks in every book. They were carried out in both horizontal and vertical dimensions in order to find a whole embedded HLT. The horizontal dimension analysis started with the topics of polynomials, algebra identities, or simplifying linear expressions, and ended with applications of quadratic equations. The vertical dimension analysis focused on tasks provided within every topic. A total of 273 pages were analyzed, covering 250 theoretical examples and 1,068 tasks.

For the horizontal dimension, the analysis was carried out in two steps (Steps 1 and 2), and for the vertical dimension, the analysis included one step (Step 3). The last analysis (Step 4) generalized both dimensions.

To find a whole HLT concerning the related topics with theoretical presentations and illustration examples in each book, the analysis of Steps 1 and 2 answered the following questions:

1. How does the textbook prepare students for learning quadratic equations? This is determined by how early the first topic appears.
2. Which topics or sub-goals of learning pre-knowledge to quadratic equations and quadratic equations are there in a whole HLT?
3. How is the topic of quadratic equations introduced?
4. How does the HLT end?
5. How are all the topics organized?
6. In which order are the topics presented?
7. What examples and related visual representations are applied in order to present algebra concepts, rules, and solution methods?

In the analysis of Step 1, all the topics concerning concepts, rules, methods, and examples including visual representations in every book were written briefly and listed according to the following categories: basic algebra concepts named as pre-knowledge, for example the concept of polynomials (Questions 1); topics before quadratic equations such as algebra identities and rules as pre-knowledge (Question 2); introduction and presentation of quadratic equations (Question 3); solution methods or application of quadratic equations as a final goal (Question 4); the order and organization of all the topics (Questions 5, 6); all examples including related visual representations within each topic (Question 7). The aim of Step 1 was to find an overall organization of a whole HLT in each book.

Step 2 aimed to summarize and compare all the HLTs of the eight books. All the HLTs were then generalized in five learning processes at most depending on different final learning goals according to the results of Step 1: 1) basic algebra concepts as pre-knowledge; 2) algebra properties and identities as pre-knowledge; 3) quadratic equations and their solution methods; 4) application of quadratic equations or finding the relationship [7] between parameters and roots of quadratic equations or discriminants; and 5) other content. The analyses of Steps 1 and 2 aimed to answer the first research question.

Step 3 was a vertical dimension analysis to answer the second research question. In this analysis, all the tasks provided for every topic in each book were categorized as types according to the four analytical criteria in Section 3.1.1: procedural, conceptual,

[7] *Relationship between parameters and roots of a quadratic equation* refers to the relationship between a constant q, a coefficient p and roots x_1 and x_2 in the common quadratic equation $x^2 + px + q = 0$. In Swedish textbooks, this relationship can be expressed as $p = -(x_1 + x_2)$; $q = x_1 \cdot x_2$.

applicational, and HLT-tasks. An example of a sequence of HLT-tasks is that students are first required to multiply two same binomials such as $(x+1) \cdot (x+1)$ and to do five similar tasks; second to discover a pattern of the results; third to square some new binomials according to the pattern at Step 2; fourth to generalize formulas of $(a \pm b)^2$ which are perfect square rules over addition and subtraction; at last discover a new rule $(a+b) \cdot (a-b)$ (Alfredsson et al., 2011, p. 75). In this example, the HLT of learning and understanding perfect square rules (as a learning goal) was explored through the development of cognitive requirements of the five tasks (as learning activities): from a simple procedure as multiplying, then to discovering a common pattern of the results, later to generalizing the rules, at last to challenging another new rule (as a learning process). Notice that procedural and conceptual types of tasks are included in this sequence.

The organization of the analysis in Step 3 entailed listing all the related topics of each book and all tasks provided within each topic according to the four types. Tasks were analyzed by examining the instructions for each task in order to translate its cognitive requirement; noting and categorizing whether it was a procedural, conceptual, applicational type of task, or a sequence of HLT-tasks; and then further analyzing in detail each type. For example, the detailed analysis of conceptual tasks was carried out by first briefly listing the cognitive demands in each book, then comparing and noting the common terms among the eight books, and finally categorizing them into different types. Its aim was to determine how every textbook offers opportunities to deeply understand algebraic thinking. The number of tasks in each type was counted in every book. Solving and computing tasks were carried out when it was difficult to interpret them directly.

After the analyses in the three steps above, and to confirm the answers to the two research questions, the analysis in Step 4, as a summary, compiled the previous analyses of both the horizontal and vertical dimensions. In the horizontal dimension, the HLTs derived from the analysis of Step 2 made up the main structure horizontally, consisting of all the analyzed topics. In the vertical dimension, the total number of tasks of each type among the procedural, conceptual, and applicational types derived from the analysis of Step 3 were listed for each book in an Excel document and were then compared among all eight books. At the same time, the different task sets were added up. A task set refers to a group of tasks for a single type of tasks within a topic. Accordingly, there are procedural sets, conceptual sets and applicational sets. A topic may consist of one, two, or all three types of task sets. A task set may contain a large number of the same type of tasks but sometimes may not. A sum and comparison of different types of task sets among the eight books can help in determining what learning opportunities a textbook provides and how widely different task sets spread out within a whole HLT. The three computations in each book were: the total number of task sets in a whole HLT of each book; the total number of all three types of task sets; and the total number of task sets with merely conceptual tasks. The results were generalized in an Excel document. HLT-tasks as the fourth type were listed separately as another category since it consists of a whole sequence of tasks containing the other three types which have already been counted in task sets earlier. Therefore, HLT-tasks were not included in the Excel document.

The basic structure of all the analytical steps is bottom-up. The repeated procedures, from the first two separate dimensions to the summed analysis, strengthen the reliability.

5 Results

This analysis has been conducted from both horizontal and vertical dimensions. The report of the results is presented according to these two dimensions. Research Question 1 regarding HLT from a teaching perspective is answered in 5.1, while Research Question 2 regarding learning tasks in every HLT from a learning perspective is answered in 5.2. Two tables (Tables 1-2), two diagrams (Figures 8-9) and six figures (Figures 2-7) are used in order to illustrate the main results. The eight textbooks are referred to as B1-B8 (Appendix).

5.1 Results in the horizontal dimension

Results show that embedded HLTs involve in three, four, or five learning processes (Table 1). All eight books present the content concerning quadratic equations primarily in four parts in horizontal order: 1) concepts of polynomials, algebra expressions, or algebra rules; 2) distributive property, multiplication of two different binomials, perfect square rules over addition and subtraction, difference of squares, factoring; 3) solving quadratic equations with the square root method, factoring, completing the square, the *pq* formula or the graphical method; and 4) the application of quadratic equations in solving real-world problems, or relationships between parameters[8] and roots of quadratic equations or discriminants[9].

All the books contain the same parts of HLT learning processes (Processes 2 and 3) concerning learning algebra identities and rules as well as solving quadratic equations in different methods, but they differ in Processes 1 and 4 as well as 5. Five of them (B1, B3, B4, B5, B7) consist of four processes, two of them (B2, B6) three processes, and one of them (B8) five processes. The HLT learning processes are the same for Processes 2 and 3. B1 does not include factoring; B3, B4, and B6 contain the most content topics in Process 2.

Table 1. A summary of five learning processes including visual representations used in examples in analysis of horizontal dimension

Books/Processes	1	2	3	4	5
B1	Polynomials GR (DP/MTB)	Algebra rules GR (DSR)	M1, 3, 4	Algebra history Application	
B2	Simplifying AE, linear expressions Solving linear equations and inequalities Showing results on number lines	Algebra rules Factoring GR (MTB)	M1-4		
B3	Polynomials Simplifying AE Solving linear equations	Algebra rules Factoring Solving QEs With DSR PSR	Introduction of QEs M1-4 (CS)	Relationship between *p*, *q* and roots	
B4	Tasks GR (DP/MTB/PSR+) Polynomials	Commutative Associative Algebra rules GR (DP/MTB)	M1-4 Algebra history GR (CS)	Application	

[8] *Parameters of quadratic equations* refer to *a*, *b*, and *c* in the general quadratic equation $ax^2 + bx + c = 0$ where *a* and *b* are coefficients while *c* is constant.

[9] *Discriminant* is an expression, $\sqrt{\left(\frac{p}{2}\right)^2 - q}$ as part of a *pq* formula. It is used to justify which types of solutions a common quadratic equation can have. A quadratic equation has one real solution when $\left(\frac{p}{2}\right)^2 - q = 0$; two real solutions when $\left(\frac{p}{2}\right)^2 - q > 0$; and no real solutions (or solutions with complex numbers) when $\left(\frac{p}{2}\right)^2 - q < 0$.

		Factoring Solving QEs with DSR PSR (DP/PSR)			
B5	Tasks GR (DP/MTB/PSR+) Polynomials	Algebra rules GR of DP/MTB/PSR- Factoring Solving QEs with DSR PSR (DP/PSR+)	M1-4 GR (CS)	Application Algebra history	
B6		Algebra identity Commutative Associative Algebra rules GR (MTB) Factoring Mixed ex (DSR/PSR-)	M1-4 GR (CS) (CS)	Relationship between *p*, *q* and roots	
B7	Tasks QF-gr (3 types of roots)	Algebra rules Factoring GR(MTB/PSR+) (PSR-)	QF-gr (intro QE) M1-4+M5 Solve QEs in complex nr GR (CS)	Relationship between *p*,*q* and roots Discriminants Tasks QF-gr	
B8	Complex nr DP Tasks GR (DP) Simplifying linear expressions Solving linear equations	Polynomials Algebra rules GR (MTB) GR (PSR+)	The concept of QEs M1, 3, 4 GR (CS) (DSR)	Discriminants Concept of complex nr Complex nr as roots to a QE Root equation Application	Factoring Solving QEs with factoring M2

Note. *AE* refers to algebra expressions.

CS refers to a solution method for solving quadratic equations and is called completing the square.

DP refers to algebra identity: distributive property $a(b+c) = ab + ac$.

DSR refers to the difference of two square $a^2 - b^2 = (a+b)(a-b)$.

GR refers to geometrical representations consisting of combinations of areas of squares and rectangles.

M1-4 refers to four solution methods used for solving quadratic equations: the square root as M1, factoring as M2, completing the square as M3, and the *pq* formula as M4.

PSR± is a short term for two algebra identities called perfect square rules over addition and subtraction $a^2 \pm 2ab + b^2 = (a \pm b)^2$.

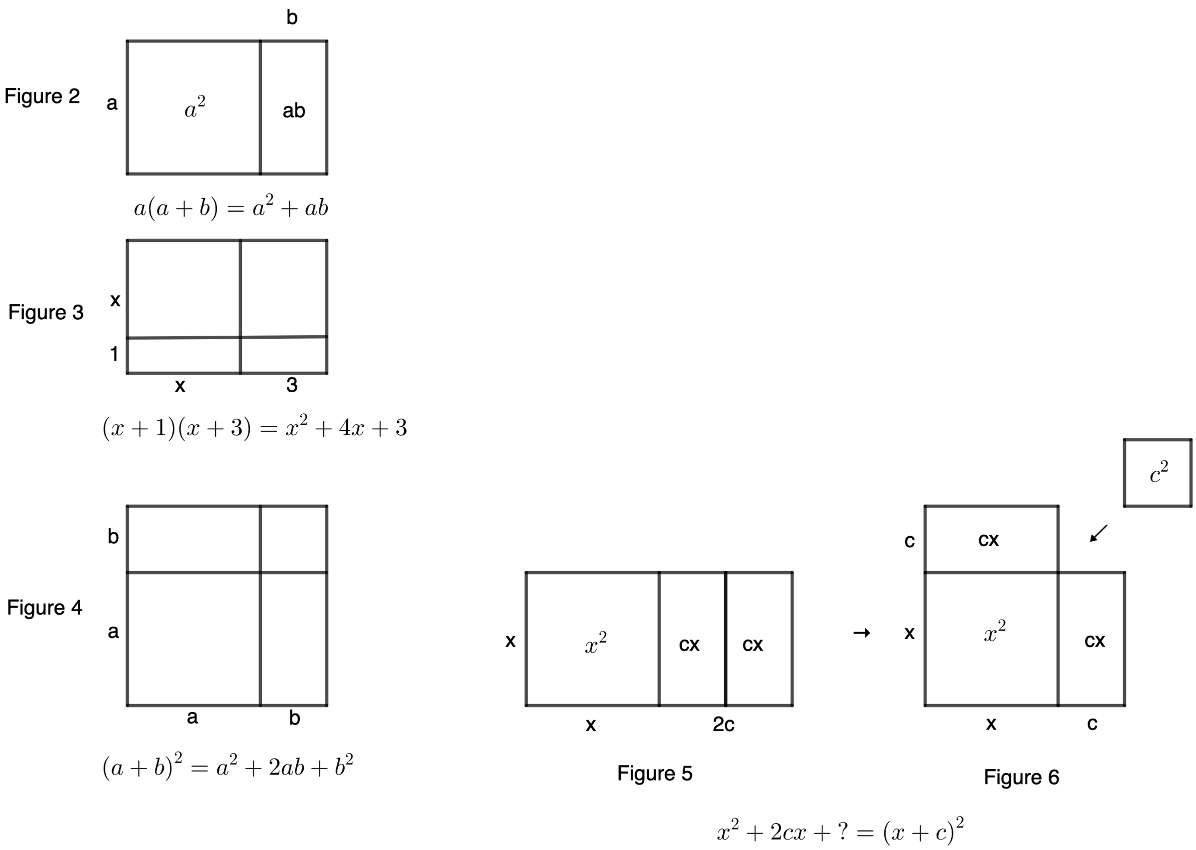
p refers to a coefficient of x in the quadratic equation $x^2 + px + q = 0$.

q refers to a constant in the quadratic equation $x^2 + px + q = 0$.

QE refers to a quadratic equation.

QF-gr as M5 refers to the approach of solving quadratic equations through searching the x-intercepts of a quadratic function graph.

Among the analyzed books, it is also found that visual representations are frequently applied in Processes 2 and 3 for illustrating distributive property (Figure 2), multiplication of two different binomials (Figure 3), perfect square rule over addition (Figure 4), and completing the square (Figures 5-6), which are often presented with geometrical representations for the sums of rectangles and squares. Another type of visual representation illustrates three types of solutions to quadratic equations, represented by three x-intercepts of graphs of quadratic functions (Figure 7).



Figures 2-6: Geometrical representations of algebra rules and completing the square method (Alfredsson et al., 2011).

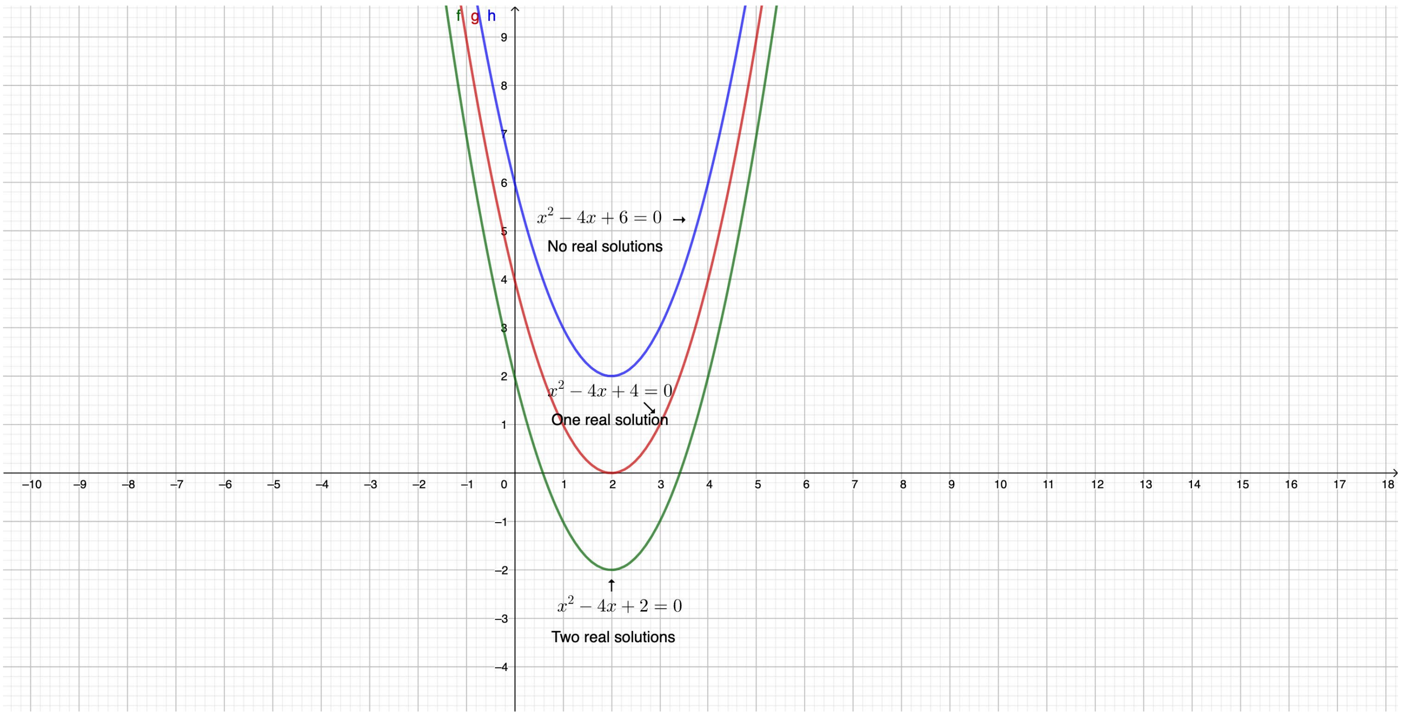


Figure 7. Three types of graphical representations of solutions to quadratic equations, represented by graphs of quadratic functions (Szabo et al., 2012).

Every book illustrates the multiplication of two binomials with geometrical representations as GR. Distributive property in B1, B3, B4, B5 and B6, the difference of two squares in B1, perfect square rule over addition in B5, B7 and B8 are illustrated with GR in Process 2. Solving quadratic equations with completing the square method in B4-8 are illustrated with GR in Process 3. Graphical representations are used in Process 3 twice in B7 but not in the other books. GRs in Process 3 are a combination of early usage of GRs in Process 2 or even Process 1, as in the cases in Figures 2-6 in B4-8, which reflects the development of an HLT. At the same time, graphical representations illustrating three different roots to quadratic equations, as in Figure 7, are applied together with GRs in B7. The combination of GRs is developed from a single figure with rectangles to three figures with squares and rectangles, with the intent of visually transforming how distributive property, multiplication of binomials, and perfect square rules are operated, and then how they are developed into a complex procedure; that is, completing a square as an approach to the *pq* formula. The emphasis on teaching concepts with the help of visual representations is explored among these five books (B4-8).

Process 3 contains content on solving quadratic equations with three, four, or five different methods: M1, M2, M3, M4, and M5 (Table 1). All the textbooks except B1 present M1-M4. Factoring method is not available in B1. In this process, M3 is illustrated with GR (Figures 5-6) as examples in five books (B4-8) just before the introduction of M4, while using quadratic function graphs to solve quadratic equations as M5 is applied in B7 (Table 1). M2 is used for solving particular quadratic equations, which can be only factorized by using algebra identities inversely. Using factoring based on Viete's formula to solve quadratic equations is not found. Processes 2 and 3 are the core learning processes of the whole HLT in every book.

Differences mainly remain in Processes 1 and 4. Process 1 is the beginning of a whole HLT in all the books except B6 (Table 1). Process 1 starts with an introduction of concepts involving polynomials in B1, B3, B4, and B5; simplifying algebra expressions in B2, B3, and B8; solving linear equations in B2, B3, and B8; doing tasks with algebra rules in B4 and B5; doing tasks relating to three types of quadratic equation roots in B7; or explaining complex numbers in B8. B2 presents more procedural knowledge in Process 1, consisting of simplifying algebra expressions, solving linear equations and inequalities, and drawing results on number lines. This beginning takes a long path in Process 1 before arriving at Process 2. It also implies an emphasis on procedural knowledge in the whole HLT in B2. B4 and B5, as the same series, have the same Process 1, providing the same tasks on understanding distributive property, multiplication of two binomials, and perfect square rule over addition with geometrical representations, before Process 2 presents these algebra rules in detail. B3 starts Process 1 with both polynomial concepts and operational procedures: simplifying algebra expressions and solving linear equations. B8 contains most of the algebra topics in Process 1 – history of complex numbers, distributive property, simplifying algebra expressions, and solving linear equations – although it introduces polynomial concepts at the beginning of Process 2. Like B2, B8 has a long Process 1 before beginning Process 2. B6 starts its HLT differently from the other books, starting with concepts of algebra identities in Process 2 instead, which means that it contains a shorter HLT, consisting of three learning processes.

Processes 4 and 5 show how an HLT ends. There are three types of endings (Table 1): ending with application of quadratic equations (QEs) to solve real-world problems and the presentation of algebra history related to solving equations to different degrees in B1, B4, and B5; ending with presenting the relationship between a coefficient, a constant, and roots of QEs in B3, B6, and B7, among which B7 includes finding the type of solution by analyzing the discriminant of a QE; ending with solving QEs using four methods in Process 3 in B2 and using factoring method in Process 5 in B8. This results in B2 having a shorter HLT like B6, while B8 having the longest HLT.

The endings with the application of QEs in B1, B4, and B5 show that these three books emphasize the use of QEs in real-world problems. The endings in the three books B3, B6, and B7 presenting discriminants and the relationship between coefficient, constant, and roots lead to an encouragement of more conceptual learning with a focus on mathematics theory.

The differences in Processes 1 and 4 provide different lengths of HLTs among all the books, depending on the number of content topics presented in these two processes. The increase in content topics in B5-8 is a result of adopting the new curriculum. For example, complex numbers are not presented in B2 but are presented in detail in B7 and B8. Root equations are also included in B8. Thus, B8 provides the longest HLT with the most content topics among all the textbooks. B1 has missed factoring as a solution method in its whole HLT. B3 and B6, as the same series, provide the shortest HLTs in three processes but do not have fewer content topics, which implies that their HLTs are effective.

5.2 Results in the vertical dimension

Tasks provided in every book were analyzed according to four types – procedural, conceptual, and applicational as well as HLT-tasks – in each HLT learning process. The result of analyzing tasks according to the first three types shows that some processes consist of all the first three types of tasks, while others do not. In each HLT, there are more procedural tasks than conceptual and applicational ones though the sum of conceptual and applicational tasks surpasses procedural tasks in B3-7 (Figure 8). Among the eight books, B6 provides the most numbers of conceptual and applicational tasks, while B8 provides the most numbers of procedural tasks. Every topic contains a number of task sets. B3 (Figure 9) provides the most sets (13) as well as the most PCA sets[10] (8), while B4, B7 and B8 provide the most conceptual sets (11). The higher number of task sets indicates that tasks are widely spread in a whole HLT but does not necessarily increase the number of tasks. B3 has less numbers of tasks than B8 but distributes the tasks among a wide range of topics in the whole HLT. With the most PCA sets, B3 emphasizes on developing all the three types of mathematics practices: procedural, conceptual and applicational.

[10] *PCA sets* refer to task sets consisting of all three types of tasks: procedural, conceptual, and applicational.

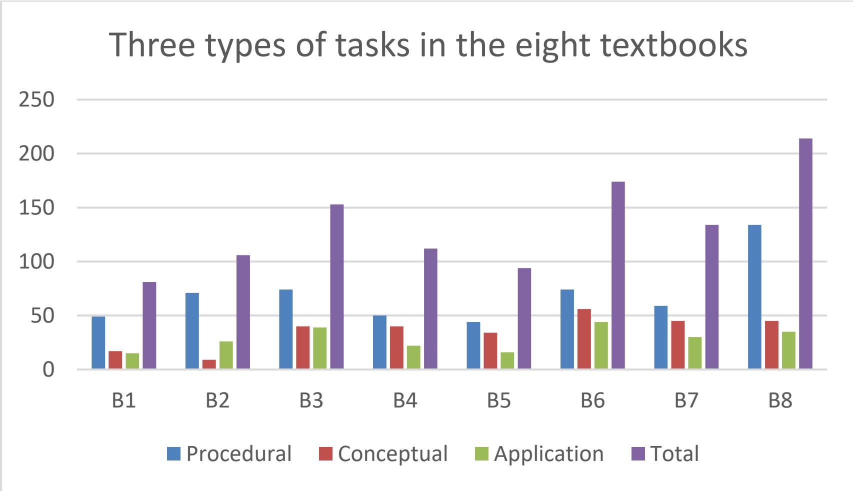


Figure 8. Three types of tasks in the eight textbooks.

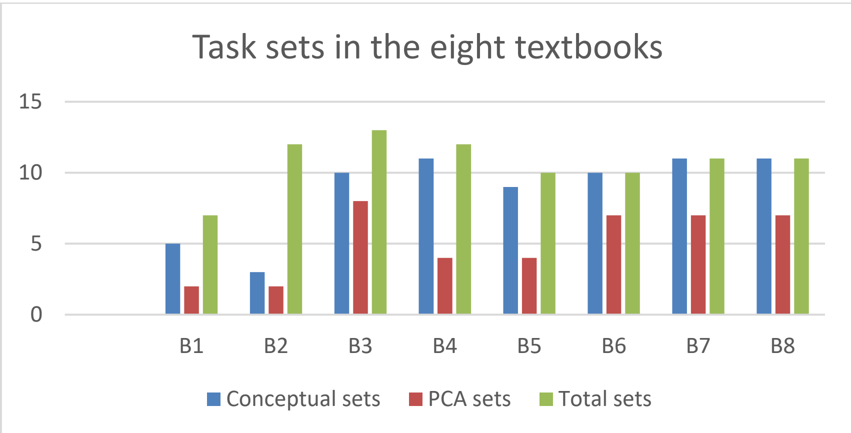


Figure 9. Task sets in the eight textbooks.

The mathematics procedures among the procedural tasks mainly involve simplifying, computing, or factoring algebra expressions; solving linear equations; solving linear inequalities; or solving quadratic equations using four different methods. Most of the quadratic equations in all the books contain integer parameters, besides a few cases with fractions or decimal numbers. The majority of the quadratic equations can be solved using both factoring and the *pq* formula. B7 presents one more solution method entailing solving QEs graphically using calculators. There is not much variation in procedural tasks among the books. The procedures for doing these tasks can be found and imitated in previously provided examples, which means that they are routine tasks (Brändström, 2005). Four of the eight books – B2, B3, B6, and B8 – provide mainly procedural tasks, with B8 providing the most. B1, B2, and B8 provide unbalanced learning opportunities, with much more procedural tasks than conceptual and applicational ones, while B6 and B7 gave a balanced task distribution (Figure 8). B3 and B6 provide tasks containing advanced procedures.

The common types of conceptual tasks (Table 2) are: finding a constant, a coefficient or an unknown *x* in an equation (B1, B3, B5-8); finding or analyzing the relationship between a coefficient, a constant, and roots (B1, B3-8); proving or explaining algebra identities such as the difference of two squares, perfect square rules over addition and subtraction, or the method for solving quadratic equations – called *Completing the Square* – with geometrical representations (B1, B3-8), especially in B7; assessing whether or justifying that calculation procedures or solutions or the equivalence of a QE are correct (B1, B3-7); reasoning why a solution or calculation procedure is wrong (B4-5, B7-8); and setting up algebra expressions (B2, B5, B8). There are also other types, such as: drawing a number line to illustrate solutions (B2); matching up graphs of quadratic functions with their QE solutions and discerning between complex numbers and real numbers (B7); and discussing an equation with no solution (B8).

B6, B7, and B8 have more conceptual tasks than the other five books, while B6 provides the most conceptual tasks (Figure 8). At the same time, B5, B7 and B8 provide more varying conceptual tasks than the other five books (Table 2). The rich conceptual tasks in B6 and B7-8 demonstrate the emphasis on conceptual learning in these books. B2 provides the least conceptual tasks.

Table 2. Different types of conceptual tasks in every textbook in the vertical dimension analysis

Textbooks	**Different types of conceptual tasks**
B1	Assess, find constant and variable, find relationship between p, q, and roots, prove equivalence, prove PSR± with GR
B2	Set up algebra expressions, draw number lines to show solutions
B3	Describe, compare, identify, analyze, assess, find constant and variable, investigate relationship between p, q, and roots, explain CS with GR
B4	Discover, interpret, reason, compute, generalize, assess, analyze relationship between p, q, and roots, explain MTB, PSR+ with GR
B5	Give examples, set up, prove, reason, compute, generalize, assess, find constant and variable, analyze relationship between p, q, and roots, explain DP, PSR+ with GR
B6	Assess, find constant and variable, find relationship between p, q, and roots, prove DSR, PSR- with GR (advanced)
B7	Describe, discern, match up, prove with the Pythagorean Theorem, reason, assess, find constant and coefficient, find relationship between p, q, and roots, prove & explain PSR-, CS with GR
B8	Describe factoring, set up, analyze, discuss, explain, prove, reason, find constant and variable, prove relationship between p, q, and roots, prove DSR with GR

Like the other two task types, applicational tasks are distributed in different learning processes of a whole HLT in every book. Seven books (B1, B3-8) provide fewer applicational tasks than procedural and conceptual ones, while B2 provides more applicational tasks than conceptual ones (Figure 8). B1 provides the least, and B6 the most, applicational tasks. Most of the applicational tasks involve solving a textual problem by setting up an algebra expression or a quadratic equation or function according to a given description and then computing some specific value; interpreting a given mathematics model such as an algebra expression, a quadratic function or equation and then calculating some specific value; or reasoning solutions. They are related to three or four topics: arithmetic (finding number sequences, even or odd numbers), economy (saving money or shopping), geometry (areas, sides, or circumferences of rectangles, squares, circles, or triangles), or physics (height of a throwing object, driving distances). A few problems are challenging and cannot be helped with previous examples or theoretical presentations, but rather require creative or high mathematical thinking. Some of them are not necessarily difficult but require thinking from other perspectives, but this type of problem is rare.

The result of analyzing the fourth type of tasks shows that only B4 and B5 provide HLT-tasks with requirements: computing (B4, B5), discovering (B4), generalizing (B4, B5), and interpreting (B4). These requirements, found in seven sequences of tasks in B4 and B5, are identified as HLT sequences, meaning that every sequence of tasks is structured from basic computing procedures to find relationships between computations and results, so that students are required to generalize abstract algebraic rules. These requirements encourage investigating and discovering activities that are more complex than other tasks, and require high-level thinking and mathematical competencies (Gracin, 2018; Niss, 2015).

Briefly, the vertical results show that B3 spreads out more exercises among a wide range of topics and includes all the first three types of the tasks most of the time in the whole HLT. B1 provides the least of tasks, while B8 provides the most of tasks. B6 encourages both conceptual learning and mathematics applications, while B8 encourages both procedural and conceptual learning. B4 and B5 encourage mathematics thinking according to an HLT progression.

To conclude the results of the analyses in both dimensions, all the embedded HLTs are similar for the presentation of core content, which is algebra identities and rules and quadratic equations. The eight analyzed textbooks are therefore similar. But they are different when it comes to how an HLT starts and ends as well as what types of tasks are provided. B1, B4, and B5 start with introducing polynomials and end with applying quadratic equations to solve real-world problems, while B4 and B5 are more similar since they introduce algebra rules by presenting tasks with GRs. B1 provides the least tasks. B2, B3, and B8 have similar starts with procedures of simplifying expressions and solving linear equations. B6 starts its HLT directly with algebra rules at Process 2 without Process 1. B7 starts with tasks related to three types of roots represented by graphical representations. B3, B6, and B7 end their HLTs by finding relationships between roots and constants, including coefficients of quadratic equations, while B2 and B8 end their HLTs by solution methods to quadratic equations. B8 has the longest HLT since it covers the most topics and provides the most tasks involving the most procedural tasks, while B2 and B6 have the shortest HLTs though B6 is more effective because of the most numbers of conceptual and applicational tasks within its HLT. B7 applies the most visual representations within its HLT. The application of geometrical representations in B4-8 visually shows the development of algebra rules within their HLTs. B4 and B5 provide similar HLT-tasks. B3 and B6 provide advanced tasks comparing to the other books. Procedural tasks are more than conceptual and applicational ones in each book but the sum of the last two types is more than procedural ones in B3-7.

6 Discussion

The results have shown similar HLTs concerning the core contents on algebra rules and quadratic equations among the eight Swedish textbooks. But they are different when it comes to how an HLT starts and ends. The explored pedagogy implies that teaching in basic algebra concepts and procedural skills is regarded as the essential pre-knowledge before teaching quadratic equations; illustrating related algebra rules or solutions methods of quadratic equations with geometrical representations is a pedagogical approach to abstract algebra and intends to develop students' conceptual understanding. Enriched through two types of visual representations, the content in B7 can be understood easily. Thus, B7 emphasizes conceptual learning. The Swedish textbooks' application of geometrical representations or geometrical models (Pieronkiewicz & Tanton, 2019) for completing the square is related to algebra history (e.g., Katz & Barton, 2007) and shows its historical aspect. The different endings of the HLTs indicate two types of intended pedagogy: learning for mastering mathematics theory, and learning for applications. A long HLT with many topics but a large number of procedural tasks in B8 implies teaching basic concepts and procedures. A short HLT in B6 without a focus on pre-knowledge implies an effective learning process for advanced learners.

The main learning opportunity provided by the textbooks is the practice of procedural tasks. This implies that teaching algebra procedures is a basic pedagogical approach. On the other hand, conceptual and applicational learning is encouraged in B3-7 especially in B6. Various conceptual tasks and the HLT-tasks in B4 and B5 intend to provide students opportunities to develop their mathematics thinking within conceptual learning progression (Simon & Tzur, 2004). Similar to algebra big ideas (Hemmi et al., 2018), this study finds categories of equivalence, expressions, equations, inequalities, variables, generalized arithmetic, function thinking in the analyzed tasks. Among these categories, generalized arithmetic and function thinking as well as inverse properties are frequently represented in the tasks. The results of the analyses in both dimensions also show an agreement with the five algebra discourses in the previous study by Palm Kaplan (2019). Geometrical, realistic and scientific discourses are common among the eight textbooks.

None of the Swedish textbooks uses graphical representations to illustrate the pq formula, as suggested by Edwards and Chelst (2019) with the purpose of helping students understand the abstract quadratic formula. The quadratic formula is not presented as the most common method for solving quadratic equations in the Swedish textbooks; while the pq formula is presented as the most common approach, it is a simplification of the quadratic formula. This result agrees with Olteanu and Holmqvist's (2012) in their study. The application of geometrical representations for completing the square intends to transform the geometrical approach to the abstract pq formula among five books (B4-8). Teaching for conceptual learning is explored. The use of the factoring method to solve quadratic equations based on Viete's formula or the AC method is not found. Therefore, the factoring method among all the textbooks is utilized limitedly for solving simple quadratic equations. The reason for this may be that quadratic equations of the type $ax^2 + bx + c = 0$ are not directly presented in any of the books, even though this type of equation appears in some tasks and has to be handled by first dividing by the coefficient of a. Consequently, the quadratic formula is not presented in the books.

This study's two-dimensional analyses with the HLT framework (Simon, 1995, 2004) and conceptual as well as procedural knowledge (Hiebert & Lefevre, 1986) have allowed for an examination of mathematics content considering both theoretical presentations with examples and representations and tasks provided widely and deeply from both the teaching and learning perspectives. Therefore, the research questions have been able to be answered. However, the analysis of tasks found a dilemma when tasks couldn't be clearly discerned as pure conceptual and applicational types since computation is a necessary step to carry out conceptual and applicational steps. In this case, identifying a task type was based on the cognitive demands of a task. In future studies, it could be interesting to examine how a specific algebra concept, rule, or solution method can be understood by students using one of the textbooks analyzed here.

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Appendix: List of mathematics textbooks included in this study

Alfredsson, L., Bråting, H., Erixon, P., & Heikne, H. (2011). *Matematik 5000 2C*. Stockholm: Natur & Kultur.

Alfredsson, L., Brolin, H., Erixon, P., Heikne, H., & Ristamäki, A. (2007). *Matematik 4000 B (Blå)*. Stockholm: Natur & Kultur.

Björup, K., Körner, S., Oscarsson, E., & Sandhall, Å. (2000). *Nya Delta matematik. Kurs A och B*. Malmö: Gleerups Utbildning AB. (B2)

Gennow, S., Gustafsson, I-M., & Silborn, B. (2008). *Exponent B (Röd)*. Malmö: Gleerups Utbildning AB.

Gennow, S., Gustafsson, I-M., & Silborn, B. (2012). *Exponent 2C*. Malmö: Gleerups Utbildning AB.

Sjunnesson, J., Holmström, M., & Smedhamre, E. (2011). *Matematik 2c*. Stockholm: Liber AB.

Szabo, A., Larson, N., Viklund, G., Dufåker, D., & Marklund, M. (2012). *Matematik origo 2c*. Stockholm: Sanoma Utbildning.

Wallin, H., Lithner, J., Wiklund, S., & Jacobsson, S. (2000). *Δ NT/a+b–Liber Pyramid. Gymnasiematematik för NV och TP, kurs A och B*. Stockholm: Liber AB.

A short list in the form of numbers:

B1: Δ NT/a+b–Liber Pyramid. Gymnasiematematik

B2: Nya Delta matematik. Kurs A och B

B3: Exponent B (Röd)

B4: Matematik 4000 B (Blå)

B5: Matematik 5000 2C

B6: Exponent 2C

B7: Matematik origo 2C

B8: Matematik 2c

Facilitating factors of scientific literacy skills development among junior high school students

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The study used causal-comparative research design to examine the scientific literacy among randomly selected Junior High School students under the Science, Technology and Engineering Program (STEP) of a National High School in the Philippines. Specifically, it investigated the factors that facilitate and hinder the students' ability to write and present scientific research. Quantitative and qualitative data were gathered from primary and secondary sources. Descriptive statistics were used to analyse the data obtained from the interviews and questionnaires. Findings showed that the scientific literacy of students in terms of writing was perceived as good while presenting the scientific research was described as fair. The study also revealed that teachers' factors, learning environment, and school administrative support affect the scientific literacy skills development of the students. Thus, the study suggested that by promoting the identified factors, the scientific literacy skills of the students will be further developed. Additionally, increase of teacher's availability during consultation hours, use of differentiated instructions, localization, contextualization, formulation of policy guidelines for the use of learning resources, plan of activities for STEP, as well as development of a module, research networks and linkages should be given importance.

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1 Introduction

Scientific literacy, which consists of the knowledge and understanding of the scientific concepts and processes required for personal decision making, participation in civic and cultural affairs and economic productivity (Mohapatra, 2013) is an important factor of development in every nation. It is an important factor of social and economic progress (Rodriguez-Espinosa, 2005). According to Dragoş and Mih (2015), scientific literacy can be classified into four categories. These include (i) *Cultural Scientific Literacy*, which is the understanding of science with average intelligence and education of a culture; (ii) *Civic Scientific Literacy*, which is the understanding of science in order to make informed decisions with regard to legislation and public policy; (iii) *Scientific Literacy Practice*, which is the understanding of science in order to solve practical problems; and (iv) *Aesthetic Literacy and Consumer Science*, which is the understanding of scientific laws and phenomena that enhances a person's appreciation of life itself through intellectual beauty of scientific ideas.

In general, science as a subject taught in every school should support the development of scientific literacy. This will prepare students for a more complex, interconnected world, with jobs that require critical thinking, teamwork and problem-solving skills. In fact, scientific literacy is one of the essential skills required in this digital age literacy (Turiman, Omar, Daud, & Osman, 2012). Thus, science education shall help the students and motivate them in pursuing careers in line with science and its application to technology and the industry.

In the process of teaching and learning science, news reports in 2017 from the Philippine Inquirer show that Filipino students exhibit high regard in science by applying the concepts through an invention or innovation and joining several science competitions in the national and international arena (Leonen, 2017).

On the contrary, it has also been reported that the state performance of Filipino students in science national and international examinations remained poor. The Philippine Department of Education (DepEd) recognizes the need for addressing this issue on scientific literacy after the country got a poor ranking in the Programme for International Student Assessment (PISA) in 2018. The PISA is a student assessment of 15-year-old learners across 79 countries done by the Organization for Economic Co-operation and Development (OECD) as part of the Quality Basic Education reform plan and a step towards globalizing the quality of Philippine basic education (DepEd, 2019). It looks into the extent to which the students have acquired key knowledge and skills that are essential for full participation in modern societies (OECD, 2018). Based on the PISA results, the Filipino students scored 357 in Scientific Literacy, which was significantly lower than the Organisation for Economic Co-operation and Development (OECD) average of 489 points.

Science Literacy, as defined in the PISA 2018 Assessment and Analytical Framework, refers to the students' ability to engage with science-related issues, and with the ideas of science, as a reflective citizen. Accordingly, a scientifically literate person is willing to engage in reasoned discourse about science and technology, which requires the competencies to explain phenomena scientifically, evaluate and design scientific enquiry, and interpret data and evidence scientifically (OECD, 2019).

The Philippines' low performance in science and scientific literacy poses a serious challenge on teachers, as they are the prime movers of education. It can be viewed that teachers possess both the privilege and responsibility in helping to address some issues in our educational system. This privilege is priceless in a sense that teachers have a direct influence on the students in shaping their minds and hence, in building the nation's future leaders. However, the price of this privilege is a greater weight of responsibility on the teachers' end.

Teachers bear the greater responsibility in the case of low performance on the National Achievement Test (NAT) in science. In essence, it is no doubt that every nation needs proactive teachers to embody and perform the goals of its education system. As reported by the National Education Testing and Research Centre of the Department of Education (NETRC-DepEd cited in Benito, 2005), on average, high school students' overall performance on NAT is improving from a mean percentage score of 46.80 in School Year (SY) 2004-2005 to 48.90 in SY 2011-2012. With these figures, however, students' performance in science was the lowest (39.49 in SY 2004-2005 and 40.53 in SY 2011-2012) among the other subjects that are included in NAT; these are Filipino, Mathematics, English, Social Studies, and Critical Thinking Skill Test.

With these results, the accomplishments of few students are overshadowed by the poor performance of many in NAT, which is in fact, the country's measure of quality education. The authors in Science Education Institute and University of the Philippines-National Institute for Science and Mathematics Education (SEI-DOST & UP NISMED, 2011) found out that, in general, Filipino students have low retention of concepts, have limited reasoning and analytical skills, have poor communication skills, and they cannot apply concepts to real-life problem-solving situation. Likewise, low performance in science of Filipino students can be associated with several factors such as the quality of teachers, the teaching-learning process, the school curriculum, instructional materials and the administrative support (SEI-DOST & UP NISMED, 2011).

University of the Philippines (UP) Board of Regents, (1997) as revealed by SEI-DOST and UP NISMED (2011) discussed the efforts of Science Education Institute of the Department of Science and Technology (SEI-DOST) and University of the Philippines National Institute for Science and Mathematics Education Development (UP NISMED) to address the low performance of Filipino students by focusing on curriculum development, conducting researches, and providing trainings for stakeholders.

However, it is necessary for other stakeholders like the schools under the supervision of the Department of Education to take part and focus on the following concerns: quality of teachers, improvement of the teaching-learning process, preparation of the instructional materials and administrative support. It is apparent therefore that National High Schools with Science, Technology and Engineering Program (STEP) should also take part in improving teaching science and research. Students under this program have an additional research subject as an elective of science subjects. Though, most of the students in STEP are not interested in doing research works, they are required to go through the scientific research process and apply it to the Science Investigatory Project (SIP). These SIPs are the direct applications of the scientific method where the students identify problems, proposed a possible solution through conducting experimentations, testing their hypothesis, and presenting their findings.

In the Schools Division of Laguna, Region IV-A, Philippines, students taking the STEP are expected to compete for the annual Science Fair and Congress. The aim of the Science Fair is to promote science and technology consciousness amongst youth. Similarly, identify the most creative and best science researches in the region (DepEd Regional Memorandum No. 270, 2016). The Science Fair features the SIPs from different participating schools competing from the following category: life science, applied science and robotics. It is held during the month of September and participated by teams or individual student from Grades 9 and 10.

In connection with this, research subjects in a National High School have been taught since 2016. For the past years, the said School was not able to compete and produce a SIP output. This only signifies how scientific literacy has been a critical issue in science education. Thus, this study looked into the scientific literacy skill of students under the STEP. The study sought answers to the following questions:

1. What is the scientific literacy of the students in terms of the level of their skills in writing scientific research paper?
2. What is the scientific literacy of the students in terms of the level of their skills in presenting scientific research paper?
3. What are the factors facilitating the student's scientific literacy?
4. What are the factors hindering the student's scientific literacy?

The study aimed to examine the scientific literacy of Junior High School students under the Science, Technology and Engineering (STE) program of a National High School in the Philippines. Specifically, the study: (i) described the scientific literacy of the students in terms of the level of their skills in writing and presenting a scientific research paper; and (ii) identified the factors that facilitate and hinder the students' ability to write and present scientific research paper in terms of the teacher (i.e., teacher's personality, teaching style, teaching procedure, teaching strategies, and classroom management), the instructional materials, learning environment, and administrative support.

2 Materials and methods

2.1 Research design

This study utilized the causal-comparative research design. According to Maheshwari (2018), it is an attempt to identify a causative relationship between an independent variable and a dependent variable. This design suggests to determine the cause or differences among the variables being studied. As cited by Salkind (2010), causal-comparative design seeks to find relationships between variables after an action or event has already occurred. It can also be termed as ex post facto research. One of the characteristics of this design is that the variables that are examined cannot be experimentally manipulated for practical or ethical reasons (Schenker & Rumrill, Jr., 2004). In this study, the independent variables were the teacher's personality traits, teaching styles, procedure, strategies, classroom management, instructional materials, the learning environment, and administrative support. These x factors were believed to have contribution to the dependent variable which is the perceived scientific literacy of three groups of Junior High School students such as Grades 7, 8, and 9. The study was conducted upon their completion of the requirements in the research subject.

2.2 Subjects of the study

The study employed a stratified random sampling in selecting the subjects. The respondents were composed of randomly selected 76 Junior High School students (i.e. 23 students from Grade 7, 31 from Grade 8, and 22 from Grade 9) under the science curriculum in particular the STEP during the fourth quarter of the School Year 2017-2018. The sample represented 81 per cent of the total population, which is 94 Junior High School students at a Public National High School in the Philippines.

2.3 Instrumentation

The study utilized the researcher-developed survey questionnaires. The research instrument is composed of two parts that determined the: 1) profile of the students as well as their perceived level of scientific literacy skills in terms of their ability to write and present a scientific research paper; 2) factors affecting their ability to write and present research paper. The students' scientific literacy in terms of writing and presenting a scientific research used five-point performance scale ranging from *needs improvement* (1) to *excellent* (5). On the other hand, factors affecting the student's ability to write and present research paper were composed of 2.a) teacher-related factors, 2.b) instructional materials, 2.c) learning environment, and 2.d) administrative support. The indicators for teacher's personality trait, teaching style, teaching procedure, teaching strategies, classroom management and administrative support used the five-point Likert scale ranging from *strongly disagree* (1) to *strongly agree* (5). Whereas the instructional materials and the learning environment indicators used another five-point frequency scale ranging from *never* (1) to *always* (5) was used.

The questionnaires were also patterned from the elements of teaching and learning, the big five personality traits, Grasha's five teaching styles, including expert, formal authority, personal model, facilitator, and delegator. Grasha (1994) describes the teaching styles as a pattern of needs, beliefs, and behaviours that teachers display in the classroom. The study also utilized the Department of Education Daily Lesson Log, and the evaluation tool for Science Investigatory Projects oral and written presentation. Experts validated the content of the research instrument. The instrument was also pilot tested and reviewed for the internal consistency of questions by conducting the test of reliability with test-retest to a group of 20 Junior High School students from another public high school. The administration of retest was three weeks after the first test. Likewise, the Cronbach-Alpha method was applied, and the result got a total test and retest scores (0.75 and 0.77) with a reliability factor of 'acceptable'.

2.4 Data collection

Quantitative and qualitative data were gathered from primary and secondary sources. The primary data were obtained from the ratings of the respondents and the interview from the teachers and the students. Secondary data such as the third quarter Grades of the students in Research I, II, and III subjects were also gathered. The result of the oral and written presentation during the conduct of Research culminating activity was also considered in the study.

After the data were gathered, an informal interview from teacher participants and a follow up interview from students were also conducted. Those students who had low ratings in some indicators were included in the interview. A total of 25 students were identified as interview participants.

2.5 Data analysis

Descriptive statistics such as mean, frequency distribution, percentage were used to analyse the data obtained from the interviews and questionnaires. The data from students' scientific literacy and the factors affecting their scientific literacy skills development were tallied, tabulated and subjected to mean analysis. To further validate the responses, the Kendall's W or Coefficient of Concordance was used to assess the agreement in the responses among students. The Kendall's W or Coefficient of Concordance for each item ranges from 0 to 1. A Kendall's W yield of zero indicates no agreement at all among students, while 1 indicates perfect agreement (Salkind, 2010).

Finally, responses to the interview were transcribed verbatim and were carefully analysed. Themes were formulated from these in order to enrich the discussion of findings on the factors facilitating and hindering the student's scientific literacy.

3 Results and discussion

3.1 Profile of the student respondents

Of the 76 respondents who participated in the study, 67 per cent were female, while 34 per cent were male. The largest percentage of the female students (29 per cent) appeared from Grade 8. Thirty-three per cent were 13 years old, 28 per cent were 14. The age group 13 to 14 belongs to Grades 7 and 8. While the age group 15 to 16 were in Grade 9.

In terms of the third grading performance in Research subjects, most respondents (42 per cent) obtained a grade from the range 86- 90 (proficient). Followed by 32 per cent who gained the Grades from 81- 85 (approaching proficiency). Majority of the respondents' academic performance fall on average.

3.2 Students' Scientific Literacy Skill in Writing Research Paper

Table 1 presents the comparison of the respondent's perceived rating on their ability to write research paper per year level. Across the Grade levels, the overall mean was described as *good* ($\bar{x}$=2.83). Respondents believed that they could write a good research paper (SIP), for it has been discussed in the class. They also underscored that the examples were available at hand. Furthermore, they were able to observe one Division competition exhibit. Among the three, Grade 9 respondents got the highest mean score of 3.35 (*good*).

Table 1. Perceived rating on research writing across grade levels

Writing a Research Paper	**Grade Level**			**Overall**
	7	**8**	**9**	**Mean**
Title				
Formulate brief and comprehensive title	3.52	2.77	3.32	3.20
Create a title relevant to the objectives of the research	3.43	2.94	3.41	3.26
Abstract				
Write an abstract that contains objectives, methodology, and results and conclusion in capsule from	2.57	2.55	3.05	2.72
Introduction				
A. Background of the Study				
Identify the origin of the problem	3.57	3.10	3.64	3.43
Indicate rational (justification) of the study	3.09	2.58	3.45	3.04
B. Statement of the Problem				
Discuss the research problem	3.35	3.16	3.59	3.37
Specify the research questions	3.09	2.84	3.68	3.20
Clearly state the research goal(s)	3.04	2.94	3.55	3.17
Evidently apply SMART Objectives	2.70	2.32	3.05	2.69
C. Significance of the Study				
Determine who and what will be the benefit from SIP	2.70	2.90	3.86	3.15
State the potential of the research for commercialization	2.78	2.52	3.32	2.87
D. Scope and Limitations				
Discuss the scope and limitations of the study	3.00	2.48	3.36	2.95
Set the time frame for conducting the study	3.30	2.32	3.18	2.94
Determine the subject and locale of the study	3.13	2.58	3.45	3.06
Review of Related Literature				

Write comprehensive RRL	2.35	2.48	2.82	2.55
Organize the RRL well	2.30	2.35	2.77	2.48
Use appropriate in text citation in RRL	2.43	2.84	2.91	2.73
Materials and Methods				
Classify the variables in the research	3.04	3.03	3.50	3.19
Explain the sample and sampling procedure in the research	2.91	2.58	3.36	2.95
Define the treatment given to the sample of the study	3.09	2.39	3.50	2.99
Discuss the research design and the data gathering procedure	3.04	2.42	3.41	2.96
State the statistical treatment of the data under study	2.87	2.16	3.23	2.75
Results and Discussion				
Present my results in an organize manner	1.39	1.90	3.23	2.17
Illustrate the results through graphs and tables with proper labels	1.35	1.97	3.36	2.23
Discuss the results completely	1.43	2.13	3.18	2.25
Ensure that the discussions of the results are relevant to the data collected	1.39	1.87	3.18	2.15
Summary, Conclusion, and Recommendations				
Write accurate summary of findings and conclusion	1.39	1.84	3.36	2.20
Formulate appropriate recommendations based on the results of the study	1.35	1.77	3.55	2.22
Literature Cited/ Bibliography				
Cite properly the sources using prescribed citation style (i.e., APA)	1.39	2.42	3.18	2.33
Grand Weighted Mean				2.80

Range: 4:.45-5.00- Excellent, 3.45-4.44- Very Good, 2.45-3.44- Good, 1.45-2.44- Fair, 1.00-1.44- Needs Improvement

The results also deemed that they were very good in writing parts of the introduction and some parts of materials and methods. When asked about their reasons, respondents answered that it was their third year doing chapters 1 to 3 and they like introduction more than the other parts of research paper. They believed it is the easiest part. Some parts of the methodology also got the highest rating, with the descriptive analysis very good. These are classification of variables ($\bar{x}$=3.50), sampling ($\bar{x}$=3.36) and treatment applied ($\bar{x}$=3.50). Grade 9 students were able to experience the whole part of the research paper.

On the other hand, the perceived rating of Grade 7 respondents ($\bar{x}$=2.62) was higher than Grade 8, which is 2.53. Grade 7 respondents believed that they were very good in the identification of the problem ($\bar{x}$=3.57), which was confirmed by their research teacher. Several parts of the review of related literature, methodology, were for the reason of having difficulty in terms of the APA format of in-text citation furthermore identifying the independent and dependent variables of the study. Results and discussions, summary, conclusion, recommendation and literature cited were described needs improvement for these were not covered by the lessons in Grade 7.

Whereas Grade 8 respondents rated good in most parts of the introduction. Similar to the reasons of Grade 7 students, Grade 8, regarded RRL and methodology as fair. In most parts of the results and discussion up to literature cited, respondents gave a rating of fair. In particular, they were relating their response to their understanding in writing a simple science paper.

On the contrary, both Research teachers believed that most of the students rating fall on needs improvement in all parts of the scientific research paper. Students under the STE lack reading habit and time management that is why their research output did not meet the highest standard. The teachers also argued that the SIP was done by group of three or by pair. The core leaders did most of the output while some members of the groups were just riders who memorized the lines for oral presentation. Furthermore, it was emphasized that it was the first time to conduct the classroom competition among Grades 9 students. Grades 7 and 8 just participated during the SIP exhibit.

Since SIP uses experimental research design, teachers underscored that students should be well aware of the classification of variables of the study as well as the treatment that will be applied (CRD or RCBD). Formulation of the hypothesis is one of the important tasks in which most of the students failed. In that case, writing of the SIP report becomes more difficult.

In terms of the in-text citation, students appear in need of more practice on paraphrasing, summarizing and direct quoting. Also, in terms of utilizing the built-in citation in Microsoft word or use a software to ease the formatting (APA) of the references.

Reading habit is an important aspect of society which helps people to develop the right mindset and create new ideas (Palani, 2012) towards skills in writing short story reviews text (Amelia, Ramadhan, & Gani, 2018). In the study of Widya, and Wahyuni

(2018), they concluded that grammatical mastery gives a significant contribution to the thesis proposal writing of English department Students at STKIP YDB Lubuk Alung, Indonesia. In the same way vocabulary mastery influence significantly the students writing ability (Abidah, Kurniasih, & Ni'mah, 2019).

In addition, time management is very important, and it may affect individual's overall performance and achievements (Nasrullah & Khan, 2015). According to Adebayo (2015), lack of proper time management on the part of the students has some impacts on certain academic activities especially in doing the assignment.

3.3 Students' Scientific Literacy in Presenting Research Paper

Table 2 on the other hand, presents the respondents' perception of their performance in presenting a scientific research paper. Contrary to the results in writing SIPs, the overall mean in presenting a research paper across all the Grade levels was 2.24 *(fair)*.

Table 2. Perceived rating on research presentation across Grade levels

Presenting the Research Paper		**Grade Level**			**Overall**
Statements		**7**	**8**	**9**	**Mean**
A. Introduction					
1. Background of the Study	Sufficiently and concisely discuss the circumstances that led to the problem	1.43	2.39	3.05	2.29
2. Statement of the Problem	Clearly and completely state the problem	1.52	2.68	3.36	2.52
3. Significance of the Study	Adequately and clearly state justification for doing the research	1.43	2.52	3.50	2.48
4. Scope and Limitations	Completely and clearly discuss the scope and limitations of the study	1.43	2.32	3.32	2.36
B. Review of Related Literature	Include relevant and adequate literature search	1.43	2.32	2.68	2.15
C. Materials and Methods	Sufficiently and concisely describe the materials and methods used in the study	1.57	2.42	3.59	2.53
D. Results and Discussions	Present and discuss the results completely	1.39	1.90	3.14	2.14
E. Summary, Conclusion and Recommendation	Present a complete summary, conclusion, and recommendation	1.39	1.97	3.18	2.29
II. Organization and Clarity	Plan my presentation well	1.57	1.94	3.36	2.29

III. Mastery of the Subject	Demonstrate thorough understanding of the subject matter	1.48	1.84	3.18	2.29
IV. Delivery	I am relax and confident during an oral presentation	1.52	1.71	2.68	1.97
V. Presentation Aid	Make use of well- prepared audio-visual materials	1.48	1.84	3.00	2.11
VI. Time Management	Finish the presentation with the prescribed time and appropriate pacing	1.43	1.81	3.14	2.13
VIII. Audience Impact	Sustain the interest of the audience most of the time	1.52	1.97	2.77	2.09
VIII. Teamwork	Work with all members of my group and share equally in handling the presentation and open forum	1.52	2.35	3.14	2.34
IX. Punctuality	Present ahead of schedule	1.48	1.94	3.14	2.18
Grand Weighted Mean					2.26

Range: 4:.45-5.00- Excellent, 3.45-4.44- Very Good, 2.45-3.44- Good, 1.45-2.44- Fair, 1.00-1.44- Needs Improvement

Several probable reasons stated by the respondents were: 1) they were not confident in speaking English when presenting, 2) they exhibited inappropriate body language when nervous, 3) they were afraid of the panels, and 4) they experienced mental blocked when ask a difficult question.

Grade 7 students seemed to have the lowest mean score of 1.48 (fair). The respondents mentioned that they have not experienced presenting the research paper but related their answers to presenting a group project or an individual report.

Grade 8 students ranked a bit higher ($\bar{x}$=2.12), which also has a descriptive equivalent as fair. It can be seen from Table 2 that parts of the introduction were rated good. This is for the reason that during their presentation, the panels gave positive feedback on the statement of the problem and the significance of the study.

Meanwhile, it can be seen from the results that Grade 9 respondents overall rated mean is described as good ($\bar{x}$=3.14). Here, the Grade 9 students believed that they have the experience in presenting the completed SIPs. It was confirmed by the respondents that the lesson learned during the presentation were worthwhile in becoming better presenters in their next SIP. Also, their view in describing the materials and methods used in the study was very good ($\bar{x}$=3.59) for it is deemed to be the easiest during the presentation.

3.4 Factors Facilitating and Hindering the Students' Scientific Literacy

3.4.1 Teachers' Personality Trait

There are two teachers handling research classes. Teacher A handles Grade 7, and teacher B handles Grades 8 and 9. Both teachers followed the curriculum guide and budget of work prescribed by the Philippine Department of Education for the STEP. Based on the result, the students' rating on the personality traits of the teacher handling research classes was high ($\bar{x}$=4.19). Wherein, the respondents *agreed* across all the variables under the teacher's personality traits.

The overall mean of the three Grade levels were 4.09, 4.16, 4.18 agree, but the highest rating was found in Grade 9. Among the personality traits depicted in Grade 7, being sociable ($\bar{x}$=4.52) *strongly agree* got the highest weighted mean followed by being responsible, calm, happy, and cooperative. The teacher who handled Grade 7, has been described by the respondents as a jolly and a very approachable person. Moreover, the teacher who handled Grades 8 and 9 was described as hardworking ($\bar{x}$=4.58, $\bar{x}$=4.64), responsible, ($\bar{x}$=4.58, $\bar{x}$=4.59), self-disciplined ($\bar{x}$=4.59), imaginative ($\bar{x}$=4.59) and intellectual ($\bar{x}$=4.55).

The respondents described their teacher as strict and serious. It was also underscored that their research teachers portray self-discipline and responsibility by sticking to the set rules and duties in the class and reprimand students with misbehaviour by counselling not by punishing. Students believed their teachers in research exert much effort in terms of preparing instructional materials since there is no module for this subject matter. However, in terms of emotional stability (unworried), the results for both teachers from Grades 7, 8, and 9 were found the lowest with the mean of 3.43, 3.13, 3.32, respectively. Respondents were uncertain if their teachers are unworried. They observed that most of the time, both teachers worry if the respondents will be able to meet the required output in every Grade level.

Moreover, the results of this study support the findings of Kim, Dar-Nimrod, and Mac Cann (2017), which explained that teacher personality characteristics such as conscientiousness (being hard-working and detail-minded), agreeableness (being sympathetic and kind), and emotional stability (having fewer negative emotions such as anxiety) are important factors in achieving students academic success. Teachers' personality traits were associated with students' confidence in achieving an academic goal. However, in the study of Mkpanang (2015) on the personality traits of teachers and the students' performance in Physics, it was found that there was a low significant relationship between teachers' personality profile and students' academic achievement in the subject.

3.4.2 Teaching Styles

Based on the students' rating regarding their teacher's teaching styles from Grasha's description as an expert, the respondents regarded their teachers being equipped with knowledge and competency they needed for the Science Investigatory Projects. As a formal authority, teacher sets rules, goals and expectations to guide the students on the track they should follow.

Correspondingly, respondents described both teachers give positive and negative feedbacks. For instance, giving merits and demerits in the class. As a personal model, guiding and directing by showing how to do things was given the mean scores of 4.52, 4.65, and 4.64 (strongly *agree*) across all the Grade levels. It is portrayed by demonstrating laboratory experiments inside the classroom using improvised and indigenous resources. Moreover, providing sample researches and SIPs was also emphasized by the respondents. As a facilitator, guiding by asking questions, exploring options and suggesting alternatives was noted with the highest mean (*strongly agree)* scores of 4.52, 4.77, and 4.73 from all Grade levels.

However, having consultation hours got the least mean score of 3.78 and 3.77 from Grades 7 and 8 (*agree*). Both teachers handling Research subjects were given other auxiliary task as the Grade level coordinator and the Grade level guidance counsellor of the school. Comparing the results across all the Grade levels on teaching styles as a delegator, it was found out that Grade 9 has the highest ($\bar{x}$=4.32, $\bar{x}$=4.41).

Respondents described their teacher being available as a resource person and allows them to work in autonomy. Grade 9 respondents indicated that they were tasked to complete the Science Investigatory Project paper and oral presentation that is the reason they were given much time to work in autonomy and via consultation with the Research teacher. Grade 7 teacher was given the lowest mean scores of 3.87 and 3.83 for the same criteria. One possible reason for that result is students were working with close supervision by the teacher. For Grade 7 is more on foundational concepts and theories that will help the students in higher year level.

The study of Frunză (2014) affirmed that effective teaching styles also depend on the students' learning styles. More so, willingness to experiment with teaching strategies will help in developing an effective teaching style for the students. However, the findings of the study deviated from the study of Stanford (2014), which revealed that the mathematical scores of students in classroom who were taught using facilitator and delegator teaching styles were significantly higher than the scores of students from an expert, formal authority, and personal model teaching styles.

3.4.3 Teaching Strategies

The teaching strategies help the teacher to engage the students in the teaching and learning process. In general, the perceived rating of the respondents towards teaching strategies used by their research teachers was 4.24 *(agree)*. Moreover, criterion from cooperative learning and technology integration got the highest mean score ($\bar{x}$=4.65).

Under cooperative learning, working by partners or peer tutoring was the highest across the Grade levels interpreted as strongly *agree* ($\bar{x}$=4.61, $\bar{x}$=4.65, and $\bar{x}$=4.68). Working with their chosen partner or the pairs selected by the teachers were allowed as long as they will accomplish the set objectives and outcomes for a certain topic. This indicates that students across all the Grade levels are willing to work with pairs and teams for they can express themselves better and progressed better results.

The findings on cooperative learning supports the study of Altun (2017), which emphasized the favourable effect of cooperative learning on students' performance. The development of students' social and personal skills can also be achieved in cooperation-based learning since it provides support and cooperation from the group (Altun, 2017).

In terms of technology integration, reporting the results of class activity through PowerPoint presentation was also high 4.77 (*strongly agree*) for both Grades 8 and 9. It is an indication that the facilities like laptop, projector, screen, and speaker are already available for the teaching and learning process, whether provided by the teachers or the school. Lessons were also delivered through PowerPoint presentation as emphasized by both teachers. The findings on technology integration verified by Weathersbee (2008), where the impact of technology and academic performance was analysed. The results specified that technology integration in the classroom increased the students' performance in science, mathematics and reading in selected public schools in Texas.

Meanwhile, criterion from the inquiry-based instruction was found highest ($\bar{x}$=4.74)- *strongly agree* at Grade 8 while the use of differentiated instruction got the lowest (3.36)- *agree* in Grade 9. The results signify that the incorporation of respondents' multiple intelligences is not highly evident in the teaching and learning process. One probable reason for that is the number of students in the class and the number of hours in teaching a particular lesson.

The findings on the use of inquiry-based instructions further support the earlier studies done by Alameddine, and Ahwal, (2016); Abdi (2014); and Bayram, Oskay, Erdem, Özgür, and Şen (2013). The studies revealed that inquiry-based instructions increase students' performance. On the other hand, the findings on the use of differentiated instruction were supported by the identified challenges on the learning curve and planning time (Stetson, Stetson & Anderson, 2018).

3.4.3 Teaching Procedure

The teaching procedure is the day to day lesson delivery of the teachers based on the Daily Lesson Log (DLL) prescribed by the Department of Education. The prescribed DLL also follows the Gagne's nine events of instructions such as: 1) gaining attention, 2) informing learners of objectives, 3) stimulating recall of prior learning, 4) presenting the content, 5) providing guidance, 6) eliciting performance, 7) providing feedback, 8) assessing performance and 9) enhancing retention (Gagne, 1997). It was found that the overall mean for all the Grade levels was high ($\bar{x}$=4.41).

The findings indicate that both teachers followed the prescribed DLL. It is interesting to note that both Grades 7 and 8, got mean scores of 4.50 and 4.55, which most of the respondents *strongly agreed*. Found in Grade 9 was the highest weighted mean of 4.77 *(strongly agree)* which indicates that the teacher always presents examples of new lesson. Respondents shared that their teacher brought a magazine, a printed journal, sample experiments, and a lot of video presentations if the resources are not available at hand.

The results of the study support Miner, Mallow, Thekee, and Barnes (2015), which revealed that Gagne instructional events enhanced teachers 'mastery, enthusiasm and effectiveness. Thus, the grades of the students increased. Moreover, the findings also support the study of Ngussa (2014), which showed that the higher the performance of the students the greater the perception on Gagne instructional events.

3.4.4 Classroom Management

Classroom management is a way in which the teacher ensures that the class maximizes learning time without disruption. It can be in a form of seating arrangement, assigning of task, creating a harmonious environment, or even delivering the lesson.

It was found out that across the Grade levels under the study, the overall mean is 4.16 with the descriptive analysis *agree*. With the same criteria across the Grade level, it was found out that the highest score with the description *strongly agree* can be seen only in Grade 9 ($\bar{x}$=4.50). Respondents confirmed that their teacher showed respect and believed that they are becoming mature that is why there is no need to scold them often. The result also shows that positive discipline reinforces positive behaviour among learners.

The use of instructional time effectively got the second highest mean score of 4.45 (*strongly agree*). However, respondents uttered that their teachers always begin with the end in mind making the class agitated and overwhelmed with ideas. The next highest score ($\bar{x}$=4.36) with the descriptive analysis agree, talks about seating arrangement that encourages an interactive teaching and learning process. It was mentioned by the respondents that seating arrangement changes from regular lecture type to circle time, U shape seat plan, by group, by pairs, and sometimes with no chairs. Seating arrangement depends on the activity prepared by the teacher.

The findings on classroom management support the study of George, Sakirudeen, and Sunday (2017), which revealed that the academic performance of students who experienced classroom management (verbal instruction, corporal punishment, instructional supervision, delegation of authority to learners) differs from those who do not. Additionally, the major findings of Ahmad and Hussain (2017) indicated that there is a positive relationship between teachers' classroom management strategies and the performance or achievement of the students.

3.4.5 Instructional Materials

Instructional materials are tools that help the teachers in facilitating the day-to-day lessons. The use of instructional materials can maximize the learning potential of the students as well as the time allotted in teaching. Findings show that the perceived mean rating on the instructional materials used by the teacher in facilitating the lesson was 3.44 described as *about half of the time*. This indicates that the Junior High School students perceived that their teachers used varieties of instructional materials as much as they could.

In general, using PowerPoint presentations, sample researches, and supplementary reading materials appeared to be the most commonly used instructional materials by the Research teachers. On the other hand, the students agreed that their teachers used learning modules once in a while, with mean ratings of 2.39, 1.45, and 2.14, respectively. They related their answers with the science learners' modules but not the research module since it was not yet developed.

Studies on the use of instructional materials revealed that that students taught with instructional materials performed better than those taught without instructional materials. The use of instructional materials generally improved students' understanding of concepts and led to high academic achievements (Olayinka, 2016; Adalikwu & Iorkpilgh, 2013).

3.4.6 Learning Environment

The status and availability of school facilities have direct and indirect impacts on the learning of students. Results revealed that the students were able to utilize all available school facilities for research-related activities *about half of the time,* with overall mean of 2.99. Classrooms appeared to be the most commonly used school facility for the conduct of the research studies of Grades 7 to 9. When asked about their reason, they explained that there is not enough time to go and avail other learning facilities. They have 10 subjects per day with different requirements.

On the other hand, across all levels, DOST Star Books facility was the least tapped resource that is available in the school, with perceived ratings of 1.57, 2.26, and 2.27 from Grades 7, 8, and 9, respectively. The respondents felt that they were not trained to operate the DOST Star Books, which are housed inside the library. They also mentioned that they were hesitant to do research works in the library since there are no clear policies or guidelines in using the resources in the library. Students also identified that the science laboratory was always closed.

Additionally, computer laboratory lacks internet access most of the time. Also, only one computer with internet access can be utilized by a class. The respondents reiterated that classes were also held at the computer laboratory that is why students opted to maximize the use of their mobile phones instead of going to the computer laboratory. In the same manner, teacher respondents agreed to the statements given by the students. They also emphasized the unavailability of teacher in charge in the laboratory and the lack of training guide to utilize the DOST star books.

The findings on the learning environment, specifically school facilities affirmed the findings of study Al-Enezi (2002) which revealed that there is a positive significant relationship exists between student achievement scores and building conditions. On the contrary, student achievement, attendance and completion rate measures were not found to be statistically significant in relation to school facility conditions as measured by the Total Learning Environment Assessment (TLEA) at the 0.05 level; second, discipline, or behaviour, was found to be significantly related to the TLEA (Mcgowen, 2007).

3.4.7 Administrative Support

Grade 7 students seemed to be *uncertain* on the support given by the school administrators during the time that they took their research subject. Specifically, the overall mean from Grade 7 students was 3.14, which described as *about half of the time*.

On the other hand, Grades 8 and 9 students both agreed that appropriate administrative support was given to them during the time they took their Research subjects. In particular, they both *agreed* that the school administration primarily encouraged research exposure visits, with means of 4.35 and 4.91, respectively.

Within each Grade level, providing moral and financial support received the least ratings. These are 3.00, 3.29 and 4.09 from Grades 7, 8, and 9, respectively. Nevertheless, the overall mean of 3.85 suggests that the research students of Grades 7 to 9 agreed that they received support from the administrators of the school.

On the contrary, the teacher respondents pointed out that financial support was not given all the time. The department raised funds to send the students for most of the competition. Funds were raised from the contributions of teachers or private individual. From the point of view of the teachers, administrative support encourages the students to achieve academic success.

Nevertheless, the findings do not support the study of Bello, Ibi, and Bukar (2016) which revealed that that 1) there were no significant relationships between principal's initiative administrative styles and students' academic performance; 2) no significant relationships between consideration structure of principals' administrative styles and students' academic performance; 3) no significant relationships between participatory administrative styles of principals' and student academic performance in senior secondary schools; and 3) among the three leadership styles, none is the best predictor of students' academic performance in Taraba State secondary schools.

Furthermore, Coefficient of Concordance revealed congruence in the perceptions of the students regarding the factors that facilitate and hinder their scientific literacy skills development. Basically, the statistical findings showed a strong to perfect agreement in the students' perceptions of all the factors examined (Kendall's W (df=8, n=76) = 0.60, p= 0.00). Table 3 presents the summary of each of these factors. The findings were all statistically significant.

Table 3. Summary of facilitating factors of students' scientific literacy skills development

Factors	Mean	SD	Kendall's W
Teaching Personality	4.23	0.38	1.00
B. Teaching Style	4.33	0.38	1.00
C. Teaching Strategy	4.26	0.37	1.00
D. Teaching Procedure	4.42	0.45	1.00
E. Classroom Management	4.16	0.44	1.00
F. Instructional Materials	3.41	0.67	1.00
G. School Facilities	3.01	0.85	0.93
Administrative Support	3.85	0.88	1.00

Kendall's W level of agreement: 0.00 No; 0.10-Weak; 0.30-Moderate; 0.60-Strong; 1.00-Perfect

4 Conclusions and implications

It can be concluded that although the results of the scientific literacy skills in terms of writing scientific research paper was good already, yet, there are several parts of the research paper that must be taken into consideration. In addition, the respondents believed that they were having difficulties in presenting the research paper. Thus, they need a lot of trainings and exposure to become better presenters.

The findings of the study also suggested that the perceived scientific literacy of students were influenced by factors primarily the teacher's personality traits, teaching styles, procedure, strategies, classroom management, instructional materials, the learning environment, and administrative support. Thus, to improve the scientific literacy skills of the students as to presenting scientific research in written and oral form, such factors should be given importance just as how scientific literacy is important in the society.

The following are the recommended strategies to improve the scientific literacy among Junior High School students:

For the Research Teachers' personality and teaching style, since the findings showed the least scores on teachers' emotional stability (unworried), it is suggested to find ways of improving teacher's communication, empathy and comfort to increase the effectiveness in teaching research as a subject. An increase in teacher's availability during consultation hours is an important avenue for giving feedback and encouragement to the students having difficulty in their identified research problems. Also, helping the students to work in autonomy will empower them to aim higher in the field of scientific research. Differentiated instruction should also be considered while utilising local and indigenous instructional materials. This will help the students to relate well to the learning activities and thus maximize their learning experiences.

For the Science Coordinator and Teachers, the formulation of some policy guidelines in the use of the library, DOST STAR BOOKS, computer laboratory, science laboratory and other learning resources will be a great help for the students as to setting directions. Developing a research learning module will help to standardise the lesson across different Grade levels. Additionally, drafting a proposal for the STEP of activities or the annual plan will also encourage teachers to perform better.

For the School Leaders, the increase of support to the Science Department in terms of moral and financial, will be a significant factor in boosting students' morale as well. Henceforth, allowing the science/research teachers to attend seminars or trainings that will improve their teaching skills and become more abreast with current research practices. Students' exposure to the field of research should also be prioritised by the educators and educational leaders. Allowing them to visit schools that already excel in the field of Science Investigatory Projects is a good way to adopt techniques and best practices. Furthermore, strengthening linkages through partnership with the Local Government Units (LGUs) and non-government organizations will help in identifying other sources of fund. This will aid the trainings and acquisition of laboratory equipment that can be utilized by the students in the development of their scientific literacy skills.

Now that the world is continuously overwhelmed by vast amount of information, the development of scientific literacy skills is becoming more important than ever. This ultimately amplifies the significant role of educational leaders and educators in promoting scientific literacy in science education. For a populace with well-developed scientific literacy can better cope with many of its problems. Hence, people will be able to make better judgements and informed decisions that will affect the quality of life beyond personal and social. And this in turn leads to the betterment of the entire nation.

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The effects of using social biographical texts of scientists on students' attitudes in science courses: A qualitative study

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Biographies of scientists are often used in the teaching environment, both in textbooks and in course contents - sections from the lives of scientists are often included to encourage students to pursue and enjoy science. This research investigated the effect of social content biographical texts of scientists on students' attitudes towards science courses. The research was a mixed-method study and consisted of 51 science teachers. The participants were determined according to a convenience sampling method. Focus group interviews, repertory grid technique, and individual interviews were used to collect data in the study. Through focus group interviews with teachers, it was discussed what kind of changes biographical texts might make to students' attitudes to science. Later, 51 teachers explained the social biographical texts to their students and observed the changes in the students. Based on their observations, they scored the repertory grids. Finally, an individual interview was held with fifteen teachers. As a result, it has been determined that social biographical texts were able to increase students' interest, motivation and questioning skills, while able to decrease their anxiety.

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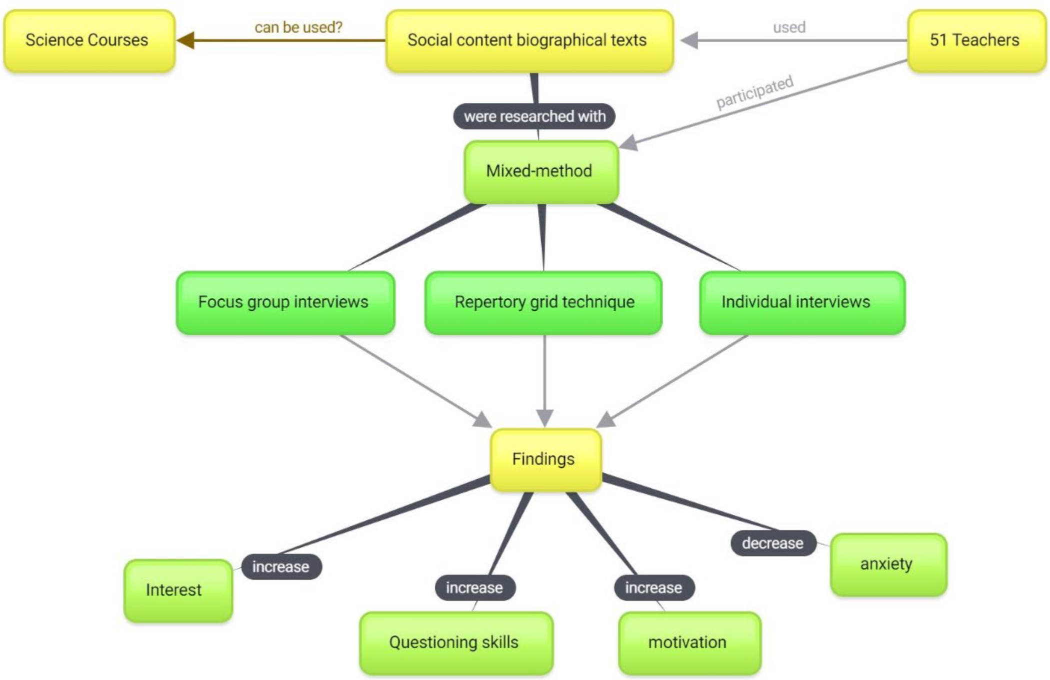

1 Introduction

Crises in science education are widely accepted, and the low rate of science literacy emerges as a troubling situation. Is it right to accept this dire result and wait for this to change without doing anything? Evolving reality has shown that science is the greatest culture that human beings take refuge in (Davies & Davies, 1984). The culture of science, which has an interesting and complex past, has brought to light a lot of information about ourselves and about the world we live in. The departure of teachers and students from this science culture, which directly and indirectly transforms both the social and natural worlds, has manifested itself as a dramatic situation (Hodson, 2008; Matthews, 1991). For example, in the mid-1980s, 600 people chose science programs every year in the USA, and 8000 people left this profession – this is compelling evidence of the moving from science classes by teachers and students (Mayer, 1987). Bown (1993) argues that students move away from science and even if these students enter university to study science, their level is very low.

There are complex economic, social, and cultural reasons for students 'distancing' themselves from science. Teachers alone are not able to solve all economic, social, and cultural problems. But at the very least, if they convey the relationship between scientists and achievement objectively, without relying too much on classical ideas, they can help correct the necessary curriculum and pedagogical deficiencies in science by breaking the prejudices that to be successful one must be born gifted (Matthews, 1992). As Lin-Siegler, Ahn, and Chen (2016) said, the mind can be improved, and this can be achieved by new experiences, not just by talent. This is of course not easy, but it is very useful to teach the techniques that achievers use to overcome difficulties and learn.

One-sided approaches to the lives of scientists when dealing with science in schools are important when understanding the perspectives of students who feel that they are 'against' science. The irrational views put forward about what people must do to become scientists are so embedded in today's curriculum that they believe that the only way for students to be successful in science is to have a solid innate intelligence (Hong & Lin-Siegler, 2012; Martin & Brouwer, 1993). Popular versions of postmodernism and constructivism in the field of education, unfortunately, reduce students' interest in science and divert their interest in other directions. Since they think that other students' successes in science are based on extraordinary abilities, their motivation towards science decreases (Lin-Siegler et al., 2016) and they consider their success as idealism and dreaminess (Matthews, 1991).

In their research, Lin-Siegler et al. (2016) asked the students whether they thought they could become scientists or not. They found that the students had difficulty even in imagining their roles in this field, and encountered answers like, “I have never thought about this before, and I cannot say that I am good at this anyway.” The research also pointed to the disconnection between students' comments about themselves and scientists, as they had participant judgments that being successful in science was only possible with talent.

This situation has resulted in students who think that innate abilities are necessary for efforts requiring high-level scientific performance, giving up without giving themselves a chance (Bandura, 1977, 1986; Dweck, 2000; Hong & Siegler, 2012). The belief in innate talent leads to the emergence of false beliefs that they will never be successful in science, even with a lot of work from the students (Dweck, 2010, 2012; Hong & Siegler, 2012). The most unexpected consequence of these false beliefs is that students' desire to pursue a career in science and technology is dulled (National Academy of Science, 2005). Worst of all, it changes students' attitudes towards science in a negative way, as well as being a serious factor that keeps them away from science and mathematics courses in high school and university (Blickenstaff, 2005; Singh, Granville, & Dika, 2002; Wang, 2013). It can be said that the perception of being successful only with innate talents is not only about students themselves, but also teachers, textbooks, and the media are effective in the emergence and persistence of this belief (Chambers, 1983; Farland, 2006; Finson, 2002; Schibeci & Sorensen, 1983).

In lessons, teachers talk about scientists' abilities and achievements rather than problems in their social lives, which reduce the motivation of students who fail in science (Shumow & Schmidt, 2014). The positive development of attitudes towards science depends not only on the special talents of scientists, but also on the feeling of not giving up despite the difficulties they experience in their social life. In this sense, one of the ways to be successful in science depends on the ability to face obstacles in a social life and to use the skills of not giving up despite everything. In this respect, especially in schools, not only should the talents of scientists but also biographies showing how they coped with the difficulties in their social lives be included (Lin-Siegler et al., 2016).

In this study, it was aimed to investigate the effects of social biographical texts on students' attitudes according to science teachers. The use of biographical texts in the classrooms depends on teachers' attitudes towards them. Therefore, these findings can contribute to science education literature. In this study, answers to the following questions were sought:

1. According to science teachers, which attitudes of students can be affected by social biographical texts?
2. How do science teachers evaluate the change in students' attitudes when social biographical texts are taught?
3. What are the opinions of teachers who argue that social biographical texts will cause changes in students' attitudes about biographical texts?

2 Theoretical framework

2.1 Biographical texts

Biography, also known as resume and life history, is a literary genre that reveals the lives and actions of important people based on documents (Oğuzkan, 2001). In other words, a biography is a document that presents the lives of people who have come to the forefront with their work and behaviour, and who have achieved significant success in fields such as science, art, literature, politics, and sports with a neutral eye (Kaymakçı & Er, 2009). On the other hand, a biography in terms of educational sciences is the task of studying people who have come to the fore in fields such as science, art, history, and politics in terms of teaching purposes (Öncül, 2000).

When it is considered that the subject of the biography is a human and what a human being has done, the biography appears as a teaching tool that can be used not only in the field of literature, but also in many other fields (Öztürk & Otluoğlu, 2003). There are many advantages to the biography method. With the help of this technique, students learn many variables such as place, time, people, war, peace, political, social, and economic situations from different angles. In addition, this technique provides the understanding of common ground in different cultures, especially by teaching critical thinking, empathy, decision-making, virtue, and that all people have similar characteristics (Levstik, 1995; Maxim, 1999; Warren, 1992; Zarnowski, 2003). In this sense, especially, Kaufman and Libby (2012) and Oatley (1999) stated that such teaching techniques should be used to confront students' beliefs, and that biographical texts strongly influence people's attitudes, beliefs, and behaviours. Miall and Kuiken (1998) argued that stories originating from biographical texts shape people's perspectives and feelings. On the other hand, Zak (2014) suggested that individuals could benefit from using biographical texts when one wants to motivate, persuade, and remember. In a sense to support this, Black and Bower (1980) stated that such biographical texts provide long-lasting knowledge, and in such texts, people either see the world as it is or create similar situations by establishing an emotional relationship between the lives of the people whose biographies are given.

As mentioned above, biographical texts can be used in many fields, as well as in science teaching (Eshach, 2009; Hong & Siegler, 2012; Martin & Brouwer, 1993; McKinney & Michalovic, 2004; Solomon, 2007). Solomon (2007) indicated that biographical texts can serve as role models for students. The fact that these scientists did not give up their studies despite the many problems they had to fight in social life leads students to the conclusion that their own struggles and the struggles of scientists are common, by revealing an emotional approach to the students (Hong & Siegler, 2012). Thus, students feel the need to reconsider their current perceptions about scientists and their beliefs by concluding that everyone has a story through biographical texts (Hong & Siegler, 2012). In this sense, Parker (1996) argued that biographical texts provide concrete satisfaction of events by shaping the pleasure of imagining. In addition, students expressed that biographical texts gave them a new way of thinking by giving them a life lesson and can bring about a change in their attitudes by focusing on the difficulties experienced by people and helping them to solve their own problems. In this context, the thought about the importance of science and attitudes towards it shows itself once again.

2.2 Science teaching and student attitudes

According to Çepni, Ayas, Johnson, and Turgut (1996), science is defined as approaching natural phenomena with critical thought and generalizing and reaching principles. According to Bloom (1979), it is an effort to make predictions about events that have not yet been observed by systematically examining natural events. Students who take science courses become individuals who can bring the mind to the forefront, think logically and make analytical analysis, communicate actively with both the environment and the world, make experiments and observations and express their results in words and maths, and become individuals (Akgün, 2004). Although all of these are possible for healthy science-literate individuals, they can be achieved by taking an interest in science topics and developing a positive attitude (Çakır & Şenler, 2007). Accordingly, it is beneficial to take the concept of attitude and look at its definition and features.

According to Bloom (1979), it is a bipolar situation - the state of liking the course and showing positive affective qualities about the course by developing positive thoughts about a course or a topic, or that has negative affective characteristics due to developing negative thoughts and disliking the lesson. Individuals primarily have prior knowledge about the object of the attitude, and then express it as an emotional response. In the last step, they transform attitudes into behaviour, and evaluate them as responding to the reactions from their environment. Köklü (1992) explains this situation as the positive and negative reactions that individuals develop against any stimulus which manifest as an attitude, by stating that most of the affective behaviours constitute the attitude. Although different meanings of attitude appear, Oppenheim (1992) made an attitude assessment in terms of education. That is, when faced with a certain stimulus, it warns that the person who reacts in a certain way to this stimulus. Küçükahmet (1997) states that students' attitudes and habits are one of the most essential factors that reveal their success. Similarly, Senemoğlu (2004) evaluates the internal state that affects the choice of individuals in their activities as attitude. Yılmaz et al. (1998) argue that attitudes are not behaviours, but are psychological variables that direct human behaviour, and state that attitudes have three dimensions. These are cognitive, affective and behavioral, and one's beliefs about attitude take place in the cognitive dimension. Cognitive beliefs create the perception that science course is difficult to learn. Affective responses of individuals about attitudes form the affective dimension. Affective dimension reveals students' feelings such as love or hate towards science course. Behavioral dimension, on the other hand, includes the actions of students regarding the subject of attitude.

When one is focussed on the attitude in terms of learning processes it can be seen that, unlike knowledge and abilities, attitudes appear in learning as a determinant and result. This is especially true of the attitude towards science concepts consists of mythos and beliefs of the person against these concepts. The emergence of these beliefs is particularly effective in choosing the science course, curiosity about scientific topics, or the development of science-related hobbies (White, 1993: cited in Atasoy, 2002). Since students' attitudes towards any course or topic are closely related to success, there are many studies on this topic. The correlation between learning and attitude reveals many dimensions related to attitude. Among these dimensions, especially self-confidence, socio-economic status, gender, age, teacher attitude, methods, and techniques used, and students' motivation coefficient are the dimension axes. It is among the results of many studies that students with low motivation towards the course will not be too successful in that course. As a result of the students losing their motivation, feelings of burnout emerge towards the lesson such as "I can't succeed", "It's too late", and "That's all from me" (Çakır & Şenler, 2007). Altınok (2004) drew attention to the close relationship between success and motivation in a study he conducted. As a result of the research, a close relationship was found between the science course and the motivation towards this course. Açıkgöz (2003) gives advice, especially for teachers. He says that it would be beneficial to know in advance the motivation levels of the students about the course which the teachers are presenting to them. He states that knowing the motivation of the students in advance can be helpful in terms of which methods and techniques the teacher will use, as well as in increasing students' attitudes towards the lesson.

Hemlick and Norland (1994) emphasize that teachers should consider these learning approaches in students because students' learning methods will occur differently. Supporting this result, Mordi (1991) found that learning and teaching approaches were higher than other parameters (socio-economic status, student characteristics, school characteristics) - a rate of 41% in the study he conducted on students' attitudes towards science. Similarly, Baykul (1990) emphasized that the attitude of students towards science decreases from the fifth grade of primary school due to the teachers' attitude towards the lessons and the content of the textbooks. In a study conducted by Bilgin et al. (2002), they stated that student-centred teaching approaches not only revealed positive results in their achievement in a chemistry course, but also brought about positive changes in their attitudes towards the chemistry course. In a study conducted by Hendley, Stables, and Stables (1996) with 190 students in terms of the emotions revealed towards the science course, they found that although the science course was the fifth most popular among 12 courses, it ranked first among the three unpopular courses.

3 Methodology

3.1 Research Design

This study was mixed-method research, in which quantitative and qualitative methods were used to determine the effects of social biographical texts of scientists on students' attitudes according to science teachers. Creswell (2006) states that the basic proposition in mixed approach should be used together with quantitative and qualitative approaches, in order to better understand the research problem. In line with this proposition, the research question in the mixed method design is both better understood and in detail by obtaining multiple data (Alkan, Şimşek, & Erbil, 2019).

This study used a sequential explanatory design, one of the mixed-method designs. The purpose of the sequential explanatory design is to begin the research problem with the quantitative stage for both data collection and analysis, and to then conduct a qualitative study to explain the quantitative results. Quantitative results present the general results of the study by providing statistical significance, confidence intervals, and effect sizes. However, this method is not enough to explain how the results are formed. For this reason, the pattern in which the qualitative stage is activated to explain the quantitative results is defined as sequential explanatory design (Creswell, 2017). In this design, qualitative results are used to help explain and interpret the findings of a quantitative study, and a sequential explanatory design can be useful - especially when an unexpected difference occurs between the results or opinions in a quantitative study (Morse, 1991; cited in Creswell, Plano Clark, Gutmann & Hanson, 2003).

Quantitative phase: The quantitative part of the research was conducted based on the survey model (Karasar, 2012), which aims to describe the existing situation as it is. In this sense, the quantitative section consists of the repertory grid technique applied to teachers in order to evaluate the change in students' attitudes as a result of explaining and teaching the social biographical texts of scientists.

Qualitative phase: The qualitative part of the research consists of two stages. At the first stage, focus group interviews were conducted with the teachers who volunteered. The findings of these focus group interviews were used to determine the constructs in the repertory grid used in the quantitative part of the study. Then, the repertory grid technique was applied quantitatively. The findings obtained from these quantitative data were used to determine the participants in the individual interviews, which was the last stage of the research. Individual interviews were conducted with teachers in order to obtain in-depth information about how social biographical texts of scientists changed students' attitudes and behaviors towards the science course.

3.2 Participants

The study group of this research consisted of 51 science teachers who worked in different schools in Erzurum in Turkey and were determined according to convenience sampling method. The teachers were informed before the study and they volunteered to participate in the study. The names of the schools were coded and the numbers of science teachers participating in the study in each school are given in Table 1.

Table 1. Participating schools and number of science teachers in these schools

Schools	Number of teachers
A	2
B	2
C	2
D	1
E	5
F	4
G	2
H	7
I	2
J	1
K	5
L	6
M	7
N	4
O	1
Total	51

3.3 Data Collection Tools

In this study, as data collection tools, the focus group interview, the Repertory Grid Technique (RGT), and a semi-structured interview form were used.

3.3.1 Focus group interview

The focus group interview is a technique of interviewing selected participants based on a previously determined topic to reveal their knowledge and opinions. Focus group interviews are usually conducted with 10-12 people at one time. The interview is conducted by an expert and is conducted in order to reveal details about the thoughts and lives of the people. The data is analyzed and a synthesis of the evaluations expressed by the participants is made. Focus group interviews are a technique that steer decisions or action plans, either alone or with the help of the results obtained from quantitative studies (Yıldırım & Şimşek, 2008).

In this study, focus group interviews were conducted with teachers in order to determine which attitudes of their students towards the science courses in which social content biographical texts were affected. Focus group interviews were held in three groups of 12 people who volunteered to participate in these interviews and lasted about 45 minutes per interview. In the focus group interviews, the participants were asked questions about the social content biographical texts of the scientists determined beforehand. The questions were chosen as questions about the difficulties faced by scientists in their social lives, rather than questions revealing the academic and intellectual aspects of these scientists. As a result of the analysis of the answers obtained from these questions, a decision was made about the structures to be included in the RGT. Table 2 shows the numbers of teachers who participated in the focus group interview.

Table 2. Number of teachers participating in focus group interviews

School	Group No	Number of teachers
A	1	7
B	1	5
C	2	6
D	2	4
E	2	2
F	3	7
G	3	4
H	3	1
	Total	36

3.3.2 Repertory Grid Technique

Personal construct psychology (PCP), which was founded by George A. Kelly in the 1950s, is known as the theory of individual and group psychological social processes, which is made to model the cognitive processes of people. The constructs that the person creates as a result of the guidance of their experiences create a subjective world, and the person act according to the structures create in this world. It is extremely important to reach these constructs created by people in order to understand a person and their actions. The repertory grid technique takes the stage at this point and aims to reach these structures. Through interviews, people question their structures and try to reveal the subjective world they have (Abazaoğlu, 2009).

Kelly says that a certain number of constructs can be reached as a result of evaluating the events that constitute one's world. Kelly defines events as 'elements' and states that the structures that individuals will have should be considered as bipolar. The constructs used in the repertory grid technique should be considered as a prediction used to present the person's world. The basis of the repertory grid technique is to create the constructs used to spread the intensity of meaning to a particular set of elements, and then score the elements of it according to the structures that are formed (Kelly, 1995).

Persons, institutions, objects, thoughts, and events are the main parameters that make up the elements in the repertory grid. The elements are generally shaped according to the subject researched by the researcher. The researcher and the participant can determine the items and structures in the repertory grid together. Constructs mostly manifest themselves as polar structures expressing similarity, contrast, and relationship between elements. The researcher directs the items to the participant in groups of two and asks about the similar and different aspects between them. The researcher reaches the constructs from the answers they receive.

The table constituting the repertory grid consists of the signs and numbers placed in the table by the participant as a result of the evaluation of the item according to the constructs and elements placed transversely and longitudinally. Jankowicz (2004) states that the repertory grid technique can be used in many areas. In terms of education, it appears more as a measurement and evaluation tool (Fatherstonhaugh, 1994; Winer & Abad, 1995; Aztekin, 2008; Abazaoğlu, 2009).

In this study, Repertory Grid Technique (RGT) was conducted to 51 science teachers to evaluate the change in students' attitudes, when social biographical texts were taught. The RGT applied in this study was a matrix consisting of six elements and eight constructs (Table 3). While the elements consisted of biographical texts with social content, the constructs were the attitudes of the students towards the science course. Focus group interviews were taken into account in the creation of the constructs. The repertory grid created for the study was completed by the participants by scoring between one and five. A total of 48 cells were scored by giving values to eight structures over six items. In the repertory grid created for this study, the maximum score of a teacher's grid was 240 (48×5) and the minimum score was 48 (48×1). As the scores of the participants increased, it meant that the participants thought that social biographical texts could affect students positively.

Table 3. Repertory grid

Constructs **Negative pole (1)**	**Blaise Pascal (B1)**	**Isaac Newton (B2)**	**Galileo Galilei (B3)**	**Nikola Tesla (B4)**	**Thomas Edison (B5)**	**Stephen Hawking (B6)**	**Constructs** **Positive pole (5)**
Increases anxiety							Reduce anxiety
Decreases interest							Increases interest
Decreases motivation							Increases motivation
Decreases academic success							Increases academic success
Increases prejudice							Reduces prejudice
Decreases the level of participation in activities							Increases the level of participation in activities
Decreases laboratory tendency							Increases laboratory tendency
Decreases questioning skills							Increases questioning skills

3.3.3 Semi-structured interviews

Interviewing is one of the research methods widely used in social sciences. Briggs (1986) states that interviewing is a data collection method frequently used in social sciences research. Stewart and Cash (1985) define the interview as an interactive communication process based on asking and answering questions for a predetermined purpose. In this sense, the interview is quite different from ordinary conversation. Patton (1987) states that the main purpose of an interview is to examine the perspective of the individual by entering the inner world. Three types of interview are mentioned in the literature. These are: 'structured interview,' 'semi-structured interview,' and 'unstructured interview.' Structured interviews include predetermined questions, while unstructured interviews include more open-ended questions (Chadwick, Bahr, & Allbrecth, 1984). Semi-structured interviews are more flexible than structured interviews. In this type of interview, the researcher prepares the interview protocol which includes the questions they plan to ask beforehand. The greatest convenience provided by the semi-structured interviews to the researcher enables systematic and comparable information to be obtained by conducting the interview according to the previously prepared interview protocol (Yıldırım & Şimşek, 2008).

In this study, semi-structured interviews were conducted with teachers (N = 15) with the highest scores in the repertory grid. In this sense, taking into account the results obtained from RGT, which is used as a quantitative measurement tool, a 'Teacher Interview Form' was prepared by the researchers. There were six open-ended questions in the interview form. The interview form was applied to the participants after the concerns about validity were eliminated as a result of both the expert opinions and the pilot scheme. The main criterion in the selection of the teachers to be interviewed was the teachers with high repertory grid scores. Table 4 shows the repertory grid scores of the teachers who participated in the interview.

Table 4. Repertory grid scores of teachers participating in the interview

Teachers	Repertory grid scores
T1	109
T2	109
T3	109
T4	109
T5	109
T6	108
T7	107
T8	106
T9	105
T10	104
T11	104
T12	104
T13	102
T14	102
T15	102

3.4 Data collection process

The lessons were carried out in the science courses with 51 science teachers. Focus group interviews were conducted within three groups of 36 science teachers, in order to find the structural equivalent of the biographical texts on social life. The interviews conducted in a suitable classroom environment took approximately 50 minutes. The interviews were videotaped with the consent of the participants. After the focus group interviews, a list of the biographical texts of the scientists selected beforehand which would be taught in the science course was given to the teachers who would participate in the study. The contents of the selected biographical texts consisted of texts that included social life and the problems encountered in the lives of the scientists rather than showing their intellectual and academic achievements. In other words, the problems faced by selected scientists in social lives and affecting their private lives constituted the main theme of the biographical texts. Before the biographical texts were applied, the opinions of two researchers who were experts in the field of education were taken. Problems with validity were corrected by the same expert group and then applied by the participants. Teachers were asked to explain social biographical texts to students over a total of 48 lessons, in science courses, and they were asked to evaluate the changes caused by the constructs included in the repertory grid in students' attitudes towards the science course. The lessons continued for three months, and for three months after the lessons the teachers observed the attitude changes in the students towards the science course. After the three-month application, the repertory grid technique was applied to the teachers in order to evaluate the changes in students' attitudes towards science from their own perspective. After the grid analysis, teachers with high RGT scores were identified and a proposal was made to interview them. Semi-structured individual interviews were conducted with 15 teachers who accepted the interview. For this, the interview forms prepared were used. The interviews lasted about 60 minutes. The interviews were recorded with the approval of the participant. The interviews were then transcribed and made ready for analysis. In Table 5, information about the application process of social biographical texts belonging to scientists is given.

Table 5. Biographical Texts Application Process

Months	SCIENTISTS						Time	Text size
	B1	B2	B3	B4	B5	B6	(minutes)	(word count)
1	X	X					960	1576
2			X	X			960	1470
3					X	X	960	1516
TOTAL							**2880**	**4562**

3.4 Data analysis

The analysis of the data in this study consists of three stages. In the first stage, focus group interviews with the participants were analyzed in order to discover the constructs to be included in the RGT. In the second step, the sum of the values given by the participants to the repertory grid applied to the constructs in the grid and their frequencies were determined. In the third stage, descriptive and content analysis was used in order to examine the results of the quantitative analysis in depth (Yıldırım & Şimşek, 2008).

3.4.1 Analysis of Focus Group Interviews

For the analysis of the focus group interviews, first of all the camera recordings were transcribed and transferred to a written environment. Later, the transcribed interview records of each group were subjected to content analysis by the researcher, and the student attitudes which social biographical texts could affect the science lesson the most were coded. Sub-categories were reached with the help of the codes obtained and constructs were obtained with the help of these sub-categories. These constructs have taken their place as the attitude situations that formed the basis for the subsequent RGT.

3.4.2 Analysis of Repertory Grids

The calculation table prepared in Excel by the researchers was used to calculate the participant scores of the RGT applied to 51 participants. With this table, the RGT scores of each participant were calculated and shown in a table. The SPSS 20 package program was used to find the frequencies of the values given by the participants to the constructs in the RGT, which consisted of six elements and eight constructs.

3.4.3 Analysis of individual interviews

Data obtained from individual interviews was analyzed using content analysis. Content analysis is defined as a systematic renewable technique in which some words of a text are summarized with smaller content categories, with codings based on certain rules (Yıldırım & Şimşek, 2008). The main purpose of content analysis is to reach concepts and relationships that can explain the collected data. In the descriptive approach, unnoticed concepts and themes can be obtained through content analysis (Patton, 1990; Miles & Huberman, 1994; Yıldırım & Şimşek, 2008). Coding and sub-categorization were carried out repeatedly by the researchers. Thus, unnecessary codings were removed by adhering to the problems and sub-problems of the research, and new codings were added where necessary. As a result, tables showing the opinions of the participants about the questions were obtained during the interviews. Sub-categories were determined by coding the data obtained from the interviews with the help of explanatory categories.

4 Findings

Under this heading, focus group interviews, RGT scores, and individual interview results are interpreted. First of all, in the focus group interviews it was determined what attitudes of the students could be revealed in the science course due to studying social biographical texts according to the teachers. Later, the RGT scores of the teachers to whom RGT was applied were calculated and interpreted. Following this, the frequencies of the points given by the teachers to the constructs in the RGT were analyzed and interpreted with SPPS. Finally, semi-structured individual interviews were analyzed and interpreted.

4.1 Findings obtained from focus group interviews

Table 6 shows the focus group interview results of all three groups. In focus group interviews, teachers stated that social biographical texts could affect students' characteristics such as anxiety, interest, motivation, prejudice, participation in activities, laboratory tendency, and questioning skills. In terms of students' attitudes towards science, the construct of interest was repeated thirteen times by the teachers in all three groups and ranked first. This was followed by the anxiety construct with 12 repetitions. These repeated constructs continued as motivation 10, prejudice 8, participation in activities 9, laboratory tendency 11, and questioning skills 10. Among these structures, the academic achievement structure showed itself as the lowest structural repetition frequency with five. This shows that although the teachers believed that biographical texts with social content will not increase or decrease the academic success of the students, they also saw it as a weak situation that may be revealed in students.

Table 6. The constructs that emerged in focus group interviews

Codes	Code Symbol	Frequency (f)	Definition of Codes	Sub Categories	Constructs
1a	1aT13	13	Like or dislike the object	Be interested in the object	interest
2b	2bT12	12	Alienating the object	Approaching the object with anxiety	anxiety
3c	3cT10	10	To demonstrate the awareness of the object	Motivation against the object	motivation
4d	4dT5	5	To make an effort against the object	Academic success in the object	academic success
5e	5eT8	8	Emotional reactions to the object	To be biased towards the object	prejudice
6f	6fT9	9	To show yourself against the object	Common behaviour with the object	participation in activities
7g	7gT11	11	Associating the object with evidence	Experimental proof of the object	laboratory tendency
8h	8hT10	10	Evaluation the object	Performing participatory roles about the object	questioning skills

T: Teacher Object: Science course

4.2 Repertory grid scores

Table 7 shows the RGT scores of each participant. Considering the scores, it was found that the teachers generally observed that biographical texts about the social life of scientists revealed positive attitudes towards science in students.

Table 7. RGT Scores of teachers

Teacher Code	RGT Score	Teacher Code	RGT Score	Teacher Code	RGT Score
T1	109	T18	101	T35	91
T2	109	T19	101	T36	90
T3	109	T20	101	T37	90
T4	109	T21	96	T38	87
T5	109	T22	95	T39	87
T6	108	T23	95	T40	87
T7	107	T24	95	T41	87
T8	106	T25	95	T42	87
T9	105	T26	95	T43	87
T10	104	T27	95	T44	87
T11	104	T28	95	T45	87
T12	104	T29	92	T46	87
T13	102	T30	92	T47	87
T14	102	T31	91	T48	85
T15	102	T32	91	T49	85
T16	101	T33	91	T50	85
T17	101	T34	91	T51	85

Table 8 shows the frequencies of scores given to the constructs in the RGT of the social biographical texts of scientists. When Table 8 is examined, teachers rated four and five points to seven constructs out of eight constructs in RGT means that teachers have observed that biographical texts have developed positive attitudes. However, within these frequencies, "increases academic achievement" construct was coded as "decreases academic success," which is the opposite structure by the participants. In other words, the teachers do not think that social biographical texts will increase the academic success of the students in the science course. Therefore, one of the interesting results of this study that the relationship between social biographical texts and academic achievement did not come out as expected.

Table 8. Frequencies of the constructs in RGT

		Frequency of scores				
Text	**Constructs**	**5 points**	**4 points**	**3 points**	**2 points**	**1 point**
B1	Y1	18	18	14	0	0
	Y2	22	22	6	0	0
	Y3	18	23	3	4	2
	Y4	0	5	15	15	11
	Y5	8	33	9	0	0
	Y6	15	21	13	0	0
	Y7	22	24	4	0	0
	Y8	21	27	2	0	0
B2	Y1	11	35	4	0	0
	Y2	15	30	4	0	0
	Y3	6	20	16	4	4
	Y4	2	6	11	19	12
	Y5	8	35	7	0	0
	Y6	10	37	5	0	0
	Y7	20	23	7	0	0
	Y8	23	25	2	0	0
B3	Y1	22	27	1	0	0
	Y2	19	28	3	0	0
	Y3	16	19	7	4	4
	Y4	8	9	10	13	12
	Y5	24	22	4	0	0
	Y6	19	28	5	0	0
	Y7	4	35	11	0	0
	Y8	10	35	4	0	1
B4	Y1	27	21	0	1	1
	Y2	19	27	4	0	0
	Y3	12	21	9	6	1
	Y4	7	6	10	15	16
	Y5	10	36	4	0	0
	Y6	13	30	6	0	1
	Y7	30	20	0	0	0
	Y8	24	25	2	0	0
B5	Y1	8	37	5	0	1
	Y2	8	36	6	0	0
	Y3	11	17	15	2	1
	Y4	5	8	8	15	14
	Y5	10	34	6	0	0
	Y6	21	27	2	0	0
	Y7	20	27	3	0	0
	Y8	16	23	9	2	0
B6	Y1	27	11	10	2	0
	Y2	24	16	8	0	0
	Y3	10	18	12	0	4
	Y4	8	7	11	17	15
	Y5	12	36	2	0	0
	Y6	12	29	8	1	0
	Y7	8	27	13	1	1
	Y8	13	28	8	0	1

4.3 Findings obtained from individual interviews

Table 9 shows the codes, subcategories, and quotations obtained from individual interviews.

Table 9. Codes, subcategories, and teacher quotations obtained from individual interviews.

Descriptive Categories	Descriptive Category Symbol	Codes	Code Symbol	Frequency (f)	Sub-Categories	Sub-Category Symbol	Quotes from teachers
Biographical text, science teaching, and importance	BTSTI	Academic success Perspective	1aT7 1bT6	7 6	The success that cannot be achieved easily	SCNAE	Science curriculum is done outside of the teacher. Biographical texts are mostly absent in teaching programs. No academic achievement can be achieved easily. Social biographies can change students' perspectives.
Biographical text and content	BTC	Struggle Success	2aT6 2bT6	6 6	Social life facts	SLF	Social life is not taught much. Social life is a fact that reveals academic success. The part that concerns us is the struggle with the difficulties and the success achieved as a result.
Academic achievement, Biographical text, and Consistency	AABTC	Bad reputation Clarity	3aT8 3bT5	8 5	Facts about life	FL	Facts about life. These facts bring up many material and spiritual consequences. Situations like this are not taught very much. These situations can provoke a bad reputation for the scientist. I think clarity should be very important in biographies.
Social life, Challenge, and success	SLCS	Attention Interest	4aT7 4bT6	7 6	Dramatized lives	DL	Many things are mentioned in biographies. Social life should be the part that needs to be told more. Social life imposes a dramatic identity on scientists. Dramatization always works. I would prefer such a method in biographies to attract attention.
Social life, Difficulty, Success, Biographical presentation and Method	SLDSM	Emotional identity Scientific skill	5aT8 5bT5	8 5	Getting away from objectivity	GAO	There are ways of presentation according to the purpose in biographical presentations. Biographical presentations about social life reveal overly emotional identities. Emotional identities distract objectivity.
Biographical text, Science and	BTSAA	Education system Culture	6aT7 6bT5	7 5	Values that increase emotions	VIE	Social biographical texts increase things like more interest and motivation.

Academic achievement	Obstacle	6cT5	5		Such biographical texts do not increase academic success. I think that academic success is more related to the education system and culture.

5 Conclusion and Discussion

In this study, it was aimed to determine the changes that may occur in the attitude of students towards the science course if they were taught social content biographical texts of scientists. Initially, focus group interviews with 36 science teachers were asked for teachers' opinions on which social biographical texts might affect students' attitudes. These attitudes were considered constructs. These constructs are determined as anxiety, interest, motivation, prejudice, participation in activities, laboratory tendency, and questioning skills. Of these constructs, the interest was the most recurring construct in focus group interviews. Similarly, in a study conducted by Aksakalli et al. (2016) using the repertory grid technique, students' perceptions of modern physics were determined. Among these perceptions, the perceptions of anxiety, interest, and alienation showed themselves as the most recurring perceptions.

The construct that increases academic achievement among the structures revealed as a result of focus group interviews has shown itself as the least repetitive structure by teachers. The fact that this construct has a low value among teachers has led to two important results. The first of these results - the thought that social biographical texts will not affect academic success positively. In a way that supports this result, teachers stated in their observations that after the biographical texts were taught in classrooms, the texts did not increase the academic success of the students. Another result revealed by this construct was that the participants did not consider the statement "social biographical texts increase the academic success of students" as an attitude.

Findings obtained from the repertory grid show that generally, teachers have a positive attitude towards social biographical texts. According to the findings, social biographical texts prevented students from approaching science with concern, but also made them more interested. In addition to this, it has shown that the rate of participation in activities increased as well as increasing their motivation. Similarly, as stated by Bandura (1986), association theory originating from biographical texts has important effects on success or failure. In this sense, it was seen as one of the results obtained in this study that biographical texts improved students' motivation to be in the laboratory, and they also go a long way in encouraging them to start asking more questions.

The results of the analysis of teachers' RGT scores indicated that among the eight constructs, seven of the eight constructs indicated a positive transformation, that is, five and four points, while one got relatively lower scores than the teachers. Teachers generally gave one or two points to the construct "increases academic achievement". The construct of "increasing academic achievement", which has the least frequency in focus group interviews, also has similar results in RGT. In other words, this study shows that teachers do not believe that social biographical texts belonging to scientists will increase the academic success of students in science courses, and that they did not observe a change in this direction in students after the applications.

Semi-structured interviews were conducted with the teachers who argued that social biographical texts would cause changes in students' attitudes. In terms of the importance of social biographical texts in science teaching, which is the first of six descriptive categories, the participants stated that such biographies would change the students’ perspectives on science courses and approach them more positively, and they stated that every aspect of scientists' lives should be explained to students. Emphasizing that success cannot be achieved easily, they stated that how scientists struggle with difficulties in their lives should be reflected to students with an impartial eye.

The descriptive category on what should be the contents of biographical texts to be used in science course has emerged as the participants should include more challenges in life. They stated that biographical texts about how they cope with the difficulties in their social life would be more effective than their academic or intellectual achievements. The other data revealed by this descriptive category is an expression of success. The achievements achieved in biographical texts to be used in science, despite the difficulties in social life, have also been participants' statements that there should be content that should be used in such biographical texts. Regarding the rationality of constantly talking about their academic achievements in the narration of biographical texts of scientists, another descriptive category, the participants thought that such expressions would create a disadvantage in terms of hiding the social lives of scientists. In this respect, they stated that biographical texts should be as transparent as possible, and include statements explaining various aspect of scientists' lives.

The teachers stated that they observed that the success of a scientist despite the difficulties in his/her social life increased the students' interest and increased their respect for the scientist. On the other hand, it can be said that teachers were in favor of telling scientists' lives without overly dramatizing them. When scientists' lives were overly dramatized, students may miss the point they really need to focus on. Emotional identities were suggested by the participants to reveal more pity than respect to scientists. In this sense, teachers emphasized that their contribution to science should also be expressed by revealing the scientific skills aspects of scientists in biographical presentations. In terms of this descriptive category, although the participants expressed difficulties in social lives, they also emphasized that objectivity should not be avoided in biographical presentations.

Teachers were sceptical about whether social biographical texts would increase students' academic success in the science course. They thought that only biographical texts will not increase academic success. They stated that various studies are required to increase academic success. They stated that the increase or decrease in academic success is closely related to the education system. In this sense, they especially emphasized that the education system should be improved in this sense. They argued that incorrectly established or inoperative education systems reduce academic achievement and underlined that biographical texts with social content will not increase academic success. They also drew attention to the relationship between culture and biographical texts.

The following recommendations on the use of biographical texts in science courses can be taken into consideration:

- Biographical presentations should be considered as one of the methods be used in the science course.
- Time should be allocated for social biographical texts of scientists in science classes.
- Biographical presentations should be included more in the science curriculum.
- By recognizing the importance of biographical presentations in terms of interest and motivation, science teachers should discuss the biographical presentation method among themselves and bring them to the fore.
- The scientific value of the social biographies of scientists and their connection with cultural values should be brought to the fore.

- Pedagogical issues should be raised, and it should be discussed that benefiting from biographical texts with social content affects students' motivation, interest, learning, and ability to do something about science.

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Linking to the real world: contextual teaching and learning of statistical hypothesis testing

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The study used experimental research design to randomly selected senior high school students in analysing their attitude and achievement in statistics when contextual teaching is implemented. In addition to structured questionnaires, semi-structured interviews with the students were also conducted to provide rich descriptions about learning experiences with contextual teaching. Results revealed that students have positive attitude towards statistics and that students learn better with contextualized instruction than direct instruction. With this, contextualized instruction must be promoted in teaching statistics. By linking it to the real world, students would be able to view statistics more than just a subject. Thus, it makes the statistical hypothesis testing learning more enjoyable and exciting.

Keywords: contextual teaching and learning, effectiveness, hypothesis testing, senior high school, statistics

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1 Introduction

In a data-driven world, statistics plays a vital role in all fields of work. Statistics being embedded within the mathematics curriculum deals with collecting, organizing, summarising, and analysing data in order generate relevant conclusions. With proper application of statistics, discoveries are made, and policies become more relevant and give answers to different social problems. These are some of the reasons why statistics has always been part of any of the school curriculum. Despite knowing these overriding contributions the statistics has on individual and society, it is hated by learners who are not into numbers or quantitative data, especially those students who are more into social sciences (Prayoga & Abraham, 2017). When they are asked to memorize formulas, they tend to pay attention less (Ramsey, 1999). Because of these formulas and rules involved in statistics lessons, students tend to develop negative attitudes and concerns towards the subject (Altintas & Ilgün, 2017). Students had misconceptions about basic statistical concepts such as in measures of central tendency and spread. Researchers have also shown that these students experienced fear and anxiety towards the subject (Zieffler, Garfield, Alt, Dupuis, Holleque, &

Chang, 2008).

Dealing with numbers for students to come up with informed decisions is just one of the major skills they have to acquire upon graduation. In the Philippine K to 12 program of the Department of Education, Statistics and Probability is offered as a core subject being taken by all senior high school students in all tracks. It cannot be denied that there are students who try to avoid mathematics, thus they are taking up a strand or program in senior high school with less mathematics. One of these strands is Humanities and Social Sciences strand. Students who are to take up education, criminology, psychology and other social science programs in college belong to this strand.

After two years of offering and teaching the subject to mostly 16-year-old Grade 11 students (i.e., First Year Senior High School), they scored below mastery level in the recorded fourth quarterly examinations. As prescribed by the curriculum guide, the fourth quarter must include the teaching and learning of hypothesis testing. In school year 2016-2017, they scored 52.6, while in school year 2017-2018, they scored an average of 49.3. Students are generally not meeting the standard of 75 percent. Moreover, when this mean percentage score is compared with the other subjects, it is one with low rank. Adding to this is the observed negligence of students in submitting requirements. The completion rates of seatwork and homework appeared lower than usual. The students truly struggle with the content and feel that they are exerting effort and that exerting effort is not productive.

In this sense, teachers must also be aware of their crucial role in facilitating the learning of their students. Recognizing the importance of teaching and learning statistics, teachers must be able to teach the subject well in the context of problem solving using real data. Garfield (1995) as cited in Larwin and Larwin (2011) suggested the use of variety of strategies in teaching statistics by providing students with activity-based and computer-assisted instructions.

To ensure quality delivery of instruction, the Philippine Department of Education promotes active learning as emphasized in the K-12 program. It has suggested some strategies to be used in different subject areas to encourage learners to be active. These strategies, in general, are student-centred, thus, encouraging learners to become active as much as possible. One of the strategies being promoted by the department is the use of contextualization. Contextualization is defined as a set of different instructional strategies designed to link content by focusing teaching and learning on “concrete applications in a specific context that is of interest to the student” (Mazzeo, Rab, & Alssid 2003) as cited in Perin (2011). With contextualization, a connection between the content and skill is taught with real-world events. Applying contextualized instruction, improvements in attitude and achievement have been discovered in several studies (Haryanto & Arty, 2019; Syahputri & Mariyati, 2019; Suryawati & Osman, 2018; Shodiq & Ihsan, 2017; Qudsyi, Wijaya, & Widiasmara, 2017)

Methods in statistics can be used in different social science situations. This makes statistics crucial to the students and graduates under the Humanities and Social Sciences strand. As teachers, they are to design and develop relevant activities to ensure learning. In this study, contextualized instruction was employed in teaching and learning hypothesis testing. The teacher used issues, events, activities, and authentic materials related to the learners to meet their needs on the subject. The study specifically sought answers to the following research questions:

1. Is there a significant difference between the attitudes (value, interest, and effort) of students towards learning statistical hypothesis testing prior and after the implementation of contextual teaching?
2. Is there a significant difference between achievement of students towards learning statistical hypothesis testing prior to and after the implementation of contextual teaching?

Generally, the study aimed to analyse the students' attitude and achievement when contextual teaching is implemented. It specifically (i) determined the attitudes (value, interest, and effort) of students towards learning statistics; (ii) compared the attitudes (value, interest, and effort) of students towards learning statistical hypothesis testing prior and after the implementation of contextual teaching; and (iii) analysed the difference in the achievement of students on hypothesis testing prior and after the implementation of contextual teaching.

1.1 Literature review

The importance of statistics has been recognized by different institutions and business organizations as defined by the American Statistical Association (ASA) – "Statistics is the science of learning from data, and of measuring, controlling and communicating uncertainty (Wild, Utts, & Horton, 2018). It is a way to understand the data that is collected about people and the world. This is why teaching statistics has been part of the school curriculum. In the Philippine education system, senior high school students are required to take it as part of any program of specialization. Statistics and Probability is taken by all senior high schools as a core subject. Before the implementation of the K-12 curriculum, statistics was taught in public high schools during the last quarter of the third year (TeacherPH, 2020). The subject has been taken for granted when most mathematics teachers cover topics involving algebra. Zieffler et al. (2008) reviewed literatures on teaching introductory statistics to college-level students and found not such good results. Students had misconceptions about basic statistical concepts such as in measures of center and spread. Researchers have also shown that these students experienced fear and anxiety towards the subject.

Recognizing the importance of teaching and learning statistics, teachers must be able to teach the subject well in the context of problem solving using real data. One of the strategies being promoted by the Philippine Department of Education is the use of contextualization.

1.2.1 Contextualized teaching and learning (CTL)

Advanced statistics and other theoretical sciences are often taught purely using the lecture format, which promotes passivity and isolation in students (Rosenthal, 1995 in Duru, 2010). Direct instruction has been the traditional way of teaching mathematics where procedures and rote memorizations are involved. Students are often bombarded with procedures and processes in subjects such as statistics after which they are given drills.

Opposing this traditional way of direct instruction is having the students engaged in activities in the context of the lesson's application to the real world. Contextualization involves strategies that link learning of a skill to its application in a context that is interesting to the students. It allows students to value and make meaning to what they learn in school (Mouraz, & Leite, 2013). Baker, Hope, and Karandjeff (2009) stated that an authentic context helps the learners see the relevance of information and creates a pathway for them to understand the material. Since students find learning relevant, with contextualization, students' confidence, enthusiasm and interest in long-term goals and education are enhanced (Bird, Livesey & Simon, n.d.).

The contextual teaching and learning (CTL) was described by Johnson (2002) as a holistic system that it is made up of several components. These components include: 1) making meaningful connections, 2) doing significant work, 3) collaborating, 4) critical and creative thinking, 5) nurturing the individual, 6) reaching high standards, and 7) authentic assessment. She argues that all these components must build a network by which students are able to make meaning and retain information. In the implementation of CTL, other elements have to be considered. In connection to this, Hidayah (2017) suggested that activation of existing knowledge requires acquiring knowledge by studying the whole world first, then paying attention to the details. Understanding, practicing, experiencing the knowledge and reflecting the strategy for developing knowledge have to be looked into when practicing CTL.

Out of these CTL principles, REACT was developed by Crawford (2001) and implemented in America. REACT (relating, experiencing, applying, cooperating, and transferring) is a set of teaching strategies that is used to make students establish their sense of interest, confidence, and a need for understanding. *Relating* is the most dominant element in contextual teaching strategy. It suggests that students' learning must happen in the context of one's life experiences or pre-existing knowledge. *Experiencing* in contextual approach is a strategy that tries to make students active learners (Misteni, & Baehaqi, 2016). *Applying* strategy can be defined as learning by putting the concepts learned to use. Through problem solving activities, students may apply what they have learned and practiced. For students to handle more complex problems, allowing them to work in groups is suggested with little outside help (Pintrich & Schunk, 1996 in Crawford, 2001). This is the strategy of *cooperating*. This strategy refers to learning in the context of sharing, responding, and communicating with other learners (Crawford, 2001). The last strategy is transferring. Transferring is a teaching strategy that allows students to use knowledge in a new context. In this case, the situation or task has to be authentic and has not been done in class.

The context that must be assumed to ensure maximized learning is crucial in CTL. Perin (2011) asserted that students' experiences are more valued in contextualized classrooms. Ambrose, Davis and Ziegler (2013) came up with a framework of CTL as follows.

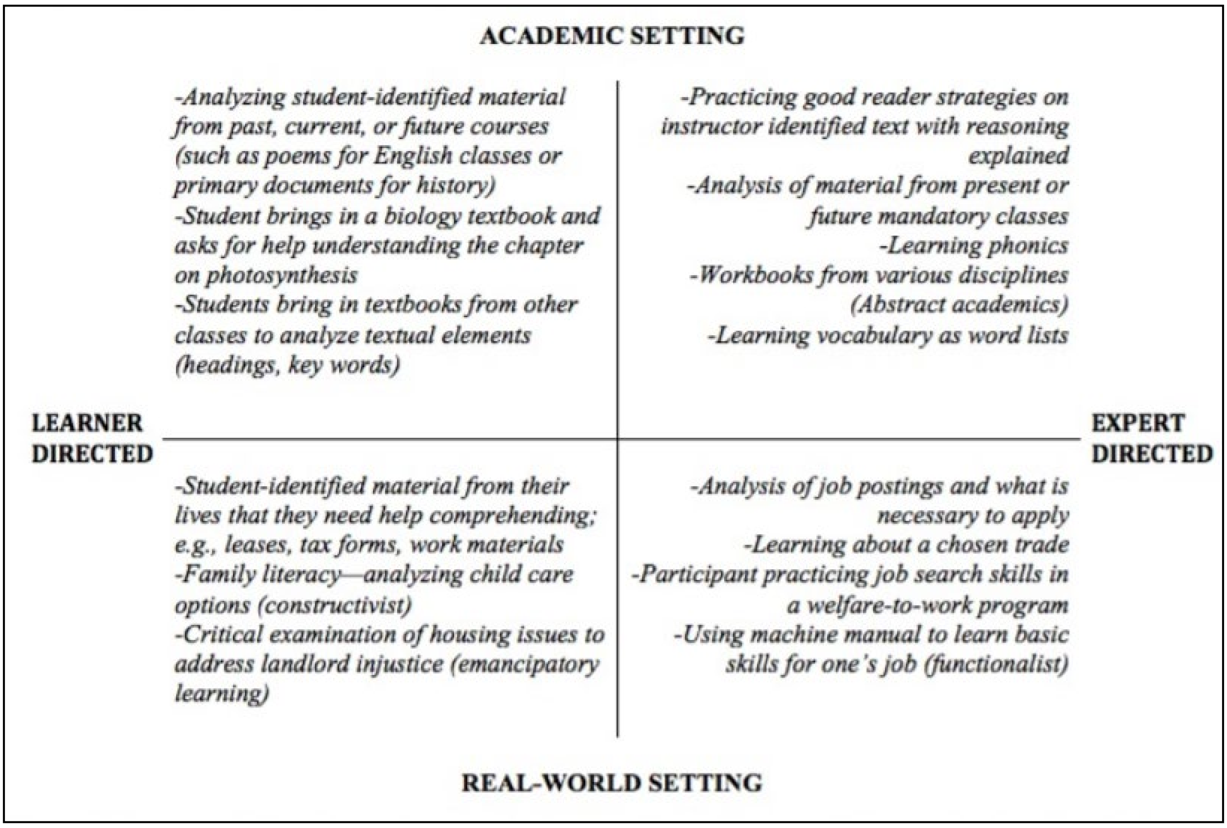

Figure 1. Contextualized Teaching and Learning Framework (Ambrose, Davis & Ziegler, 2013)

The framework suggests that contextualized teaching and learning is done when materials that are student-led or identified are used in the classrooms. Another way is to offer a program that focuses on real-world settings and integrates academic content into it. In designing the teaching guide, the continuum shown above was considered.

1.2.2 Attitudes towards mathematics

Besides ability, achievement is also influenced by attitude. Attitude as a concept has implications to both the learner and teacher. Researchers showed that positive attitude leads to better performance. Mensah, Okyere, and Kuranchie (2013) stated that through positive experiences and reinforcement, a student develops positive attitude towards mathematics.

Furthermore, attitudes of students regarding statistics must be given attention for several reasons as stated by Gal, Ginsburg and Schau, (1997). These attitudes have role in the teaching and learning process in influencing student statistical behaviour and its influence on whether a student will pursue further statistics courses. In this sense, teachers should be aware that student's attitudes towards the subject is something that they should take into account. It is important for teachers to assess the student's attitude towards statistics for further research and improvement of instruction.

Attitudes such as effort, value and interest of learners are important for them to achieve more in mathematics. To achieve this, one is required to exert effort. Students' effort is critical to their learning (Carbonaro, 2005 in Karabiyik & Mirici, 2018).

Both interest and value predict mathematics achievement (Kim, Jiang & Song, 2012). Students perform better when they are into a task and when they find it useful. Between the two, it was found out that interest has greater predictive power of achievement than value. High achievers also develop more positive attitude towards mathematics (Mata, Monteiro & Peixoto, 2012).

Attitude towards mathematics is caused by other factors such as positive experiences students have in class (Mensah, Okyere, & Kuranchie, 2013). Among the second- and third-year classes in Senior High School, it was found out that attitude and performance are positively correlated. Hence, it is recommended that teachers must provide activities that are appealing and interesting to the students. Once positive attitudes are developed, it will then lead to better performance.

1.2.3 Effectiveness of CTL

Contextualization has already posed several advantages to student achievement and attitude in different subject areas. Using CTL approach against direct instruction, it was found out that students learn better when instructions are contextualized (Selvianiresa & Prabawanto, 2017). In terms of attitude, learning science becomes more interesting, challenging, and cooperative (Suryawati, Osman, & Meerah, 2010). From three secondary schools in Indonesia, results showed that problem solving was improved through contextualized instruction. Through hands-on activities provided, students learn to apply concepts in real life (Suryawati, Osman & Meerah, 2010).

Studies also showed positive results of the use of CTL in mathematics. Representational ability may also be improved using CTL modules (Surya & Saragih, 2017). In the study, a CTL module was developed based on the context of Aceh culture, a unique Indonesian culture. As a result, students responded positively to the material developed, which arouse their interest in learning which then made teaching effective. Moreover, results showed that there was an increase in the representational ability of the students in topics in algebra.

The results of a multiple case study conducted by Valenzuela (2018) illustrated the effectiveness of using contextualized curriculum in college. Considering statistics, it may be taught in different contexts. One of which is teaching statistics in the context of biology. Hypothesis testing and estimation were included in the beginning of the semester making students motivated by seeing the importance of statistics in the science or scientific method (Seier & Joplin, 2010). With this, activities that motivate students were easier to find. In this case, teachers are said to benefit from it.

Lastly, the study of Mam, Domantay, and Rosals (2017) revealed that the use of contextualized and localized teaching increased the scores of the students in statistics. They suggested the development of an authentic, contextualized and localized instructional material in statistics subject to improve the performance of the students.

2 Materials and methods

2.1 Research design

An experimental research design was utilized in the study. This design aims to test the cause and effect of specific treatment by comparing end results between a control and an experimental group (Fraenkel, Wallen, & Hyun, 2011). With this design, a group was randomly assigned as the experimental group that received contextual teaching of statistical hypothesis testing. The researcher manipulated the independent variable by providing different learning activities to two groups of students. One received the contextualized instruction, while the other was taught using direct instruction. In the end, its effect was determined by comparing the post-test scores of the respondents.

2.2 Subjects of the study

From the total population of 166 Grade 11 students, 72 students were randomly selected for the study. They were under the Humanities and Social Sciences program, a strand that offers subjects with specialization on Social Sciences. The students were divided into two groups, each group was composed of 36 students who have the same level of performance in Statistics. Match pairing was based on their third-quarter grade in Statistics. Basically of the 72 respondents, 36 students were assigned to the experimental group and another 36 students were assigned to the control group.

2.3 Instrumentation

In order to meet the objectives of the study, research instruments were developed such as the researcher-made test and the survey questionnaires for determining the attitudes.

Pre/Post Test on Statistical Hypothesis Testing. Using the competencies expressed in the Senior High School Statistics and Probability Curriculum Guide, a table of specification was made by the researcher which served as a guide in the development of the pre/post-test (see Appendix). An item was created for each competency. The researcher-made test was checked by the principal and the master teacher in mathematics. The prepared test was validated by checking the item against the corresponding learning competency in the curriculum guide. The researcher also conducted test of reliability with test-retest and Cronbach-Alpha method and the result got a total degree (0.80) with a reliability factor of 'good'.

Determining Student's Attitudes towards Statistics. To measure the attitudes of learners towards the subject, the Survey of Attitudes Towards Statistics (SATS-36) by Schau, Stevens, Dauphinee, and Vecchio (1995) as used by Hommik and Luik (2017) was utilized. Only three of the six aspects included in the survey were adopted. These three aspects including interest, value, and effort were found appropriate in measuring the attitude towards statistics using localised and REACT strategies under CTL. Under each aspect are nine, four and another four items respectively. Each item was rated using a seven-point response scale. Higher scores correspond to more positive attitudes. Negative words and statements in the survey were modified. The adopted and modified test was pilot tested on 30 senior high school students. Results were tested for reliability through Cronbach's alpha. The value of Cronbach's alpha greater than 0.7 is considered acceptable and reliable (Panayides, 2013). Results of the pilot testing showed an acceptable Cronbach's alpha values of 0.74, 0.88, 0.79 for the value, interest, and effort values respectively. Short semi-structured interviews with the students were also conducted to provide rich descriptions of their learning experiences with a contextual teaching.

Intervention: *Designing and Contextualizing Teaching Guide on Statistical Hypothesis Testing.* ADDIE model was adopted in creating the instructional material. Syatriana (2013) described each phase which includes analysis, design, development, implementation and evaluation.

Firstly, background information about the students were gathered. Secondly, the design phase involved the identification of CTL on which the teaching guide was based. This follows the 5 E's constructive instruction model which stand for engage, explore, explain, elaborate/extend, and evaluate (Bybee, 2014).

The development of teaching guide involved the use of real data, actual findings and reports downloaded online. The guide was developed and validated by two mathematics teachers from the public high school and another two mathematics teachers from a private high school. The developed teaching material was pre-implemented to another group of students, who were not part of the subjects of the study.

Finally, it was implemented in the experimental group. Feedback was collected through interviews with the students under the experimental group. The short interview took a minimum of three and maximum of 10 minutes per student. It determined the students' individual perceptions based on their experience with the contextualised instruction used in Statistics. Also, the effectiveness of the developed teaching material was evaluated through a post-test.

The Contextualized Teaching Guide. Using the prescribed lesson plan format in designing lesson plans (DepEd Order No. 42, s. 2016), and elements of REACT (relating, experiencing, applying, cooperating, transferring), the following teaching guide was created (Table 1). The lessons started with identifying objectives and end with evaluation.

Table 1. Summary of the guide for contextual teaching

Lesson	Description of Activities (REACT strategy used)
1.Introduction to Hypothesis Testing	Engage (*Relating*): 1. Have the learners watch the video clip, "Bad Effect of Social Media". 2. Have them grouped into 5. Allow them to share their insights about the video clip and answer some questions. Explore (*Relating, Cooperating*): Present a STAT report. In groups, make learners identify the steps or processes must be done to come up with a decision. Explain (*Cooperating*): Answers will be published and put together. Discussion follows. Elaborate: 1. Discussion follows about the three methods used to test hypotheses, the general steps in hypothesis testing and steps in hypothesis testing – traditional method. 2. With the steps given, let the learners identify key concepts they know a little, know a lot and totally do not know about.

	Evaluate: Learners will answer question about the meaning of hypothesis testing, the steps involved in hypothesis testing, as well as as its importance.
2.Statistical Hypothesis	Engage/Explore (*Relating*): My Mobile Legend Stats 1. Learners will have some sharing about their ML stats and answer some questions: Win Rate, KDA = (Kills + Assists) / Deaths, MVP Rate = (mvp + mvp from losing team)/matches*100% 2. In groups, allow students to make a CLAIM about the average win rate of CSHS students on ML. They must also answer, *"How may hypothesis testing help you provide evidence for their claims?"* 3. The steps in hypothesis testing will be reviewed. Explain (*Relating, Cooperating*): 1. Definitions of null and alternative hypotheses will be discussed in the context on claims on the average win rate of CSHS students on ML. 3. Examples about absenteeism in senior high school, killings in the Philippines will be given. 4. Q and A Extend (*Applying*): The average number of students who come to school late in a week is 43. Suggest a policy that would make a significant change in this number. State your hypotheses when this policy is implemented. Evaluate (*Relating, Applying*): Let the students state the null and alternative hypothesis for a given situation. Also Identify the type of test involved.
3.Error and Significance Level	Engage (*Relating*): Is he Mr. Right? The review on statistical hypothesis will be done. Questions will be asked. Discussion follows. Ask the students, "What mistakes may you have in deciding whether to reject Mr. X or not? Explore (*Cooperating*): Based on the photos given, definitions of Type I and Type II errors will be formulated by groups. Explain (*Relating*): Discuss errors further. Significance level will be discussed by giving the trial decision error example. Students shall be asked to volunteer as judge and defendant. Extend (*Relating*): Another example will be given in the context of opening a food business. Evaluate (*Applying*): SAFETY.

4.Test Value	Engage/Explore (*Relating*): Reflection about the last post on Facebook. Quick Survey: What claims may be done using the data that we have? What values may be computed out of the data gathered? How can these be used in coming with a conclusion or decision? *Explain (Experiencing)*: 1. Formulas will be discussed. 2. Examples on absenteeism and killings will be given. Stat tester app will be used. Extend (*Experiencing, Cooperating*): In the social media use activity done in class, which test must be used? Compute for the test statistic. Evaluate (*Applying*): Let the students identify the appropriate statistical test then compute for the test statistic for a given situation such as: watching tv, sleeping hours, and use of phones.
5.Critical Values and Rejection Regions	Engage (*Relating*) Post a statement. Ask the learners: From whom do you often here this? What does this mean? Explore (*Cooperating*): In groups of 5, allow students to recall among themselves how to use the z-table. Each group will be provided with the areas under the normal curve. They must look for the closest z-value for the corresponding areas. Each group must present the values they got. Explain (*Cooperating*): Allow groups to discover where to get critical values given a specific problem or situation. Ask the students to: (i)State the hypotheses. (ii) What type of test is appropriate for this? (iii) Identify and illustrate the critical value/s and the rejection region/s. Extend: Illustrate the rejection regions in each situation. Evaluate (*Applying*): Given a situation, the students will be asked the following (i) Write the hypotheses. (ii) What type of test must she use based on the hypotheses? (iii) What significance level must she use? (iv) Draw the diagram showing the critical or rejection regions.
6. Making the Decision	Engage (*Cooperating*): A relay game will be played. After 30 seconds, students of each group will be transferring from one station to another.

	Explore (*Relating, Cooperating*): The average score of STEM students will be given. In groups, learners will then formulate hypothesis about the average score of Humanities and Social Sciences students as compared to STEM. Each group must then discuss the steps that must be done until the computation of test statistic and determination of critical values. Allow learners to construct the idea that the critical and test values will be compared. Explain (*Cooperating*): Using the previous problem presented, allow groups to make the decision by just posting a statement and ask some questions about hypothesis testing and making decision. Processing follows. Extend (*Applying*): Post a situation. Then Q and A. Evaluate (*Applying, Cooperating*): Students must get the average number of absences per day during the first semester from their records. They shall then make a claim about their average number of absences per day during the second semester as compared to the first semester. The process of hypothesis testing will be conducted.
7. Conducting a Hypothesis Test for One Sample Mean	Engage (*Relating*): Smile Watch “Just Smile”. Students will be asked some questions. Explore (*Experiencing, Applying, Cooperating*): 1. Quick survey on the number of times the students smile each day. 2. Conduct a hypothesis test given this set of data. Explain/Extend (Relating): Another example will be given. The theme is about killings in the Philippines. Evaluate (*Applying*): Improving Sales. Q and A. Test hypothesis at a 5% significance level.
8. Conducting a Hypothesis Test for One Sample Proportion	Engage (*Relating*): Group activity 1. The learners will be asked to group themselves in terms of: a. sex, b. favourite TV channel, c. favourite past time: TV viewing, internet use, mobile phone use Explore (*Relating*): 1. Learners will be asked to give comments about the given news report. 2. Ask the learners. 3. Let the learners analyse the answers of others.

	Explain (*Cooperating*): Given the formulas, definitions given to them, they must be able to explain to the class the similarities and differences in conducting a hypothesis test for one sample mean and one sample proportion. Extend (*Experiencing*): Consider the claim made by the students in the TV viewing report. Conduct a hypothesis test at 5% significance level. Evaluate (Cooperating, Applying): The infographics showing smoking statistics in the Philippines will be presented. In groups of 5, plan a hypothesis test for one sample proportion using one of the values given. The following steps must be followed: (i)Define the population under study. (ii) State the hypotheses that will be investigated. (iii) Give the significance level. (iv) Select a sample from a population. (v) Collect data. (vi) Perform the calculations required for the statistical test. (vii) Make a conclusion.
Final Task	Evaluate (*Transferring*): Let the students identify a specific phenomenon. With this, search for studies or survey results that relate to it. The topic they will be choosing must be related to their preferred career in the future. Have their initial hypothesis given what they have researched. Then, conduct the appropriate test. Following the same suggested steps in evaluate part. The process involved and the results of their study must be presented through an infographic. This shall be presented orally to the class.

As for the control group, direct instruction was implemented. Direct instruction focuses on a sequenced and incremental mastery of curriculum-based competence and a capacity to apply generalizable skills (Liem & Martin, 2013) in teaching concepts. It relies much on explicit delivery of lecture by the teacher. The study followed the seven steps in the implementation of DI (Liem & Martin, 2013). These included setting objectives, assessment of prior knowledge, presentation of lesson through clear instruction, checking of understanding by giving examples, guided practice, processing of performance, and finally independent practice is given (see Table 2).

Table 2. General structure of the direct instruction implementation

Activity	Description
Giving the objectives of the lesson	The objectives which are derived from the curriculum guide are presented to the class.
Review of previous lesson	Review of concepts, formulas and definitions are done through recitation.
Presentation of Content	Definitions, concepts and steps are directly given to the students. This is also when demonstration by teacher is done.
Guided Practice	A problem or example is solved by the class with help of the teacher. Then solutions are presented.
Individual Practice	Another example is solved by the class without the assistance of the teacher. Then solutions are presented.
Evaluation	Problems that are similar to the practice problems are given. Then solutions are presented.
Final Task	Quiz

The experimental and control groups received contextualized and direct instructions respectively. Examples and problems included were derived from a Statistics book by Prentice Hall. Moreover, there were no group activities done in class, while all sessions were more of a lecture type of instruction. They were given formulas which they applied to a word problem. Unlike with contextualized instruction when a performance task was conducted, with direct instruction, a quiz was administered.

2.4 Data analysis

Descriptive statistics such as mean, median, standard deviation and percentage were used in analysing the attitudes and achievement of the students. To get the SATS scores, the average of the ratings given by the students in each item under each aspect was obtained.

Attitudes and achievement were analysed and compared to the results in the pre-test using the Wilcoxon Sign test, Mann Whitney U and independent and dependent t tests respectively. A sign test makes one determine the difference in the number of times one group score higher than those in the other group (Fraenkel et al., 2011).

To analyse and compare the pre and post test scores, t-test for dependent samples was utilized. The mean percentage scores of the experimental and control groups were analysed by utilizing t test for independent samples.

Finally, responses to the interview were transcribed verbatim and were carefully analysed. Themes were formulated from these in order to give a summary of the general attitude of the students towards the topics taught to them.

3 Results and discussion

3.1 Pre-test results of students' attitudes towards learning statistics

In general, prior to the implementation of instructions involving hypothesis testing, students from both the control and experimental groups have positive attitude towards statistics in terms of value, interest and effort as shown in Table 3. Furthermore, effort as an attitude had the highest mean for both control and experimental groups with mean values 5.48 and 5.68 respectively. This is followed by interest ($\bar{x}$=5.25; $\bar{x}$=5.39), while value got the lowest ($\bar{x}$=5.12; $\bar{x}$=5.14). Students believed that they have been exerting effort towards the subject such as in doing their tasks, exercises and attending class. From the interviews conducted, a student mentioned, "I do what has to be done and it's okay". Other positive response includes how they show interest by saying, "I like statistics because it was taught well, and I like computing."

Table 3. Attitude of students towards statistics

Attitude	Control Group		Experimental Group		Overall		
	Mean	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Description*
Value	5.12	0.80	5.14	0.83	5.13	0.81	positive
Interest	5.25	0.68	5.39	0.77	5.32	0.73	positive
Effort	5.48	0.79	5.68	0.83	5.58	0.81	positive

Range: 0.00–3.50 Negative attitude; 3.51–4.49 Neutral; 4.50–7.00 Positive attitude

Overall, only one student had negative attitude towards statistics in terms of value, interest, and effort. Seventy-two percent valued statistics positively. Majority were also positive in terms of interest and effort with 82 percent and 94 percent respectively. These results agree with the study of Coetzee and Merwe (2010) in South Africa where students displayed high level of interest in understanding and learning, valued statistics and were willing to work hard for the subject. Despite these students perceived statistics as a difficult subject, they were willing to put some effort to become successful in the subject.

Results showed that majority of the students valued statistics. This means that students valued the subject as it is useful in life. They also believed that skills and knowledge taught to them may be relevant to their future jobs. Majority of the students were also positive about their interest towards the subject. They were interested in learning, understanding, and using the concepts in statistics. Finally, the students showed a positive attitude towards the subject in terms of effort. They completed requirements and gave ample time for studying statistics.

The students from both groups were also handled by the researcher prior to the conduct of the study. This may indicate that the students have already built a positive attitude towards the subject previously.

Other factors that could have influenced the students' attitudes were predicted by motivation-related variables where teachers and peer support were found to be highly significant in understanding these attitudes (Mata et al., 2012). These factors cited may have influenced the respondents of the study prior to the implementation of the developed instruction.

Hypothesis testing or statistics can be too abstract. Therefore, it is taught considering it as a means, not an end. This indicates that it must be taught in the context of real world. With direct instruction, elements of constructivism and collaboration were not present but, in both instructions, applications were done. This may have contributed to the similarity of the attitudes towards statistics by both groups.

Other than type of instruction, several factors affect student attitudes. Some of which are teacher characteristics and attitudes, pressure, and peer support (Mata et al., 2012). The teacher's attitude shown in class greatly affects students' attitudes towards the subject. The same amount of support was provided to both groups. Students feel more positive when support from the teacher and peers is present.

3.2 Students' attitudes towards statistics after the implementation of contextual teaching

Table 4 shows that the attitudes of students in the experimental group remained positive in terms of value, interest and effort as indicated by the median scores. SATS post scores in interest and effort have increased. Effort remained to be the highest (Mdn = 5.72, R = 2.8), followed by interest (Mdn = 5.48, R = 3.5) and value (Mdn = 5.29, R = 3.0).

Table 4. Pre and post-test results of students' attitude towards statistics

	Pre Test		Post Test		z	p
Attitude	Mdn	R	Mdn	R		
Value	5.00	3.34	5.29	3.0	1.12	0.26
Interest	5.45	2.76	5.48	3.5	1.07	0.29
Effort	5.65	4.42	5.72	2.8	0.28	0.78

*Significant at $\alpha = 0.05$

Wilcoxon Signed Rank Test results showed that the difference in the pre and post SATS median scores were not significant in terms of value, interest and effort. This means that there was no significant increase nor decrease in the SATS scores of students who were taught using contextualized instruction. It must be noted that prior to the implementation of contextualized instruction, students had already positive attitude towards statistics. Hence, a small increase is found to be not significant. This supports the findings of Suryawati and Osman (2018), who tested the effectiveness of CTL and found no difference in terms of scientific attitude in Natural Science among Junior school students in Pekanbaru, Indonesia.

Statistics has been disliked by non-statisticians. Memorizing formulae is a potential reason that students have for disliking statistics (Altintas & Ilgün, 2017; Ramsey, 1999). What happens is they are asked to memorise formulae to be used in problems provided by the teacher. This is what the CTL is trying to avoid. Instead of focusing on the memorization of formulae, teachers activate the previous knowledge of students in order to create a new one through group activities. After which, the students use newly learned or constructed concepts in examples and exercises which are relevant to them.

With the implementation of the CTL guide, a student commented, "It's okay that formulae are not memorized, so we can focus with the computations."

These students in the experimental group were provided with meaningful and collaborative activities using contextualized instruction. Through contextualized instruction, students maintained positive attitudes towards the subject. In the interview, students said, "When teachers give examples, we can relate. The more we can relate with the examples, the more we learn." With examples that they can relate with such as social media and online games they get attracted to listen; as mentioned by another student, "Others were attracted to listen."

They valued and were interested in learning statistics. From the interviews, it was found out that the decision-making topic for them is the one that they think is very important in real life. For them, this skill is crucial in the future. Students find the topics interesting because they can relate to them. One example is the use of social media. They also mentioned about the examples being informative. They tend to learn more aside from the topics in statistics with the examples and activities given to them. Through the contextualized instruction, students value and make meaning to what they learn (Mouraz & Leite, 2013). In giving examples, hobbies and preferred college course were considered, thus making the students more interested in class. With assessment, Baker, Hope, and Karandjeff (2009) stated that an authentic context helps the learners see the relevance of information and creates a pathway for them to understand the material. This was done through actual data gathering and analysis regarding the topic of their interest. This contributes to the positive attitude they have towards statistics.

They also exerted effort towards the accomplishment of tasks. These results agree with Suryawati, Osman and Meerah (2010) that as learning becomes more cooperative and challenging, students exert effort in completing the tasks. The students find the performance task of actual data gathering and analysis activity interesting at the same time challenging. They enjoyed going after respondents to be surveyed. Moreover, they get to know and get along with students from other classes.

In terms of cooperative learning as one of the underlying principles of CTL, they find group activities positive as they get to help and be helped by their classmates. As in Suryawati, et al (2010), learning becomes more cooperative. Elements of cooperation were found to be evident while they learn in groups and when they communicate with each other.

3.3 Students' achievement in hypothesis testing before the implementation of contextual teaching and learning

Prior to teaching hypothesis testing, the students had a very low achievement in terms of mean percentage score. On the average, the mean percentage score is 18.4 which was *below mastery* level as shown in Table 5.

Table 5. Pre-test results of students' achievement in hypothesis testing

Group	Mean	SD	Description
Control	17.7	6.50	Below Mastery
Experimental	19.1	8.40	Below Mastery
Overall	18.4	7.50	Below Mastery

Range: 60 and below-Below Mastery; 61-79-Moving Towards Mastery; 80 and above- Mastered

The topic was taught only during the fourth quarter; thus students have very limited knowledge about it. Hypothesis testing is a unit covering topics involving one sample mean and one sample proportion. Required prior knowledge on reading z and t values from tables had been discussed but were also part in the unit of hypothesis testing. Other than these topics, concepts were relatively new to the students.

3.4 Students' achievement in hypothesis testing after the implementation of contextual teaching

Results of paired t-test for dependent samples revealed a significant difference in the mean scores of experimental group students between the pre-test and post-test (Table 6). In particular, the students significantly scored higher in the post-test (M = 61.2, SD = 8.65) than in the pre-test (M = 19.1, SD = 8.40), t (36) = 18.3, p =0.00). Using the teaching guide developed by the researcher, the student achievement had improved to *moving towards mastery* level. Although findings in the control group also showed significant difference in the mean scores of the students between pre-test and post-test, but the increase was not that high and still in the *below mastery* level as compared to the increase observed in the experimental group. This affirms the findings of Surya, and Saragih (2017) who found positive results in the use of CTL in mathematics; Haryanto and Arty (2019) and Suryawati and Osman (2018) who tested the effectiveness of CTL and found significant difference on the achievement in Natural Science; Syahputri and Mariyati (2019) who found significant improvement of the students' achievement in reading comprehension by applying CTL; Shodiq and Ihsan (2017) who found significant improvement in the achievement of students in Basic Grammar Class; and Qudsyi, Wijaya and Widiasmara (2017) who in the same manner found CTL as an effective way of improving the students achievement in Cognitive Psychology Course.

Table 6. Pre and post test scores of experimental group

Mean Percentage Score	M	SD	t	p
Experimental Group				
Pre-Test	19.1	8.40		
Post Test	61.2	8.65	18.3	0.00**
Control Group				
Pre-Test	17.7	6.50		
Post Test	39.9	14.0	2.03	0.00**

**Significant at 0.00

The post test results in the researcher-made test were compared. Results of the independent t test (Table 7) reveal a significant difference in the mean scores between the and experimental and control groups ($t(72)=3.75$, $p=0.00$) In particular, students who were taught using contextualized instruction scored higher ($M = 61.2$, $SD = 8.65$) than the students who were taught using direct instruction ($M = 39.9$, $SD = 14.0$).

Table 7. Post test scores of the control and experimental group

Mean Percentage Score	M	SD	t	p
Control Group	39.9	14.0		
Experimental Group	61.2	8.65	3.75	0.00**

**Significant at 0.00

Results agree with Selvianiresa and Prabawanto, (2017), where students learn better through contextualized instructions than direct instruction. Both results show that CTL learning can be more effective when there is collaborative interaction, and connection to real-world contexts. One example is the use of business example as mentioned by a student. In the future, they already have the idea if they wish to start a business while learning errors in statistics. Another student mentioned that the more they can relate to the topic, the more they learn. Other related studies show similar results and conclude that with contextualized instruction, students learn better. Valenzuela (2018) suggested that students understand problems successfully when they can relate with them. Finally, Mam, Domantay, and Rosals (2017) also concluded that students learn statistics better when students are exposed to contextualized and indigenized teaching.

4. Conclusions

This study provided evidence on how contextual teaching promotes the learning of statistical hypothesis testing. Though the students already have a positive attitude towards statistics even before the implementation of contextual teaching, such attitude was heightened after the implementation of contextual teaching. They find the experience interesting and challenging at the same time. Students learned hypothesis testing through contextual teaching with low mastery. Moreover, students learn better with contextualized instruction than direct instruction.

With these, contextualized instruction must be promoted in teaching statistics. Instead of teaching students using plain lecture methods, more relevant and constructivist activities must be provided. Applications must also be done in context where they can relate with.

The study affirmed the potential of the implementation of contextual instructions in teaching and learning statistics, thus the REACT (relating, experiencing, applying, cooperating, transferring) strategies used are recommended. In designing lesson plans, the following strategies are suggested to be intensified:

1. *Use of real-life examples*. Examples used in class must be within the context or background of the students so that they may be able to relate to them. These could be their hobbies, preferred course or future careers. Data to be used in class must also be timely, thus reports found online may be utilized.
2. *More hands-on activities*. Students must also be given hands-on activities through real-life exercises where they can apply the concepts learned. They must be given set of exercises which are in context of real-life situations. With this, actual data may be gathered through quick surveys in class. This may then be analysed or used for students to practice the skill being taught to them.
3. *Collaborative activities*. There must also be activities that promote collaboration. Providing performance task where students are involved in real data gathering and analysis activities are also recommended. Students must be allowed to think of their own topic of interest to be investigated. In this case, students are given opportunities to negotiate and interact with their learning partner or group mates.
4. *Authentic assessment*. Finally, an authentic assessment is recommended in statistics. In this case, it may be connected to a research activity where students get to test hypothesis based on a topic of their interest. In addition, they are to experience the rigor of data collection.

Studying and learning statistics is quite challenging, especially for students who find no affinity in numbers. But no one can deny its importance as everyone lives in this information world, and much of this information is determined mathematically by statistics. Guided by this reality is a responsibility for subject teachers to find better and efficient ways on how learning statistics would be enjoyable and exciting. By linking it to the real world, students would be able to view statistics as more than just a subject. Thus, teachers will find their way to succeed in facilitating learning in statistics.

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Categories of intuitive reasoning and GeoGebra 3D: an experience with Brazilian students

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This work presents the result of the application of a didactic sequence designed to understand the concept of the Cavalieri's Principle, supported by the GeoGebra application in its version for mobile phones - 3D Calculator. For this study, the Theory of Categories of Intuitive Reasoning, by Efraim Fischbein, was used as a conceptual basis. The objective of this work was to elaborate and develop a didactic sequence aiming to subsidize the learning of the Cavalieri's Principle from GeoGebra, as a way to help the student in the construction of geometric reasoning, through visualization, perception and intuition. The methodology of this work is qualitative research, exploratory type, being carried out from a didactic sequence developed in two meetings remotely, due to the scenario of the COVID-19 pandemic. The target audience of this research is a group of students aged 15-17 years from a public school in Fortaleza - CE, Brazil. In summary, it is pointed out that the intuitive reasoning categories mobilized from the use of GeoGebra have great potential to stimulate the evolution of the student's geometric thinking, through the development of perception, intuition and geometric visualization.

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1 Introduction

Geometry - Plane, Spatial and Analytical - is part of the curriculum component of High School, being a requirement for the cognitive evolution of students in this stage of teaching. According to the Common National Curriculum Base (BNCC) the skills necessary for a global understanding of Geometry refer to the interpretation, construction of models, solving and formulating mathematical problems involving notions, concepts and spatial procedures (Brazil, 2018). Geometry, even with its visual and concrete character, as several everyday objects involve it, is still an area in which the high school student faces difficulties in assimilation, and this is pointed out by authors as Alves & Borges Neto (2011), Alves & Borges Neto (2012), Oliveira & Leivas (2017), Cunha & Aguiar (2019).

Regarding the mathematical object under study in this article - the Cavalieri's Principle - there is an existing problem in the teaching and learning process, where this subject is presented in a ready-made, mechanized way, not exploring the visualization and perception necessary for its understanding, in some occasions due to lack of depth on the part of the teacher. According to Paterlini (2010) the Cavalieri's Principle is commonly presented without demonstration, as a way to avoid the obstacles of showing this theory early. Difficulties are concentrated in a single assertion, which is admitted as plausible upon good argumentation and explanation by the teacher.

Starting from this premise and taking into account the importance that the Cavalieri's Principle has for the student's understanding of the calculation of volumes of geometric solids, how the student can understand and develop geometric concepts about volumes using the Cavalieri's Principle, from a visualization perspective and intuitive reasoning? To this end, the contribution of GeoGebra was used, as it is a software that is easy to access and use, being efficient for teaching Geometry, among other areas.

GeoGebra is a resource that comes to add to the teacher and facilitate their practice, being efficient in the presentation of complex assimilated content, where the virtual environment allows the visualization and manipulation of its elements and constructions. Mariotti & Fischbein (1997, p. 221) bring that "the definitions of basic geometric figures are not merely conventions in the field of purely arbitrary facts; elementary geometry and geometric concepts are deeply rooted in common experience". Thus, the software has the potential to stimulate the student's intuition and geometric reasoning, enabling deduction and interaction through content experimentation. In addition, according to Breda, Trocado and Santos (2013), with 3D functionalities, GeoGebra makes the representation of elements in space more accessible, contributing to understanding through visualization.

The objective of this work was to elaborate and develop a didactic sequence aiming to subsidize the learning of the Cavalieri's Principle from GeoGebra, as a way to help the student in the construction of geometric reasoning, through visualization, perception and intuition. To this end, this work brings the Theory of Intuitive Reasoning Categories, by Efraim Fischbein, which guides the observations raised from the applied didactic sequence, which seeks to understand how the student's geometric reasoning is consolidated.

The methodology of this work is qualitative research, exploratory in nature and had as target audience a group of students aged 15-17 years, high school students in a public school in Fortaleza - CE, Brazil. The two meetings held for the application of the didactic sequence and data collection took place virtually, due to the scenario of the COVID-19 pandemic, which changed the format of classes and the routine of schools around the world.

In the next sections, historical, epistemological and didactic aspects about the Cavalieri's Principle, the Theory of Intuitive Reasoning Categories associated with GeoGebra and its relevance to this article, as well as the applied methodology with their respective results and considerations of the authors, will be addressed.

2 Historical, epistemological and didactic aspects about the Cavalieri's Principle

Bonaventura Cavalieri (1598-1647) was an Italian mathematician and priest, who brought several contributions to the development of Mathematics, in the areas of Trigonometry, Astronomy, Geometry and Optics, being considered one of the precursors of Integral Calculus (Hoffmann, 2018). However, the core of this work focuses on the object of study that bears its name, the postulate known as the "Cavalieri's Principle", commonly used in problems involving the calculation of the volume of solids.

The statement of Cavalieri's Principle states that "Two solids, in which every secant plane, parallel to a given plane, determines surfaces of equal areas (equivalent surfaces), are solids of equal volumes (equivalent solids)" (Dolce & Pompeo, 2005, p. 165). It is noteworthy that although the demonstration of the Cavalieri's Principle is not easily understood by high school students (Cunha & Aguiar, 2019), the presentation of practical examples should be considered to illustrate it and prove its veracity, providing the development of geometric reasoning in an intuitive way, based on visualization.

Plane and Spatial Geometries are directly related, as the postulates of one serve as a basis for understanding the nature of the other. Based on this relationship, Alves & Borges Neto (2011) point out that the subject (student) relies on mental images, experienced in their daily lives based on objects from the physical world to understand Geometry. On the other hand, when this subject undergoes some formal training, it is expected that he will manifest geometric perceptions such as linearity, regularity, depth of the figures. Regarding depth, this deserves to be highlighted, because, despite displaying an intuitive bias, in general, in the teaching of Spatial Geometry, the representations are displayed on the plane, conveying an illusory impression of belonging to three-dimensional space.

From an epistemological point of view, it is known that many students have difficulties in understanding geometric problems (Oliveira & Leivas, 2017; Cunha & Aguiar, 2019), for not having well-developed visualization and perception skills of geometric representations. Oliveira & Leivas (2017, p. 110) state that "Geometry requires the activity of the gaze with the understanding that an image drawn on a plane is the representation of a three-dimensional object". Thus, it is noted that there is an obstacle to overcome, regarding the articulation between the two-dimensional (2D) and three-dimensional (3D) dimensions, that is, the articulation of the figure in space and its representation.

Cunha & Aguiar (2019) mention that the excess of algebraic thinking within geometry ends up creating cognitive barriers for the student, as such reasoning can often be disconnected. It is important to consider that geometric visualization is also an exercise that develops the student's reasoning, helping him to understand geometric properties and solve problems. Also, according to the authors, the Cavalieri's Principle is not well explored in textbooks, being insufficient in several aspects, such as the presentation of ready-made formulas, for example.

Didactically, by exploring the Cavalieri's Principle, students could develop a different perspective on space and form. By understanding Cavalieri's idea geometrically and by accepting these principles as obvious, intuitively, many measurement problems that normally require more advanced calculation techniques can be solved (Cunha & Aguiar, 2019 as cited in Eves, 2011).

Considering these aspects, it was decided to develop a didactic sequence to work on the student's geometric reasoning based on visualization and intuition about the Cavalieri's Principle, using the dynamism of GeoGebra. For this development, the following section presents an overview of the categories of intuitive reasoning and makes a parallel of its mobilization from the use of GeoGebra software, as a support to the understanding of the student's geometric reasoning from the visual field.

3 Theory of Intuitive Reasoning Categories and GeoGebra

According to Pais (1996) there are four fundamental elements that directly influence the learning of Euclidean Geometry, whether flat or spatial, which are the object, the concept, the drawing, and the mental image. Regarding these four elements, it is crucial to add the semantics present in geometric language within problems. Thus, still according to the author, such objects and their respective representations by drawing interfere in the procedural reasoning and in the construction of the student's geometric knowledge.

Fischbein (1993) points out that geometric objects have two essential components, which are the concept and the image, which conceive the learning of geometry in a considerable way. Furthermore, the passage from the experimentation stage to the abstraction requires a balance between such components, which in turn can be provided by the use of mathematical software, as is the case of GeoGebra, presented in this work.

Alves & Borges Neto (2011) also point out about Fischbein's perspective that geometric figures constitute a mental entity, elaborated from geometric reasoning, in which a figure is different both from its formal definition and from its mental image and in turn it is supported by a sensory perception of a particular given representation. The authors still bring that:

> [...] we can conceive and compare the course of the evolution of a student's reasoning with a teacher. The first, when knowing a subject for the first time, does not yet have sufficiently developed familiarity to deal with formal definitions of this content. Thus, it will rely predominantly on intuitive reasoning. During evolution and in the successive advancement of their mental stages of learning, the student, gradually, generalizes, systematizes, and synthesizes the fundamental ideas involved in that subject. In this way, the subject advances to logical and mathematical reasoning. (Alves & Borges Neto, 2011, p. 42).

That is, the student's reasoning about a new subject initially starts from intuition and based on his perceptions, he starts to conjecture his ideas, formalizing them in a line of reasoning that makes sense to him. Still with regard to intuition, Fischbein & Gazit (1984) state that the term 'intuition' means, “basically, a global, synthetic, not explicitly justified assessment or prediction. Such global cognition is felt by a subject as self-evident, self-consistent, and hard questionable.” (Fischbein & Gazit, 1984, p. 2). Thus, as far as Geometry is concerned, the existing theorems are statements that can be proved, in which their veracity (or not) is guaranteed by a sequence of logical inferences, supported by the structure that starts the model and by other theorems previously demonstrated.

For Fischbein (1982), intuition or intuitive reasoning in geometry can occur in problem solving, since the student is encouraged to analyze, experiment, evolve, abstract, and systematize in order to build their mathematical knowledge, with intuitive structures being essential components of all. form of active understanding and productive thinking.

In this sense, Fischbein (1983) classifies intuition into categories, considering the relationship between intuition and solution, being divided into affirmative, conjectural, and anticipatory intuitions, described below in this author's perspective.

Affirmative intuitions refer to a representation, an explanation or an interpretation directly accepted by people as something natural, evident, intrinsically significant, for example, if someone asks a student what a straight line is, he will most likely try to draw a straight line or he will show the example of a well-stretched line (Fischbein, 1983).

With respect to conjectural intuitions, the solution aspect is explicit, but it is not clearly involved in a resolution effort. They represent statements about future events or about the course of a certain event. This category represents a preliminary, global view that precedes an analytical and fully developed solution to a problem (Fischbein, 1982). The author exemplifies in his work Intuition and Proof:

> It seems intuitively clear that the diagonals in a rectangle are equal, that the shortest path between a point and a line is the perpendicular drawn from the point to the line, and so on. At the same time, these claims can be proven, although no proof is needed and, in fact, it seems quite superfluous. (Fischbein, 1982, p. 11).

With regard to anticipatory intuitions, Fischbein (1982) points out that this category of intuition provides a global understanding of a possible way to solve a problem and, thus, influences and directs the stages of search and construction of the solution. In this case, the student is "in the phase of concrete application of strategies, use of formulas, elaboration of drawings that effectively help to identify a solution" (Alves & Borges Neto, 2011, p. 44).

Fischbein (1987) in his studies thoroughly analyzes the teaching and learning process by stating that students often face difficulties in their learning, understanding and problem solving at more advanced levels, as their reasoning techniques and strategies are guided by implicit models, sometimes inadequate. Thus, the teacher must seek to identify such models and provide support to the student in correcting their mental models, so that their reasoning is built properly.

In this sense, the GeoGebra software has great potential to develop the student's intuition and geometric reasoning through visual perception. As it is a dynamic geometry software with a 3D window available in its interface, it allows the visualization of figures in an xyz plane. According to Alves & Borges Neto (2012), the exploration of GeoGebra as a technological instrument enables the visualization of unimaginable situations, when restricted to pencil and paper.

To enable the construction of geometric reasoning, Hohenwarter and Jones (2007) claim that GeoGebra provides a closer connection between the symbolic, manipulation and visualization capabilities of CAS - Computer Algebra Systems -, as well as the dynamic mutability of DGS - Dynamic Geometry Systems - within your interface. “GeoGebra does this by providing not only DGS functionality (where the user can work with points, vectors, segments, lines and conic sections), but also CAS (where equations and coordinates can be entered directly and functions can be defined algebraically and then dynamically changed), for example" (Hohenwarter & Jones, 2007, p. 127).

Mariotti and Fischbein (1997) argue that there is a link between geometry and reality, but even so, geometry is not an empirical science. However, geometry needs reality to serve as a model to demonstrate its various aspects.

It is a fact that Geometry is present everywhere, which reinforces the need for its understanding to understand the world around us. Thus, according to Oliveira & Leivas (2017), it is opportune to work with learning situations that encourage the student's thinking to establish relationships between spatial figures and their flat representations, seeking to develop, from their observations, different points of view, building and interpreting their representations. Breda, Trocado and Santos (2013) point out that the alternative of a three-dimensional visualization is presented as a way to facilitate the understanding of concepts by the student, which favors their learning.

From what is exposed in this section, the relevance of the study about the relationship between mathematical intuition and Spatial Geometry is based, since the perception of geometry, through visualization, has the potential to develop the student's geometric reasoning, helping him in construction of mental models suited to their cognitive evolution. The next topic illustrates the application of the didactic sequence, the methodology of this work, which exposes such ideas in a practical way.

4 Methodology

For this article, an exploratory research methodology was adopted, delineated by a case study, as a way to observe the applied experiments and to anchor data that allow us to understand how GeoGebra contributes to the mobilization of intuitive reasoning categories in students. According to Gil (2002) the results of a case study are presented as hypotheses and not as conclusions.

The research was applied with a group of twenty students, aged 15 to 17 years, attending high school, and coming from a public school in Fortaleza – CE, Brazil. The application took place in two meetings during extra-class time, in which students were invited to participate in a moment of experimentation with the use of dynamic geometry through the GeoGebra software. The meetings took place remotely, due to the current scenario of the New Coronavirus pandemic (COVID-19), using the Google Meet platform. The structuring of the didactic application followed the steps:

1. Presentation of the didactic sequence and establishment of the didactic contract[1];
2. Availability of constructions in GeoGebra;
3. Manipulation of constructions by students, in search of solutions and construction of knowledge about Cavalieri's Principle.

For data collection, two electronic forms were used, an initial questionnaire to survey the class about their knowledge of GeoGebra and a final questionnaire to capture their impressions and reflections on their learning in the meetings, video recording file of the moment application and photographic record. To preserve the identity of participants in this application, students will have their names represented by Student 1, Student 2, and so on.

The constructions in GeoGebra were made available to the students, as an alternative to optimize time and focus on the manipulation and understanding of geometric properties that would help in the construction of the concept of the Cavalieri's Principle.

4.1 Application of the didactic sequence

In the dynamics of the first meeting, a didactic contract was established with the class, presenting the set of guidelines for carrying out both moments. According to Brousseau (2008), the didactic contract consists of a set of expected behaviors from the teacher and students, mediated by knowledge.

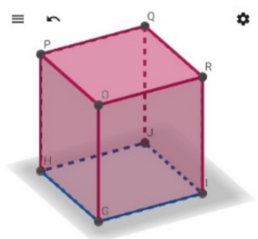
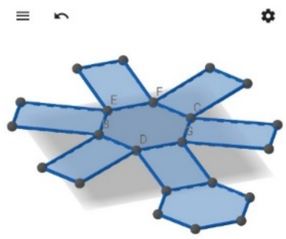
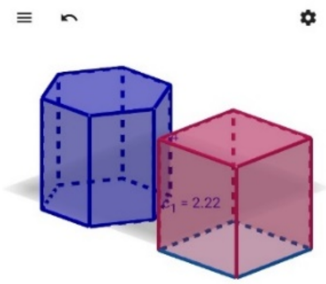
In the didactic contract with the class, it was explained that they would solve a didactic sequence, a component of a research, reinforcing that the participation of all

[1] The didactic contract, according to Brousseau (2008) is a verbal contract that determines the role of the subjects - teacher and student -, places and functions of everyone involved in the didactic situation, in a system of obligations whose reciprocity is necessary, with this relationship being mediated by knowing.

was of great importance. The group was asked to record the encounters through photographs or print screens from the cell phone screen, both from the calculations noted in their notebooks, and from manipulations in GeoGebra. Students were instructed to perform the calculations in their notebook and using the application as a way to compare and reflect on the answers. In addition, they were instructed to use the platform chat or microphone to dialogue, ask and/or validate their answers. Furthermore, it was explained that they would answer two forms, an initial and a final questionnaire, and it was established that everyone would receive constructions 1, 2 and 3 as GeoGebra files (.ggb) via the WhatsApp group.

Then, the didactic sequence was presented, as shown in Table 1:

Table 1. Proposed didactic sequence

Question 1: Given construction 1, determine what is required in the items below: **Construction 1**  a) Identify edge measurements. b) Calculate the base area and volume of this construction. c) Present handwritten calculations and then using GeoGebra tools, verifying your findings.	**Question 2:** Given construction 2, determine what is required in the items below: **Construction 2**  a) Identify edge measurements. b) Calculate the base area and volume of this construction. c) Present handwritten calculations and then using GeoGebra tools.	**Question 3:** Analyzing constructions 1 and 2: **Constructions 1 and 2**  a) Manipulate constructions using GeoGebra tools. b) What do these constructions have in common?

The didactic sequence sought to work on the development of skills such as geometric perception and intuition through the visualization and manipulation of constructions in the GeoGebra - 3D Calculator application. According to Breda, Trocado & Santos (2013, p. 64) “The possibility of viewing in 3D seems to be a way to facilitate the apprehension of concepts and to favor the students' learning”.

To get the class acquainted with GeoGebra, the teacher projected her cell phone screen (Figure 1) in the remote classroom, presenting the 3D Calculator application in its smartphone version. Thus, it showed some basic functionality of the application, the algebra window, and the 3D window.

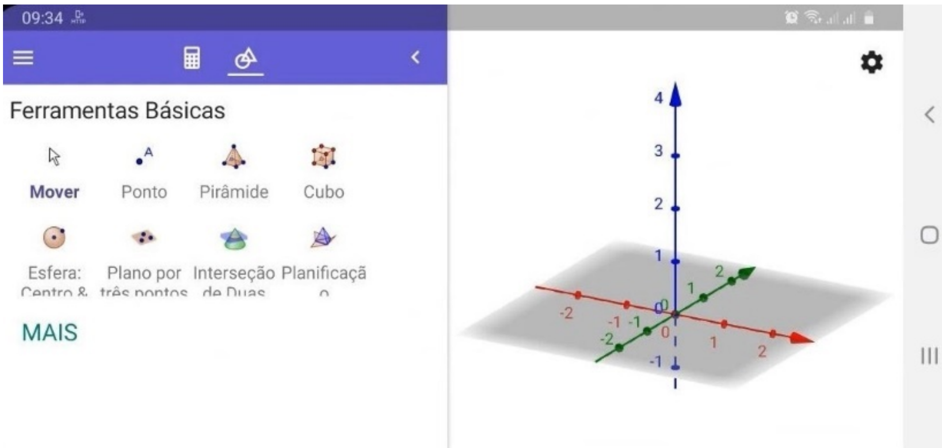


Figure 1. Presentation of the 3D Calculator app for smartphones

Figure 1 shows the initial interface of the 3D Calculator application, explored by the class. Next, the first question of the didactic sequence was proposed, where construction 1 was made available in (.ggb) format for the group of students.

At first, the students manipulated and explored construction 1 presented within the 3D Calculator application, through rotation, enlargement and reduction, seeking the necessary information to solve the proposed question.

When asking the class: "Which polyhedron is this?", the initial, possibly intuitive, response was to say, "it's a cube, teacher!", even without the side measurements being presented. After provocations and during manipulation, some students in the group realized that it was not a cube. The clipping of some lines was:

> *"It is not a cube, as the 'sides' are not all the same" (Student 1)*
> *"Now I saw that it is not a cube, as the height is different from the base" (Student 2)*
> *"It looks like a cube, but it isn't. It's a cobblestone" (Student 3).*

According to Fischbein (1982), mental experience is a reproduction of the practical process based on goal-oriented trial and error. The perfect proof has no meaning for the natural empirical way of thinking, justifying why the students referred to the construction as a cube immediately, based on their experiences, without resorting to a demonstration. "To really understand what a mathematical proof means, the student's mind must undergo a fundamental modification." (Fischbein, 1982, p. 17).

By manipulating the construction, using the command "distance, length or perimeter", the students were able to find the measurements of the edges, both of the base polygon and of the lateral edges, which correspond to the height of the prism. This can be seen in Figure 2, which corresponds to the presentation of Student 4:

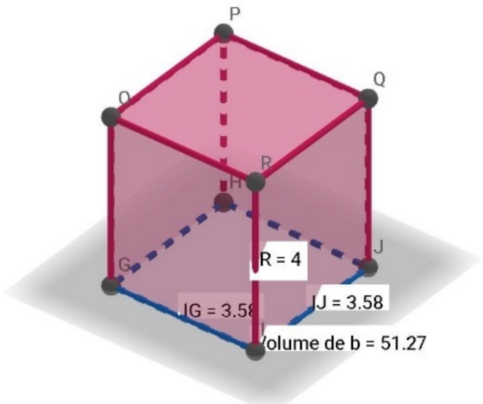


Figure 2. Student 4 Presentation

From the measurements found in Figure 2, the students started to look for the results of the base area and the volume of the construction, based on their previous knowledge and perceptions about the construction.

After dialogue and participation of the class, some students in the group shared their ideas with those present, presenting their perceptions, conjectures and their notes about the construction, listening to the others in an environment conducive to the construction of knowledge.

It is noteworthy that the validation presented by Student 2 was wrong, as he presented the area of the base of the construction using one of the edges of the base and the measure of the lateral edge ($3{,}58 * 4 = 14{,}32$). Even proving the value of this area in GeoGebra, it was not the answer requested by the question. Fischbein (1993), in his perspective, points out that in mathematical reasoning, material objects - solid or drawings - are only materialized models of mental entities with which the mathematician deals. Thus, a geometric figure is not a mere concept, but a visual image. Therefore, Student 2's mistake was based on an erroneous observation of the image.

After discussion, Student 3 designed his screen, showing the top view of the building in GeoGebra, proving that the base is a square and exposing the respective measurements, as shown in Figure 3:

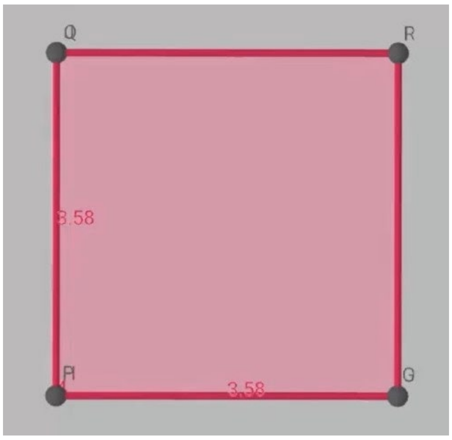


Figure 3. Top view of construction 1 presented by Student 3

Figure 4 shows the comparison between the calculations in the notebook and the proof of the answer based on manipulation in GeoGebra:

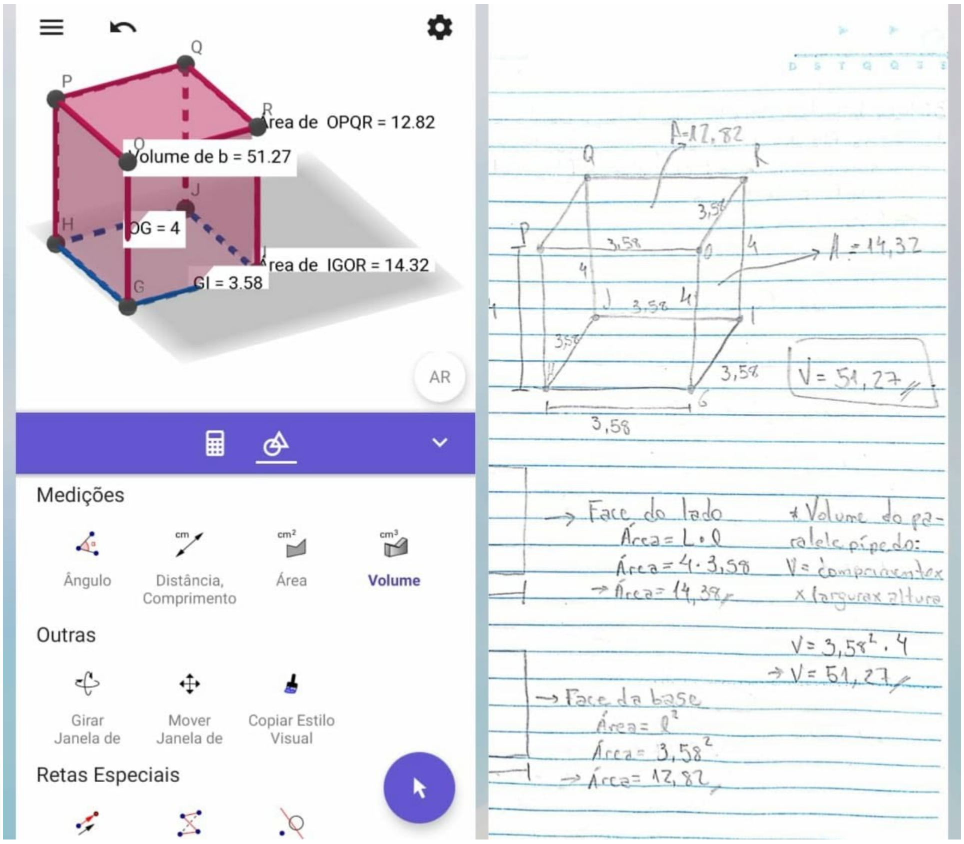


Figure 4. Comparison between manual and GeoGebra responses - Student 3

In Figure 4, it is noted that Student 3, in order to perform the calculations manually, felt the need to visualize the image of the cobblestone of construction 1, so much so that he scribbled a drawing for better understanding. In this case, according to Fischbein (1982) a level of intuitive acceptance occurs in Figure 4, referring to the fact of understanding the universal validity of the statement as guaranteed and imposed by the validity of the proof.

Regarding the second problem proposed, as the class was already familiar with GeoGebra, the process of manipulating the construction and solving the proposed situation were optimized. The discovery of the planning tool aroused enthusiasm and interest. One of the students used the expression "What a genius!" as a demonstration of the reaction when seeing the construction in motion, when opening and closing the plan. In Figure 5 and Figure 6 there is a record of this movement, as well as the recognition of the solid formed:

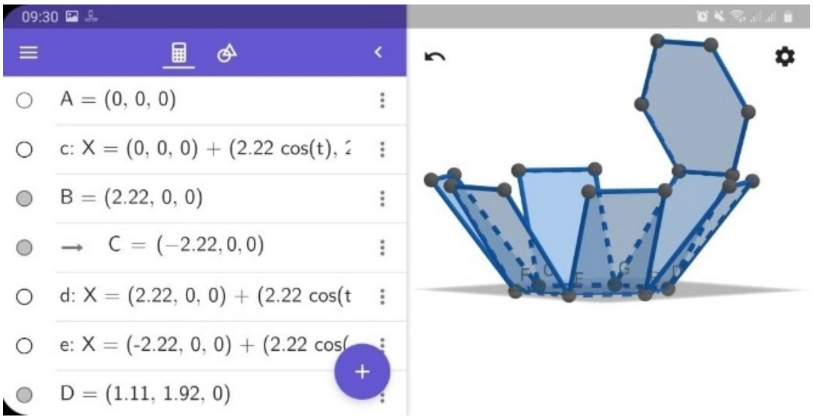


Figure 5. Construction planning movement 2

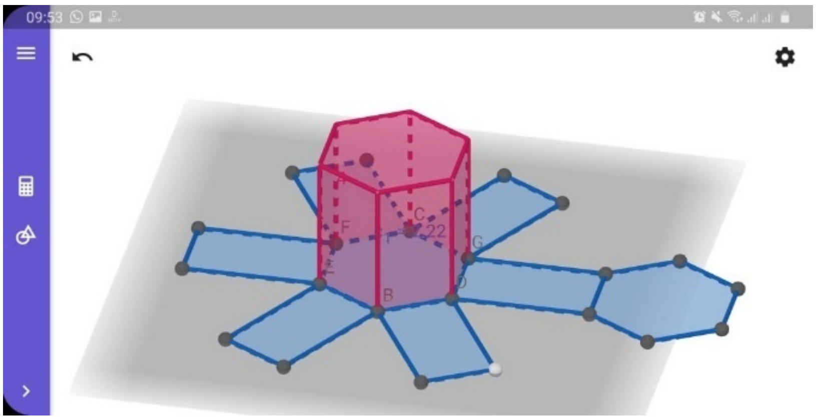


Figure 6. Construction of the flat prism and its 3D sketch

When manipulating construction 2, the students recognized it as a hexagonal base prism and performed the same procedures performed in the situation in question 1, where they discovered the measurements of the base edges and the side edges. Thus, some students in the group have already resorted to what Fischbein (1987) classifies as anticipatory intuition, where there is a notion of the steps to be taken to solve the problem and the strategy to be used for that.

However, in the formulation, when elaborating the conjectures to carry out the manual calculation procedures, the students did not remember how the area of a hexagon was calculated. Until a conjectural intuition from Student 2 facilitated the group's insight:

> *"A hexagon is formed by six equilateral triangles. If I know how to calculate the area of an equilateral triangle, I just multiply the area by 6" (Student 2).*

Student 2's speech is marked by the elaboration of conjectures. From the perspective of Fischbein (1987) it is inferred that Student 2 presented a conjectural intuition, as the elements necessary for the solution were already explicit in the construction, but the student still cannot visualize the solution.

After discussion and an intuitive structural understanding of the mathematical formulas for calculating the area of the equilateral triangle and, later, the area of the hexagon, the students proved the veracity of their manual calculations with the calculator (because it contains decimal approximations) and interacting with the construction in GeoGebra, as seen in Figure 7:

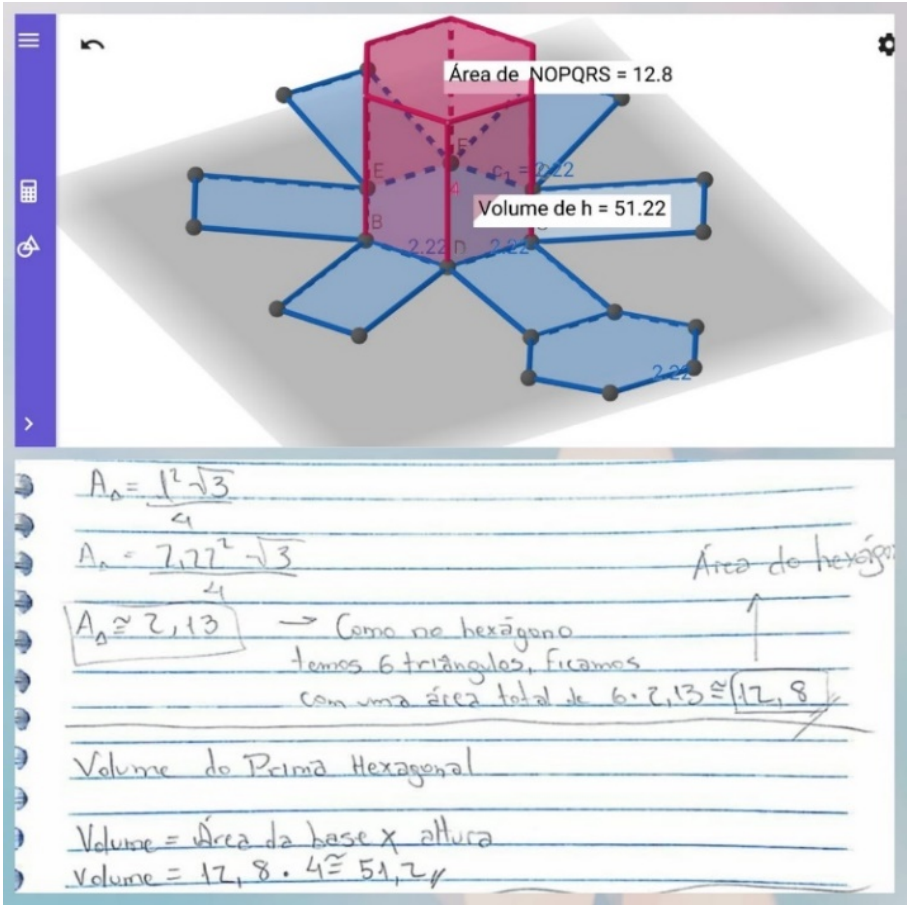


Figure 7. Comparison between manual calculation and result in GeoGebra - Student 3

Fischbein (1993, p. 141) points out that “the properties of geometric figures are imposed by, or derived from, definitions in the domain of a given axiomatic system”. From this point of view, a geometric figure also has a conceptual nature. In Figure 7, conceptually assuming that the construction corresponds to a hexagonal base prism, verifying the base and lateral edge measurements, we found, through a set of procedures, base area and volume values, as numerically equal results, considering the rounding of decimal places.

And finally, the third question of the didactic sequence was presented, which seeks to stimulate the conception of the generalization of the Cavalieri’s Principle. When viewing constructions 1 and 2 (side by side), the students were instructed to observe and manipulate them, modifying the base and height edge measurements, always giving the same value to these edges in both constructions and watching your results. See the example of this modification in Figure 8, where Student 2 modifies the height:

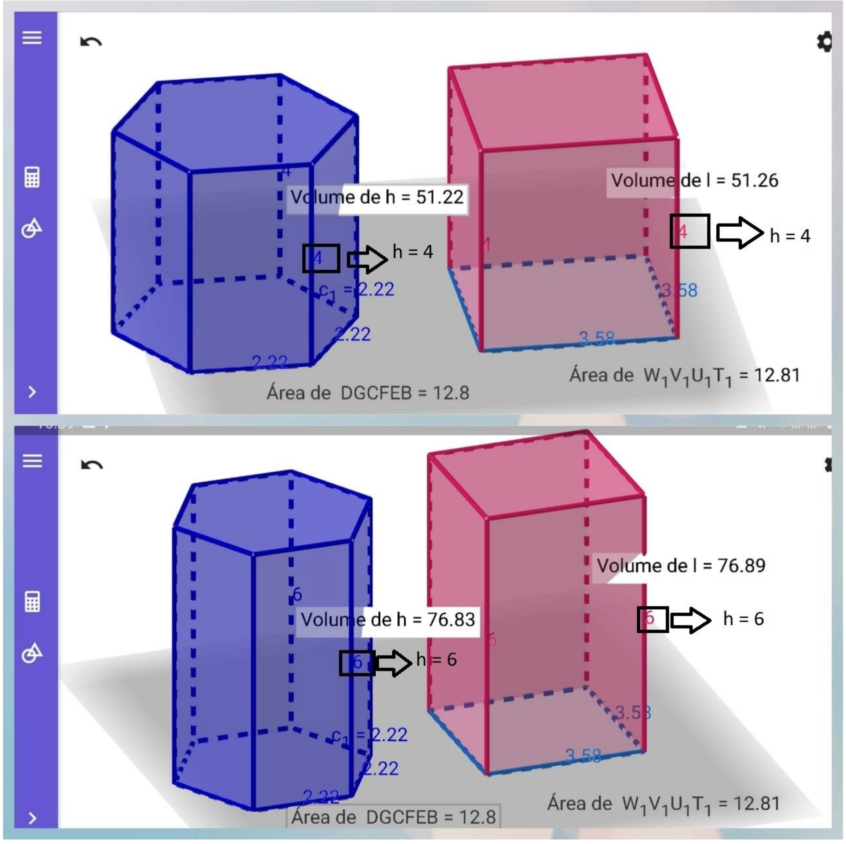


Figure 8. Manipulation of constructions 1 and 2 by Student 2

In Figure 8, the students observed that when manipulating the height measurement in both constructions, but leaving the same measurement in the two solids, the volume was also the same, with minimal differences arising from the truncation of decimal places. The question "what do these constructions have in common?", asked by the teacher, was answered by some participants:

> "Teacher, the base area is the same" (Student 1).
> "Ah, I remember that in the last class we had done the calculations and the height was the same" (Student 2).
> "Teacher, the base area is the same, the height and the volume too!" (Student 3).

After requesting the manipulation and modification of the measurements of the two constructions, the students reached the conclusion that "the volume always remains the same". After verifying these observations, a moment of conceptualization carried out by the teacher began, in which it was sought to verify whether the objective of the activity was actually achieved. At this point, the teacher significantly synthesized everything that was exposed in the previous steps, formalizing the mathematical character of what was validated by the students.

The teacher presented a formal definition about the object proposed in the question, which in this case was the Cavalieri Principle. This definition was taken from the textbook adopted by the school, *Conexões com a Matemática* collection - volume 2, illustrated in Figure 9:

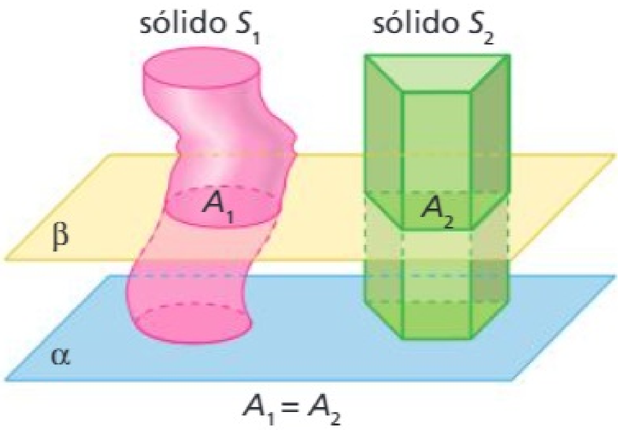


Figure 9. Illustration of the Cavalieri's Principle, by Leonardo (2016, p. 117)

The definition given in the book says the following: "Given two solids S_1 and S_2, supported in a plane α and contained in the same half-space, they will have the same volume V if every plane β, parallel to α, sections the two solids according to plane regions of the same area (A)" (Leonardo, 2016, p. 117).

After presenting the definition of the Cavalieri's Principle, the teacher related it to the proposed didactic sequence, considering that the intention of this application was to provide an opportunity for an investigation, experienced by the students, through the visualization and perception made possible by GeoGebra in the construction of geometric reasoning and understanding of this subject.

5 Results and discussion

The results presented in this section were collected from the application of the didactic sequence carried out in the meetings, characterizing the students' impressions about the use of GeoGebra and its usefulness for the understanding of Geometry, culminating in learning about the Cavalieri's Principle. After acceptance, the initial questionnaire was applied, which provided us with the following results:

- In the question "Do you know GeoGebra?" 76.2% of participating students said yes, against 23.8% who said they did not know;

- When asked if they had ever used GeoGebra to solve math exercises, 76.2% said they had never used it, while 23.8% had ever used the software;
- 81% of participating students said they had difficulties in drawing spatial figures by hand and in understanding them without a drawing or visual representation;
- 100% of participants said they are interested in studying geometry in a more dynamic way.

Student responses show us that a significant number know or have heard about GeoGebra. However, a significant number of students have never used the exercise solving software. According to Fischbein (1993, p. 141) “The material objects - solids or drawings - are only materialized models of the mental entities with which the mathematician deals”, so if these mental models are not well developed and linked to their mathematical concepts, there is difficulty in their understanding and transposition of the mental representation to paper, which was characterized in the students' speech. The answers show an interest in learning geometry in a more dynamic way.

Regarding the final questionnaire, we sought a subjective view of manipulation in GeoGebra. When collecting the students' opinion, we found that 75% of the students marked the option “I found it very interesting”, while 25% marked “I found it reasonable”. No student marked the alternatives “I found it very difficult” and “I found it uninteresting”. Thus, it can be inferred that GeoGebra was a resource that caught the attention and aroused the interest of a significant portion of the participating students. When asking them to justify their answer about this manipulation, we bring the clipping of some lines:

> “I found it a great subject, a tool that helps a lot” (Student 1).
> “It produces dynamic knowledge, which makes learning effective” (Student 2).
> “I found it a little difficult, as I had never used it” (Student 3).
> “It's a little difficult to use the cell phone” (Student 4).
> “I really enjoyed the class and managed to absorb several things, I learned a lot” (Student 5).
> “It is very practical and simple, very easy to use and very useful too” (Student 6).
> “I found it very interesting to discover areas, volumes, edges and base” (Student 7).
> “I had never used it, I thought it was AMAZING” (Student 8).
> “I think that after you learn how to handle yourself, it is very practical, and it certainly helps and encourages mathematical studies a lot” (Student 9).
> “I found it very simple, but I need more time using the program to improve it was something new, it was interesting after the training” (Student 10).

The report of these students shows the importance of considering subjective elements such as attention, interest, emotion and even memory in the learning process, constituting their cognitive context. In this sense, the cognitive processes in student learning were constituted as theoretical assumptions arising from the student's own mobilization in learning. Thus, as a way to enhance learning conditions, it is important to encourage active student participation, seeking to develop their role in the construction of their knowledge.

With this application and data collection, the mobilization of intuitive reasoning categories helped in the construction of this new knowledge, which was acquired from the student's geometric perception and visualization, when investigating the solution of the didactic sequence through their mathematical knowledge and reasoning intuitive geometric design. About the content reviewed or learned in this application, some of the most common responses collected were:

> “I reviewed the area of the equilateral triangle, prism volume and also learned the Cavalieri’s Principle” (Student 1).
> “Cavalieri’s principle, area of the equilateral triangle and much more” (Student 2).
> “Area, volume, spatial figures, planning” (Student 3).
> “Handling GeoGebra” (Student 4).

In a subjective analysis of their speeches, it can be seen that GeoGebra enabled the development of the student's reasoning ability, as this student thought, simulated and elaborated strategies, verbalizing ideas and conjectures and learning from their own perceptions. It is understood that the intuitive categories were mobilized and, in a future perspective, can be improved as the use of GeoGebra is encouraged.

Upon concluding that the volumes of any prism, with equal base area and height, were also equal, the students reached the goal of the didactic sequence. Thus, the search for a solution generated a satisfactory result, providing the construction of knowledge.

6 Final considerations

With this application we can analyze that the didactic sequence presented and explored with the GeoGebra - 3D Calculator application enabled the construction of knowledge about the Cavalieri Principle, being an approach with satisfactory results. The questions explored enabled an evolution of the students' geometric perception of the mathematical content in question through visualization, reaching the research objective, as the students were able to build new knowledge from the exploration of the association between GeoGebra and the questions presented.

In addition, the testimonies of the students confer legitimacy with regard to the contribution that GeoGebra made possible, through the manipulation of constructions, allowing the experimentation and exploration of concepts within Plane and Spatial Geometry, showing itself as a dynamic and interactive tool.

It is noteworthy that the categories of intuitive reasoning were mobilized from a mediation in a oriented way, where the support offered by GeoGebra with regard to the visualization and manipulation of constructions allowed the student to structure conjectures and explore the knowledge that composed the identification of insights verified in the development of the application.

Thus, GeoGebra in its smartphone version - 3D Calculator - is an alternative resource that is more accessible to students and that can be promoted in schools as a contribution to teaching, not only on the Cavalieri Principle, but also on various topics related to Mathematics.

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Children's perceptions of scientists, and of themselves as scientists

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In rapidly developing countries such as the United Arab Emirates (UAE), where this study took place, having a body of competent, dedicated key workers in STEM fields is critical to growing national economies. This, in turn, requires motivated, well-qualified graduates of STEM degrees. School students' perceptions of science, scientists and science careers have been shown in some research to affect uptake of science degrees later on. How much of their science classwork students experience as authentically 'feeling like scientists' is less understood, yet important. This study took place in upper primary science classrooms in the UAE. Immediately following a science lesson, children were interviewed in focus groups (n=66, with an approximately even gender split). Broad questions were explored, such as whether they felt like 'real scientists' when they 'performed' science in the classroom, whether they enjoyed science, and their science career aspirations. 83% of students stated enjoying science, while 61% would like to have a career involving science in the future. The interview data revealed that, overall, children mostly disagreed that their classroom science was reflective of work a 'real scientist' would do, chiefly due to perceptions of a lack of discovery element in their work, which suggested to them a lack of authentic science exploration, and of the work not being *dangerous* enough. Students frequently reported feeling that they were 'following steps' because the teacher 'already knew the answers', which was different from the work of a scientist. The implications of these findings to classroom practice are discussed.

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1 Introduction

In rapidly developing nations such as the United Arab Emirates (UAE), having a highly skilled and qualified STEM workforce is paramount to lofty national ambitions of a strong Knowledge Economy by 2030 and of "science and technology [forming] the pillars of a knowledge-based, highly productive and competitive economy"[1]. Having this kind of currency in workforce is dependent upon universities graduating highly motivated, competent science and technology graduates. This, in turn, is heavily dependent upon students having opted for those degrees in the first place. An

[1] https://www.vision2021.ae/en/uae-vision/list/united-in-knowledge

examination of students' perceptions of science is therefore prudent in Gulf countries such as the UAE, but also internationally, as countries strive to meet the growing needs of 21st century society.

Trujillo and Tanner (2014) explain the importance of taking an "inventory of our students' science identities" (p.13) in order to encourage students towards careers in science. This study explores the ways in which students' science identity - their perceptions of themselves as scientists - in the classroom compare with their thoughts about the work which 'real' scientists perform and their thoughts on science career aspirations. We also wished to explore their enjoyment of science and identify particular areas of enjoyment, or reasons for lack of enjoyment, to provide a context within which to explore their 'possible self' science identities. It is thought that by focusing on primary age children, it may still be early enough to influence perspectives on classroom science and these 'possible selves' with reference to 'real' scientists, in order to positively impact science career perceptions and outcomes.

2 Literature Review

It is often reported that students' positive experiences in the science classroom affect multiple outcomes, including positive science identification, science aspirations and later uptake of science in higher education (Archer et al., 2010; Aschbacher et al., 2010; Bennett & Hogarth, 2009). It is also true that negative science classroom experiences can, conversely, dampen science career aspirations and pursuit of science in higher education (Archer et al., 2020; Lyons, 2006). The way in which children experience science in their classrooms is complex and a factor of multiple variables.

2.1 Students' Perceptions of themselves 'Doing Science'

Research into how primary school students think of themselves as 'doing science' in school has generated a diversity of findings, such as being involved in hands-on investigations, completing their workbooks, learning from their teacher, working alone and doing 'dangerous' things (Zhai et al., 2014). The students in the Zhai et al. (2014) study also reported themselves 'acting like a scientist' when they were doing experiments. Students' understanding of authentic science experiences was explored by Hsu and Roth (2010) in the form of documentation of high school students' experiences of science internship. 'Authentic' was explained as denoting "forms of engagement that have a considerable degree of family resemblance with what individuals in science-related fields really do and experience" (p. 292). However, Archer et al. (2010) found that students who reported engaging in informal science activities outside of school, such as experiments with household chemicals in their kitchens, were not necessarily extending this engagement to their classroom science. Instead, they often posited these experiences as being 'naughty'. Boys were seen to be particularly prone to using these descriptions to perform hegemonic masculinity, while girls often reported more formal interactions with science outside of class, such as reading science texts. These interactions did not necessarily lead to stronger engagement with classroom science.

Schinske et al. (2016), whilst researching science identity and stereotype threat, chose to focus on the concept of 'possible selves' over 'role models', arguing that the concept of these 'possible selves' could be more useful than a more typical role modelling concept. They and we, as a team of researchers, felt that the students would be more likely to identify with themselves as scientists if they saw their own possible selves reflected. Referring to Schinske et al.'s work, this meant in practice that we chose to focus on how students thought of themselves as scientists, rather than exploring their perceptions of others as scientist role models. To enable this, it was critical to explore students' ideas of how their own work and activity in science class compared with what they thought scientists did, rather than limiting the study to their perceptions of scientists' work only. This is a subtle yet important difference, and this comparative 'possible self' reflection is hopefully a more effective means of examining the likelihood of students being inclined towards science careers.

2.2 Student's Science Identity and Self-Efficacy

Science identity and self-efficacy are both seen to positively impact 'using and doing' science (Williams & George-Jackson, 2014). Many studies have shown disparity between genders in science self-efficacy and identity (e.g. Archer & DeWitt, 2015; Banchefsky et al., 2016), suggesting that classroom practices are critical to both genders. This is perhaps even more poignant for nations where one gender under-performs in school-level attainment, as in the case of boys in the UAE (Schleicher, 2018). Positive science identity and career aspirations have also been found to be developed by science research experiences, mentoring and community involvement (Aschbacher et al., 2010; Chemers et al., 2011). One's perception of self as being good at school science and one's sense of having an identity as a scientist are of course not necessarily conflated. Some students do perceive themselves as being 'good' at science yet would not necessarily consider themselves to be classroom scientists. To encourage students' commitment to science careers, researchers must look carefully at how students' science identities play out within the science classroom setting itself (DeWitt et al., 2013). Children's science identity impacts upon how they place themselves within the classroom arena. Students with strong science identity, for example, may exhibit greater confidence in taking initiative with unfamiliar or challenging science activities, including practical activities (Kim, 2018). By contrast, students who do not identify themselves as being able to do science, being 'good' at science, or 'doing' science in the classroom at all, are not considered to have strong science identity. This may manifest itself in a number of ways, such as hesitancy or a lack of participation in the class (Carlone et al., 2014).

2.3 Students' comparisons of their work with that of 'real' scientists

"Am I Like a Scientist? I feel like a scientist when I do dangerous things"

Student response (Zhai et al., 2014)

Students are more likely to embody being a scientist when they believe what they are doing in class closely resembles what a 'real' scientist does. The way in which science is 'done' in the classroom, therefore, may strongly impact students' views and attitudes towards science, making a distinction between classroom science and 'real' science. Jaber and Hammer (2016) argue that there is much to be learned from the emotions and feelings experienced by students within science, for example the excitement and engagement in learning that a student feels when they experience a new idea. A science teacher's positioning may also act to nurture students' identity in science by encouraging perseverance and providing opportunities for students to become excited about science (Kim, 2018). Various research about stereotypes of science and science learning, such as science being filled with hard and dry content, laboratory experiments and male-dominated work environments, have resulted in feelings of distance from science in students' minds. This affects inclinations towards following careers in science and their likelihood of enjoying school science (Archer et al., 2020; Aschbacher et al., 2010; Bøe et al., 2011). The premise of this logic is that if students enjoy lessons more, they may be more likely to be drawn towards science subjects and possibly science careers later. This enjoyment may be developed by a variety of means such as particular forms of learning; inquiry based, problem-based, project-based learning, more of a hands-on approach, and integrating technology with learning (Jaber & Hammer, 2016). For example, a multi-level path analysis using data from 54 countries included in the PISA 2015 cycle demonstrated that inquiry-based science instruction was associated with significant improvement in students' enjoyment of science. Additionally, analysis of this data for students in the UAE (Cairns & Dickson,2021), Taiwan and Australia (Wang et al., 2021), showed that students who enjoyed science were significantly more likely to aspire to enter a STEM career than students reporting lower levels of enjoyment.

2.4 Study Purpose

The purpose of this study was to explore aspects of the science classroom experience in which children are likely to 'feel' like scientists. Vice versa, if they did not feel like scientists, we wished to explore why, and whether this was a factor of having a strong identification with, and enjoyment of, science in the first place. We also explore the students' perceptions of what 'real' scientists do.

The underlying justification for carrying out this work is that in countries such as the UAE, there are shortages in national citizens within their STEM workforce (which is currently mostly outsourced to expatriates), and yet the National Vision and indeed plans for future economy are built upon the predication that this will change in the near future. However, in order to have cohorts of students who select STEM subjects in higher education (and take up STEM careers following this), students need to have the interest, self-efficacy, and identifiably positive perceptions of the work which scientists do in the first place. This study therefore seeks to identify preconceptions which students hold, which could potentially affect this uptake, but which could be over-turned too. The study has novelty in that we are not aware of any qualitative work in the Middle East and Near Asia (MENA) region specifically looking at students' comparative perceptions of the classroom work they do with the work of scientists. Internationally, much science attitudinal or dispositional research is undertaken using quantitative fixed response data. Qualitative data, particularly group interview-based, is known to be a relative rarity even internationally, aside from a few high-profile studies, such as the longitudinal ASPIRE project led by Louise Archer, and to which we refer in this article.

2.5 Research Questions

The study aimed to answer the following research questions:

Q1. What areas of enjoyment of science do students report?
Q2. How do primary students perceive themselves to be 'doing' science in the classroom?
Q3. What perceptions do students have of the work of scientists, and how does this compare with their own science classroom experiences explored in Q2?
Q4. What science career aspirations do students report?

3 Methodology

In this section, we describe the methodology used in this qualitative study. We first describe the participants, the recruitment methods used, the methods for carrying out the focus group interviews, the way in which the interview data was analysed, and finally the ethical considerations of the study.

3.1 Participant Recruitment

The participants were selected via stratified, convenience sampling in terms of the site choice, due primarily to researcher familiarity with administrators in local schools who had expressed keen interest in participation in research studies. Both public and private schooling options are available in the UAE, with public schools following the UAE's Ministry of Education curriculum, and private schools following a variety of international curricula. All schools are supervised and monitored by local regulatory bodies. For this study, three different private school sites were involved, all co-educational, and each following a different international curriculum (American, British, Canadian). The 66 participants were all in Grade 5 or 6, aged approximately 10-11 years old. The gender split of the sample group was approximately 50%, with 34 girls and 32 boys. Due to the confidential nature of the participation in the research, we were not aware of the nationalities of the students, but each school had a proportion of national (Emirati) students within its student body, which ranged between 30% and 90%. In each school, all children in Grade 5 or 6 were invited to participate in the study in order to give a reasonable reflection of attitudes within those grade levels.

Although all three schools were co-educational in theory, one of the three schools was segregated by gender from grade 6 onwards, and so some single-gender classes were included in the study. Grade 5 students were taught science by their homeroom teacher, who also taught other core subjects, and science lessons took place within the general classroom. Where students were Grade 6, they were taught by a science specialist teacher and utilized a dedicated science lab for lessons. At the time of the study, students were learning about topics related to chemistry and earth sciences.

The interviews took place immediately following a science lesson. These were focus group interviews consisting of between three and five children, so a total of fifteen interviews. Each interview lasted between 20-30 minutes. This allocation of time was discussed within the research team, and was aimed for as a compromise of a suitable duration for children of this age, and being able to gather reasonably in-depth data. By having only five questions, this ensured that the children were not conceptually over-loaded during the interview duration. These were planned in cooperation with the school administration and class-teachers so that the interviews would take place at a time most convenient and least disruptive. For some schools, this meant selecting a lesson from the schedule, which backed onto a break, or for others this meant that the interviews took place during the second lesson of a double science lesson. One interview took place per group, in a quiet area close to the science classroom. The interviews were audio-recorded and later transcribed.

We chose to adopt the focus group approach, as opposed to one-on-one interviews, in part due to the known propensity of this method when working with vulnerable populations, such as students, to help participants feel more at ease. We hoped that they would therefore be more likely to speak authentically, and so provide valid and reliable data, albeit with a requirement of careful and skillful interviewer guidance (Lewis, 1992). It also allowed for a combination of interviewing techniques to be utilized, such as use of 'group interview' techniques when individual standpoints or opinions were sought (e.g. do you enjoy science? Why/why not?), and also for 'group discussions' whereby the researcher's prompts lead students to discuss questions (Savin-Baden & Major, 2013). By interviewing the children in their normal school and classroom setting, we reasoned that thoughts of science would be fresh in their minds. In this way, we attempted to create a setting for naturalistic enquiry in as far as possible, in other words: "obtaining data in as natural a setting as possible" so as to "minimize influence of an unrealistic research environment" (Newby, 2010, p. 117). We utilized in-depth interviewing techniques, exploring the 'why' question frequently, and adopted a semi-structured approach based upon the interview guide framework. See appendix for interview schedule. This allowed for flexibility to allow participants to elaborate upon issues they considered to be important where appropriate.

The study was reviewed and approved by our IRB, and we also received approval from the educational authority council in Abu Dhabi to approach the schools in order to recruit participants to the study. Both student assent and parental consent were sought for their participation in the study. Verbally, we used child-friendly language to explain to students at the beginning of the interview that they could refrain from answering any question they wished to, and that they could withdraw at any time. We also explained that their names or schools would not be identifiable from any published work in relation to the study.

3.2 Data Analysis

The data transcripts were first read repeatedly by each member of the research team. The analysis of the interview data was predominantly based upon thematic analysis whereby themes and sub-themes were defined, where particular themes clustered in alignment to research questions (Robson, 2011). Initially, preliminary coding and the process of categorizing common codes took place (Johnson & Christensen, 2014) which were then carefully analysed for repetition and saturation, so that themes emerged such as 'more exciting', 'more dangerous' and 'philanthropic'. Within each of these themes, sub-themes were also developed. We were aware of the criticisms surrounding the use of ICR (intercoder reliability), such as that it is sometimes seen as an "unwarranted attempt to import standards derived for positivist research" (O'Connor & Joffe, 2020, p. 4). However, we also believed that a system of coding where we would be accountable to one another as team members was more likely to lead to consistency, subscribing to the view that this self-awareness of our own coding would incentivize to code to a high standard (O'Connor & Joffe, 2020).

We adopted the strategy often used by researchers whereby the principal investigator carried out the initial segmentation based on conceptual breaks, and generation of relevant codes (Campbell et al., 2013). These were then shared with the team, followed by a group discussion of overlaps and divergences (Thomas & Harden, 2008) and adjusted on the basis of these discussions and initial analyses. We also worked with a coding frame which included possible examples of text which would align with this code, and also exclusions to that particular code (Roberts et al., 2019). This helped to reduce the coding variation between the team members. Our ICR calculation, using Cohen's kappa coefficient, was 0.81. This is deemed by various researchers to be a figure indicative of acceptable and sometimes substantial levels of agreements (Burla et al., 2008; Lombard et al., 2002).

By using narrative analysis, we explored the individuals' spoken statements of their accounts of what they have experienced in the science classroom, and their personal voice describing where their possible selves as scientists in the classroom sit with their perceptions of the work scientists do. In narrative analysis, this individual voice reflects a combination of students' attitudes, concerns and priorities (Newby, 2010) and provides insight into their perspectives. Additionally, codes pertaining to science enjoyment and science classroom experiences as per the research questions, were extracted and categorized. By following this process systematically, we were satisfied that the codes were saturated to these classifications. This process of coding and iterative sub-coding was undergone with each subsequent interview analysis until it appeared that new theories ceased to emerge, but served to support previously established themes (Bryman, 2012). An example of this would be in relation to students' ideas about differences between the science which they did in the class, compared to the science which they perceived scientists to be. We also exercised reflexivity through critical and continual self-reflection about our predispositions, biases and the potential impact of these elements in our interpretation of research findings and conclusions (Johnson & Christensen, 2014).

4 Findings

In alignment with our research questions, the findings are reported here in relation to students' enjoyment of science, the perceived work of scientists, and the differences between classroom science and the work of scientists. Finally, the science career aspirations of students are reported.

4.1 Students' Science Enjoyment

We asked the students to state whether or not they enjoyed science, and if so, what specifically they liked about it, or conversely, if not, what they disliked. The vast majority of children (55/66, 83%) stated that they enjoyed science. Most of their explanations for this enjoyment connected to ideas about the opportunities which science provided to carry out experiments, in particular those with unexpected results or visual drama. Responses related to this include:

> P1: I like doing experiments that amaze me!
>
> P5: That's the best thing. To make an explosion!
>
> P23: Explosions, bombs! Cause I like mixing them.

In the same vein, science was described by some students as being particularly appealing where there were elements of danger. One student described a favourite lesson where they had created 'elephant toothpaste':

> P19: It's dangerous … it's like this stuff that's really, really hot, and it can burn your skin off

Likewise, science activities and experiments which dramatically repulsed were often cited as favourites, such as:

> P42: I like the bread experiment … we were trying to figure out how mould grows on bread… It was disgusting!

Perceived fun and entertainment aspects of science seemed to feature frequently in responses, as these examples illustrate:

> P13: I like it cause it's entertaining.
>
> P27: Experiments are fun.
>
> P51: You might have some fun with learning science and other chemicals and other things.

Students frequently referred to the appeal of novelty in science experimenting, including the appeal of things that they could not do in other subjects. This was particularly the case when a variety of science experiments were offered to them. This added to the appeal, since students appeared to relish not knowing what to expect:

> P63: I like doing experiments – things that I didn't do before.
>
> P18: Science is interesting, it's fun. I think that like there's always something new to find.
>
> P1: I like it because you get to experience more things that you haven't known in your life.

One student referred to enjoying achieving results which were new to themselves and to others, as this comment shows:

> P57: My favourite thing is like, when you do your experiments and you get your results, but everyone gets different results.

Several comments referred to the mixing of chemicals as particularly 'fun'. There were also references to other branches of science, such as biology or astronomy:

> P32: Like, if we learn about digestive and nerve system, I loved it.
>
> P44: I like to see bones. Microscopes and bones.
>
> P 12: I like animals and I like studying about animals, that's a strong factor.
>
> P 35: I love science a lot, because the first time when I've been taught, I learned about the planets and space, and I want to be an astronaut when I grow up. And when I knew that Hazza[2] is the first astronaut in Emirates, I was so happy for him.

There were references to liking science, in particular, when compared with other subjects, namely mathematics, where students felt they did not have opportunities for hands-on learning:

> P17: I like science since I think you get to do a lot of different things. Cause in math, I just feel like it's torture and all you do is just sit, and add, and add, and add.
>
> P35: We get to experiment with stuff, unlike maths.
>
> P2: I love when we do experiments. I don't love when we do math.

Students who stated that that they did *not* like or enjoy science (11/66, 17%) tended to do so for three main reasons; either that they considered it 'boring', or that it was perceived to be a difficult subject which was somehow out of their reach. They also stated not enjoying science because even when they did do experiments, these were considered to be too safe to be exciting, as one student emphatically stated:

[2] Emirati astronaut, Hazza Al Mansouri, made a voyage to the International Space Station as the UAE's first astronaut during the weeks these interviews took place.

P65: I want it to be dangerous!

Those that considered science to be boring, shared the following:

P13: I don't like it when we do reviews. Yeah, it's good for us, but we already know it, why do we study it again?

P39: We watch videos and we write, and then another video and then we write. We take a lot of notes.

A small number of students described their lack of science enjoyment as being chiefly due to their perceptions of science being 'hard' and almost unobtainable, as these two students explain:

P27: I just think, like, science is really hard, and I, like, never get it. Like, what's a pure substance? Even if I study a lot, I'll still not get the best mark.

P59: I'd say no, because sometimes … I don't get it, there's a lot of stuff to remember, like how, okay – so there's oil and water and you have to remember where does the oil go where does the water go, how do you separate them, all those stuff. And for science you have to like know the degrees and all those stuff are really hard.

4.2 Students' Perceptions of Scientists' Work

Students were asked to describe the kinds of work which they believed scientists do, which fell into three main categories. Firstly, that scientists tend to perform work which leads to discovery, trying and failing, or by venturing to unfamiliar and unknown places, as these statements show:

P42: Try to discover new things, no-one ever knew about.

P12: Undiscovered places that no-one has been to.

Interestingly, whilst students had referred to themselves liking science when working with chemistry experiments such as mixing and exploding things, scientists were frequently referred to in these direct responses as working within a diverse range of fields such as geological, meteorological, and pale ontology types of work. For example,

P48: Like, they discover, like, minerals. Like, why natural disasters happen, and why chemical reactions happen.

> P11: They research, like, why the weather changes, different weathers.
>
> P60: I think scientists discover things, like new animals, minerals, and also do experiments with bugs. And they also do experiments, like animals, do experiments with plants, different types of things.
>
> P21: They find bones, like bones from us, and animal bones, they find these.

Scientists' doing good' and contributing positively to global issues was also a theme in the students' responses, from researching cures for illnesses such as cancer, to testing for food safety, for example:

> P27: I think scientists conduct experiments, that people have been trying to find, like to solve it, to solve like cancer.

Some students spoke negatively of the work which some scientists are involved in, such as animal testing for spurious purposes, and explained their feelings of towards this ab/use of science:

> P63: And they make experiments on frogs or about them.
>
> P13: Using creams, they start with animals and then they – if it's good, they put it out for people to use. Many animals die because of the experiments. It's not good.

Some students also talked about ideas of scientists doing 'difficult work' and solving problems, as the following statement demonstrates:

> P18: I think scientists do work that includes a lot of things that humans cannot acquire with special equipment, cannot acquire with just a certain brain – not anybody can be a scientist, you have to be qualified. You have to go through hours of learning and memorizing to actually become a scientist.

Statements such as this one strongly implied a sense of intellectual 'elitism', in who was capable of becoming a scientist.

4.3 Students' Perceptions of themselves as Scientists

Students were asked to discuss whether there was a difference between the work which scientists did, and their own work in the classroom, thereby prompting a reflection of scientist as 'possible self', as referred to earlier. Responses were mixed, though more tended to think that the work was not the same, for example:

> P23: Yes [there is a difference], because the one that we do is basically not a real scientific work ... because we didn't know as much as the scientist workers, they took a long period of years to learn about it, and worked on it, so we can do less than what they can.

An over-riding theme in the responses, to explain why their classwork was not like 'real' scientists, was that scientists chiefly performed work which was much more intellectually or conceptually challenging than their own on a much larger, more complicated scale. On comparative reflection, this led many students to explain the science they do in the classroom as being childish, less important, and less 'difficult', as these statements show:

> P33: You kind of do all the small things, that a scientist does in the classroom, but when you're a scientist you do harder things, very much harder.

> P13: Most scientists are adults. So they are more responsible and they get to do more ... stuff. They get to do more complicated experiments.

> P14: Yeah we do childish stuff and they do more bigger and better things. And they have like different chemicals words, like not child-friendly ones.

Key to the differences between what the students perceived as their own and scientists' work were the students' perceptions of a lack of discovery element in their classroom work. Students frequently referred to the fact that their classroom work generally involved pre-determined steps, ones which it was obvious their teacher had previously set out, removing the 'unknown' element of their work. Some examples of representative statements are:

> P5: Yes, when scientists work they don't know the answer, they are trying to discover something. But in the classroom the teacher already gave you the plan to learn. So you have to find that out but you already know the steps from the teacher.

> P32: I think that scientists are more advanced in their experiments than us doing things in schools. When they're doing the experiments and working, they're going freely, not following ... if someone tells you – we're going to be working on this, you have to follow this and you have to do that

Students also frequently referred to scientists as performing much more 'dangerous' work than them, which pertained to them, therefore, being less like a scientist:

> P24: Of course we're not going to use chemicals or something. The most dangerous thing we did in science was the teacher using a lighter. I think when you're a scientist you use more dangerous chemicals, and more effective stuff to humans.
>
> P56: They do, let's say, dangerous stuff, but we do safe stuff, so we don't get hurt. [Why do you think we don't do anything dangerous at school?] Cause we're still young. If we got hurt it would be the school's responsibility cause if we do something hurt, or dangerous … the school might shut down.
>
> P19: Real scientists would risk their life for doing things in real science. They do things which are … more dangerous than we do.

These descriptions of danger were frequently set against their own experience of 'safe' science, such as:

> P51: They have like safety gadgets, but we don't. We don't need it, we're already safe cause we're doing safe projects.
>
> P48: Because we're just little kids and not sensible.
>
> P5: They (scientists) use goggles and safety things.

Another key theme in students' perceptions of differences in work was that scientists were also more likely to require complex materials and equipment to perform their work, but this was also where students thought there were similarities between their classwork and a scientist's work:

> P28: And [as a scientist] you get to, you have to use all the equipment … they use these … machines that can find, for example, the temperature and these things.
>
> P37: Like, you guys use equipment and stuff, we don't... us, we don't, we just learn and we do projects and we take papers.
>
> P50: I think we don't do the same thing in school, because scientists use more professional things to separate substances, and we use like not very good things to separate substances, because we have like plastic items and like not really stable items, but you guys have like more stable items and better glasses than us.

4.4 Students' Science Career Aspirations

Finally, we asked students about their own career aspirations with regards to being a scientist and whether it was a career they would have considered. Of the 66 students interviewed, 61% stated that they would like to either be a scientist or have a career which used science in some ways. Of those students who answered categorically no, that they did not want to work in science or as a scientist, responses are categorized according to themes such as everything already having been discovered (e.g. *I think they discovered everything in life!)*. Others explained their reticence, regardless of whether they enjoyed science, they worried that (again) they did not have the intellectual capacity, or worried somehow about how 'hard' the subject would be:

> P38: I feel like I would never be qualified to be a scientist because I have a really bad memory. Like a terrible memory.
>
> P50: No. You have to be really good at it.

Those who answered 'yes' explained this either as wanting to again discover new things or places, or cure illnesses, much like the earlier descriptions of scientists' work. Some also expressly wanted to become scientists due to the perceived propensity of the work to provide opportunities to carry out '*dangerous experiments'*. Some students also felt inspired by the possibility of making discoveries of new things, not for discovery itself, but more for the fame that this discovery would bring, e.g. "*if I invent something new, new for the planets then maybe I'm gonna be famous!"*. Many of those who stated 'yes', then elaborated that they were interested in careers which used science, such as engineering or medicine, while others stated that while they liked science and enjoyed science class, they simply had passions in other areas such as teaching: "*I love science, but I don't want to be a scientist. I don't know why, I just love many other things"*. Two mentioned that they would like to be the type of scientist *"who works with animals"*.

5 Discussion

Overall, the findings were positive, with more students than not enjoying their science lessons and some reporting an interest in pursuing a career that involves science. However, qualifying statements by the students suggest that these positive findings are true only when certain criteria are met. We now discuss this, and consider the implications for classroom practice.

5.1 Science Enjoyment

As reported in the previous section, many students referred to chemistry experiments as being synonymous with enjoyable science. There were references to a greater diversity of science work beyond chemistry, such as enjoying lessons in biology, anatomy ('bones') and astronomy. That 83% of the students reported enjoying science is a positive finding, though it was often dependent upon the particular science lesson. It could be perceived as problematic that students particularly warmed to dangerous science examples, since it indicates limited views of what science is, and what scientists do, and in turn of children's sense that they can indeed become scientists. The other aspect of enjoyment was in the concept of novelty and examples of science which were unfamiliar both to themselves and to others. The expectation (or experience) of science as a subject with opportunities for hands on learning meant that it sometimes 'trumped' other subjects deemed to be less interactive. The hands-on element of science lessons has been found to be a common reason that students enjoy the subject (Archer et al., 2010; Osborne & Collins, 2001). There was a strong sense of expectation that science, as a supposedly exciting and active subject, perhaps should not involve such mundane activities as writing and note-taking.

5.2 Scientists' Work

That the children often described the work of scientists as revolving around discovery is not new, and this idea prevails across literature even as far back as the 1960s. This is not necessarily inaccurate. Whether it is the chief, omnipresent work of scientists is debatable though, and poses the question as to why students would not be experiencing a sense of discovery in their own science classrooms. This is explained in students' responses as being due to the fact that teachers not only already know the answers to set problems, explaining why students feel that there is a lack of discovery element in the work. A further key reasoning for why classroom science is not the same was the fact that scientists' work was considered to be dangerous. Many students thought that inside the classroom, teachers had a responsibility to protect children from dangers, so they could not perform 'unsafe' work (again, not untrue). This did however, result in children feeling that what they performed in the classroom did not leave them 'feeling like scientists'.

Some interesting, and positive, perceptions of scientists' work which arose included finding solutions for global problems such as natural disasters and diseases, but also negative work such as testing medicines and cosmetics on animals. Just as their enjoyment depictions related to more diverse science topics than the previously found predicted chemistry related experiences (e.g. Chambers, 1982; Kane, 2016) so too did the descriptions of scientists' work surpass this. A diversity of science work was referred to such as geology, ecology, and animal- and weather-related work. Finally, the idea that science is something difficult and challenging, and outside of the reach of many, was referred to quite often, and was consistent with previous studies (see for example, Archer et al., 2010; Archer et al., 2013). This was positioned within narratives explaining why they themselves did not feel like scientists, for example the student who described science as being something one "*cannot acquire with just a certain brain*". Where comparisons were drawn and seen between classroom science and 'real' science, as some students termed it, they tended to be in relation to processes such as the scientific method ("*we learn, they learn, they hypothesise, we hypothesise'*). A final distinction was in the associations with equipment, which many felt scientists used ('*gadgets*'!) and children in schools did not.

5.3 Science Career Aspirations

Analysis of the 2015 UAE PISA data showed that a one-point increase in enjoyment of science was related to a 19% increase in the probability that a student would aspire to enter a STEM career (Cairns & Dickson, 2021). However, our study did not particularly reflect this, since fewer students were interested in a career either as a scientist, or in which science was involved (61%), than had stated they enjoyed science (83%). In other words, articulation of enjoyment of science in the class did not necessarily indicate a desire or aspiration towards a science career. Reasons for this included perceptions of other career choices as being more appealing, and ideas of science being only for the 'brilliant'. Various work has shown that girls are more susceptible to stereotypes associating genius with science (the 'mad, brilliant scientist!'), which discourages and dissuades many from entering STEM degree programs (Bian et al., 2017; Leslie et al., 2015). Although we do not focus specifically in this paper on gender comparisons, we note that both boys and girls were equally likely to make these kinds of comments in our study. Three-quarters of the students who reported enjoying science did express an interest in science-related careers. The reasons for this interest included being able to indulge in danger, and again the opportunity to discover, and perhaps become famous too along the way.

6 Implications for Classroom Practice

Criticisms of the gap between school science and 'real' science are not new, and calls continue to be made to increase the "real-world" relevance of science to better engage young people (e.g. Archer et al., 2010). The concept of danger appeared, as with many other studies, to be the all-important appeal of science in the current study. It could be worthwhile for science teachers to emphasize elements within science curricula where there are opportunities for visual drama or danger, and which require some level of protection in a strongly monitored and structured environment. However, as mentioned earlier, this favouring of 'dangerous' science does indicate a narrow conception of science. We suggest that science classrooms can instead be framed expansively as spaces that engage students in the ways of talking and reasoning of scientists, such as in the raising of questions, the noticing of inconsistencies, the sense-making about everyday phenomena, presenting those practices as central to the work of science and to sense-making, therefore broadening this view of science as ‘danger’.

Pragmatically, many primary schools do not have access to labs or equipment, and because of this perhaps, science may become infantilized or reduced in form. Whilst we fully appreciate that good science education can take place without the need for labs or indeed expensive equipment, we are anecdotally aware that this often results in teachers performing rather superficial experiments, or simply avoidance of implementing science curriculum where this is not tightly governed in schools. This avoidance might of course be connected to teachers' own science self-efficacy, and may explain some of the disconnect in perceptions between classroom science and ‘scientists’ science’. We suggest that, if it is possible to sometimes expose children, particularly in upper primary levels, to lab environments and where possible to bring elements of those labs (such as boxes of equipment) into the classroom setting, this is more likely to result in students feeling indeed like ‘real scientists’. This may also help teachers; in a study correlating the beliefs and practices of Abu Dhabi's private school teachers of science, over 90% of all participants self-reported beliefs in line with current best practice (for example; group work, learning through inquiry, and time for reflection on learning), but also reported that they do not have enough physical space that enables them to teach science (47%), nor have enough materials and resources (42%) (Kadbey et al., 2015).

The idea of scientists being discoverers can definitely, we feel, be addressed in primary science classrooms. Since it appears that the clear following of steps somehow spelled out for students how predictable their work was, and how therefore 'not' like a scientist it was, teachers can work to develop the element of suspense in lessons, perhaps providing students with more opportunities to plan, and develop learning opportunities such as paths of inquiry learning, and being given flexibility with the way they choose to carry out some investigations such as what they are investigating (choices of topics, or equipment), choice of variables to investigate and control, and so on. Less prescriptive, inquiry focused learning activities that are similar to the work of scientists (used as a context for reflection) have been shown to improve conceptions of the nature of science, conceptions that, in turn, are reported to strengthen students' science identities (Lederman & Lederman, 2014). For example, a study involving year 5 and 6 students in Australian schools participating in an inquiry focused science community of practice, revealed that students' perceptions of science and the work of real scientists were transformed by this approach (Forbes & Skamp, 2019). Students' prior views about science (teacher-directed with limited opportunities for exploration) changed during the ten week project such that they described their current science lessons as collaborative, creative, challenging, and perhaps most importantly as an effective way of understanding "how the world works" (Forbes & Skamp, 2019, p. 480). The possibility of discovering a range of solutions to scientific problems and gathering data through a range of methods also responds to the point students made the present study regarding the appeal of good science lessons that involve features of inquiry as being in novelty; or as they put it, "feeling that your results were different from anyone else's". These inquiry elements of their science classrooms; doing experiments, finding different results, using equipment, are where the students in this study reported feeling *most* like scientists. As such, these are the activities in which students are most likely to develop positive science identities and see their possible selves reflected, and therefore should be emphasized within the classroom. In other words, primary science lessons should be designed in such a way to "offer students avenues to see themselves as people who can do science and see science as a way to engage with the world so they might find their own place in it and make their own contributions" (Kane, 2016, p.115). A further implication of this is relevant to teacher education in the UAE, since this would be important to reflect upon, review existing courses and professional development for pre- and in-service teachers and include this in the curriculum.

Indeed, science identities have been shown to be positively impacted in primary science classrooms where students are referred to as scientists, are encouraged to act like scientists who raise questions and seek answers through research, and in which collaboration, making mistakes, and accidental findings are expected (Kim, 2018). Beyond the primary classroom environment, authentic, inquiry focused science experiences were offered to marginalised grade 10-12 students in an urban high school in the south-eastern US. Students carried out research-related activities for a local biofuels research group and overall developed more diverse perceptions of science and scientists and improved recognition of themselves as scientists (Chapman et al., 2017).

However, when developing students' views on science inquiry in order to encourage the development of their science identities, it should be noted that students (Concannon et al., 2020) and teachers (Cigdemoglu & Köseoğlu, 2019) often have naïve views regarding the purpose of science inquiry processes. Although we would recommend students participate in inquiry-based learning experiences, simply *doing* inquiry in the classroom is not sufficient to develop an *understanding about* inquiry. For example, students may be very good at controlling variables in an experiment but may have little understanding as to why they are doing so. Studies that employ the Views About Scientific Inquiry (VASI) questionnaire indicate that naïve views relating to scientific inquiry can be further improved by explicit-reflective instruction about scientific inquiry (Lederman et al., 2014).

A positive shift in science identities is likely to, at least in part, address the references to science as being hard, challenging, and by implication, out of reach for many. The perception that science is a particularly challenging subject echoes the findings of Archer et al. (2010). They reported that the sentiment of the challenge was appealing to some, causing primary school children to "imagine[d] that the science they would encounter in secondary school would be even harder and that this would be "a good thing" because it would require them to "use our brains more" (p.13). We did not see evidence in our study of this sense of difficulty as inspiring children to rise to the challenge. This implication, that science was accessible to a select few is prohibitive to students making choices, not only long-term in science subjects and degrees, but also in every day participatory choices in the science classroom. Archer et al. (2010) also noted that the perceived necessity of being 'brainy', 'geeky', and 'smart' was perceived by some students as attractive, and perhaps in itself providing appeal: "the hard or difficult nature of science was something that many of the students reported as attractive." (p. 12). Again, we did not see evidence of this potentially positive side to perceptions of difficulty in our study. Therefore, opportunities for small successes in science, to boost self-efficacy, may serve as a starting point. Furthermore, some research has shown effective use of life story documentation, focusing on the life stories of key scientists, and emphasizing elements such as some who had not enjoyed, or were not perceived to be 'good' at science at school, but also those who had struggled with ideas or recognition for many years, had some success at over-turning students' views on the need for brilliance in science (Schinske et al., 2016). Whilst this study was carried out in a community college, involving teenage students, this approach can be emulated with age-appropriate resources at primary and secondary science level too. A focus on a 'cool', but familiar, relatable local personality, such as UAE astronaut Hazza Al Mansouri, may work well in the current context. Learning collaborations in science communities of practice involving practicing scientists and engineers visiting and mentoring primary school children, in the previously mentioned study by Forbes and Skamp (2019), resulted in children expressing a sense of pride, excitement and an increased sense of "awareness" regarding the work of scientists. Such approaches allow for the development of more realistic perceptions of science and scientists and should considerably strengthen science identity in primary aged children.

The research by Kim (2018) indicating the way in which science teachers' positioning can help to develop students' identity, excitement and perseverance, is also important in this context. Since the students in our study refer to a lack of discovery and danger as key reasons for not identifying classroom science with 'real science', teachers can deeply reflect and examine the way in which they position work which should genuinely have elements of inquiry and discovery. For example, the teacher in the Kim (2018) study positioned herself as a scientist, modelling curiosity, excitement, confusion, perseverance; but also as a teacher, learner, researcher, and problem solver alongside her students, to positive effect. The teachers variously positioned the students into these roles as well.

Lastly, to complement the implementation of more authentic science learning experiences, primary science teachers could also develop students' capacity for developing their own experiments, within frameworks of student-led risk assessments. If students are to choose equipment and methods for investigating science phenomena, they need to become competent in identifying, assessing and mitigating risks in the science classroom thus allowing for the safe use of a wider range of substances and equipment. As students develop this capacity the range of experiments they can design and safely execute will increase, thus reducing the gap between school science activities and the practices of working scientists. If we are to provide authentic experiences in terms of discovery and inquiry for children in the science classroom, then we must also provide them with "training" that simulates the health and safety considerations that scientists regularly employ in the workplace.

7 Conclusion

Our study shows that even where students enjoy science, there is still considerable resistance to them identifying their 'possible selves' with scientists. We see this clearly in the way that many students do not identify the work they perform as 'real science', which appears to be remote from the work of the 'real scientist' for many. This study adds to the growing evidence that attitudes towards scientists, and by extrapolation students' science career aspirations, are often steeped in perceptions of elements of genius and brilliance being required. Similarly, to other research in different international regions, the work of a scientist is categorized as chiefly discovery and danger based, leading many students not to see what they do in the science classroom as being 'like a scientist' at all. This favouring of 'dangerous' science does indicate a limited notion of science. However, since these attitudes are likely to impact on their 'possible self' identity as scientists, we consider that we have a responsibility as science educators to provide opportunities for a broader view of 'danger' that incorporates questioning, sense-making, and discovery of the unknown as a vehicle to engage students in science. Unfortunately, the use of danger can also reinforce stereotypes, and this need be handled with care. However, the excitement which danger generates in students could also be inspired through more discovery and exploratory types of science inquiry.

One positive feature of our study findings is that much enjoyment of science lessons is experienced, yet this too is conditional upon particular characteristics. We believe that schools can adopt a two-pronged approach to addressing these issues. One of these is to provide children with more advanced, realistic conceptions of science and scientists by increasing the exposure that students have to 'real' scientists; divulging not only the diversity and relative safety of their work but also the ways in which it is conducted, highlighting collaboration and work in contexts beyond the stereotype of a lone scientist working in a chemistry lab. The second approach we recommend is for teachers to consider adapting science learning experiences to include the elements that would allow students to 'feel more like a scientist'. These might be, for example, the inclusion of specialist equipment even in the general classroom setting, satisfying the proclaimed need for 'gadgets', and to closely review ways in which planned lessons can incorporate elements of inquiry learning which are perceived to be missing, for example by allowing students to make more choices, plan more independently, be assigned different tasks, and not simply *'already know the steps from the teacher'*. Further research should increase the diversity of schools included in order to include a greater number of UAE national student participants.

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Appendix 1. Student Focus Group Interview Schedule

1. What kind of work do you think scientists do? Can you describe this?
2. Do you think there is a difference between the work a scientist would do as a job, and the kinds of science you do in the classroom?
3. Do you like science? (if yes, *, if not, "why not"?) Would you like to have a job where you would use science in the future?
4. * What is your favourite thing about science?

Student teachers' knowledge of students' difficulties with the concept of function

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An important part of the mathematics syllabuses at the secondary school level in most countries is the concept of function. However, secondary school students often experience difficulties with this concept. These difficulties are well-known in the research literature. The study applies the mathematical knowledge for teaching (MKT) framework, including the category knowledge of content and students (KCS). Teachers' ability to anticipate students' difficulties is one aspect of KCS. The aim of this study is to investigate secondary mathematics student teachers' KCS regarding the concept of function. Ten mathematics student teachers participating in a Supplementary Teacher Education Program answered a questionnaire about fictive secondary school students' various difficulties with the concept of function. Follow-up interviews were conducted with four of the respondents. Compared to the findings of previous research on students' difficulties with the concept of function, the respondents in the study sometimes provide reasonable suggestions about the sources of students' difficulties. Some of the respondents demonstrate an aspect of KCS when they suggest that students can reason that a function must be defined by one algebraic expression only, and that students only know about continuous functions. However, no respondent suggests that one source of students' difficulties with a constant function with an implicit domain is the missing domain. In addition, some respondents take for granted that students can interpret the algebraic representation of a piecewise-defined function and translate it into a graph.

Keywords: The concept of function, teacher knowledge, student teacher, mathematical knowledge for teaching (MKT), knowledge of content and students (KCS)

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1 Introduction

The concept of function is an important part of mathematics (Freudenthal, 1983), and of mathematics syllabuses at the secondary school level in most countries (National Council of Teachers of Mathematics, 2017; Swedish National Agency for Education, 2012). However, this concept is difficult to master. Secondary school students often experience difficulties with, for example, constant functions, piecewise-defined functions, and with the one-valuedness property of a function (Clement, 2001; Hatisaru & Erbas, 2017; Tall & Bakar, 1992; Vinner & Dreyfus, 1989). Teachers' knowledge about students' misconceptions, and how to overcome them, is one aspect of *pedagogical content knowledge* (PCK) (Shulman, 1986). Teachers' PCK has a positive effect on students' learning gains (Baumert et al., 2010).

Ball, Thames and Phelps (2008) conceive *mathematical knowledge for teaching* (MKT) as a development of PCK. There is a positive correlation between teachers' MKT and student achievement gains (Hill, Schilling & Ball, 2004; Hill, Rowan & Ball, 2005). Teachers' ability to anticipate and resolve students' errors and misconceptions regarding the concept of function, ability to interpret students' incomplete reasoning, and to anticipate what tasks students will experience as difficult are aspects of *knowledge of content and students* (KCS) which in turn is part of MKT (Nyikahadzoyi, 2015). This knowledge influences the teacher's decision on how to respond to students' questions (Even & Tirosh, 1995). Hence, student teachers' need to develop their level of KCS in order to enhance students' understanding of the concept of function. Therefore, it is valuable to investigate student teachers' knowledge of the sources of secondary students' difficulties with the concept of function.

1.1 Research question

What *knowledge of content and students* (KCS) do the participating secondary mathematics student teachers demonstrate regarding the concept of function? In particular, suggestions about students' difficulties in recognizing constant functions and piecewise-defined functions, difficulties regarding the one-valuedness property of a function, difficulties related to the various representations, and students use of prototype examples are considered.

2 Background

2.1 The concept of function

In this study, we define the concept of function as follows: Let D and S be two nonempty subsets of the real numbers. A function from D to S is a rule that assigns *exactly one* number in S to each number in D. The last part of the definition is referred to as the one-valuedness property of a function. The two sets D and S are called domain and codomain.

Now we give a summary of how this concept is defined in Swedish upper secondary school mathematics textbooks. The concept of variable is usually defined as a letter in an algebraic expression that can assume different values. Two of the most frequently used Swedish mathematics textbooks for upper secondary school (Sönnerhed, 2021) define a function as a relationship between two variables that satisfy the one-valuedness property:

> If the relationship between two variables x and y is such that each x-value, according to some rule, gives a unique y-value, we say that y is a function of x. (Alfredsson, Bråting, Erixon, & Heikne, 2011, p. 288).

> A function is a relationship or a dependency between two variables. It is said that y is a function of x, if for each value of x there is a unique value of y. (Szabo, Larson, Viklund, Dufåker, & Marklund, 2011, p. 162).

In this study, x and y are referred to as the independent and the dependent variable, respectively. The textbooks define the domain of a function as all the values that the independent variable can assume. The codomain of a function is defined as all values of the function when the independent variable is selected from the domain. The domain of a function is most often implicit, that is, not explicitly stated. When the domain is implicit, it is a convention to assume that it is the largest set of real numbers for which the rule of the function makes sense (Adams, 1995).

A function can be represented in different ways, for example, with an algebraic expression, a table of values, a graph, a verbal description (Chang, Cromley & Tran, 2016), or an arrow diagram (Markovits, Eylon, & Bruckheimer, 1986).

2.2 Students' difficulties with functions

The research question concerns student teachers' knowledge of secondary students' difficulties with the concept of function; therefore, a review of previous research literature is presented on such difficulties. In this study, we have chosen five difficulties with the concept of function which are well known from the previous research literature. We have investigated if student teachers know about the sources of these difficulties by requesting their suggestions about fictive students' possible reasoning about the concept of function.

The first example of such difficulties is the constant function with an implicit domain. Students can have difficulties recognizing constant functions with an implicit domain because they expect an independent variable in the algebraic representation of a function; when there are no independent variables in the algebraic expression, they do not regard it as a function (Hatisaru & Erbas, 2017; Tall, 1992; Tall & Bakar, 1992; Vinner & Dreyfus, 1989). Tall and Bakar (1992) ask secondary school students if a horizontal line in a coordinate system represents a function. Almost 50 % of the responses state that it does not represent a function, and about 70 % of the students answer that the corresponding algebraic expression does not represent a function. Thus, the choice of representation of the constant function was critical for the students' misconceptions. Also, secondary mathematics student teachers can have difficulties recognizing constant functions (Viirman, Attorps, & Tossavainen, 2010).

The second difficulty we have chosen to include in this study is that of piecewise-defined functions; that is, functions defined by different expressions on different subdomains. Such functions present difficulties for secondary school students (Hatisaru & Erbas, 2017; Tall, 1992; Tall & Bakar, 1992; Vinner & Dreyfus, 1989). In a survey study, Vinner and Dreyfus (1989) investigate college students' *concept images*, i.e. "the total cognitive structure that is associated with the concept" (Tall & Vinner, 1981, p. 152). The students were supposed to identify the graphs of two piecewise-defined functions — one continuous and one discontinuous — in a questionnaire. Some of the students propose that a function, which is represented with a graph, must be continuous and that it cannot be defined by different expressions on different subdomains (Vinner & Dreyfus, 1989).

The third difficulty we have included in this study is that of the one-valuedness property of a function. This property presents difficulties for secondary school students (Tall, 1992; Tall & Bakar, 1992; Vinner & Dreyfus, 1989), and also for secondary mathematics student teachers (Viirman et al., 2010). About two-thirds of the secondary students in the study of Tall and Bakar (1992) propose that a circle is the graph of a function. The authors' explanation for this misconception is that students' reasoning about functions rely on properties of familiar examples, such as circles or polynomials, and this familiarity evokes the concept of function. Thus, students do not check the one-valuedness property.

The fourth difficulty we have included in this study is that of students' ability to use multiple representations and the ability to translate between representations, for example, an algebraic expression or a graph. This ability develops a better conceptual understanding (Chang et al., 2016; Even, 1998). However, students encounter difficulties when translating between representations of functions (Bossé, Adu-Gyamfi, & Cheetham, 2011; Hitt, 1998).

The fifth difficulty we have included in this study is that of students use of *prototype examples*. Schwarz and Hershkowitz (1999) assert that when students try to understand a concept, some examples are more central in understanding the concept than others. Students use these *prototype examples* to decide whether other examples can be considered to belong to the given concept. Some students use linear and quadratic functions as prototype examples instead of using the definition of the concept (ibid.). In a survey study, Markovits et al. (1986) include a task in a questionnaire with two given points in a coordinate system and instruct the secondary school students to draw graphs of a function that passes through the two points. About half the students only drew the straight line, which is determined by the two points. The authors conclude that "there was an excessive adherence to linearity" (p. 24), and that this may have been caused by the time spent studying linear functions in algebra teaching.

In connection with solving equations, x is often called a variable; Kilhamn (2014) emphasizes that x should be named unknown instead of variable in this context.

2.3 The MKT framework

Teachers need knowledge of the sources of students' difficulties with the concept of function in order to improve students' achievements (Hatisaru & Erbas, 2017; Tasdan & Koyunkaya, 2017). Therefore, a literature review on teacher knowledge is presented.

The prevailing conceptions of teaching among policymakers and teacher educators who were contemporaries of Lee Shulman were that general pedagogical knowledge and some content knowledge was sufficient for teaching. Shulman (1986) argued that this ignored the complexities of teaching; instead, he emphasized the role of content in teaching. Hence, Shulman (1986) proposed a content-specific teacher knowledge referred to as *content knowledge for teaching,* including *subject matter content knowledge, curricular knowledge* and *pedagogical content knowledge* (PCK).

PCK includes the most useful forms of representing the content in a way that make it comprehensible to students. PCK also includes teachers' understanding of students' preconceptions of various topics. If these preconceptions are misconceptions teachers need knowledge of how to identify and overcome them (ibid.). PCK is the most influential of these three categories of knowledge (Ball et al., 2008).

Several well-proven extensions of Shulman's framework *content knowledge for teaching* have been developed with the aim of measuring teachers' knowledge for teaching mathematics (Kaarstein, 2014); for example, *professional knowledge of secondary school mathematics teachers* (Baumert et al., 2010), *teacher education* *and development study in mathematics* (Tatto et al., 2008), and *mathematical knowledge for teaching* (MKT) (Ball et al., 2008).

Ball et al. (2008) conceive MKT as a refinement of two of Shulman's (1986) categories of knowledge: *subject matter content knowledge* and PCK. It consists of six categories of knowledge: C*ommon content knowledge* (CCK) is mathematical knowledge not unique to teaching; it is needed by teachers and non-teachers. *Specialized content knowledge* (SCK) is "the mathematical knowledge and skill unique to teaching, for example, finding an example to make a specific mathematical point" (Ball et al., 2008, p. 400). *Horizon knowledge* is an awareness of the relations between the mathematical topics included in the curriculum.

In *knowledge of content and students* (KCS), knowledge of common student conceptions and misconceptions is combined with knowledge of the content; for example, teachers need to predict whether the students will find the content easy or difficult and they also need to interpret students' incomplete reasoning. A teacher who has seen a misconception of a certain concept before in her teaching is able to recognize it without effort when she encounters the misconception again. In *knowledge of content and teaching* (KCT), knowing about teaching is combined with knowledge of the content, for example, how to sequence the content in the teaching. *Knowledge of curriculum* is self-explanatory. The framework MKT has been derived primarily from elementary school teachers' practices. A summary of the six categories of *mathematical knowledge for teaching* (MKT) is given in Table 1 below.

Table 1. A summary of the components of *Mathematical knowledge for teaching* (Ball et al., 2008).

	Categories of knowledge	**Description**
Subject matter content knowledge	Common content knowledge (CCK)	mathematical knowledge not unique to teaching
	Specialized content knowledge (SCK)	mathematical knowledge and skill unique to teaching
	Horizon knowledge	awareness of the relations between the mathematical topics included in the curriculum.
Pedagogical content knowledge	Knowledge of content and students (KCS)	knowledge of common student conceptions and misconceptions combined with knowledge of the content
	Knowledge of content and teaching (KCT)	knowledge of teaching combined with knowledge of the content
	Knowledge of curriculum	knowledge of curriculum

Nyikahadzoyi (2015) provides two examples of students' difficulties with the concept of function: To translate between representations and to interpret symbols related to functions. Teachers should be aware of the sources of the misconceptions associated with the use of certain representations, such as the function box that can lead to the misconception that all functions can be expressed with a formula. Identifying functions with the algebraic representation only can cause students to perceive functions as rules with a certain regularity, where a change in the independent variable causes a change in the dependent variable. One consequence may be that some students do not recognize constant functions with an implicit domain.

2.4 Teacher knowledge

Even and Tirosh (1995) examine 162 prospective secondary mathematics teachers' knowledge of students' conceptions, and also the sources of students' misconceptions related to functions. The authors use an open-ended questionnaire with fictive students' erroneous answers and misunderstandings of the concept of function. The prospective teachers were supposed to respond to the fictive students' erroneous answers. The authors conclude that several of the prospective teachers did not understand the sources of the students' misconceptions related to functions.

Hatisaru and Erbas (2017) investigate two secondary school teachers' levels of KCS regarding the concept of function with the use of two tasks in a test, where the teachers are asked to provide suggestions on the sources of a fictive student's difficulties regarding the concept of function. One of these tasks concerns a fictive student's difficulty with six different representations of six different functions: an arrow diagram representing a non-injective function, the graph of a discontinuous function, the algebraic representation of a piecewise-defined function, a verbal description of a function, a constant function with an implicit domain, and a set of ordered pairs of numbers. The authors' other task uses two given points in a coordinate system, where the fictive students were supposed to draw graphs of a function that pass through the two given points. This last task is also used by Markovits et al. (1986) with the purpose of investigating whether students use linear functions as *prototype examples* of functions. One of the two teachers demonstrate an aspect of KCS when she says that "the student may have thought that a function should be given by *one* rule only" about the algebraic representation of the piecewise-defined function in the test, and also when she says “it does not involve x”, about the constant function with an implicit domain in the teacher test (ibid, p. 13).

Tasdan and Koyunkaya (2017) investigate prospective secondary mathematics teachers’ MKT regarding the concept of function. The authors’ findings indicate that the three participating prospective teachers had limited knowledge of how to anticipate what students will find difficult, and how to interpret students’ incomplete thinking about the concept of function.

3 Method

3.1 Instruments

A questionnaire and follow-up interviews were used to collect data. Combining these two instruments to collect data about teachers’ knowledge for teaching mathematics is a well-tried method (e.g. Even & Tirosh, 1995; Hatisaru & Erbas, 2017). A questionnaire was designed, including open-ended tasks which are about fictive secondary school students who have various difficulties with the concept of function. The tasks can be found in the Results chapter below. The intention of the questionnaire was to investigate secondary student teachers’ level of KCS regarding the concept of function by requesting their suggestions about fictive students’ possible reasoning. During the design of the questionnaire, inspiration for Task 1 and Task 2 in this study was taken from the tasks concerning students’ difficulties with constant functions and piecewise-defined functions in the studies of Tall and Bakar (1992) and Vinner and Dreyfus (1989). In addition, inspiration for Task 6 was taken from a task in Hatisaru and Erbas (2017) with an arrow diagram representing a non-injective function, and for Task 8 from the task in Hatisaru and Erbas (2017) with two given points in a coordinate system, where fictive students were supposed to draw graphs of a function that pass through the two points.

A semi-structured interview (Bryman, 2012) was used with the purpose to further investigate the student teachers’ knowledge of the sources of students’ difficulties with the concept of function. During the interviews, follow-up questions based on the respondents’ written suggestions in their questionnaires were asked task-by-task. The individual interviews were held in a seminar room and lasted approximately an hour each. All interviews were recorded using a digital audio recorder and transcribed verbatim.

3.2 Participants

The participants of this study were in the middle of a one-year teacher education program at the University of Gothenburg, referred to as the *Supplementary Teacher Education Program* (*Kompletterande pedagogisk utbildning*, KPU) *with increased study rate*. The program was designed for university students who have already completed a bachelor's degree in biology, physics, chemistry, mathematics, or technology, and wanted to become certified teachers in Swedish secondary education. During the clinical training (VFU), they talked about their teaching experiences at the university and at school. These talks took place in the form of dialogue seminars. During the training, their own recorded lessons were used as a basis for discussion and analysis. (Kompletterande pedagogisk utbildning, Ma/Nv/Tk, förhöjd studietakt, 2018).

Thirteen student teachers participated in a seminar entitled "An Introduction to Mathematics Education" which was a part of their teacher education. This was the very first time in their teacher education that they had formal training in mathematics education at the university. The questionnaire was distributed to the student teachers after the seminar. Ten of the thirteen student teachers answered it. They received the following pseudonyms: Bo, Dan, Eric, Fredrik, John, Patrick, Rickard, Sven, Tom, and Viktor. Four of them gave consent to be interviewed: Dan, John, Patrick, and Sven. Six of the ten respondents had only a short teaching experience, and the other four had none, before they were enrolled in the program. During their first semester of the *Supplementary Teacher Education Program,* they gained experience in teaching in clinical training (VFU) at about half time.

The participating student teachers had strong *subject matter knowledge* of the concept of function when they were enrolled in a teacher education program. This was assessed with the aid of a questionnaire concerning fictive secondary school students' erroneous statements about some examples of functions (NN, 2017, p. 93-95). This questionnaire was distributed to the participating student teachers the very first day of their teacher training at the university. At the same time, some background information was collected from the participating teacher students. A summary of the participating student teachers' ECTS points in mathematics, academic degree and teaching experiences is presented in Table 2 below.

Table 2. The participating student teachers' ECTS points in mathematics, academic degree and teaching experience.

Student teacher	ECTS points in mathematics	Academic degree	Teaching experience (month)
Bo	120	Master of Science	No
Dan	90	Master of Science	1-3
Eric	60	Master of Engineering	1-3
Fredrik	60	Master of Engineering	one semester
John	60	Master of Engineering	No
Patrick	220	Master of Science	1-3
Rickard	60	Master of Engineering	No
Sven	45	Master of Engineering	1-3
Tom	60	Master of Engineering	No
Viktor	60	Master of Engineering	one semester

3.3 Method of analysis

Qualitative content analysis is a research method for the analysis of texts aiming at an objective, systematic and replicable account of the content of the text. The method is applicable to different forms of information, for example, transcripts of semi-structured interviews (Bryman, 2012).

The respondents' suggestions, on how the fictive students in the questionnaire may have reasoned, were analysed task-by-task. After reading the questionnaires, quotes were identified where it was clear that the respondents suggest how the fictive students in the tasks may have reasoned about the sources of students' difficulties with the concept of function. Similar quotes were grouped and categories were formulated task-by-task. Hence, the categories emerged during data analysis through qualitative content analysis of data, that is, the categories were not given in advance in the questionnaire. Then the categorizations were validated by two of my supervisors.

The analysis of the questionnaires and the transcripts of the interviews were focused on the respondents' suggestions concerning students' difficulties in recognizing constant functions and piecewise-defined functions, difficulties regarding the one-valuedness property of a function, difficulties related to the various representations, and students use of prototype examples. Two tasks (Task 3 and Task 7) in the questionnaire were excluded from the analysis because they were not considered to contribute to answering the research questions.

Because the questions in the questionnaire were inspired by previous studies, the validity of the present study was improved. Although there were few respondents in the study, combining questionnaire and follow-up interviews to collect data about student teachers' knowledge for teaching mathematics improved the reliability of the study.

4 Results

The results are presented task-by-task. The respondents' suggestions on how the fictive secondary students in the questionnaire may have reasoned are presented in different categories. The various categories are illustrated with one representative quotation of the student teachers' responses. The numbers in parentheses show the number of suggestions in the respective category. In addition, interviews with Dan, John, Patrick, and Sven are presented. A summary of the respondents' suggestions on the tasks in the questionnaire is attached in Appendix A.

Task 1

To a question from the teacher, whether $y = 4$ is a function, Ahmad answers no.

How can Ahmad have reasoned? Please give several possible explanations!

A: Ahmad expects an independent variable in an algebraic expression representing a function. (8)

> He would like to see a dependency on a variable. John

Eight respondents suggest that Ahmad expects an independent variable in the expression $y = 4$. Two of the respondents did not recognize this difficulty.

B: Ahmad reasons that the expression $y = 4$ is an equation with one unknown. (5)

> The student thinks that y is an unknown number in an equation. Patrick

Five respondents suggest that Ahmad may have perceived the expression $y = 4$ as an equation with one unknown instead of a function.

All the four student teachers who were interviewed suggest that Ahmad expects an independent variable in the expression $y = 4$. Patrick suggests that the teacher can write $y = f(x) = 4$ instead of $y = 4$, where f denotes the function. In this way, it becomes clearer that a function is represented, and Ahmad's misconception can be avoided. However, the teacher raises the level of difficulty if she writes $f(x) = 4$ instead of $y = 4$ because students may have difficulty with parentheses in connection with algebraic expressions, according to Patrick.

Dan, Patrick, and Sven also suggest that Ahmad perceives the expression as an equation with one unknown, and that one should distinguish between a variable in connection with functions and an unknown in connection with an equation. Patrick puts the reasoning slightly forward when he suggests that there is a causal relationship between Ahmad lacking a dependence between two variables, and that he interprets the expression as an equation with one unknown instead of a function.

Sven suggests an exercise he calls "guess my rule". He suggests using, among other examples, a rule that always gives the value four[1]. This should give a rewarding discussion among the students about what a function is, according to Sven.

Task 2

> To a question from the teacher, whether $y = \begin{cases} x - 3, if\ x \leq 0 \\ x + 3, if\ x > 0 \end{cases}$ is a function, Benjamin responds no. How can Benjamin have reasoned?

A: Benjamin may have difficulty interpreting this representation. (4)

> Benjamin can have reasoned that a function must be represented with one expression only. Patrick

Four respondents suggest that Benjamin may have difficulty interpreting the algebraic representation of this piecewise-defined function.

B: Benjamin reasons that a function must be continuous. (8)

> There is a jump in the curve. Tom

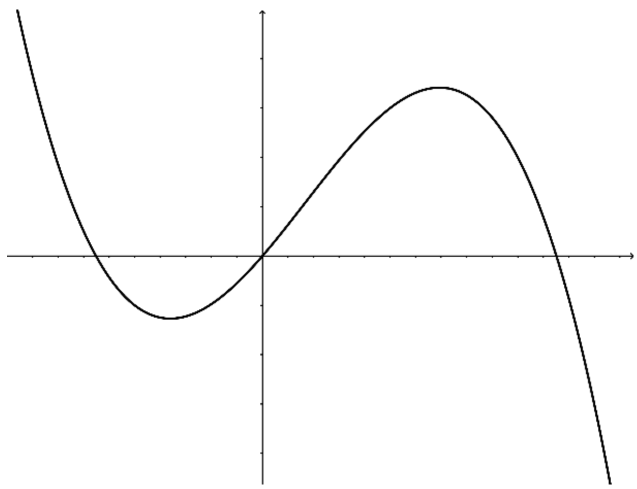
[1]"Guess my rule" is a game in which one student (or the teacher) gives examples of some unknown rule, and the other students try to discover the rule based on the given examples.

Eight respondents suggest that Benjamin can interpret the algebraic representation of the piecewise-defined function, translate it to a graph, discern a jump in the graph and conclude that it is not continuous.

During the interviews, Dan, Patrick, and Sven mention that the difficulty with seeing that a function is represented is that it is written on two lines with two algebraic expressions, using "if". Patrick also suggests that Benjamin can believe that this represents a system of linear equations with two equations and two unknowns. Another suggestion from Patrick is that Benjamin can translate this algebraic representation to a graph, and he concludes that it is not continuous *since there is a jump in the graph*. Because Benjamin takes for granted that a function must be continuous, he draws the erroneous conclusion that this does not represent a function.

Task 4

To a question from the teacher, whether the graph below represents a function, Daniel answers yes. How can Daniel have reasoned?



A: Daniel reasons that the curve looks like the graph of a function; therefore, it represents a function. (6)

> It's similar to the graphs of functions that you work with; for example, a third-degree function. Viktor

Six of the respondents suggest that Daniel reasons that the curve looks like the graph of a function because he recognizes the curve.

B: Daniel reasons that the curve is one-valued and therefore concludes that it represents a function. (7)

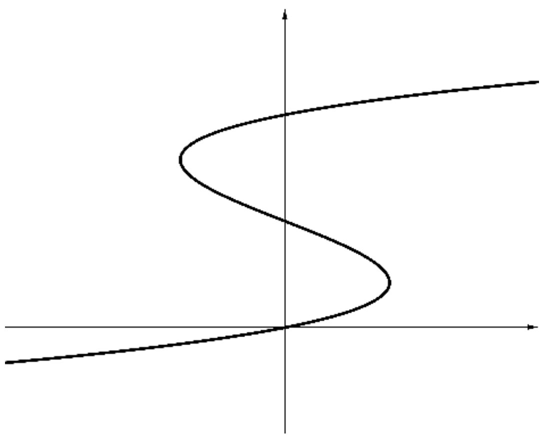
> There is one and only one value of the function to every value of x in the domain.
> Dan

Seven respondents suggest that Daniel may have reached his conclusion using the one-valuedness property of a function.

All student teachers who were interviewed suggest that Daniel recognizes the curve in Task 4 as a third-degree curve; therefore, he does not need to consider the definition of the concept. On the other hand, John, Dan and Sven also suggest that Daniel uses the definition to determine if the curve represents a function: *There is one and only one function value corresponding to each value of the variable.* Daniel may have used the definition of the concept, but it is unusual for students to do so, according to Sven.

Task 5

To a question from the teacher, whether the graph below represents a function, Emilia answers yes. How can Emilia have reasoned?



A: Emilia may have reasoned that all curves are graphs of a function; therefore, this curve represents a function. (8)

> Curves always express functions. Patrick.

Eight respondents suggest that Emilia reasons that all curves are graphs of a function.

B: Emilia reasons that *x is a function of y*. (3)

> The value of the function is on what is usually called the x-axis and the variable is on what is usually called the y-axis. John

Three respondents suggest that Emilia correctly reasons that *x is a function of y*.

C: Emilia assumes that to one value of x there may correspond several values of y. (1)

> Probably she does not have knowledge of the definition of function and thinks that it is perfectly ok that for a given value of x there are several values of y. Dan

One respondent suggests that Emilia does not have knowledge about the definition of the concept, because she supposes that for a given value of *x* it can correspond several values of *y*.

All interviewed student teachers suggest that Emilia reasons that *all curves you can draw without lifting the pen are graphs of functions*. This erroneous reasoning may be due to the fact that all curves Emilia have met in school have been graphs of functions.

According to Marton's theory of variation, one must see a colour other than green to understand what green is[2]. Therefore, the student must see something that is not a function in order to understand what a function is, Patrick concludes.

Dan suggests that Emilia can reason that *if you rotate the S-shaped curve a quarter of a turn, it looks like an ordinary third-degree curve; therefore, it is a function*. Dan compares this to the fact that when you rotate a triangle you get a congruent triangle.

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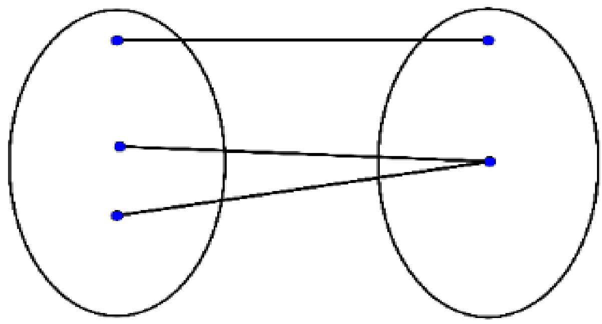
[2] The respondents have had a literature seminar on Ference Marton's book "Necessary conditions of Learning" before the interviews were conducted.

Task 6

To a question from the teacher, whether the diagram below represents a function, Faiza answers no. How can she have reasoned?



A: Faiza does not recognize this representation of a function. (5)

> This is an unusual representation of a function in schools. Students do not meet this visualization very often in secondary schools. I do not think Faiza understands it. It does not look like a graph. Viktor

B: Faiza can read the diagram from right to left because the lines in the diagram lack direction. (5)

> She may have read from right to left and assumed that the domain is on the right, which does not have to be wrong since the teacher has been vague. Eric

All respondents suggest that Faiza may find it difficult to interpret the diagram, either by reading it from right to left, or that she does not recognize this representation at all.

C: Faiza assumes that a function must be injective and hence the diagram does not represent a function. (2)

> She thought that functions must be injective to be called functions. Dan

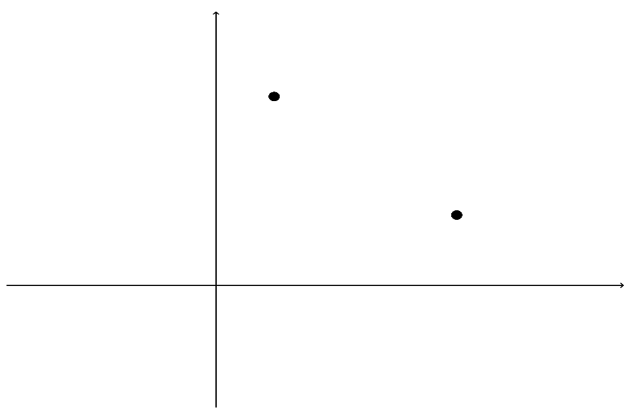
Two respondents suggest that Faiza equates *functions* and *injective functions*; therefore, the diagram does not represent a function.

During the interview, Patrick suggests that Faiza does not know that a function has a domain and a codomain; therefore, she cannot interpret the two ovals in the diagram. Dan, John, and Sven suggest that Faiza has read the diagram from right to left; thus, she has changed places of domain and codomain. Dan suggests that this can be explained by that Faiza has an Arabic origin.

Dan and Patrick suggest that Faiza may reason that the diagram does not represent a function because two elements in the domain are connected to one and the same element in the codomain. During the interview, Dan expresses Faiza's possible reasoning as *maybe she thought that there must not be two of something; maybe it is not allowed to be two different x.*

Task 8

Helena is a teacher of mathematics in upper secondary school. She teaches the course mathematics 1c. During a review on functions, she asks the students how many graphs of a function one can draw through the two given points in the coordinate system below. What possible mistakes do you think the students will make? Why do you think the students will make these mistakes?



A: The students will only draw a straight line through (or between) the two points. (9)

> They will only draw straight lines. They see two points and they are used to connecting them. John

Nine respondents suggest that the students will only draw a straight line through (or between) the two given points.

B: The students will also draw the graph of another function, other than a straight line. (2)

> They will say that there are two functions: linear and quadratic, or a few more.
> Dan

Two respondents suggest that the students will also draw a graph of another elementary functions, such as a quadratic function.

C: The students will draw curves that do not represent functions. (2)

> They will draw a curve with several values of y corresponding to one value of x.
> Fredrik

Two respondents suggest that students will draw curves that do not represent functions, such as an S-shaped curve, similar to the curve in Task 5.

During the interviews, John, Patrick, and Sven suggest that students will only draw a straight line through the two given points because linear functions are the only examples of functions they have met in school.

Dan suggests that students will draw graphs of two classes of functions, linear and quadratic, because the students' image of the concept only consists of the examples they have met in school. Instead of seeing the concept of function as a general concept, the student reasons that the only functions that exist are some classes of elementary functions, according to Dan.

Dan describes a lesson where the purpose was to problematize the concept of function: I drew a circle in a coordinate system on the whiteboard and asked my students: "Does the circle represent a function?" A student responded that it could not be a function because there are no points on it. I thought that I must examine how the student reasons here. Therefore, I constructed the graph of a linear function using three points which I marked strongly and then I drew a straight line through the three given points. The student thought it was a function because there were points on it. Then I formulated the following hypothesis for myself. The student perceives the three points as the function and the line as a filling in between the points. I explained that the line consists of infinitely many points, including the three marked points. My conclusion is that you should erase the marked points, after you have constructed the graph.

5 Discussion

The present study investigates secondary mathematics student teachers' level of KCS regarding the concept of function; in particular, the respondents' suggestions about secondary students' potential difficulties recognizing constant functions and piecewise-defined functions are investigated. Also, suggestions about students' difficulties regarding the one-valuedness property of a function, difficulties related to the various representations, and students use of prototype examples are considered. The results of this study are now discussed in relation to previous research.

5.1 Constant function with an implicit domain

Task 1 in the questionnaire concerns students' potential difficulties in interpreting the algebraic expression $y = 4$. Eight of the ten respondents suggest that students expect an independent variable in an algebraic expression representing a function; therefore, students conclude that the expression $y = 4$ does not represent a function. The respondents' suggestions are consistent with the findings of Tall and Bakar (1992), who report that some students do not recognize constant functions with an implicit domain because they expect an independent variable in the algebraic representation of a function.

The suggestions from the respondents in the present study are also consistent with the findings of Hatisaru and Erbas (2017), who propose that teachers demonstrate an aspect of KCS when they suggest that students may have difficulty in recognizing constant functions with an implicit domain.

The algebraic expression $y = 4$ can be interpreted as a function with an implicit domain, for example, the real numbers. Making this interpretation is not at all obvious; instead, you must learn how to do it. It is an aspect of KCS to recognize the difficulty with an implicit domain of a function; however, no respondent in this study explicitly suggests this in connection with Task 1.

Five respondents in this study suggest that students may perceive the expression $y = 4$ as an equation with one unknown instead of a function. Dan, Patrick, and Sven also suggest that one should distinguish between a variable in connection with functions and an unknown in connection with an equation. This suggestion is consistent with the conclusions of Kilhamn (2014).

5.2 Piecewise-defined function

Only four respondents suggest that students may have difficulties in interpreting the algebraic representation of the piecewise-defined function in Task 2. The domain of the piecewise-defined function is split into two parts. However, no respondent in this study explicitly mentions the difficulty with a split domain, although Patrick suggests that students may reason that *a function must be represented with one expression only*. His suggestion about students' erroneous reasoning about piecewise-defined functions is consistent with the findings of Vinner and Dreyfus (1989), who report that some college students propose that a function cannot be defined by different expressions on different subdomains. This result is also consistent with the findings of Hatisaru and Erbas (2017). Therefore, we conclude that it is an aspect of KCS to know about the difficulty with a split domain of a function.

Some of the college students in Vinner and Dreyfus (1989) propose that a function must be continuous. A teacher in the study of Hatisaru and Erbas (2017) suggests that students think that the graph of a function must not be disconnected. These findings are consistent with the suggestions of eight respondents in this study who suggest that students only know about continuous functions. Nevertheless, these respondents take for granted that students can interpret the algebraic representation of the piecewise-defined function, translate it to a graph and discern a jump in the graph. However, if students can interpret the algebraic representation of a piecewise-defined function, then they have probably studied such functions before; therefore, they presumably know about discontinuous functions too. One difficulty in interpreting the algebraic representation of a piecewise-defined function is what a rule of a function is. Knowledge of this difficulty is an aspect of KCS; however, no respondent in this study suggests this.

5.3 The graph of a function

Seven respondents in this study suggest that the student reasons that the curve in Task 4 is one-valued; therefore, the student concludes that it represents a function. On the other hand, six respondents suggest that the student reasons that the curve in Task 4 looks like the graph of a function, for example, a third-degree curve; hence, the student concludes that it represents a function without checking the one-valuedness property. Eight respondents in this study suggest that students suppose that every curve is the graph of a function. Hence, students draw the erroneous conclusion that the S-shaped curve in Task 5 is the graph of a function.

Six respondents in this study suggest that the curves in Task 4 and Task 5 look like the graph of a function. Their suggestions are consistent with the findings of Tall and Bakar (1992), who report that students – when they try to decide whether a given relation represents a function – rely on familiar examples, such as polynomials. This familiarity evokes the concept of function (ibid.).

5.4 Arrow diagram

The diagram in Task 6 can be interpreted as representing a function or as a relation that is not a function, depending on whether you read it from left to right or vice versa. It is drawn without arrows in order to open up for the possibility to read it from right to left, as five respondents in this study suggest that students may possibly do.

All three parts of a function — rule, domain and codomain — are visible in an arrow diagram. All the respondents in this study demonstrate an aspect of KCS when they propose that students may have difficulties interpreting the diagram in Task 6, for example interpreting the two ovals. Patrick also suggests that some students do not even know that a function has a domain and a codomain; for them it is impossible to interpret the diagram. Also, Dan, John, and Sven discuss the possibility that Faiza has changed places of domain and codomain and read the diagram from right to left. Patrick, Dan, John, and Sven demonstrate an aspect of KCS when discussing the difficulty of identifying the domain of a function.

If students can interpret arrow diagrams, they are useful for illustrating the one-valuedness property of a function. With this use of arrow diagrams in mind we can interpret what Dan means when he writes *Faiza thought that functions must be injective* in his questionnaire. We interpret his statement as communicating an idea to the interviewer between two mathematicians. Dan does *not* take for granted that Faiza has acquired the concept of injective function; instead he is describing a source of Faiza's misconception when he says that Faiza reasons that *it is not allowed to be two different x*. In this quotation Dan is referring to the two points in the left oval that are connected to one and the same point in the right oval. In these two quotations he describes the confusion of the idea of injectivity with the one-valuedness property of a function as a possible source of the difficulty in interpreting an arrow diagram. With this interpretation Dan demonstrates an aspect of KCS.

5.5 Prototype examples

Some students use *prototype examples* of functions instead of the definition when they try to decide if a given example represents a function (Schwarz & Hershkowitz, 1999). The purpose of Task 8 in the questionnaire is to investigate whether student teachers know this. Nine respondents in this study suggest that the students in Task 8 will only draw a straight line through the two given points in the coordinate system. These respondents demonstrate an aspect of KCS, because their proposals are consistent with the conclusion of Markovits et al. (1986) who report that secondary school students drew mostly linear functions in connection with a similar task.

6 Conclusions and implications

Teachers' ability to anticipate students' misconceptions regarding the concept of function, is an aspect of KCS, which in turn is part of MKT (Nyikahadzoyi, 2015). Therefore, teachers need KCS to help their students resolve misconceptions about the concept of function.

Compared to the findings of previous research on the sources of students' difficulties with the concept of function, some of the respondents sometimes provide reasonable suggestions about the sources of students' difficulties regarding the concept of function. For example, all the four interviewed respondents demonstrate an aspect of KCS when they suggest that students may have difficulties identifying the domain of a function in connection with interpreting an arrow diagram as a function. Or, for example, when eight respondents suggest that students only know about continuous functions.

However, some of the respondents' level of KCS regarding the concept of function is not sufficiently developed, for example: Two of the respondents never suggest that students can expect an independent variable in an algebraic expression representing a function. No respondent suggests that one source of students' difficulties with a constant function with an implicit domain is the missing domain. As many as six respondents take for granted that students can interpret the algebraic representation of a piecewise-defined function and translate it into a graph. Only six respondents demonstrate an aspect of KCS when they suggest that students do not always check the one-valuedness property of a function when determining whether a given curve represents a function; instead students use familiar examples of functions.

Teachers need to understand students' ways of thinking in order to help and guide them in their knowledge construction (Even & Tirosh, 1995). Because some of the respondents' level of KCS regarding the concept of function is not sufficiently developed, they may face difficulties in helping and guiding students in their future teaching. Therefore, teacher education needs to facilitate the development of student teachers' level of KCS. This can be achieved by including dialogue seminars in teacher education where the sources of students' misconceptions can be discussed. This may allow student teachers to develop the knowledge required to help students to overcome their misconceptions, for example, that the algebraic representation of a function must include an independent variable, or that a function must be represented with one expression only.

Researchers assume that the difficulties with the rule and the domain of a function manifest themselves in connection with, for example, piecewise-defined functions and constant functions with an implicit domain (e.g. Hatisaru & Erbas, 2017; Vinner & Dreyfus, 1989). Only four respondents in this study suggest that students can have difficulties interpreting the algebraic representation of a piecewise-defined function, because they assume that a function must be represented with one and only one expression. Students may interpret this representation as several rules, and not *one* rule (Vinner & Dreyfus, 1989). Therefore, it is important – in teaching and in teacher education – to emphasize that a rule cannot always be defined by one algebraic expression only. Also, Even (1993) emphasizes the arbitrariness of a rule of a function.

Furthermore, when representing functions algebraically, teachers can make an implicit domain visible, just by explicitly defining a domain. This is especially important for constant functions with an implicit domain, because the independent variable is not present in the algebraic expression. However, no respondent in this study explicitly suggests the difficulty with an implicit domain in connection with the algebraic representation of a constant function with an implicit domain.

It is reasonable to assume that in-service teachers continue to develop their level of KCS while teaching mathematics to students, therefore, we propose further research on in-service teachers' level of KCS regarding the concept of function. Because teachers' KCS and KCT are interrelated, we also propose further research on in-service teachers' *knowledge of content and teaching* (KCT) regarding the concept of function; in particular, teachers' choices of appropriate representations and knowledge of advantages and disadvantages of the various representations as well as how to sequence the content in the teaching can be investigated.

Since the construct MKT is mainly derived from elementary school level, concerns about how transferable it is to secondary school level have been raised (Speer, King, & Howell, 2014). Therefore, we propose these concerns regarding transferability as a topic for further research.

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Appendix A

A summary of the respondents' suggestions on the tasks in the questionnaire.

	1	2	4	5	6	8
Bo	B	B	B	A	A	A
Dan	AB	B	B	BC	BC	AB
Eric	AB	A	B	A	B	AC
Fredrik	B	B	A	A	B	AC
John	A	B	AB	B	B	A
Patrick	A	A	AB	A	A	A
Rickard	A	B	A	AB	A	-
Sven	AB	AB	AB	A	BC	A
Tom	A	B	B	A	A	A
Viktor	A	AB	A	A	A	AB

Effect of the REACT strategy on senior high school students' achievement in molecular genetics

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Molecular genetics, a key concept in biology, is found to be very difficult for students at the senior high school level. A situation largely blamed on teachers' instructional approaches. Since the Relating, Experiencing, Applying, Cooperating and Transferring (REACT) strategy is reported to be an effective pedagogical approach for improving students' understanding of science concepts, in this paper, we sought to explore its effectiveness on Senior High School students' achievement in molecular genetics in Ghana. To do this, the embedded mixed methods research design was employed. Two intact biology classes selected through simple random sampling were assigned as experimental and control groups and taught with the REACT strategy and the conventional approach respectively. Quantitative data were obtained with pre-test-post-test control group design and analysed with Independent sample t-test and ANOVA. The qualitative data on students' perception of learning with the REACT strategy was obtained through interviews and analysed thematically. The findings showed that the REACT strategy was more effective for teaching molecular genetics compared with the conventional approach. Although REACT could not bridge the gap between low and high achievers in that group, the performance of low achievers in the REACT group was at par with high achievers in the conventional group. Students perceived the opportunity to search and share information as well as relate new concepts to prior learning provided by the REACT strategy to have facilitated their understanding of concepts in molecular genetics. It is recommended that biology teachers use the REACT strategy to teach concepts students find problematic.

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1 Introduction

Molecular genetics, which is thought to be the cornerstone of modern biology (Rotbain, Marbach-Ad & Stavy, 2005; Gericke & Wahlberg, 2013), continues to be one of the problematic topics for students, especially those at the senior high school level (Marbach-Ad & Stavy, 2000; Tsui & Treagust, 2010; Thörne & Gericke, 2014; Kılıç & Sağlam, 2014; Aivelo & Uitto, 2015; Casanoves, Salvadó, González, Valls, & Novo, 2017). This has been reported in a myriad of studies which showed students at the senior high school level demonstrate a poor understanding of fundamental issues related to molecular genetics (Knippels, Waarlo, & Boersma, 2005; Kılıç & Sağlam, 2014; Solé-Llussà, Casanoves, Salvadó, Garcia-Vallve, Valls, & Novo, 2019). As a result, many biology educators are worried considering the impact knowledge of molecular genetics have on science policy decisions such as genetic screening and genetically modified foods (Duncan, Freidenreich, Chinn, & Bausch, 2011; Freidenreich, Duncan, & Shea, 2011; Aivelo & Uitto, 2019; Solé-Llussà, Casanoves, Salvadó, Garcia-Vallve, Valls, & Novo, 2019).

In Ghana, senior high school students are no exception to poor understanding of fundamental concepts in molecular genetics. This is because the West Africa Examinations Council's Chief Examiners' reports for Senior High School (SHS) elective biology (WAEC, 2011; 2013; 2015; 2016, 2017; 2018), continue to highlight students' poor achievements in concepts related to molecular genetics. Key to students' poor understanding of molecular genetics is the instructional approach adopted by teachers to teach it (Eklund, Rogat, Alozie & Krajcik, 2007; Thörne & Gericke, 2014; Kılıç, & Sağlam, 2014). Therefore, various instructional strategies are suggested in the biology education literature to help address students' difficulties in understanding concepts related to molecular genetics (Rotbain, Marbach-Ad, & Stavy, 2005; Knippels, Waarolo & Boersma, 2005; Gericke & Hageberg, 2010; Gericke & Smith, 2014; Todd & Kenyon, 2015; Casanoves, Salvadó, González, Valls, & Novo, 2017; Nichols; 2018; Solé-Llussà, Casanoves, Salvadó, Garcia-Vallve, Valls, & Novo, 2019). Examples of which include the use of inquiry games (Casanoves, Salvado, González, Valls, & Novo, 2017), teaching genetics with developmental biology lens (Stern & Kampourakis, 2017), using the History and Philosophy of Science ([HPS] approach (Gericke & Smith, 2014), employing the historical models (Kinnear, 1991; Gericke & Hagberg, 2010), using computer animation and illustration activities that mirror real-life situations (Marbach-Ad, Rotbain, & Stavy, 2008), sequencing of genetics content from macro- to micro-levels (Knippels, Waarolo, & Boersma 2005) and employing drawing-based activity (Rotbain, Marbach-Ad, & Stavy, 2005). Despite the suggested instructional methods, molecular genetics continue to be challenging for both teachers and students (Gericke & Wahlberg, 2013; Gericke & Smith, 2014; Stern & Kampourakis, 2017). Given the light advances in molecular genetics shed on our daily lives, other instructional approaches with the potential to enhance understanding must be explored.

Consequently, the current Ghanaian Senior High School Biology curriculum prescribes constructivist-based instructional approaches for teachers to foster students' understanding and, thus, their achievement in concepts related to molecular genetics (Curriculum Research and Development Division [CRDD], 2012). Though the Biology curriculum indicates that teachers should employ constructivist-based instructional approaches, no specific one is recommended. This leads to a situation where teachers end up employing the conventional approach whereby they take centre stage of the teaching and learning process. Although students may be engaged through the conventional approach, such engagement emanates from the teacher and it is relatively minimal, which does not promote effective conceptual understanding and learning (Preszler, Dawe, Shuster, & Shuster, 2007). Therefore, it is interesting to explore how a constructivist-based instructional approach will help improve students understanding of molecular genetics in the Ghanaian context.

Since the REACT strategy is reported to be an effective instructional approach for improving students understanding (Günter, 2018; Karsli & Yigit, 2017; Ültay, Durukan & Ültay, 2015), it is important that it is tried with Ghanaian senior high school students to find its effects on their achievements in molecular genetics. In our effort to identify the efficacy of the REACT strategy in the Ghanaian educational system, the focus was not paid only to the achievement of students but also their perception of the teaching strategy. The exploration of students' perceptions with regards to the new approach was pertinent because it will bring to the fore students' views on the approach (Beatty & Albert, 2016), which can affect its acceptance and also aid in planning effective instructional activities (Yoon, Suh, & Park, 2014). This is because the perceptions students have on instructional approaches are relevant to the teaching and learning process (Tudor, Penlington, & McDowell, 2010; Johnson, 2016) since students' academic achievement is dependent on the perceptions they have on the teaching and learning strategies employed in the classroom (Knight, 1991; Uiboleht, Karm, & Postareff, 2019). As elaborated by Ferreira and Santoso (2008), learners tend to perform better when they have positive perceptions about instructional strategies employed in the classroom.

Hence, in this paper, we describe an attempt to improve students' achievement by empirically exploring the efficacy of the REACT strategy on senior high school students' achievements in molecular genetics. In doing so, the REACT strategy was compared with the conventional approach of teaching. To this end, two null hypotheses and a research question were formulated to guide the study:

1. There is no statistically significant difference between the achievement scores of students taught with the REACT strategy and those taught with the conventional approach.
2. There is no statistically significant difference between the achievement scores of high and low achievers within the treatment groups.

3. What are students' perceptions about the REACT strategy?

1.1 Theoretical framework

Our theoretical framework is grounded in social constructivism, which emphasizes understanding/learning through knowledge construction based on social interaction (Kukla, 2000; Gredler, 2009; Vygotsky, 1978). Social constructivism essentially emphasizes the role of collaboration among learners and the experiences they bring to their learning environment (Lave & Wenger, 1991; McMahon, 1997). Instructional approaches hinged on social constructivism are largely student-centred and foster student collaboration to enhance achievement through peer collaboration, problem-based instruction, and learning with others (Schunk, 2000). Though there are various instructional approaches based on social constructivism, one that has the potential to impact science learning is the REACT strategy (Gökalp & Adem, 2020; Günter, 2018; Ültay & Alev, 2017; Center for Occupational Research and Development (CORD) 2016).

The REACT strategy is reported to be very effective in teaching and learning science concepts that students find very difficult (Crawford, 2001; Günter, 2018). It was introduced by the Center for Occupational Research and Development (CORD) in the United States of America (CORD, 2016). The name is derived from the first letter of the various stages. 'R' is for the relating stage, 'E' is for the experiencing stage, 'A' is the applying stage, 'C' for cooperating and 'T' is for transferring stage.

The first stage of the REACT strategy is relating. Here, learners learn in the context of their life experiences or prior knowledge. The learners' attention is drawn to everyday life experiences, and these experiences are then related to new concepts to be learned or a problem to be solved (CORD, 2016). After learners' previous knowledge have been assessed, an enabling environment and the needed materials are given to them to explore. This occurs at the experiencing stage, which is the second stage of the strategy. Crawford (2001) explained that the experiencing stage is to allow learners to experience activities that are related to real-life occurrences as they learn in the context of discovery, exploration and invention. As opined by CORD (2016), the experiencing stage is regarded to be the core of contextual learning and hence, gets students to be interested in learning with respect to text or audiovisual-based activities or both.

After the experiencing stage comes the applying stage, where opportunity is given to learners to apply what they have learned. During the applying stage, learners apply the new concepts they have learned or information they have obtained in useful context through class activities, laboratory and project works (Ültay, Güngören, & Ültay, 2017). At this stage of contextual learning, guidance is given to learners to apply the new knowledge they have obtained in everyday occurrences (CORD, 2016). Since students learn best through collaboration, the teacher must ensure groups are formed for the students to learn in the context of collaborating with their peers. This is done at the cooperating stage, where students learn by sharing, responding and communicating with other learners (CORD, 2016). Learning in groups may result in some students not participating in the process and thus, the purpose of cooperative learning may not be achieved. Consequently, Crawford (2001) noted that teachers should follow the guidelines for cooperative learning for every learner to get involved.

The transferring stage, which comes after cooperating is the final stage of the REACT strategy, where students learn in the context of utilizing the newly learned concepts in a novel setting (CORD, 2016). As reported by Günter (2018), transferring of learning can be done by learners through building upon new concepts they have learned which are familiar to the novel concepts or topic that is to be learned. A critical consideration of all the stages of the REACT strategy reveals that the context-based REACT strategy hinges on social constructivism. This is because the approach takes into consideration learners' prior knowledge, exploration, cooperative learning and transfer of learning which are advocated by proponents of the social constructivist theory.

Various studies have sought to determine the effectiveness of the REACT strategy by comparing it to other instructional approaches. Ültay and Alev (2017) investigated the effect of REACT strategy on pre-service science teachers' learning in collision, impulse and momentum concepts. The study revealed that REACT strategy was significantly more effective than the conventional teaching model in terms of achievement and eliminated students' misconceptions in the concepts taught. In a similar study by Günter (2018) which investigated the effect of REACT strategy on students' achievement in solubility equilibrium, students instructed through the REACT strategy performed better than those who were taught with the conventional approach. A similar finding was obtained by Gökalp and Adem (2020) when they compared the effect of the 5E model supported with REACT strategy and a computer-assisted 5E model on basic school pupils' achievement on Acids, Bases and Salts. The findings of their study revealed that those taught with the REACT outperformed those exposed to the computer-assisted 5E model. It could therefore be concluded that the REACT strategy is effective for improving learners' academic performance in science.

Students usually come to the classroom with different conceptions about the new concepts to be learnt, hence some studies have focused on the efficacy of the REACT strategy to help students learn correct concepts and do away with their misconceptions or alternative concepts. A study by Ültay, Durukan and Ültay (2015) brought to light that the REACT strategy was effective in eliminating students' alternative conceptions about concepts on solutions. The effectiveness of the approach in eliminating learners' alternative conceptions is also reported by Karsli and Yigit (2017). They employed a one-group pretest-posttest design to investigate the effect of the REACT strategy on correcting 12th-grade students' alternative conceptions about Alkenes. Their study revealed that the REACT approach to learning was efficient in correcting the learners' misconceptions about alkenes. Günter (2018) confirms the effectiveness of REACT strategy in remedying learners' alternative conceptions about science concepts when she investigated the effect of the REACT instructional approach on students' learning of solubility equilibrium. Her study brought to light that context-based REACT strategy reduced the learners' alternative conceptions about concepts on solubility equilibrium.

Aside from its effect on the learning outcomes, students have been found to have positive perceptions of the REACT strategy as a teaching and learning approach (Günter, 2018; Karsli & Yigit, 2016). Karsli and Yiğit (2016) conducted a semi-structured interview with 12th-grade students after they had been taught concepts on alkanes using a worksheet developed based on REACT strategy. The results of the content analysis of the study showed that the students perceived the alkane worksheet based on REACT strategy to have connected school knowledge with daily life situations, made chemistry lessons interesting, appealing and motivating. A similar result was obtained by Günter (2018) when she investigated the effect of REACT strategy on students' achievement in concepts on solubility equilibrium and then conducted a structured and semi-structured interview on the students' perception about the REACT strategy.

There is enough evidence in the literature to suggest that the REACT strategy would be effective for teaching concepts that are problematic to students. This is because it is reported to be effective in the cognitive domain (Karsli & Yigit, 2017; Ültay, Güngören, & Ültay, 2017) and the affective domain (Crawford, 2001; Günter, 2018; Karsli & Yigit, 2016). Thus, our attempt to investigate the effectiveness of REACT strategy in the learning of concepts in molecular genetics.

2 Materials and methods

2.1 Research design

As the study aimed to explore the effectiveness of the REACT strategy on senior high school students' achievement in molecular genetics, an embedded mixed methods research design (Creswell, Plano Clark, Gutmann, & Hanson, 2003) was adopted. In this design, a qualitative component was embedded in the main quantitative pre-test-post-test non-equivalent control group design. The qualitative aspect was to identify students' perceptions about the REACT strategy to complement the quantitative experimental approach which was used to obtain information on the effect of the strategy on achievement in molecular genetics. The rationale for using the research design was to gain insights into how the REACT strategy could influence Ghanaian senior high school students' achievement in molecular genetics as well as their perception of the approach as an instructional strategy (Hanson, Creswell, Clark, Petska, & Creswell, 2005; Creswell 2012).

2.2 Participants

The participants of this study consisted of 57 senior high school second-year elective Biology students. To obtain the participants, two schools out of five public senior high schools in a school district (Ajumako-Enyan-Essiam) in the Central region of Ghana were selected using a simple random sampling technique. From each of the two selected senior high schools, one intact second-year Biology class was selected using a simple random sampling technique for the study. The two science classes were then randomly assigned to experimental and control groups. The experimental group had 27 students and were instructed with the REACT strategy, while the control group, with 30 students, were taught using the conventional approach on the concepts of molecular genetics. Lesson plans on molecular genetics were developed for the two groups based on their assigned teaching and learning approach (See Appendix 1 & Appendix 2). Both groups were instructed by the same teacher. The rationale was to make the effect of teacher personality on students' performance constant for the two groups (Huang & Moon, 2009). The teaching and assessment of students, as indicated in the study, took six weeks.

For the qualitative part, 12 students from the experimental group were selected through the stratified random sampling technique for interviews to gauge the students' perception of learning through the REACT strategy. In selecting the students, consideration was given to above average, average and below-average based on their performance in the subject of biology using their biology teachers' classroom assessments. To ensure equal representation of both sexes in the interviews, two males and two females were selected for each of the three categories.

2.3 Instrument

To obtain quantitative data for this study, two achievement tests (pre-test and post-test) consisting of 30 multiple-choice items with four answer options were constructed based on the biology syllabus for Ghanaian senior high schools. The pre-test was based on "diversity in Living things" which students in both groups had already treated. The posttest items were based on the molecular genetics content treated during the experiment. A semi-structured interview guide was used in collecting the qualitative data (see Appendix 3 for the instruments used in the study).

2.4 Pilot testing of instrument for quantitative data

The achievement tests were administered to students in a Senior High School in the Cape Coast Metropolis to determine their reliability. The school used for the pilot testing of the instrument was part of the target population but was not part of the main study. Thirty second-year students took the pre-test and post-test. The students took approximately 60 minutes to complete the tests and the question papers were collected from the students just after the test. Students' scores for the items in the pre-test ranged from 10 to 28 and that of the post-test ranged from 8 to 25. The reliability coefficient for the pre-test was 0.73 and that of the post-test was 0.71. The KR-20 formula was used to calculate the reliability coefficients. KR-20 was used because the test items were of different levels of difficulty and they were also scored dichotomously.

2.5 Quantitative data collection procedure

Students in the assigned groups were first pretested on “diversity in Living things”, which they had already treated. The pre-test was conducted to assess whether the academic achievements of students in the two groups were at par before administering the intervention. The students’ scores on the pre-test were also used to categorize the members of each group (i.e., the experimental and control) into low and high achievers. In this study, students whose scores in the pre-test were above the group mean score were classified to be high achievers and those below, designated low achievers. The stratification was to offer an opportunity to gauge whether the REACT could help bring the low achievers up and, thus, bridge the gap between them and the high achievers. Since the conventional instructional approach mostly employed to teach science in Ghanaian classrooms (Yeboah, Abonyi, & Luguterah, 2019) had not impacted desired learning outcomes and brought lower achievers up (Nwagbo, 2006), we hoped a constructivist approach with attendant cooperating and collaborative tendencies could help low achievers to improve their achievement. After the pre-test, students were instructed molecular genetics with the REACT strategy for the experimental group and the conventional approach for the control group, which lasted for six weeks. The students in both groups were post-tested two weeks after they were taught molecular genetics with the assigned instructional models. Thus, in this study, the content that was taught (molecular genetics) was the same for each group. However, the instructional approach was different for the two groups. The experimental group was instructed through the REACT strategy, while the conventional group were instructed through teacher-centred teaching strategies.

2.6 Qualitative data collection procedure

Qualitative information on the perceptions about instruction with the REACT strategy was obtained through a semi-structured interview with the 12 students from the experimental group. The selected students were interviewed for an hour.

2.7 Quantitative data analysis procedure

The quantitative data were analyzed using independent samples t-test and one-way analysis of variance (ANOVA). Data for testing the first hypothesis was analyzed with independent samples t-test to ascertain if there was any significant difference between the students in the REACT group and those in the conventional group. The effect size was also calculated to identify the practical significance of the statistical result for the difference between the effects of REACT and conventional teaching approaches.

The second hypothesis was tested with a one-way ANOVA to identify if there were differences in the scores of students categorized as high and low achievers in the two groups. Thus, in this hypothesis, there were four groups whose scores were compared against each other.

2.8 Qualitative data analysis procedure

To determine the perception of students about the efficacy of the REACT strategy, a semi-structured interview was conducted with the students. The responses obtained were recorded and transcribed. An initial reading was conducted by the researchers independently to categorize the responses into themes. Researchers met to synchronize their categorizations. Four themes emerged ultimately, under which results were presented and discussed.

3 Results

3.1 Results for the analysis of quantitative data

The first hypothesis sought to determine if there was any statistically significant difference in the achievement scores of students exposed to REACT strategy and the conventional teaching approach. An independent sample t-test was used to analyze the scores with the results presented in Table 1. The results in Table 1 show that there was no statistically significant difference in the mean score of pre-tests between the students chosen for the two groups. Thus, the performance of the students selected to be in the REACT group and those for the conventional group were at par before the experiment. However, on the post-test scores, the REACT group performed better compared with the conventional group. This indicates that after the experiment, students instructed with the REACT strategy performed better than those instructed through the conventional approach. Therefore, the null hypothesis that there was no statistically significant difference in the achievement scores of students exposed to REACT strategy and the conventional approach was rejected. An effect size index of 1.52 was obtained, which according to Cohen (1988) indicates a large effect size for the difference between the post-test scores of the REACT and the conventional groups.

Table 1. Results of Independent Sample T-test for pre-test and post-test Scores of the REACT and Conventional Groups

	Group	N	Mean	T	Df	P
Pre-test	REACT	27	18.15	0.66	55	.947
	Conventional	30	18.07			
Post-test	REACT	27	16.48	5.647	55	.001*
	Conventional	30	11.50			

*Significant @ p < 0.05.

The second hypothesis sought to find out if there was a statistically significant difference in the scores of the achievement levels (high and low) within the groups. That is, a comparison was made among the high achievers in REACT, low achievers in REACT, high achievers in the conventional group and the low achievers in the conventional group. To be able to ascribe any difference to the treatment, the initial comparisons were made before the experiment. The pre-test scores of the achievement groups were compared using one-way ANOVA. As shown in Table 2, there was a statistically significant difference in the pre-test scores among the groups (F (3, 53) = 30.279, p < .001). Post-hoc comparisons using the Bonferroni test indicated that there was a statistically significant difference in achievement scores between the REACT low achievers (M = 14.57, SD = 3.502) and REACT high achievers (M = 22.00, SD = 2.677, p < .001). There was no statistically significant difference in achievement scores between REACT low achievers (M = 14.57, SD = 3.502) and conventional low achievers (M = 14.29, SD = 2.758, p = 1.000). There was a statistically significant difference in achievement scores between REACT low achievers (M = 14.57, SD = 3.502) and conventional high achievers' group (M = 21.38, SD = 2.473, p < .001) and similar difference was seen in achievement scores between high achievers in the REACT group (M = 22.00, SD = 2.677) and the low achievers in the conventional group (M = 14.29, SD = 2.758, p < .001). There was no significant difference between the achievement scores of high achievers in the REACT group (M = 22.00, SD = 2.677) and the high achievers in the conventional group (M = 21.38, SD = 2.473, p= 1.000). However, there was a significant difference between high achievers in the conventional group (M = 21.38, SD = 2.473) and low achievers in the same group (M = 14.29, SD = 2.758, p = .001). The results of the pre-test revealed that before the experiment, the high achievers in the REACT and conventional groups were better in terms of achievement scores as compared to the low achievers in both groups. However, the performance of high achievers in both groups was at par. A similar non-difference in performance was observed between the low achievers in both groups.

The one-way ANOVA was again used to compare the posttest scores of the groups and the results presented in Table 3. As seen from Table 3, the ANOVA test was statistically significant ($F(3,53) = 25.749$, $p < .001$). The null hypothesis which stated that there is no statistically significant difference in the scores of the achievement groups after the treatments is therefore rejected. To identify where the difference lies, post-hoc comparisons using Bonferroni test was conducted. The Bonferroni test indicated that there was statistically significant difference in the achievement scores between the REACT low achievers (M = 14.07, SD = 2.702) and the REACT high achievers (M = 19.08, SD = 2.362, $p < .001$). There was statistically significant difference in achievement scores between REACT low achievers (M = 14.07, SD = 2.702) and conventional low achievers (M = 10.14, SD = 3.134, $p = .002$). There was no statistically significant difference in achievement scores between REACT low achievers (M = 14.07, SD = 2.702) and conventional high achievers (M = 13.69, SD = 2.600, $p = 1.000$). The difference between REACT high achievers (M = 19.08, SD = 2.362) and conventional high achievers (M = 13.69, SD = 2.600, $p < .001$) was significant. Again, there was a significant difference between REACT high achievers (M = 19.08, SD = 2.362) and low achievers in the conventional group (M = 10.14, SD = 3.134, $p < .001$), with similar difference between high (M = 13.69, SD = 2.600) and low achievers (M = 10.14, SD = 3.134, $p = .008$) in the conventional group.

Table 2. Results of One-way ANOVA for Pretest of REACT Low, REACT High, Conventional Low and Conventional High Achievers' Groups

Sources	df	Sum of Squares	Mean Squares	F	p
Between Groups	3	747.333	249.111	30.279	.000*
Within Groups	53	436.036	8.227		
Total	56	1183.368			

Table 3. Results of One-way ANOVA for Posttest of REACT Low, REACT High, Conventional Low and Conventional High Achievers' Groups

Sources	df	Sum of Squares	Mean Squares	F	p
Between Groups	3	569.874	189.958	25.749	.000*
Within Groups	53	391.003	7.377		
Total	56	960.877			

*Significant @ $p < 0.05$.

3.2 Result for the analysis of the qualitative data

Students were asked to express their views about the REACT strategy, after which their responses were analysed. Four themes emerged from the responses obtained viz understanding of the concept, searching for information, relating concepts to prior learning and sharing of information.

Understanding of Concept

The students viewed the REACT to have made them understand the concepts they learned. Students responded that it helped to improve their understanding. Student C argued that *"we understood the topic"*. The approach seems to have also enhanced the confidence and enthusiasm of the students as Student G stated, "*I understood the lesson very well and if they give me any test on it, I can answer*". Students were so much enthusiastic about the REACT to the extent that they wished it could be used for other topics by their regular teachers, as noted by Student H, who asserted that *"I wish all the topics are taught using this approach"*. These quotes from the students buttress the efficacy of the REACT strategy to improve students' understanding of concepts in molecular genetics, as seen in their achievement scores.

Search for information

Another aspect of the REACT that students reported on was the ability to search for information. The students asserted that the approach moved away from the teacher-led teaching they were used to where the information on the concepts was provided by the teacher. "We obtained a lot of information before we went through the lessons and it helped us to have some idea about the topic" Student B. This was further explained by Student F that *"because we were told to find information on the topic, I searched for a lot of information about the topic…and I got everything taught"*. Students finding the exploration for information useful is a good trait required for the 21st century. Thus, teachers should build on this to foster inquiry skills and life-long learning in students.

Relating concepts to prior learning

Students expressed the opinion that they found the idea of linking new concepts to prior learning was very helpful. "*In the course of the lesson, what we have already been taught, which is the 'parts of the cell' made the topic easy*" Student B. A similar view was expressed by Student I that *"we have already learnt the Cell so when you introduced it here we easily understood what you were teaching"*. Although students were not told the stages of the REACT strategy explicitly, they were able to decipher what went on. The ability to relate prior and new learning is a good trait that enhances the transfer of learning. Thus, the REACT strategy seemed to have that innate ability to ensure that students' learning will not be isolated but rather linked and transferred to new situations.

Sharing of information

Again, the students perceived the REACT strategy to had facilitated information sharing among students. For instance, Student J pointed out that "*everybody in my group brought their ideas for us to combine them and so we got more information*". Student E also asserted that "*being in the group helped us to join our hands together through discussion and had solution to the questions that you gave to us*." This means that the REACT strategy allowed the students to take charge of their learning. As a social constructivist strategy, the REACT emphasizes sharing of ideas among students. In view of that students were grouped during the instructional delivery process, and this seemed to have facilitated the sharing of information leading to the maximization of their learning.

The students did not raise any adverse issues regarding their exposure to the REACT strategy. They asserted that "*we couldn't see any negative thing about the lessons"* Student D. Student F also noted that "*all I saw good (sic)"* and went on to say that "*the teaching style was perfect. There was nothing wrong with it*." Student K also stated that "*I did not see anything bad about it. It was good, I hope we will be taught other topic using this method*". Although students generally were impressed with the REACT approach, they expressed concern about the duration of the lessons. Student J noted that "*the lessons took much time so I don't think my teacher will teach this way*". "I don't know if we can cover all the topics if we learn this way" Student L noted. Aside from these misgivings about the duration, the students' perception of the REACT strategy was generally positive which shows that the REACT is an effective strategy that biology teachers could employ to teach concepts in molecular genetics.

4 Discussion

The results of the study showed that students in the REACT group performed better than their colleagues in the conventional group on the post-test implying that the REACT strategy can improve Senior High School students' performance in molecular genetics. This finding is similar to those of (Günter, 2018; Karsli & Yigit, 2017; Ültay, Durukan, & Ültay, 2015), who reported that students who were exposed to the REACT strategy performed better than those who were exposed to the conventional approach. Also, Ültay and Çalik (2016) compared the effect of REACT strategy, 5E learning cycle and traditional approach on Turkish pre-service science teachers' learning of acid and base. Their study revealed that REACT strategy was the most efficient among the three approaches in retaining concepts that have been learned in long term memory.

Effective teachers seek to improve the performance of all students. The conventional approach, which has characterised classroom instruction in Ghana (Yeboah et al., 2019) has been found to consistently improve the performance of high achievers leading to an increasing achievement gap between students. The REACT strategy, being a constructivist approach with its associated elements of cooperation and collaboration, was anticipated to possess the ability to prop up the achievements of the low achievers. However, in this current study, the REACT strategy could not bridge the gap between the low and high achievers within that group. This is similar to the findings of Jelatu, Sariyasa and Ardana (2018) when they found that the REACT strategy led to the higher achievement of high ability students compared with low ability students in the of understanding concepts in geometry.

Although the low achievers in the REACT group could not close the achievement gap between them and their high achieving counterparts in the group, they were able to match the performance of students in the conventional group. The evidence points to the fact that the performance of the low achievers in the REACT group was at par with that of the high achievers in the conventional group after the treatment. This is a tremendous improvement since before the intervention, the high achievers in the conventional group had higher achievement scores than the low achievers in the REACT group. Since these groups of students differed only in the interventions provided, it is not farfetched to attribute the improvement of performance in the low achievers in the REACT group to the approach used to instruct them. Thus, it seems the REACT approach helped the low achievers in that group as compared to the influence of the conventional approach on low achievers.

Various aspects of the REACT strategy can help low achievers to understand the concepts being taught better, thereby leading to improved learning outcomes. For instance, the cooperating phase of the REACT strategy comes with inherent properties whereby students mediate and prop up colleagues' learning ensuring that all succeed (Johnson & Johnson, 1999; Slavin, 1995). Moreover, cooperating in the classroom among learners comes with positive interdependence and individual accountability, which tends to improve students' learning (Johnson & Johnson, 1999; Slavin, 1995). Thus, in general, students cooperating during learning tend to achieve better scores (Kagan & Kagan, 2009).

It could, therefore, be concluded that the REACT strategy is effective in improving the overall achievement of students compared with the conventional approach. Although both approaches could not help in lowering the gap in achievements between low and high achieving students concerning learning of concepts in molecular genetics within each group, there seemed to be an improvement of the achievement of low achievers in the REACT group when compared to high and low achievers in the control group. Thus, the REACT approach has the potential of improving the performance of low achievers more than the conventional approach.

Students' perceptions about learning Biology in one way or the other influence their achievement in the subject. Since Ghanaian students' attitudes towards Biology have not been encouraging (Yawson et al., 2016), any teaching model that is capable of arousing students' interest in learning the subject can be employed in the classroom. The current study found that the REACT strategy made molecular genetics lessons interesting, as it helped students to understand the concepts well (Karsli & Yigit, 2016; Ültay, Durukan, & Ültay, 2015), facilitated collaboration amongst the students as well as improved students' skills for information search. This finding is similar to that of Günter (2018) when students noted that the REACT strategy made them understand the concepts taught, related the new concepts to what they already know and aided them to share information.

5 Conclusion and implications

5.1 Conclusion

Based on the findings of this research, it can be concluded that the REACT strategy is more effective compared with the conventional approach in improving students' academic achievement in molecular genetics. It is also concluded based on the findings that the REACT strategy could not bridge the gap between the achievement of low and high achievers, just like the case for the conventional approach. The inability of the teaching approaches to bridge the gap between the performance of low and high achievers could be due to the difference in the individual construction of knowledge and the difference in the innate ability of students to achieve academically. Again, it can be concluded that students had positive perceptions about the REACT strategy despite the duration it took to complete the lessons.

5.2 Implications for practice

The current study has revealed another constructivist-based model of teaching that can be used in the Ghanaian context to teach concepts in molecular genetics at the Senior High School level. As a result, the REACT strategy can be prescribed in the Biology curriculum for teachers to willfully employ in their instruction to improve students' achievement in the subject.

Further, the REACT strategy could be used as a means to increase the declining interest of students to pursue Biology and its related courses since it is capable of increasing students' motivation to learn the subject. Moreover, since the strategy improved collaboration among students, teachers could use the REACT to help develop collaboration skills among students.

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Enhancing the performance of students in chemistry through flipped classroom with peer instruction teaching strategy

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This study examines the implementation of flipped classroom with peer instruction teaching strategy to Grade 7 public high school students in Laguna, Philippines. To analyze the effect of flipped classroom with peer instruction on Chemistry achievement, a two-group quasi-experimental pretest-posttest research design was used. In addition, student perception and participation were conducted using a post-implementation survey. In the flipped classroom with peer instruction, the students were introduced to the lesson using the science courseware developed by the Department of Science and Technology and YouTube videos as pre-class activities. The in-class activity was focused on answering concept questions through peer instruction. Findings show that the two groups of students significantly increased their Chemistry achievement after the implementation of the teaching strategies. However, the students in the flipped classroom with peer instruction had higher Chemistry achievement, high level of participation, and wide acceptance of the teaching strategy than the control group. With this teaching strategy, the students were able to complete their assigned tasks on time, show cooperative and supportive attitude during classroom discussion and activities, share ideas in class, and show respect for the opinion of others. On the contrary, students in the traditional classroom with peer instruction setup performed poorly on these aspects of classroom participation.

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1 Introduction

Engaging students in the learning process has always been a challenge for teachers. With the students' increased access to technology such as computers, gadgets, and the Internet, many teachers have recognized the potential of these tools in facilitating their classes (Ifenthaler et al., 2018). One strategy that highlights technology integration in teaching and that has become popular nowadays in the Philippines is blended learning (Custodio, 2020).

The flipped classroom, a blended learning approach, is a model of teaching where students get exposure to instructional content using readings, videos, or other learning resources outside the class. Students perform pre-class activities asynchronously. This approach allows the teacher and the students to do active learning activities inside the classroom. More importantly, students get immediate support and feedback from the teacher and fellow students (Srinivasan et al., 2018). Some active learning strategies employed in flipped classrooms include problem-based learning, simulation, debates, and think-pair-share (Gilboy et al., 2014), knowledge sharing, contests, brainstorming, group discussions, practical work, and presentations (Shih and Tsai, 2017), among others. In the Philippines, teachers use active learning activities such as group workshops, worksheets, engage and explore activities, and exercises and problem sets (Camiling, 2017; Gayeta, 2017; Malto et al., 2018).

Research studies on flipped classroom showed that many teachers were satisfied and would recommend this teaching modality. They also observed improvement in academic achievement (Goodwin & Miller, 2013; Rivero, 2013). For instance, studies conducted on the effects of flipped classroom in the Philippines showed a significant increase in student performance in biology (Malto et al., 2018), physics (Cagande and Jugar, 2018), and trigonometry (Calamlam, 2016; Segumpan and Tan, 2018). In addition, the flipped classroom revealed positive effects in different grade levels starting with science process skills for elementary, junior high school, and college students (Camiling, 2017).

The flipped classroom teaching strategy used in this study integrates peer instruction as a form of active learning activity in teaching Chemistry for Grade 7 students. Peer instruction, developed by Eric Mazur in Harvard University, is an active learning activity which centers on collaborative work (Chou and Lin, 2015). The discussion is initiated with a question which requires application of previously acquired knowledge on a principle in a specific course content. During peer instruction, the teacher monitors and corrects any misconception or issues. With the structure of peer instruction, students acquire more problem-solving skills than what they can develop alone (Morice et al., 2015). Peer instruction keeps even the passive students engaged and see multiple approaches to problems, increases comprehension, and creates a lively classroom atmosphere (Lucas, 2009; Morice et al., 2015). According to Crouch et al. (2007), peer instruction provides immediate feedback to students. In this strategy, the teachers became more satisfied with the increased engagement of the students in class. Students were found to be more satisfied with the course delivery and had higher retention in courses taught using peer instruction.

2 Theoretical and conceptual frameworks

The theoretical framework of this study lies in the context of not using classroom time for lectures. Jean Piaget's theory of learning focused on student-centered teaching (Piaget, 2008). He pointed out that learning progresses as a factor of student's inherent capabilities and of the learning environments where he acquires new information and skills. Environmental factors include the role of the teacher and how actively engaged a student is in the classroom. As implied in Piaget's theory, the classroom should be a place for active learning where the teacher is the facilitator. Moreover, learners can reconstruct "truth" with other learners. Following the constructivist theory of learning, teachers should not simply lecture. Instead, teachers should encourage them to work in groups to think about issues and questions which facilitate cognitive growth and learning.

Learning with peers is also advocated by Lev Vygotsky in his social learning theory. According to his theory, important learning occurs when a student interacts and discusses with peers and/or tutors (Berk & Winsler, 1995). Flipped classroom with peer instruction spurred from Vygotsky's zone of proximal development. Vygotsky (1978) defines the zone of proximal development as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers". Students interacting with competent peers effectively develop competencies and skills. Students learn from the explanation and approach of the more competent peer; in return, the more competent peer also learns as he applies the concept to different situations.

In addition, this new learning model is also anchored on the key findings of the National Research Council's article on "How People Learn?" (Bransford et al., 2000):

1. A deep understanding of factual knowledge, skill in contextualizing facts and ideas and organizing them for retrieval and application is necessary for students to develop competence in an area.
2. Students who were taught using metacognitive approach are more likely to develop autonomy in learning. They define their learning goals and monitor their progress related to the set goals. Thru peer instruction, students can monitor their own understanding. This activity will lead to adaptive expertise (Baroody, 2003).

The learning theories and information on recent technological advancements serve as anchors for the implementation of flipped classroom with peer instruction.

The schematic diagram of the conceptual framework for this study is shown in Figure 1. This study is founded on the assumption that when students come to class equally prepared, they can participate and be more engaged in class. In effect, teachers can better aim for higher level of intended learning goals. Figure 1 shows how the flipped classroom with peer instruction requires individual/independent learning before class and interaction with teacher and peers during class. The pre-class activities using the Department of Science and Technology (DOST) science courseware and YouTube videos prepare students for active engagement in class. Since students learn at different paces, it is expected that some students will learn and grasp lessons ahead of others. This results in an increase in the gap between *what should be taught, what is actually taught,* and *what the students learn* in the lesson. In the flipped classroom with peer instruction approach, students learn and understand the lesson outside the classroom. Class hours will be devoted to higher levels of cognition as the students apply, analyze, evaluate, and create from the information they have acquired in their reading assignments. Through the flipped classroom, students are given the chance to prepare ahead and the teacher can plan the lesson with student capabilities in mind. Improving the performance of the students before class, that is, in a flipped classroom set-up, will likely improve their performance in an active learning activity in class like the peer instruction.

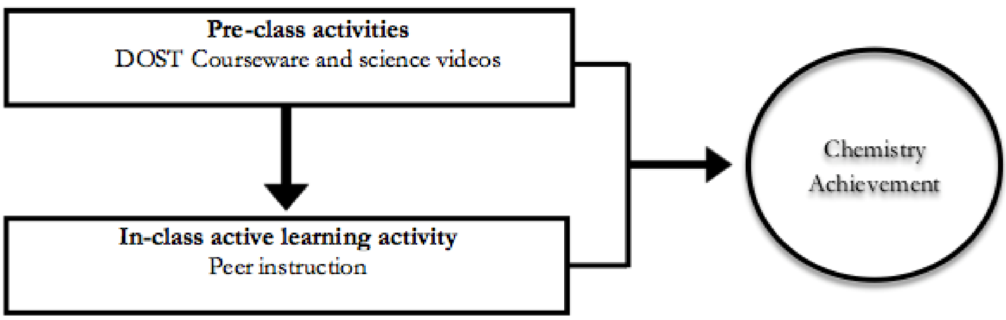

Figure 1. The conceptual framework

This study sought to answer the following questions:

1. Is there a significant difference in the Chemistry achievement of students exposed to the flipped classroom with peer instruction (FCPI) and traditional classroom with peer instruction (TCPI)?

2. What is the level of student participation in the two classroom set-ups?
3. What are the perceptions of students on the two classroom set-ups?

3 Methodology

3.1 Research design

This study used a two-group quasi-experimental pretest-posttest control group research design (Rogers and Revesz, 2019) to compare Chemistry achievement among Grade 7 junior high school students exposed to FCPI (experimental group) and TCPI (control group). A 75-item Chemistry achievement pretest and post-test on the topics discussed during the implementation of the study was administered.

Table 1. The research design.

Intact Groups	Pretest	Treatments	Posttest
Experimental	O1	FCPI	O2
Control	O3	TCPI	O4

In addition, student participation and perception on peer instruction and flipped classroom method were examined through a survey. Students' journal entries were studied to validate and complement the quantitative findings of the study.

3.2 Participants and content of the study

The study was conducted in a public high school situated in a rural area in Sta.Cruz, Laguna, Philippines. Majority of the participants belonged to families with annual income below USD2004.91. Overall, 59 Grade 7 junior high school students (34 males and 25 females) were included in the TCPI and 49 students (28 males and 21 females) in the FCPI. The TCPI set-up had more students with "Satisfactory" Grade 6 science grades ranging from 80 to 84, while the FCPI had more students with "Fairly Satisfactory" Grade 6 science grades ranging from 75-79. The number of participants in the FCPI was lower because a few days after the implementation of the study, six students in the FCPI set-up were dropped from the list because they either transferred to another school or incurred excessive absences. The equivalence of each group was established using two one-sided test (Lakens, 2017; Lewis and Lewis, 2005). Using independent samples *t*-test with equal variances, the equivalence test was significant

(p-value = 0.00799), given equivalence bounds of −2.611 and 2.611 (on a raw scale) and $\alpha = 0.05$. Based on the equivalence test and the null-hypothesis test combined, the observed effect is statistically not different from zero and statistically equivalent to zero.

The implementation of the study was conducted for four (4) months in the first quarter of SY 2019-2020. The topics covered in the first quarter Grade 7 Department of Education science K-12 curriculum were components of a scientific investigation, properties of unsaturated or saturated solutions, concentrations of solutions, properties of mixtures and substances, elements and compounds, properties of acidic and basic mixtures, and metals and nonmetals.

3.3 Treatments

The FCPI teaching strategy was the intervention used in this study. The participants did not have exposure to flipped classroom and peer instruction prior to the conduct of the study. In the flipped classroom setting, the students were introduced to the lesson using the science courseware developed by DOST and from selected YouTube videos before class. Due to a lack of reliable internet connection at home, students access these materials offline using desktop computers and laptops available in the school. In class, the teacher focused on the discussion of concept questions which were answered using peer instruction and Plickers app.

For the control group, students were exposed to TCPI. The same content from the DOST science courseware and YouTube videos were discussed with the students. The lessons were enriched with various activities to cater to the different types of learners in the classroom. However, some activities were given as assignment in the TCPI due to limited class time. Both classrooms were taught by the same teacher and used PowerPoint presentation and LCD projector in delivering instructions in class. Daily lesson logs containing the scheduled laboratory demonstrations, activity sheets, board works, visual aids, recitations, and other activities served as a guide of the teacher in handling the classes. The activities for each group are described in Figure 2 and Figure 3.

Peer discussion was implemented as follows:

- Step 1: A question was posted by the teacher.
- Step 2: The students answered the questions individually. They raised their Plickers™ card representing their answers. The teacher collected on-the-spot student answers using a camera phone. Plickers is an assessment tool

which allow quick assessment of student understanding of the lesson (Plickers, 2019).

- Step 3: When all students have given their answers, the teacher showed the percentage of students who got the correct answer.
- Step 4: The students were prompted to discuss their answers with their preferred partner.
- Step 5: After the peer discussion, the students were allowed to change their answer.
- Step 6: The teacher gave the correct answer and the explanation.

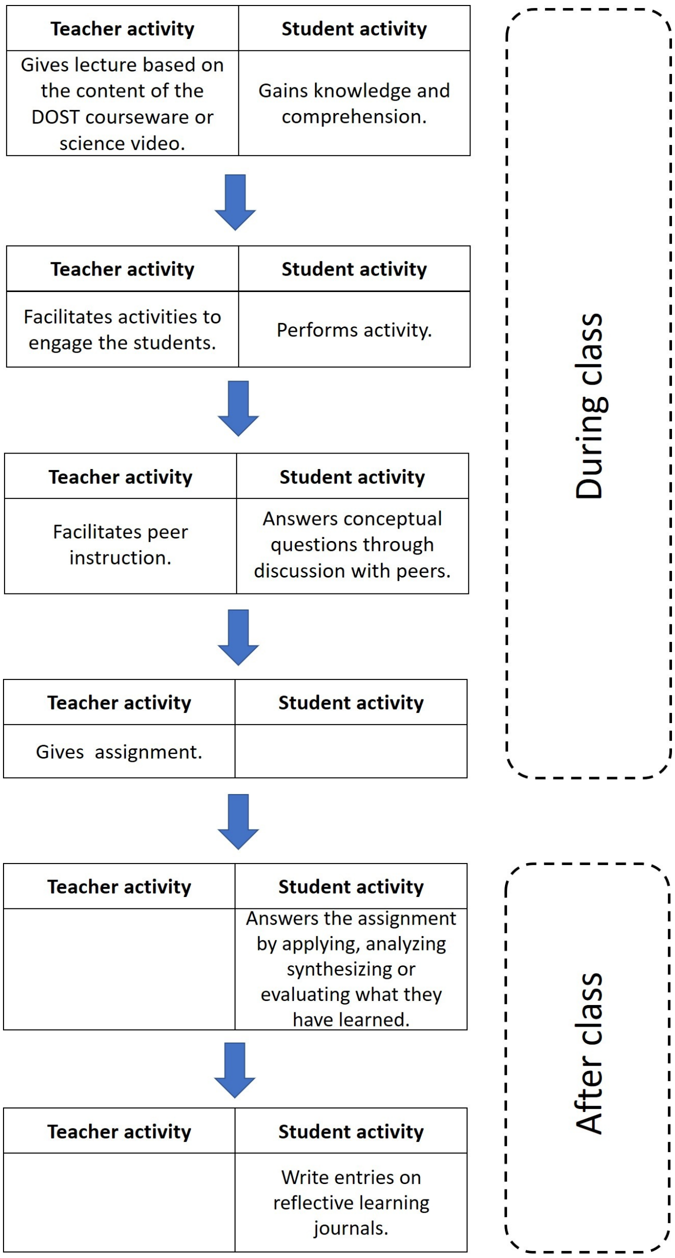

Figure 2. Flow of activities in TCPI.

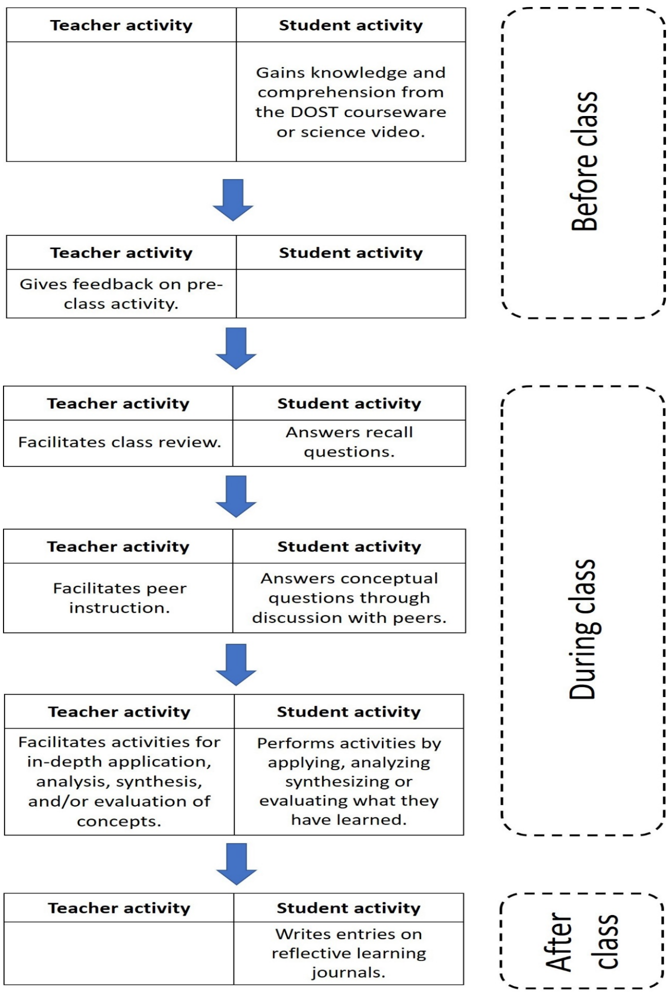

Figure 3. Flow of activities in FCPI.

3.4 Instruments

Chemistry achievement test

Participants in both groups took a 75-item multiple-choice type pre- and final tests in Chemistry covering topics on scientific investigation, solutions, concentration of solutions, substances, mixtures, elements, compounds, acids and bases, and metals and nonmetals. Validity evidence from test content was established by a panel of five science high school teachers, and pilot test to 111 Grade 8 junior high school students who have already taken the enumerated science competencies. Validity evidence based on internal structure was collected using exploratory factor analysis (Arjoon et al., 2013). The Kaiser-Meyer-Olkin (KMO) test was used to measure the sampling adequacy and determine if the data is suitable for factor analysis (Kaiser, 1974). According to Kaiser's guidelines, a suggested cutoff for determining the factorability of the sample data is KMO ≥ 60. The overall KMO is 0.74, indicating that the data is suitable for factor analysis. The result of the KMO test is also supported by Bartlett's Test of Sphericity (p-value = 3.213864e−12). The parallel analysis suggests that the number of factors is 1. The assumption of tau-equivalence as a requirement for Cronbach's alpha was met. The reliability coefficient of the achievement test was estimated at 0.708 using Cronbach alpha.

Post-implementation surveys

Survey questionnaires on student participation in class and perception of the treatments were given to the students at the end of the implementation (Centeno, 2016). Validity evidence from test content was established by a panel of five experienced high school teachers. Similar evidence based on internal structure was gathered for the post-implementation surveys. The overall KMO is 0.66 and 0.86, respectively, indicating that the data is suitable for factor analysis. The result of the KMO test is also supported by Bartlett's Test of Sphericity (p-value = 2.849352e−11 and p-value = 1.635812e−86, respectively). The parallel analyses of the post-implementation surveys do not satisfy tau-equivalence, thus, Omega Total (ω_t) coefficient for estimating reliability was used (McDonald, 1999). The Omega Total was computed to be 0.68 for the participation instrument and 0.91 for the perception instrument.

3.5 Data Collection

Data were collected from the scores of the students in the pre- and final tests for the quantitative research analysis. Survey questionnaires on the level of participation in class and perception of students were given after the implementation of the treatments. Students wrote their learning experiences, learning activities, and other related insights about the teaching strategy in a journal. Data collected from the learning journals were used in validating and interpreting the results of statistical analysis.

3.6 Data Analysis

All statistical analyses were set at a significance level of 0.05. Data collected from the scores of students in the pre- and final tests were analyzed using STATA-IC 12.1. For the hypothesis on the effect of TCPI and FCPI on Chemistry achievement of students, the assumptions of normality and homogeneity of variance were satisfied, thus, the parametric ANACOVA was used to test if there was a difference in the Chemistry achievement of students in the two classroom set-ups. The results were triangulated using journal entries. Data gathered from the post-implementation survey were analyzed using percentages.

4 Results and Discussions

4.1 Chemistry Achievement of Students Exposed to the Classroom Set-ups

Comparison of pretest and post-test scores within groups

Data gathered from the TCPI (p-value = 0.9892) and FCPI (p-value = 0.8545) satisfied the assumption of normality; thus, a parametric t-test was used. The tabulated results of the paired t-test showed sufficient evidence to say that the students in the TCPI and FCPI had an increase in Chemistry achievement as measured using the post-test after the treatment.

Table 2. Chemistry achievement of students in the TCPI and FCPI.

Treatments	**Paired t-test p-value**	**Estimated increase (%)**	**Standard error (%)**
TCPI	0.0022	3.67	1.14
FCPI	0.0016	4.99	1.46

The use of the peer instruction resulted in an increase in the Chemistry achievement of the students. It is worth noting that the use of the flipped classroom contributed to an improved Chemistry achievement for students. This could be attributed to the difference in the level of student participation in each classroom and their perception of the science classroom they were in. As can be seen in succeeding discussions in the section *Student Participation*, the students in the FCPI were more engaged and motivated. Students recognized the value of the activities they did in their science class, and the rapport they had with their classmates and teacher. This indicates that students in the FCPI were seen to show good performance in a sustained manner. The flipped classroom environment greatly affected the performance and participation of students in class (Goss and Sonnemann, 2017). This was reflected in some of the journal entries (JE) of the students in the FCPI:

> "I realized that learning is fun." - *JE1*
> "I learn to value the work of others." - *JE2*
> "The things we need to do in science were not always easy...but I still finished them." - *JE3*
> "Science is my favorite subject." - *JE4*

This study supports the findings of Unal and Unal (2017), which showed high student satisfaction with the flipped classroom model. The same experience was reported by the students who participated in the study of Shih and Tsai (2017) on students' perception of a flipped classroom in a marketing course. The students in Shih and Tsai's study also regarded the flipped classroom strategy to be engaging, interesting, and unique. The students already understood the lesson during class because they watched and studied the video before coming to class. The class hour became more interesting as the students learned by interacting with their classmates and teacher.

Comparison of pretest and post-test scores between groups

Data gathered from the pretest scores of the TCPI and the FCPI satisfied the assumption for normality and homogeneity of variance, thus t-test with equal variance was used for comparison. Results of the statistical analysis showed no sufficient evidence to say that the pretest scores of the students from the two classrooms differ (p - value = 0.4155).

Table 3. Comparison of pretest and post-test scores of students in the TCPI and FCPI.

Classroom set-up	**n**	**Mean score**		**Standard deviation**		***p*-value**	
		Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
TCPI	55	18.3	20.6	0.05	0.09	0.4155	0.4158
FCPI	47	18.8	21.0	0.06	0.10		

Data gathered from the post-test scores of the students from the TCPI and FCPI failed to satisfy the assumption for normality, thus the nonparametric Mann-Whitney U test was used for comparison. Results of the statistical analysis showed no sufficient evidence to say that the post-test scores of the students from the two classrooms differ

(p - value = 0.4158). Further, using two one-sided test using Mann-Whitney Test, a 90% confidence interval is set at [−2.000013, 2.000032]. Since the confidence interval is within the equivalence bounds of −4.945 and 4.945 (on a raw scale), thus, the equivalence test was significant. Based on the equivalence test and the null-hypothesis test combined, the observed effect is statistically not different from zero and statistically equivalent to zero.

Further statistical analysis using ANACOVA with Grade 6 science grade as covariate was conducted to compare the two classrooms in terms of Chemistry achievement. Before proceeding, the assumptions for ANACOVA were checked and satisfied: normal data (p-value = 0.5575), homogeneity of variance (p-value = 0.8952), equality of Grade 6 science grades of students in both groups (p-value = 0.4461) and achievement in Chemistry are linearly dependent with Grade 6 science grade (p-value < 0.0001). Analysis using F-test (p-value = 0.5993) showed no sufficient evidence to say that there is a difference in the Chemistry achievement of students subjected to the two classroom set-ups. Though the FCPI is as good as the TCPI in its ability to increase the Chemistry achievement of the students, the students in the FCPI showed consistent good perception of the teaching strategy. They also showed improved behavior towards learning as reflected in their level of participation in class.

Sufficient evidence on the effectiveness of the FCPI over the TCPI may be gathered through a) longer exposure to the teaching strategy, b) allowing the students to bring home the pre-class activities for them to watch at a longer time or on a more frequent schedule, and c) taking into consideration other factors which can affect academic achievement like delivery of lesson, learning tools used, and teaching modality, among others.

4.2 Student participation

The two classroom set-ups were compared in terms of the level of student participation. The succeeding discussion highlighted the strengths and weaknesses of each classroom set-up in terms of resulting student participation. The summary of results is shown in Table 4.

Table 4. Level of participation of the students in the TCPI and FCPI set-ups.

Manner of Participation	**Traditional Classroom with Peer Instruction (n = 55)**				**Flipped Classroom with Peer Instruction (n = 47)**			
	Never	Seldom	Some-times	Always	Never	Seldom	Some-times	Always
	%	%	%	%	%	%	%	%
1. I attend classes regularly.	38.2	0	5.5	56.4	29.8	0	21.3	48.9
2. I complete assigned tasks on time.	38.2	0	34.5	27.3	29.8	0	21.3	48.9
3. I contribute meaningfully in class discussion by answering questions or asking relevant questions.	38.2	0	50.9	10.9	29.8	0	51.1	19.1
4. I attentively listen to the classroom discussion.	38.2	0	7.3	54.5	29.8	0	8.5	61.7
5. I show no disruptive behaviour in class.	45.5	0	20	34.5	38.3	4.3	19.1	38.3
6. I participate in class activities.	38.2	1.8	9.1	50.9	29.8	4.3	4.3	61.7
7. I show cooperative and supportive attitude during classroom discussion and activities.	43.6	3.6	20	32.7	29.8	4.3	8.5	57.4
8. I share my ideas in class and show respect for the opinion of others.	47.3	1.8	32.7	18.2	29.8	0	21.3	48.9

Attendance in class

The results of the survey showed that more students in the TCPI (56.4 %) attended class as compared to the FCPI (48.9 %). However, it was also evident that cutting classes and absenteeism is a problem in both classrooms. A portion of the students from the TCPI (38.2 %) and FCPI (29.8 %) “never” attended class regularly. An informal interview with the students who were observed to cut classes revealed that some of the students helped their households by working as laborers in the construction of their homes in the afternoon, while some were sickly and lack financial resources to come back to school after eating lunch at home. Majority of the students were cutting classes to spend time with their schoolmates. These students were either repeaters or students who stopped schooling for a long time and then went back to school (balik-aral students). With their situation, it is possible that they got easily bored with the school activities thus decided to skip class.

Completion of assigned tasks. It is noticeable that more students in the TCPI 'never' completed assigned tasks on time (38.2 %). On the other hand, in the FCPI, almost half of the students indicated that they 'always' completed assigned tasks on time (48.9 %). There was an observed big difference between the percentages of students that chose completing assigned tasks on time in the two-classrooms. As reflected in the journal entries of the students in the FCPI, they were eager to finish the assigned activities because they know that it will help them participate in class. In fact, they became worried when they were not able to finish the pre-class activities on time. They were anxious that their classmates will know more than them and that they will not be able to answer the recall questions during class.

Students in the TCPI perceived assignments differently. Assignments served as additional practice for the lesson discussed in class and were given to the students after a formal discussion of the topic was done in class. The assignments had minimal bearing on their performance in the next lesson. However, though this was not fully explored in this study, it is possible that some students just do not like assignments at all (Unal and Unal, 2017). This aspect requires further investigation.

Classroom discussion. Both classrooms have low percentages of students who always participate in classroom discussion by answering or asking relevant questions. Only 10.9 % of the students in the TCPI 'always' participate in classroom discussion by answering or asking relevant questions. More students in the FCPI are always participating in classroom discussion (19.1 %). This shows that the design of the FCPI motivated students to participate and contribute meaningfully in classroom discussions. Their prior knowledge about the lesson allowed them to contribute meaningfully during the classroom discussion.

It is worth noting that although the students in the FCPI were already exposed to the learning materials, they still attentively listened to the classroom discussion (61.7 %) compared with TCPI (54.5 %). Active listening was a specific indicator of student engagement in class as seen from the perspective of the teacher (Nyman, 2015).

With regard to student behavior, more students in the TCPI showed disruptive behaviour in class (45.5 %) than in the FCPI (38.3 %). The disruptive behaviours reported in the journal entries were bullying, making noise, coming into class late, and cheating.

Class/Group activities

Majority of the students in the TCPI (50.9 %) and FCPI (61.7 %) always participated in-class activities. Hands-on, minds-on activities capture the student's interest especially in learning science. Some of the classroom activities done in class were crossword puzzles, role playing, matching types, demonstration of laboratory equipment, and taste tests for acids and bases. This was also in agreement with the characteristic study habits of the students who actively participate in group works and ask for help when they do not understand something in class.

Another criterion in which the two classroom set-ups differed greatly was the cooperative and supportive attitude in classroom discussion and activities. A larger percentage (43.6 %) of students in the TCPI 'never' showed cooperative and supportive attitude during class discussion and activities. Only a smaller percentage (32.7 %) showed 'always' cooperative and supportive attitude during class discussions and activities. On the other hand, cooperative and supportive attitude during classroom discussions and activities (57.4 %) was observed in the FCPI. The students saw their classmates as instruments for their learning during group activities and peer instruction. They also saw the strengths, and weaknesses of their classmates through the different activities in class. According to Vygotsky in his social learning theory, important learning occurs when a student interacts and discusses with peers and/or tutors. Students interacting with competent peers effectively develop competencies and skills. Students learn from the explanation and approach of the more competent peer; in return, the more competent peer also learns as he applies the concept to different situations. This was captured by Porter et al. (2011) from the statements of his students who participated in flipped classroom with peer instruction saying that the bits of information thrown during peer instruction led them to the right answer, deeper understanding of the concepts and the different approaches to a problem.

As reflected in the result of the classroom observations in the FCPI, the students supported each other, and no atmosphere of competition was seen. This conducive classroom environment brought satisfaction to the students. Students in the FCPI wrote in their journal entries:

> "I am happy when me and my classmates show cooperative attitude during group activities." - *JE5*
> "I learn many things during group activities." - *JE6*
> "I learn from my classmates during group activities." - *JE7*
> "Me and my partner helped each other to get the right answer." - *JE8*

The two classrooms differed by 30.7% in students' sharing of ideas in class and showing respect for the opinion of others. More students in the TCPI (47.3 %) 'never' shared ideas in class nor showed respect for the opinion of others. In the FCPI, 48.9 % of the students developed sharing of ideas in class and showed respect for the opinion of others. The first step to develop sharing ideas and showing respect for others in a classroom setting is to help students understand what is expected of them (Hannah, 2013). Once they have a clear idea of how the discussion will proceed, they will be more accountable of their actions. The constant presence of group activities in the FCPI seemed to train the students in sharing ideas and impressed on them the importance of respect for the opinion of others in order to accomplish a task. The peer instruction and group activities conducted in the TCPI could be too brief and not frequent enough to have an effect on student participation.

4.3 Factors that prevent students from participating in class

Based on the self-reported factors which prevented students from participating in class (Table 5), health problem was high in both the TCPI (29.1 %) and FCPI (31.9 %). This could be one of the reasons for frequent absences of students in class. The students in both TCPI (21.8 %) and FCPI (31.9 %) experienced personal/family problems which also prevented them from participating in class.

Table 5. Factors that prevent students from participating in class.

Factors	Traditional classroom with Peer Instruction n = 55		Flipped Classroom with Peer Instruction n = 47	
	**f*	%	*f*	%
1. Health problems	16	29.1	15	31.9
2. Being not in good terms with peers	15	27.3	16	34
3. Laziness	14	25.5	2	4.3
4. Personal/Family problem	12	21.8	15	31.9
5. Discussing things not related to science with my peers	5	9.1	6	12.8
6. The physical environment (too hot or too cold)	4	7.3	11	23.4
7. Lack of resources and school supplies	4	7.3	0	0
8. Being occupied with activities and requirements for other subjects	4	7.3	6	12.8
9. Using gadget inside the classroom	3	5.5	5	10.6

*Frequency of students whose participation in class were affected by the given factors.

Being not in good terms with peers can also prevent students from participating in class, 27.3 % in the TCPI and 34.0 % in the FCPI. It was noticeable that this factor greatly affected the students in the FCPI. It was just logical because with the nature of the FCPI implemented in this study, being focused on group activities and peer instruction, it would be very difficult for a student to participate and perform in class if he is not in good terms with his classmates. Student-student relations play an important role in establishing a classroom environment. This relation can be seen whenever peers praise one another, smile at each other, and exchange personal stories and experiences (Barr, 2016). Aside from its effect in the classroom environment, positive student-student relation motivates students to learn and participate in class (Frisby & Martin, 2010).

However, it was noteworthy that laziness ranked last in the factors that prevent students from participating in the FCPI (4.3 %), while it ranked third in the TCPI (25.5

%). This supported the succeeding discussions that the students in the FCPI were more engaged in class based on this survey. The students in the FCPI were more affected by the physical environment (23.4 %). This should be a special consideration in implementing the FCPI. The classroom should be spacious and well ventilated to be conducive for conducting group activities that require extra movements among the students. This could also be applied to the computer laboratory, which they use during pre-class activities.

4.4 Student perception of the classroom set-ups

The post-implementation survey aimed to determine the perception of students on the implemented teaching strategies, including the materials used in class, the peer instruction (for the TCPI set-up), science videos and courseware (FCPI set-up), assignments and practice exercises, feedback from the instructor, and the technology used in class. The results of the survey are summarised in Table 6 for TCPI and Table 7 for the FCPI.

Table 6. Perception of students of the implemented TCPI set-up.

Criteria	Strongly disagree		Disagree		No opinion		Agree		Strongly agree	
	*f	%	*f	%	*f	%	*f	%	*f	%
1. The materials discussed in class helped me understand the lesson.	12	21.8	0	0	0	0	31	56.4	12	21.8
2. The peer instruction using Plickers allows me to communicate more with my classmates.	4	7.3	9	16.3	0	0	21	38.2	21	38.2
3. The peer instruction using Plickers allows me to solve more problems in class.	0	0	5	9.1	2	3.6	28	50.9	20	36.4
4. I enjoy answering the assignment and practice exercises.	7	12.7	4	7.3	4	7.3	26	47.2	14	25.5
5. My teacher provided me feedback on my assignments.	11	20	5	9.1	5	9.1	27	49.1	7	12.7

6. I was able to solve and analyse practice problems with my classmates during class.	5	9.1	0	0	4	7.3	35	63.6	11	20
7. The peer instruction using Plickers allows me to monitor my progress independently.	5	9.1	5	9.1	2	3.6	29	52.7	14	25.5
8. The peer instruction using Plickers encourages me to work with small groups.	11	20	4	7.3	2	3.6	27	49.1	11	20
9. My instructor connects the assignments to the activities we had in class.	7	12.7	12	21.8	4	7.3	25	45.5	7	12.7
10. I easily adopted to the peer instruction and the technology used.	11	20	4	7.3	7	12.7	28	50.9	5	9.1

*Frequency of student responses describing their perception of the implemented TCPI.

Students in the TCPI agreed that the materials discussed in class helped them understand the lesson (56.4 %). The use of an LCD projector and laptop saves time in displaying instructional materials in class. Also, the size of the visual can be easily adjusted and illustrations that are difficult to draw on the board can easily be displayed in the PowerPoint presentation.

Only 49.1 % of the students in the TCPI agreed that the teacher provided them feedback on their assignments. The teacher was more focused on preparing for the next lesson rather than giving feedback on the previous lesson. In case feedback on an assignment is given, it is likely to be a little late because assignments were submitted after two days, and the feedback will be given the following science meeting. This was supported by the lower percentage (45.5 %) of students agreeing that the teacher was able to connect the assignments to the activities they had in class.

As a consequence, only 47.2 % of the students agreed that they enjoyed their science assignments and practice exercises. Equal percentages of students in the TCPI agreed (38.2 %) and strongly agreed (38.2 %) that the peer instruction using Plickers allowed them to communicate more with their classmates. The students also agreed (50.9 %) that peer instruction allowed them to solve more problems in class. During peer instruction, the students were prompted to discuss their answers with their peers. This is consistent with the findings of Buchart et al. (2009) in his study of peer instruction in a philosophy class. This happens before showing the correct answer for an item. Though this communication lasted for less than a minute, their frequent communication with their peers in class removed their shyness and developed cooperation and participation.

In addition, the peer instruction using Plickers allowed the students to monitor their progress independently (52.7 %). Most of the students in the TCPI perceived that the implemented peer instruction helped promote problem-solving and analysis among classmates (63.6 %). Students also agreed (49.1 %) that the peer instruction using Plickers encouraged them to work in small groups. Discussion among peers is a salient feature of peer instruction activity. This discussion is not limited to just telling the students' answer to the concept question. The students state why they chose that answer, why they think it is the correct answer, or why they think it should be changed.

Though peer instruction and the use of Plickers in class was something new to the students, 50.9 % of the students easily adapted to the peer instruction and the technology used.

Table 7. Perception of students of the implemented FCPI set-up.

Criteria	**Strongly disagree**		**Disagree**		**No opinion**		**Agree**		**Strongly agree**	
	*f	%	*f	%	*f	%	*f	%	*f	%
1. The instructional videos and science courseware I watch before the class help me understand the lesson.	1	2.1	0	0	3	6.4	3	6.4	40	85.1
2. Watching instructional videos and science courseware prepares me to communicate more with my classmates during class.	0	0	0	0	3	6.4	22	46.8	22	46.8
3. Watching instructional videos and science course-ware allows me to solve more problems in class.	0	0	0	0	0	0	6	12.8	41	87.2
4. I enjoy answering the practice exercises in the courseware and video at my own pace.	0	0	0	0	4	8.5	3	6.4	40	85.1
5. My instructor provided me feedback on my pre-class practice problems.	0	0	0	0	0	0	10	21.3	37	78.7

6. I was able to solve and analyse practice problems with my classmates during class.	0	0	3	6.4	6	12.7	25	53.2	13	27.7
7. Watching science videos and science courseware allows me to monitor my progress independently.	0	0	3	6.4	0	0	4	8.5	40	85.1
8. Watching science videos and science courseware encourages me to work with small groups.	0	0	0	0	1	2.1	3	6.4	43	91.5
9. My instructor connects the instructional videos and courseware to the activities we had in class.	0	0	1	2.1	0	0	9	19.2	37	78.7
10. I easily adopted to the flipped classroom and the technology used.	1	2.1	1	2.1	1	2.1	10	21.3	34	72.3

*Frequency of student responses describing their perception of the implemented FCPI.

Though not all students were familiar with using computers, laptops, and tablets, 72.3% easily adapted to the flipped classroom with peer instruction set-up and the technology used. In fact, their journal entries revealed that their limited exposure to such technologies influenced them to participate in the FCPI.

> "The leveled up mode of learning made me more interested to learn." – *JE9*
> "I don't have my own tablet. I was excited every time we use tablets in science." – *JE10*

The students in the FCPI 'strongly agreed' that the implemented strategy was helpful in making the students understand the lesson (85.1 %), allowing students to communicate more with classmates (46.8 %). They also enjoyed both the pre-class activities, and in-class activities (91.5 %). The majority of them strongly agreed that the FCPI allowed them to develop independent learning as they monitor their progress (85.1 %) and learn at their own pace (85.1 %). The students were given guide questions to be answered after watching the science courseware/video. This procedure helped students think about what they had learned (Miller, 2012). The place of the teacher was also recognised in giving feedback on pre-class assignments (78.7 %) and making connection between the pre- and in-class activities (78.7 %). The connection between the pre-class activity and the science lesson is an important consideration in implementing flipped classroom (Erhke, 2016).

With the content delivered online in a flipped classroom, the students were divided into small groups and engaged in meaningful learning activities in class (Danker, 2015). There was also increased participation and time for feedback in the flipped classroom. Working in small groups also initiate participation and lessen intimidation among students. The flipped classroom also helped the students connect new and previous knowledge. Students are actively engaged physically and cognitively in a flipped classroom (Butt, 2014; Gaughan, 2014). Students who are actively engaged tend to perceive the content delivered to be more meaningful (Bormann, 2014). Students look forward to classroom activities and the things they will accomplish for the day.

An aspect of FCPI which needed improvement was the ability of the students to solve and analyse practice problems with their classmates. Only 27.7 % of the students strongly agreed to this, while 53.2 % agreed. To address this, some basic problem solving practice problems could be included in the pre-class activities.

The high level of agreement of the students in the FCPI on the criteria for evaluating the implemented strategy implied that the pre-class activities were found useful and were properly matched and connected to the in-class activities. The success of implemented FCPI was highly reliant on how the pre-class activity prepared the students for in-class activity. The pre-class activity in the implemented FCPI met the targets mentioned by Ehrke (2016): a) equip students with requisite content to follow in class lectures and activities, b) spark learner interest in the lesson, and c) encourage students to become effective note takers. The perception of students in the TCPI was seen to be two sided with the constant presence of students disagreeing on the given statements describing the aspects of the classroom. Also, the highest level of response of the students was at the 'agree' level only. The FCPI design was perceived well by the students compared to the TCPI.

4.5 Student regard of the technology used in the flipped classroom

Use of laptops and tablets to view the science courseware and videos. The students easily adopted to the use of laptops and tablets in learning science. They were engaged in watching the science courseware and videos. This became one of their favorite activities in class. They noted that the videos and courseware were educational and directly related to the science lessons. However, sometimes they cannot understand the content of the video because it was in English and when their classmates were noisy while viewing. Also, it was noteworthy that none in the FCPI reported that the pre-class activities were too much to study or too complicated. Several studies conducted on flipped classroom reported otherwise (Shih, 2017; Szparagowski, 2014).

Regarding the use of technology in class, the students in this study wrote the following in journal entries:

> "The computer activities were exciting and fun." – *JE11*
> "The science videos were easy to understand." - *JE12*
> "I learn from the computer activities." - *JE13*
> "I was amazed by the technology we used in science class." - *JE14*
> "Using computer to learn science was my favorite, especially when I got the correct answer." - *JE15*
> "I was able to complete my notes while viewing the science courseware and videos. - *JE16*

Use of Plickers app during peer instruction. Many students in the FCPI liked the use of Plickers card during peer instruction. They easily adapted to this technology. They also took advantage of the second chance feature of the implemented peer instruction to get the correct answer. In the process, the students developed the skill of choosing the correct answer. Later on during the implementation, students got 100% correct answer during the peer instruction. This motivated them to do more in class. They wrote in their journal entries:

> "I enjoyed the science class because of the Plickers activity (used during peer instruction)." - *JE17*
> "My thinking skills were developed during peer instruction." - *JE18*
> "I accept it when I got the wrong answer, I just do my best in the second answer." - *JE19*
> "My day is complete when we (the class) got 100 % correct answer during Plickers (peer instruction) activity." – *JE20*

4.6 General evaluation of the science class

After focusing on the individual teaching strategies, the students were asked to give a general evaluation of the science class. The answers of the students from the two classroom set-ups are summarised in Table 8.

Table 8. General evaluation of the science class.

	Traditional Classroom with Peer Instruction		**Flipped Classroom with Peer Instruction**	
	Responses	%	Responses	%
Did you feel that you had sufficient support to learn during the course?	Yes	96.7	Yes	100.0
	No	3.3	No	0.0
Were you given sufficient opportunity to practice concepts in class?	Yes	100.0	Yes	100.0
	No	0.0	No	0.0
How?	Assignment	31.3	Through quizzes	53.8
	Seatwork	25.0	Through the problems solved during class	38.5
	Quiz	18.8	During class review	15.4
	Boardwork	12.5		
	Peer instruction using Plickers card	12.5		
	Through the problems solved during class	12.5		
Were you given sufficient opportunity to clarify concepts in class?	Yes	100.0	Yes	100.0
	No	0.0	No	0.0
How?	The teacher explains concepts which the students do not understand	80.0	The teacher explains concepts which the students do not understand	100.0
	The teacher explains as she moves around to check	20.0		

The whole class of the FCPI felt they had sufficient support to learn during the course. The students were provided with well selected pre-class activities aligned with the in-class activities. The class was also enriched with group activities where the students can apply and reinforce what they have learned during the pre-class activities. In terms of opportunity to practice in class, both classrooms got 100 % agreement among the students. The students in the TCPI practiced concepts through assignments (31.3 %) and seatwork (25.0 %), quizzes (18.8 %), board work (12.5 %), peer instruction (12.5 %), and problems solved during class (12.5 %). In the FCPI, the students identified quizzes (58.3 %), problems solved during class (38.5 %), and class review (15.4 %) as the activities where they practiced concepts. It was noticeable that the students in the FCPI can already apply concepts which they learned in the pre-class activities during class review; thus, making the class time interactive. The students in the FCPI were equally equipped with knowledge which they applied in activities and discussion. On the other hand, the students in the TCPI applied concepts in their assignments. As previously discussed, there was less opportunity for such application of concept to be checked or reinforced by the teacher since the class proceeded to the next topic in the following science meeting. The pre-class activities in FCPI prepared the students before coming to class thus maximising the use of class time with high order activities as indicated in the framework of this study.

All the students in both classroom set-ups were given the opportunity to clarify concepts in class. However, in the TCPI, 80.0 % of the students were able to clarify concepts which they did not understand through the teacher's explanation as she delivered the lesson, while the remaining 20.0 % were able to clarify the concepts only when the teacher moved around to check their understanding. In this scenario, it is possible that the students were not able to resolve the questions when the teacher failed to move around. In the FCPI, all concepts were clarified during the class review. Since the class already performed a pre-class activity, the teacher can focus her discussion on concepts that were unclear to the students during the pre-class activity. In addition, since the students already had a background of the lesson, they were able to easily and immediately raise their questions for clarification during the class review.

Another interesting aspect of the implementation of the two teaching strategies were the activities listed by the students they did during science class. This enumerated activity gave a picture of student productivity and student engagement in the two set-ups.

Table 9. Activities of the students during the science class.

Traditional Classroom with Peer Instruction		**Flipped Classroom with Peer Instruction**	
Responses	%	Responses	%
Writing notes	66.7	Peer instruction using Plickers card	73.7
Listening to the teacher	22.2	Listening to the teacher	57.9
Peer instruction using Plickers card	18.5	Studying	26.3
Reading	18.5	Watching science courseware and videos	31.6
Group activity	3.7	Group activity	10.5
Playing around	40.7	Recitation	10.5
Making noise	29.6	Quiz	10.5
Using cellphone	7.4	Review	5.3
Bullying	3.7	Journal writing	5.3
		Cooperating	5.3
		Copying notes	5.3
		Making noise	15.8

The top 4 activities the students in the FCPI do during science class were peer instruction using Plickers card, listening to the teacher, and studying. The students in the FCPI maximised the benefit of the peer instruction because they were prepared before coming to class. This was validated by their responses in the open-ended questions that the pre-class activities support their learning because without it they cannot answer the questions in class. The student responses also mentioned that the flipped classroom strategy allowed them to do more activities in class, allowed them to know what will be discussed in class and that the contents of the science courseware and videos were reinforced in class. They attentively listened and followed the discussion of the teacher. This is an assurance that the role of the teacher in the flipped classroom was strengthened and not threatened.

Other academic-related activities included in their list were: group activity, recitation, quiz, class review, journal writing, cooperating, and copying of notes. The only non-academic activity listed by the students in the FCPI was making noise. The combination of flipped classroom and peer instruction truly made the use of class time productive and academically inclined.

On the other hand, the top 4 activities listed by the students in the TCPI were not so academically inclined: writing down notes, playing around, making noise, and listening to the teacher. Managing disruptive behaviours in class consumes much of the class time instead of the teacher teaching (Guardino and Fullerton, 2010). As a matter of fact, 30 % of the learning time was lost because of disruptive behaviours in class (TALIS Executive Summary, 2009). Though an active learning activity was present in the TCPI, the class time was not maximised as it was too focused on delivering the content to the students. Due to time constraint, the application and practice of the content were commonly done as take home assignments. The students in the TCPI were not able to fully maximise the benefit of the peer instruction and only had limited participation because they only relied on the content to be delivered by the teacher during class.

5 Conclusions

The students in the TCPI and FCPI had an increase in Chemistry achievement as measured by their scores in the post-tests. The gain in learning of the students in the FCPI was evident and they showed observable classroom participation. However, further statistical analysis showed no sufficient evidence to say that these scores are significantly different.

The students in the FCPI showed observable classroom participation than the students in the TCPI. The design of the FCPI implemented in this study made students complete their assigned tasks on time, show cooperative and supportive attitude during classroom discussion and activities, share ideas in class, and show respect for the opinion of others. The students in the TCPI were not able to show evidence of these aspects of classroom participation. Similar level of participation and attitude was seen in the flipped classroom and traditional classroom groups in an English class in Malaysia (Muniandy, 2018).

Students highly accepted the strategies used in the FCPI. Majority of the class 'strongly agreed' that modality aided the students in understanding the lesson, communicating with classmates in class, solving problems, answering practice exercises at their own pace, receiving feedback on the pre-class practice problems, monitoring progress independently, working with small groups, providing connection between instructional videos and courseware and activities in class, and adapting to the flipped classroom and the technology used. The students in this group 'agreed' that the implemented FCPI allowed them to solve and analyse practice problems with their classmates in class. On the other hand, TCPI was accepted by the students as indicated by the number of students who 'agreed' on the criteria used in the study.

The findings of this study highlighted the importance of giving students prior exposure to the learning materials in science whenever appropriate. In this way, the students can prime themselves with the learning content and be prepared to apply or clarify the concept in class. The use of technology in teaching and learning was proven effective in engaging and enticing students to learn. As shown in this study, having limited exposure to digital material didn't hinder the students from adjusting to a certain teaching strategy, instead, it excites them more to learn using gadgets and technology.

Not all teachers are 'digital natives'. Many would be hesitant to try including technology in class. However, this study shows that the use of animations and technology-supported media was highly recommended for abstract Chemistry lessons.

Teachers should not be afraid aim for higher level of learning goals in class. The implemented FCPI approach provided students with sufficient support in terms of materials, technology, scaffolding, and social interactions in class which enabled them to be independent learners. When students are prepared, they know and believe that they can perform in class; and once a small goal is achieved, they will continue trying until they reach our standard.

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Drawing out classroom social climate: The use of participant-produced drawings in research on psychosocial classroom learning environment in the context of school mathematics

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Over the last twenty years, visual methods in childhood research have become more mainstream across social science research. Through this paradigm shift, children became active agents in the research process. Participant-produced drawings in particular allow a constructive process of thinking in action, rather than seeing drawings as simple representations of the participants' worldviews. In this paper, we use participant-produced drawings as a window into students' perceptions of the mathematics classroom learning milieu from a social perspective. The goals of this report are threefold: (1) to conceptualize the complex and multifaceted construct of classroom social climate from the standpoint of primary grade students by using a qualitative research approach (i.e., participant-produced drawings), (2) to evaluate the extent to which participant-produced drawings can be used when researching the construct of classroom social climate, and (3) to provide two analytical tools that can be used in qualitative inquiry on classroom social climate in different mathematics lessons. To conclude, versatile recommendations for theory and practice are discussed regarding the employed methodology (i.e., participant-produced drawings as a visual research method) as well as some possible future directions.

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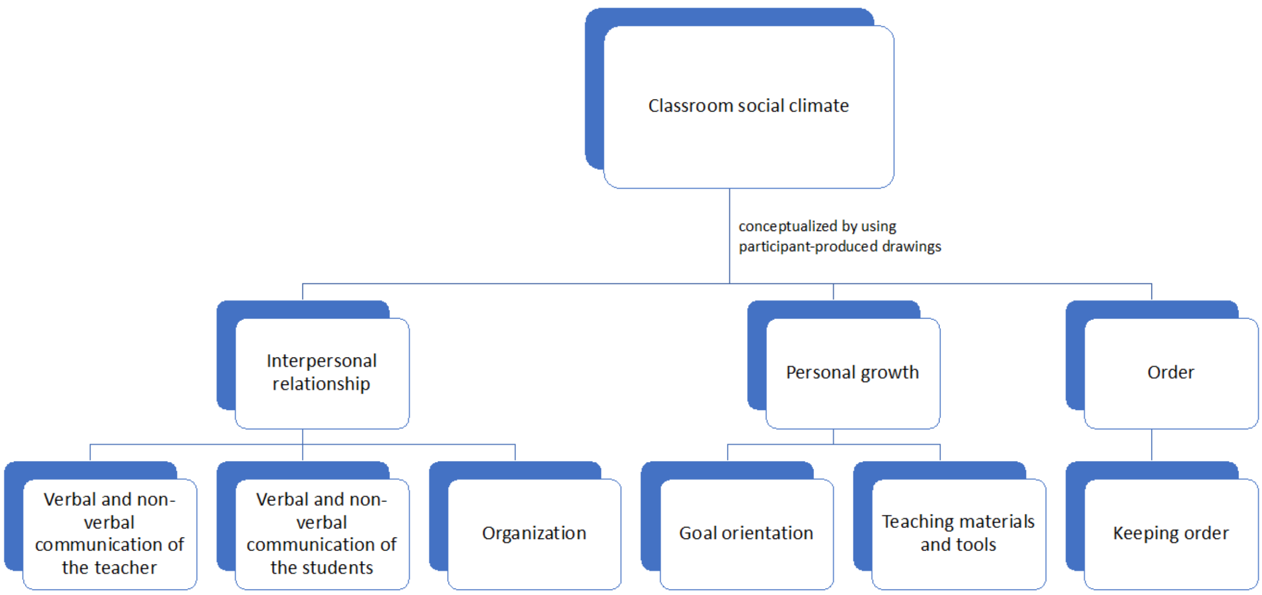

1 Introduction

The classroom is a significant social environment in the multifaceted development of children. It shapes students' essential perceptions, and it allows each child to acquire new concepts and procedures (Ahtee et al., 2016). During their time at elementary school, students spend an average of 20-30 hours a week in the classroom (OECD, 2019). In this time, the teacher has responsibility over the classroom activities, guides, and accompanies these as well as the related learning processes (Ahtee et al., 2016). Research on psychosocial classroom learning environments has a strong tradition due to the early discovery of a relationship between positive classroom climate and academic performance and motivation, engagement, participation, and attitude towards school and teaching (e.g., Trickett & Moos, 1973). Moreover, classroom climate influences students' growth and their academic, social, and emotional development (Evans et al., 2009). The classroom climate in a broader sense may also include physical environments (e.g., school building, classroom furniture) that likewise affect the learning and teaching of mathematics (Fahlström & Sumpter, 2018). Yet, only recently has attention turned to the rich concept of classroom climate, and more research is needed in this context (Evans et al., 2009). Likewise, new methodological approaches are emerging with respect to studying activities in mathematics teaching, evaluating teaching quality, teacher-child relationship quality, and school and administrative adjustment (Ahtee et al., 2016; Harrison et al., 2007; Kearney & Hyle, 2004; Lodge, 2007; Pehkonen et al., 2016).

Questionnaires have mainly been employed to research the complex construct of classroom social climate, with the focus most often on middle and secondary school (e.g., Bülter & Meyer, 2015; Eder, 2002). Little research has been done at the elementary level, which included Grade 4 and higher (Bülter & Meyer, 2015). Younger children, in particular, may have difficulties with reading and understanding survey items and expressing themselves clearly in writing or within interview contexts where they have to talk to an often relatively unfamiliar researcher, or providing verbally rich answers to questions they do not consider relevant (Pehkonen et al., 2016). Furthermore, both methods are particularly time-consuming and accompanied by partially unreliable student answers (Ahtee et al., 2016). Thus, these methods have shown not to be always reliable due to the participants' young age (e.g., Einarsdóttir, 2007; Pehkonen et al., 2016).

In recent decades, however, childhood research has experienced a paradigm shift that has had a comprehensive impact on qualitative research design and methods.

While it was common to view children as objects by using methodologies such as direct observations, interviews, questionnaires, and test procedures, the shift has led to children being increasingly viewed as subjects in the research process (Hill, 1997). Among other things, this shift has led to the increased use of participatory and visual methods and processes in childhood research, such as photography, video, or drawing (Literat, 2013; Veale, 2005). Thus, there is an increased focus on methods in qualitative research that are primarily designed to engage and emphasize children's experiences, perspectives, and understandings, making them active agents in the research process (Einarsdóttir, 2007; Hill, 1997; Veale, 2005). The use of visual methods in the research process is not new in itself, however, children's participation is becoming increasingly important, and with it the research inquiry is becoming more participatory.

The work presented in this paper aimed at evaluating if and to what extent participant-produced drawings can be used to conceptualize and research the complex and multi-faceted construct of classroom social climate from the standpoint of primary grade students[1] (Grades 3-6). First, this paper provides theoretical and empirical foundations on drawings as a visual and participatory qualitative inquiry method in general and in mathematics education and on classroom social climate. After presenting the research process, a new classroom social climate model emerging from the students' drawings of their mathematics classroom is presented and exemplified on three drawings. Lastly, the power of participant-produced drawings concerning the conceptualization of the classroom social climate construct is discussed from a methodological, theoretical, and practical perspective.

2 Participatory and visual methods in the research process with children: Drawing and drawings

Drawing is a creative method based on inventive and imaginative processes with drawings as a research tool having the function of capturing children's individual experiences (Veale, 2005). Drawings as a visual method have been recognized as an alternative form of expression for (young) children. For children, drawing is much more than a simple representation of what they see before them; rather it can be

[1] In the federal states of Berlin and Brandenburg (Germany) primary education covers Grades 1 to 6.

understood as one way in which they are making sense of their experiences (Anning & Ring, 2004). According to Lucquet (1913, 1923, in Anning & Ring, 2004), the act of drawing can be generally regarded as a skill that can be acquired by all children up to a certain level. Lodge (2007), for instance, showed that even very young children (6 to 7-year-olds) developed a wide range of ways to represent learning in the classroom, such as drawing the common perceptions of learning in classrooms (e.g., dependence on the teacher, individual and separate learning activities), and social relationships with their teacher. Thus, already with young children this method provides a lens for research designs on classroom climate (Anning & Ring, 2004; Malchiodi, 1998).

In contrast to classical data collection methods, the use of students' drawings showed significant benefits in qualitative inquiry, such as familiarity with the act of drawing, non-verbal expression (i.e., language mediation, language barrier), which is particularly beneficial when working with young students (Ahtee et al., 2016), and through simple alternation by quickly adding or deleting elements in the drawing (Einarsdóttir, 2007). Additionally, they can help students better recall and express more details about the events they depicted (Einarsdóttir, 2007). For instance, Barlow et al. (2011) stressed that the drawing process gives the child suggestions to talk about particularly relevant occurrences and events that are related to the situation depicted in the drawing. Furthermore, thought and speech bubbles can be used as an additional visual representation to facilitate children's description of their thoughts (Wellman et al., 1996). As such, verbal and drawn aspects together provide a deeper insight into the classroom climate (e.g., feelings, attitudes, values, norms, activities, communication) (Ahtee et al., 2016). Lastly, Kearney and Hyle (2004) found that using participant-produced drawings was more likely to accurately represent participants' experiences and emotions. Here, the participatory approach is characterized by establishing a rapport between the researcher and the participant as well as by a shift in the power (im)balance in the researcher-participant relationship, with a less researcher-imposed structure (Kearney & Hyle, 2004). In other words, drawings function as a catalyst, helping participants to articulate their feelings, emotions, and lived experiences. Most importantly, they avoid adults interpreting children's drawings other than intended by the child (Einarsdóttir, 2007). Consequently, the participant approach allows for depth of discussion, the participant's shaping of agenda, and encourages collaborative meaning-making as well as reliable and trustworthy data (Kearney & Hyle, 2004).

3 Conceptualizing classroom social climate

3.1 Classroom social climate construct and relevance

The classroom is an environment in which students develop both interpersonal and academic skills (Trickett & Moos, 1973). Furthermore, it is a social context for learning, which with time develops a distinct *social climate* or feel having certain demand characteristics (e.g., Evans et al., 2009; Moos & Moos, 1978; Trickett & Moos, 1973). To date, there is no uniform definition of the construct *classroom climate*, but it is described through its fundamental supporting elements (Eder, 2002). In a general sense, Eder (2002) emphasized that classroom climate defined the quality of social relationships within a classroom. Within different disciplines various approaches to the conceptualization and assessment of environments have been used. One often-applied approach is based on the concept of the so-called *perceived environment* (e.g., Eder, 2002; Moos & Moos, 1978; Trickett & Moos, 1973). This approach is based on the contention that the environment of a particular setting is defined by the shared perceptions of its members along with several *environmental domains* over a longer period (Moos & Moos, 1978). According to Trickett and Moos (1973), nine dimensions of classroom climate can be used in conceptualizing the individual dimensions characterizing diverse psychosocial environments. These fall under three general conceptual domains or categories: (1) Relationship, the degree to which individuals in the environment help and support each other, and to which they are involved in the class and its activities (i.e., involvement, affiliation, teacher support); (2) Personal Development, the degree to which self-enhancement can occur (i.e., task orientation, competition); and (3) System Maintenance and System Change, the degree to which the environment is orderly, clear in expectations, maintains control, and can change (i.e., order and organization, rule clarity, teacher control, innovation). On the other hand, Evans et al. (2009) conceptualized classroom climate as a function of three different components: academic, referring to the pedagogical and curricular elements of the learning environment; management, referring to discipline styles for maintaining order; and emotional, referring to the affective interactions within the classroom.

The pedagogical goal of schooling is to enable and support the development of students' cognitive, social, and practical skills which generally includes imparting knowledge, but also fostering the social climate among students (Radatz & Rickmeyer, 1991). In this regard, the classroom is a significant social environment in children's development (e.g., Evans et al., 2009; Moos & Moos, 1978; Trickett & Moos, 1973). In particular, communication between the teacher and her/his pupils is central to pupils' formalization of mathematical concepts and procedures (Ahtee et al., 2016). Furthermore, Meyer (2019) outlined ten criteria of good teaching, which also included a climate conducive to learning. Thus, in that manner, the classroom climate has broad effects, ranging "from an increase in the joy of learning or the reduction of school disenchantment and (performance) anxiety, the improvement of classroom discipline, the increase in the willingness to exert effort and cooperation, to the improvement of cohesion and self-esteem" (Bülter & Meyer, 2015, p. 25). A growing literature points to the importance of classroom social climate as one of the determinants of students' academic performance and motivation, engagement, participation, and attitude towards school and teaching. However, little attention is given to classroom social climate in the context of the mathematics learning milieu, and if so, studies only provide insights into specific aspects of the classroom social climate during mathematics lessons, such as activities of classroom protagonists (Ahtee et al., 2016) or their communication (Pehkonen et al., 2016).

3.2 Measuring classroom social climate

Depending on the grade level, different quantitative instruments were developed to measure the classroom social climate, such as Moos' Classroom Environment Scale (CES), which laid a foundation in school and classroom climate research at the secondary level (Evans et al., 2009). Concretely, the CES included 90 items evenly distributed across nine dimensions which consisted of (a) Involvement, (b) Affiliation, (c) Teacher support, (d) Task orientation, (e) Competition, (f) Order and organization, (g) Rule clarity, (h) Teacher control, and (i) Innovation (Fisher & Fraser, 1983a; Fraser, 2012; Trickett & Moos, 1973; Trickett & Quinlan, 1979). The dimensions are explained as follows: (a) *Involvement* examines the extent to which students show attentive interest, participate in discussions, complete extra work, or enjoy being in class; (b) The extent to which students help others, try to get to know others better, and enjoy working together are all part of the *affiliation* dimension; (c) *Teacher support* reflects the extent to which the teacher helps, trusts, and shows interest in students; (d) *Task orientation* examines the extent to which the classroom activities are centered around the achievement of specified academic objectives; (e) *Competition* examines the students' competitive behavior among each other in terms of grades and recognition; (f) O*rder and organization* examines how students interact with each other (i.e., behaving in an orderly, polite, and quiet manner) and how overall classroom activities are organized; (g) *Clarity of rules* considers the degree to which the rules of conduct are clearly understood, and the degree to which the teacher consistently deals with rule violations; (h) *Teacher control* examines the amount and the extent of rules governing students' behavior in the classroom; (i) In terms of *innovation*, the extent to which the teacher plans new, unusual, and varying activities and techniques as well as students' contribution to classroom planning and creative thinking is examined (Fraser, 2012; Trickett & Moos, 1973). These nine dimensions fall into the three main categories, namely Relationship (a-c), Personal Development (d-e), and System Maintenance and System Change (f-i) (Fisher & Fraser, 1983a; Trickett & Quinlan, 1979). Thus, by using the CES, it was possible to capture the essence of the psychosocial classroom environment, to obtain systematic data on classroom social climate (e.g., teachers' behavior, teacher-student interactions, interactions among students), and to determine and understand the effects of socialization in a wide variety of classrooms as perceived by different individuals in the same setting (Fisher & Fraser, 1983a).

Only one instrument, namely the My Class Inventory (MCI) was developed on the basis of the CES to measure the perception of an actual environment by elementary grade students (8 to 12 years of age) (Fraser & Fischer, 1983). It contained only five scales, namely, Satisfaction, Friction, Cohesiveness, Competitiveness, and Difficulty. In terms of placement in Moos' (1974) schema, the first three affiliate with the relationship category, and the latter two with the personal development category (Fraser, 2012). Furthermore, the wording of items was simplified to enhance readability, and instead of a 4-point Likert scale, a 2-point scale was used. However, only a handful of studies used the MCI starting at the earliest in Grade 4, but still mainly in middle school classes.

There are many other instruments in addition to the ones mentioned (see Bülter & Meyer, 2015; Fraser & Fischer, 1983), the comparison of which showed that the basic concept of classroom climate is broad but generally homogeneous. On the one hand, it comprised relationship characteristics (i.e., teacher-student, student-student), and, on the other hand, teaching characteristics (e.g., choice of teaching methods, internal differentiation, design of the learning environment) (Bülter & Meyer, 2015). Thus, the classroom climate is a multi-faceted construct "made up of a large number of components, which can be reduced to factors in a variety of ways" (Evans et al., 2009, p. 141) depending on the type of environment (Fraser, 2012).

Although a broad focus on the concept of classroom social climate makes perfect sense, in an extreme form, it would amount to having to include all aspects of good teaching (Bülter & Meyer, 2015). From a research perspective, it is more of an advantage to start from a narrow concept of classroom social climate since its different facets are by no means uniform.

In recent years, it has been shown that the use of creative methods, such as drawings, provide a multi-dimensional and holistic view of young students' latent experiences, ideas, and perceptions in the classroom concerning communicative and social aspects of mathematics teaching (e.g., Ahtee et al., 2016; Glasnović Gracin & Kuzle, 2018; Pehkonen et al., 2016). For instance, Ahtee et al. (2016) focused on developing a method to determine teachers' and pupils' activities during a mathematics lesson. As a result of the analysis of students' drawings two inventories emerged. The first inventory contained 14 separate items organized into six groups that included diverse teacher activities (e.g., giving information on mathematics, giving feedback, asking questions), whereas the second one focused on students' activities that were organized into five groups that included altogether 22 items (e.g., activities of a single student, student-student discussion on mathematics, student-teacher discussion on mathematics). In that manner, from a research and practical perspective both inventories opened a window into students' perceptions of their teacher's and their classmates' activities in mathematics lessons, and how different aspects change over time (Ahtee et al., 2016). Glasnović Gracin and Kuzle (2018), on the other hand, conducted an exploratory case study with four elementary school children with the goal of capturing the social dimension of the classroom climate in geometry lessons. In their analysis, they combined the inventories of Pehkonen et al. (2016) and Ahtee et al. (2016) by focusing on the teacher's communication (i.e., poses questions, gives a task, gives instructions, teaches, gives feedback, maintains order, quietly observes students working) and students' communication (i.e., answers the teacher's question, makes/asks/thinks a remark/question in connection to teaching, solves a task, asks for help, discusses something with other student(s), makes/thinks an improper remark, keeps order, works quietly without communicating with other students). Though Glasnović Gracin and Kuzle (2018) reported on the benefits of using drawings to capture students' perception of the classroom social climate, the two scales were imposed on the data analysis. Furthermore, the sample was too small to capture other aspects of the classroom social climate, and in that manner, it was not possible to develop a comprehensive analytical tool. Thus, the general utility of the inventory as a research tool appeared to be insufficient in its current form in terms of gaining a thorough insight into the classroom social climate using participant-produced drawings.

3.3 Purpose of the study

To date, it has not been possible to make an area-wide statement about the classroom social climate in primary grade mathematics (Eder, 2002). One reason for this is the lack of a suitable instrument since the previous studies mainly employed quantitative methods (e.g., Bülter & Meyer, 2015). In order to obtain meaningful information concerning the classroom climate from visual research methods, such as drawings, viable models and tools need to be developed that focus exclusively on the social aspects (e.g., actions of the teacher and the students, classroom activities, norms in the classroom). Studies in this area using students' drawings (e.g., Ahtee et al., 2016; Glasnović Gracin & Kuzle, 2018; Pehkonen et al., 2016) could not provide a comprehensive picture of what was happening in the classroom. Although Moos' CES instrument (1973, after Trickett & Moos, 1973) provides an important basis for creating such a model and tools, it requires adaptation with respect to the used method (i.e., students' drawings), breadth of scales (i.e., other aspects of classroom social climate), and the participants' age (i.e., elementary grade students).

This being said, the paper's overarching goal is to answer the question of how and to what extent the model can capture the many aspects of classroom climate in different mathematics lessons using participant-produced drawings. Concretely, the purpose of the study was – by using an explorative qualitative research design – to (1) develop analytical tools that can be used in a qualitative inquiry on classroom social climate when using participant-produced drawings in the context of school mathematics (arithmetic and geometry[2] lessons), (2) evaluate the extent to which the participant-produced drawings can be used when researching the complex and multi-faceted construct of classroom social climate using participant-produced drawings from the standpoint of primary grade students (Grades 3-6), and (3) present a modification and further development of existing classroom climate models from the standpoint of primary grade students by using participant-produced drawings. The following research questions guided the study:

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[2] Geometry lessons refer to two standards: space and form, and measurement.

1. What different aspects of classroom social climate emerge through participant-produced drawings?
2. To what extent do different classroom social climate characteristics differ depending on the context of mathematics lessons?

4 Research context

4.1 Research design and subjects

For this study, an explorative cross-sectional qualitative research design using participant-produced drawings was chosen. The study participants were 227 elementary school students (Grades 3–6) from two federal states in Germany. This age group was optimal as they have already gathered enough experience in school mathematics, and their drawing skills are already solid to high enough. Typical case sampling as a type of purposive sampling was utilized as a way of collecting rich and in-depth data (Patton, 2002).

4.2 Data collection instruments and procedures

The research data were collected in a one-to-one setting between a student and the first author of the paper which consisted of (a) audio data, (b) document review, and (c) a semi-structured interview. The audio data (a) were composed of the students' unprompted verbal reports during the drawing process, and prompted verbal reports after the drawing process. For the document review (b), each student was given a piece of paper with the following assignment: "Dear ___________, I am Anna and new to your class. I would like to get to know your class better. Draw two pictures of your mathematics lessons. The first drawing should show what your arithmetic lessons are like and how you view them. The second drawing should show what your geometry lessons are like and how you view them. Include in each drawing your teaching group, the teacher, and the students. Use speech bubbles and thought bubbles to describe conversation and thoughts. Mark the student that represents you in the drawing by writing "ME". Thank you and see you soon! Yours Anna." The drawings were then used as a catalyst for a semi-structured interview (c) as suggested by Kearney and Hyle (2004). During the interview both a free description of the drawing on the part of the child were given (e.g., "Describe your picture to me."), and specific questions based on the child's description were posed (e.g., "You said/drew that your teacher stands at the blackboard/sits at the table a lot. How does this change in the course of the lesson?", "I see you only drew one child, where are the other children?", "How does the position of you students change during the lesson?", Can you tell me what you did in the lesson?", "What do the other students say when someone is not paying attention?"). Multiple data sources were used to assess the consistency of the results, and to increase the validity of the results as was suggested by Einarsdóttir (2007) when employing visual research methods.

4.3 Data analysis

The drawings were analyzed after all the data had been collected. As suggested by Patton (2002), multiple stages of the analysis using an analytic approach were performed with a focus on developing two inventories (arithmetic, geometry) to determine different facets of classroom social climate in the students' data. This process contained the following steps: transcribing audio data, analysis of drawings with respect to Moos' conceptualization of classroom social climate (i.e., relationship, personal development, system maintenance and system change), confirmation of the interpretation and coding of other facets included in the students' data, and developing dimensions, subdimensions, and scales for each general category of classroom social climate by clustering similar concepts. The first author transcribed the audio data and analyzed the drawings separately with another researcher using Moos' conceptualization of classroom social climate. Specifically, we started with a deductively created coding manual that provided descriptions of each general category of the CES (Fisher & Fraser, 1983b; Trickett & Moos, 1973). This allowed us to assign a particular general category to items that emerged in the students' data. However, given the design (i.e., participant-produced drawings, study sample) and implementation context (i.e., mathematics lessons), we needed to revise Moos' model by structuring and expanding it with the goal of developing multi-faceted inventories. Here, each general category, as well as descriptions of each general category, were re-examined, refined, or expanded based on the students' data taking into account different expression forms. From this theoretical basis, the three categories of the coding manual, (1) personal relationship, (2) personal growth, and (3) order, evolved. Thus, the general categories were adapted to data emerging through participant-produced drawings. Afterward, the same researchers focused on separately developing two inventories with dimensions, subdimensions, and scales for each general category of classroom social climate by going through all the drawings starting with Grade 3 and ending with Grade 6 using both deductive (e.g., CES, MCI) and inductive approaches. For this purpose, the drawings were viewed one by one and the passages in the interviews that referred to the social aspects or the passages that defined situations or objects in the drawing more clearly were marked. The inventories were discussed to obtain full agreement. Concretely, the nature of each dimension, subdimension, and scale was discussed, which allowed the refining of each descriptor, and new dimensions, subdimensions, and/or scales emerged from the students' data. If a descriptor was not given, the researchers discussed the nature of the descriptor before developing a new dimension, subdimension, or scale, and extending the analytical tools. All procedures and decisions were recorded in an audit trail, which also ensured trustworthiness and rigor (Patton, 2002). Lastly, both authors validated the developed inventories through an iterative process of coding the drawings once again, and constant comparison in order to obtain full agreement as suggested by Creswell and Miller (2000). Here, analyst triangulation contributed to the verification and validation of qualitative analysis (Creswell & Miller, 2000; Patton, 2002). Consequently, this allowed the development of very detailed and refined inventories to analyze students' perceptions of the classroom social climate in mathematics lessons.

5 Rethinking the construct of classroom social climate using participant-produced drawings

5.1 Classroom social climate as seen in the students' drawings of mathematics lessons: an emerging model

On the basis of the analysis of the students' data from both arithmetic and geometry lessons, we conceptualize classroom social climate as a function of three conceptual categories, namely *Interpersonal Relationship*, *Personal Growth*, and *Order*. Each of these is described through its dimensions, subdimensions and scales.

The first category (see Table 1) *Interpersonal Relationship* refers to nature, the intensity of personal relationships, and the mutual influences of the teacher and the students within the classroom, including social, pedagogical, and mathematical aspects. Verbal and non-verbal communication of the teacher, Verbal and non-verbal communication of the students, and Organization are conceptualized as interpersonal relationship dimensions. The first dimension is specified through the teacher's position in the classroom and teacher's support. The second dimension is specified through the students' position in the classroom, participation, and affiliation. The third dimension is specified through the working method and classroom seating arrangement.

Table 1. Description of the "Interpersonal Relationship" category

1. Category: Interpersonal Relationship		
Dimension	Subdimension	Scale
Verbal and non-verbal communication of the teacher	Position in the classroom	In front of the blackboard, Among students, At the desk, Somewhere in the classroom
	Support by the teacher	Assistance, Positive feedback, Negative feedback, Mathematics-related question, Mathematics-related statement, Observation, Non-mathematical comment, Passive
Verbal and non-verbal communication of the students	Position in the classroom	At the blackboard, At the table, Next to the teacher, In front of the blackboard, Amongst other students, Somewhere in the classroom
	Participation	Working on assignments at the table, Working on assignments on the blackboard, Listening, Responding, Questioning, Asking for assistance, Review, Discussion, Positive expression, Negative expression, Non-mathematical comment, Passive
	Affiliation	No communication with other students, Student-student communication, Student-student encouragement, Student-student help request, Student-student support, Negative comments towards other students
Organization	Working method	Teacher-centered instruction (frontal), Individual work, Group work, Working with a partner, Work/discussion while sitting in a (half-)circle
	Classroom seating arrangement	Traditional classroom arrangement, U-shaped arrangement, Mixed arrangement, (Half-)circle arrangements, Group tables

The second category *Personal Growth* refers to the goal orientation and clarity of the lesson objective. A lesson goal can be represented by mathematical content or an assignment on the backboard, the teacher identifying the goal of the lesson or students working on their assignment. Alternatively, the lesson objective can be pursued by using different teaching materials specific to geometry (e.g., geometric forms, models, tools) and arithmetic (e.g., inch-worms, number line, place value board), which can be utilized by classroom protagonists (teacher, students) (see Table 2).

Table 2. Description of the "Personal Growth" category

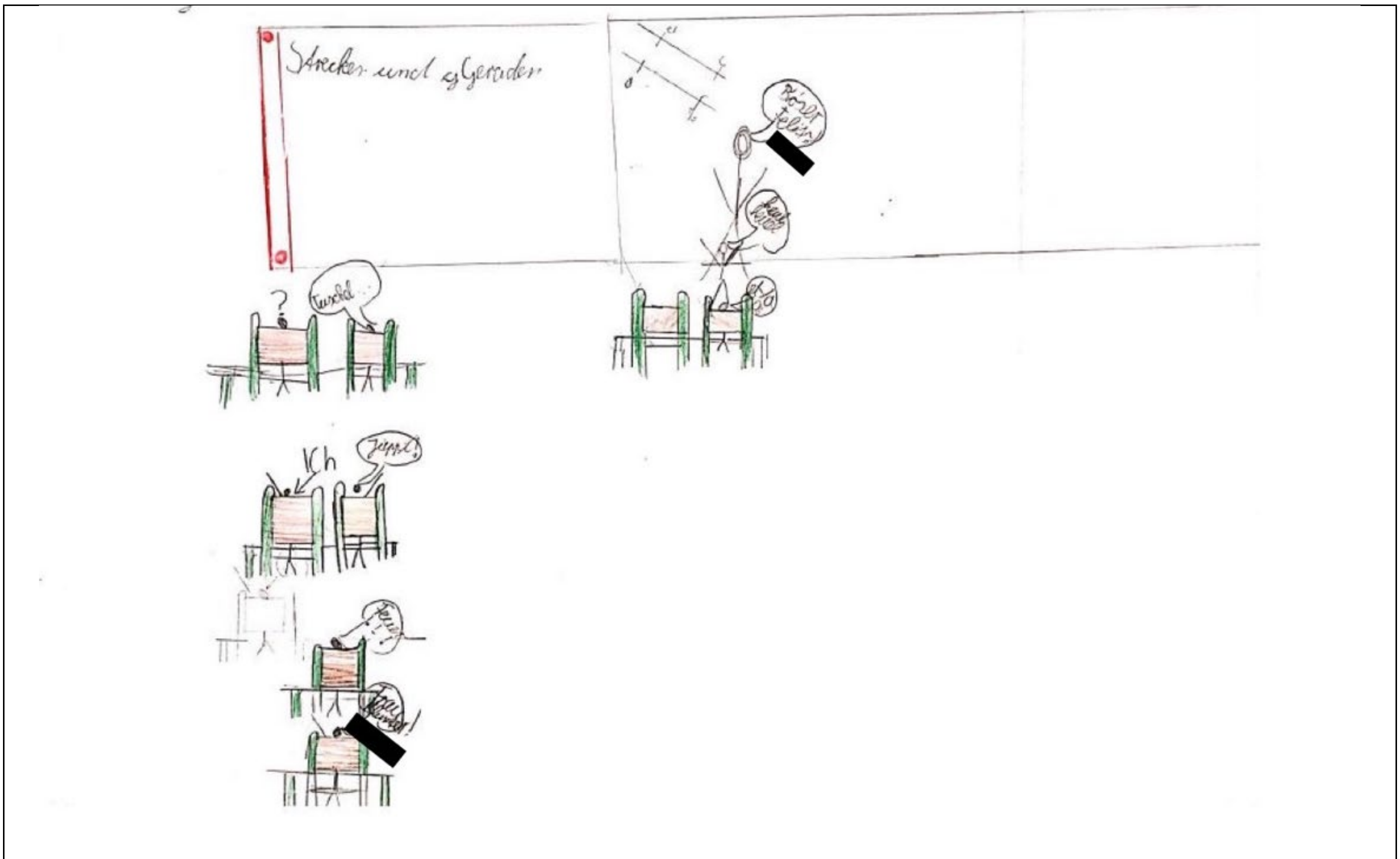
	2. Category: Personal Growth
Dimension	Scale
Goal orientation	Goal of the lesson, Presence of mathematical content, Teacher's identification of the mathematical content, Students working on the assignment
Teaching materials and tools	Geometry: 2D-shapes and models, 3D-solids and models, geometric tools (e.g., ruler, protractor, compass), poster Arithmetic: number line, place value board, poster

The third category *Order* refers to the social norms and maintenance of order in the classroom. We understand social norms as shared principles of behavior that are considered acceptable in a group. Here, not only the teacher, but also the students are responsible for proper conduct, keeping order, and behaving properly. Whether behavioral prompts need to be made by the teacher or the students suggests the extent to which rules are established, order and behavior prevail in the classroom, and the teacher is in control of the class (see Table 3).

Table 3. Description of the "Order" category

3. Category	
Dimension	Scale
Keeping order	Student led, teacher led

The following three figures (Figures 1, 2, and 3) illustrate the coding of the students' drawings. Figures 1 and 3 illustrate geometry lessons, and Figure 2 illustrates an arithmetic lesson. In that manner both the similarities and differences between the two analytical tools can be discerned. The drawings do not represent a prototypical drawing, but rather have been selected on the basis of data richness and versatility. For example, all three drawings contain speech or thought bubbles whereas some of the drawings were very simple, having only a schematic picture of the classroom with the students substituted by their desks or represented by stick figures. In the description of the drawings, we used the coding system presented in Appendices A and B (e.g., D = domain, letters A to C = dimensions, ordinal numbers = subscales, T = teacher, S = student). The number in brackets gives the number of drawn persons who fall into this category. For example, code D1A.1.T means the following: D (abbreviation for the word 'domain'), 1A (Which dimension is considered?), 1 (ranking of the dimension in the domain, here: first domain and dimension with subdimension 'position in the room'), T (Is teacher T or student S considered?). That is, we consider dimension 1A ‘verbal and nonverbal communication of the teacher’. It is the first scale ‘in front of the blackboard’ in the subdimension ‘position in the room’ (see Appendices A and B). Here the teacher is considered. For us, this means in summary that the teacher is located in the classroom in front of the blackboard. It is recommended to refer to the coding manuals while reading the following explanations (see Appendices A or B).



The teacher is standing in front of the blackboard. She is making a non-mathematical comment (“Bad Felix!”). In total eight students are illustrated in the drawing. Seven of them are sitting at their desks. One child is standing in front of the blackboard. Three students are raising their hands to participate in a discussion. Six students express non-mathematical content (e.g., “Fire!!!”, “Ha”, “Yippie!”). One student is making a negative comment to another student by saying “cry, cry”.

In the classroom the teacher is standing in front of the class, and teaching a lesson on line segments and rays which are illustrated on the blackboard. The heading on the blackboard also makes the goal and the content of the lesson clear. The tables are arranged in rows.

The teacher is keeping order by admonishing a student by saying “Bad Felix!”

Coding of 1st domain “Interpersonal Relationship”: D1A.1.T: Position in the classroom; in front of the blackboard. D1A.13.T: Support by the teacher; non-mathematical comment. D1B.2.S(7): Position in classroom; at the table. D1B.4.S(1): Position in classroom; in front of the blackboard. D1B.16.S(3): Participation; discussion. D1B.19.S(6): Participation; non-mathematical comment. D1B.28.S: Affiliation; negative comments towards other students. D1C.1: Working method; teacher-centered instruction. D1C.7: Classroom seating arrangement; traditional classroom arrangement.

Coding of 2nd domain “Personal Growth”: D2A.1: Goal orientation; the goal of the lesson is clear. D2B.1: Teaching materials and tools; 1D-objects.

Coding of 3rd domain “Order”: D3A.2: Keeping order; led by the teacher.

Figure 1. A Grade 3 student’s drawing of a geometry classroom with codes.

In the classroom the teacher is standing in front of the class and teaching. The tables are arranged in rows.

Three mathematical problems (e.g., a subtraction task, two divisional tasks) are illustrated on the blackboard. The goal of the lesson is clear. The teacher is making the mathematical content clear by asking a question about it. The students work on the assignments orally. The subtraction task is to be solved in writing.

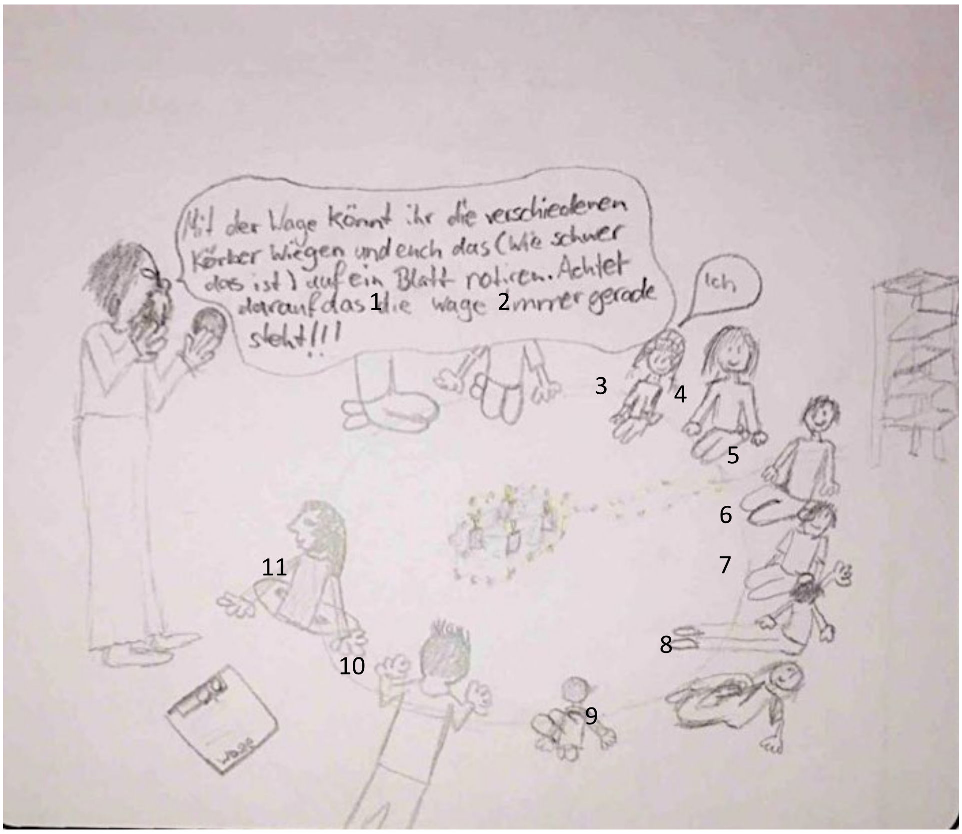
There are no behavioral demands on the part of the students or the teacher in the drawing or the interview.

Coding of 1st domain "Interpersonal Relationship": D1A.1.T: Position in the classroom; in front of the blackboard. D1A.10.T: Support by the teacher; mathematics related question. D1B.2.S(14): Position in the classroom; at the table. D1B.9.S: Participation; working on the assignment at the table. D1B.12.S(1): Participation; responding. D1B.17.S(1): Participation; positive expression. D1B.18.S(1): Participation; negative expression. D1B.23.S: Affiliation; no communication with other students while working on the assignments. D1C.1: Working method; teacher-centered instruction. D1C.7: Classroom seating arrangement; traditional classroom arrangement.

Coding of 2nd domain "Personal Growth": D2A.1: Goal orientation; the goal of the lesson is clear. D2A.3: Goal orientation; the teacher shows the mathematical content. D2A.4: Goal orientation; students work on their assignment. D2B.2: Teaching content/materials and tools; subtraction task. D2B.4: Teaching content/materials and tools; division task. D2B.7: Teaching content/materials and tools; calculating strategies.

Coding of 3rd domain "Order": D3A.3: Keeping order; unavailable.

Figure 2. A Grade 4 student's drawing of an arithmetic classroom with codes.



The teacher is standing amongst students who are sitting in a circle. The teacher is explaining the task and at the same time providing help by giving hints for better handling of the task: "Make sure that the (balance) scale is always straight!" The teacher is making a mathematically related statement: "With the (balance) scale you can weigh different objects (how heavy they are) and write it down on a sheet of paper." The students are sitting on the classroom floor in a circle. Students 3-5 and 8-11 are listening to the teacher. The pairs of students 1 and 2 and 6 and 7 are talking to each other about non-mathematical topics while the teacher is explaining the task. Students 6 and 7 have their faces directed towards each other. Students 1 and 2 have no faces to be seen but their legs are directed towards each other, whereupon one can assume that they are talking to each other since student 1's legs are pointing towards student 2. The content of their discussion is not known.

The teacher is explaining the work assignment. The goal of the lesson is clear. She is identifying the mathematical content (i.e., weighing objects by using a balance scale) by giving explanations. The teacher is holding two objects in her hands. In the circle there are different objects that the children are supposed to weigh.

There are no behavioral demands on the part of the students or the teacher in the drawing or the interview.

Coding of 1st domain "Interpersonal Relationship": D1A.2.T: Position in the classroom; amongst students. D1A.7.T: Support by the teacher; assistance. D1A.11.T: Support by the teacher; mathematics related statement. D1B.6.S(11): Position in the classroom; somewhere in the classroom. D1B.11.S(7): Participation; listening. D1B.19.S(4): Participation; non-mathematical comment. D1B.24.S(4): Affiliation; student-student communication. D1C.5: Working method; work/discussion while sitting in a circle. D1C.10: Classroom seating arrangement; circle arrangement

Coding of 2nd domain "Personal Growth" D2.1: Goal orientation; the goal of the lesson is clear. D2.3: Goal orientation; the teacher identifies the mathematical content. D2.8: Teaching materials and tools; 3D-models.

Coding of 3rd domain "Order": D3A.3: Keeping order; unavailable.

Figure 3. A Grade 5 student's drawing of a geometry classroom with codes.

The results presented in Figures 1–3 exemplify again the structure of the model. Furthermore, the fine-grain features of the model are obvious with different subdimensions and scales emerging from the students' data. The subdimensions divide the dimensions in the coding manual into much more concrete aspects (i.e., scales). For instance, using only a subdimension without scales, it would have only been possible to record whether the teacher is in the room or not. However, through the scales assigned to the subcategories “position in the room” both regarding the teacher and the students, the exact position could be identified. Thus, it can be determined whether the teacher is, for example, at the blackboard, at the teacher’s desk, among students or somewhere else in the room. In that manner, the developed inventories allow the obtaining of rich information about the classroom social climate from the students’ perspective in the mathematics classrooms.

5.2 A comparison of the analytical tools: geometry and arithmetic lessons

In terms of commonalities, the analytical tools for geometry and arithmetic lessons show an identical structure in the form of a table in which the domains, dimensions, subdimensions, and scales are presented. Furthermore, they do not differ with regard to the three main domains and the associated dimensions with subdimensions. Differences exist only in the scales themselves or their description (i.e., specific aspects assigned to the dimensions and/or subdimensions).

In particular, a significant difference lies in the scales of dimension 2b, namely *Teaching materials and tools*, which is assigned to the second domain *Personal Growth*. While the subcategories in the coding manual for geometry instruction refer primarily to plane figures and shapes, geometric solids, angles, distances, and lines, and working with geometric tools and 2D as well as 3D models, the coding manual for arithmetic instruction focuses in particular on basic arithmetic operations. Here, in addition to addition, subtraction, multiplication and division tasks, arithmetic procedures or calculating with variables and fractions can also be coded. In addition, the analysis tool for arithmetic instruction refers to aids such as the number line or the place value table. The reason for this is that the teaching materials and tools in geometry instruction differ from those in arithmetic instruction. While the scales in the analytical tool for geometry instruction refer primarily to geometry specific materials and tools (e.g., 2D- and 3D-models), the scales in the analytical tool for arithmetic instruction focus on arithmetic specific materials and tools (e.g., number line, place value table). However, both analytical tools include the scales *Other content* and *Other tools*. For instance, in geometry instruction content such as angles, distances, 1-D objects (e.g., lines, line segments) were illustrated, whereas in arithmetic instruction content such as addition, subtraction, multiplication and division tasks, arithmetic procedures or calculating with variables and fractions. Thus, the scale was developed in order to be able to capture contents and tools that are drawn less frequently or to capture all content, and materials that do not have a specific subcategory which would otherwise distort the results of the evaluation.

Furthermore, minor differences can be found in the explanations of the scales. For example, looking at the scale “Mathematics-related question” (D1A.10.T) within the subcategory *Support by the teacher*, in the explanation for geometry instruction, the following example can be found “What is the name of the solid?”, whereas in the explanation for arithmetic lessons “What is the result of the addition task?”. Similarly, the description of the scale *Mathematics-related statement* (D1A.11.T) differed between the two manuals. In both cases the teacher expressed a mathematical statement or gave students a particular assignment. However, the nature of the two differed; in each case they both reflected the content of the lesson type. For instance, in the case of geometry lessons, statements and/or assignments mostly dealt with geometrical shapes and solids (e.g., “This shape is called a parallelogram.”, “Draw a diagonal please.”). In the case of arithmetic lessons, statements and/or assignments dealt with different topics from “Numbers and Operations standard” (e.g., “Numbers 2, 3, 5, … are examples of prime numbers.”, “Please continue the fifth row of multiplication tables.”).

As noted, the coding manuals are similar in all listed domains, dimensions and subdimensions, except in the above mentioned two cases regarding the scales or their description. The reason for this is that, for example, although the teacher’s position may differ between geometry and arithmetic instruction, it may be fundamentally the same. The same is true for the subdimensions *Support by the teacher*, *Position in the classroom* of the students, *Participation*, *Affiliation*, *Working method*, *Classroom seating arrangement*, and the dimensions *Goal orientation* and *Keeping order*. In all dimensions, significant differences could be discovered in the drawings. For instance, in Figure 2 the teacher asked the question “What is the answer to 630 : 7?” (D1A.10.T) with one student responding “90.” (D1B.12.S). In the geometry lessons, such questions and responses could not be found, but rather “What solid can you identify?” (D1A.10.T) with a student response “I can see a cube.” (D1B.12.S). Nevertheless, there was always the possibility that the same subdimensions will be identified. This would have been less likely in the dimension *Teaching materials and tools*, which is why the associated scales are very different from each other in that the learning content and materials have been adapted to the field of mathematics.

6 Discussion

In our study, we used participant-produced drawings as a data source pursuing three goals, namely (1) conceptualize the complex and multifaceted construct of classroom social climate from the standpoint of primary grade students, (2) evaluate the utility of drawings when researching the construct of classroom social climate, and (3) provide two analytical tools for researching classroom social climate in the context of mathematics lessons.

As it was not obvious whether existing models of classroom social climate, especially Moos' (1999) model of classroom social climate, worked for the approach of using participant-produced drawings, in the first step we were concerned with clarifying whether and how this framework can be understood in this context. For that reason, the qualitative inquiry process was guided by three principles. First, consistency with literature describing the characteristics of the mathematics classroom. Secondly, individual insider characterization of the classroom through students' eyes by using participant-produced drawings. Thirdly, the age-appropriateness of the model without sacrificing its depth. Since the model turned out to be suitable, it was used as a basis for developing multi-faceted inventories which both refined and expanded Moos' (1999) model of classroom social climate on the basis of produced data. These have evolved from the basic principles of the CES (e.g., Fisher & Fraser, 1983a, 1983b; Trickett & Moos, 1973; Trickett & Quinlan, 1979), but still reflecting its multifaceted nature made up of a large number of components. Specifically, each domain is divided into dimensions, dimensions into subdimensions with accompanying scales to capture different aspects from the students' data (i.e., drawings, semi-structured interview). Based on the qualitative analysis of participant-produced drawings, we proposed a possible further development of existing classroom climate models reported in the literature (e.g., Bülter & Meyer, 2015; Eder, 2002; Evans et al., 2009; Fraser, 2012; Fraser & Fisher, 1983; Trickett & Moos, 1973) to better understand structure, functions, and processes in a mathematics classroom. The final version of the analytical tools for eliciting, describing, and analyzing the classroom social climate in elementary school mathematics lessons can be found in Appendices A and B.

The model and with it both inventories elicit many similarities, but also differences with existing classroom climate models, such as the CES. Regarding the first domain *Interpersonal Relationship*, in comparison to Moos' CES (Fisher & Fraser, 1983a, 1983b) where the domain Relationship is described through three dimensions (i.e., involvement, affiliation, teacher support), our model is wider and more versatile with respect to its different aspects (see Table 1). On the other hand, the domain *Personal Growth* has a different character than Personal Development in the CES. In our model, the focus is more on the academic and pedagogical aspect of teaching, especially by adding the dimension *Teaching materials and tools*. Task orientation in the CES is rather narrow and was expanded in our model. No data relevant to the CES dimension Competition emerged from the students' data and this dimension is not part of our model. Lastly, compared to Moos, who described the domain System Maintenance and System Change with four dimensions (i.e., order and organization, rule clarity, teacher control, innovation) (Fisher & Fraser, 1983b), the qualitatively obtained data revealed only one aspect, that is, who is in charge of keeping order, which can conceptually be understood as a combination of Moos' first three dimensions (see Table 3). Thus, the domain is named *Order*. Here the participant-produced drawings did not allow a more fine-grained analysis to distinguish between Moos' different dimensions.

The students' drawings did not only reveal social aspects of the classroom learning milieu but also the physical environment of both lessons. For instance, the students drew classroom furniture (e.g., tables, chairs, blackboard, whiteboard, shelves, storage racks). Thus, the students perceive not only the teacher, other students, and mathematical content (i.e., arithmetic, geometry) as a part of a teaching and/or learning situation, but also the physical environment itself (Fahlström & Sumpter, 2018).

With respect to the two coding manuals emerging from that data, there were more commonalities (i.e., identical structure: domains, dimensions, subdimensions, scales as well as content of these) than differences. The two differed only in the scales themselves or their description. Because of the many similarities between the coding manuals, it would then be interesting to see how arithmetic instruction and geometry instruction differ from each other from the students' perspective. How are the desks arranged in the geometry classroom? How are the desks arranged in the arithmetic classroom? What are the working methods of the students in the geometry classroom? What are the working methods in the arithmetic classroom? Or, do the students help each other in the geometry class? Do they help each other in the arithmetic lessons? Due to the similarities of the instruments, it would be possible to draw a direct comparison between the students' perception on arithmetic and geometry lessons from a social perspective.

7 Concluding comments and recommendations

The findings created by the use of participant-produced drawings in this study, together with those reported in the literature, led to a number of observations about the application of this methodology. These findings provide areas of consideration for other researchers who are considering using visual research methods in their work with (young) students on classroom (social) climate in mathematics education or in educational settings in general. These recommendations and associated explanations are given in the following lines.

Participant-produced drawings create a window into students' perceptions of mathematics classroom learning milieu from a social perspective, making them viable tools for researchers who seek access to this research area. In mathematics education research, drawings and the processes by which they are made open a new way of gaining insight into the classroom social climate in mathematics lessons without imposing the researchers' perspective (e.g., Ahtee et al., 2016; Glasnović Gracin & Kuzle, 2018; Pehkonen et al., 2016). On the basis of the data from participant-produced drawings, two inventories were developed. These were then used as a window into students' perceptions of geometry and arithmetic classroom learning milieu from a social perspective. As outlined earlier, the drawings provided meaningful information that would not have been evident through simple interviews or observations (Ahtee et al., 2016). Since our model of classroom social climate is divided into subdimensions with accompanying scales to capture different aspects of the student's data, it also enables researchers to precisely capture the classroom social climate reflecting versatile behaviors, actions, situations, and experiences that were available in the participant-produced drawings. However, some of these can be combined into bigger entities, depending on the research interest, thus if the fine-grained analysis is needed or not. For instance, subdimensions of the dimension *Participation* could be clustered in active and passive, if the focus is only on its nature, and not on different types of students' participation during the mathematics lessons.

The students' perceptions of the classroom social climate could only be considered complete with additional interpretation and discussion of the drawing by the participant. As Blumer (1969) noted, the analysis of drawings is understood as interpreting the meanings that the students had given to the situations and objects they had presented. Thus, in order to avoid the coder's own interpretation, not only analyst triangulation is needed, but also methodological triangulation such as participant-produced drawings (Kearney & Hyle, 2004), allowing each student to interpret his or her own drawing, which consequently allowed an in-depth understanding of what the student had drawn. In the analysis presented in this paper, the students' drawings were combined with a semi-structured interview. This means that the research object (i.e., students) was approached from two methodological perspectives by using different approaches to capture the classroom social climate in mathematics teaching in elementary school. This has the advantage that the perspectives in the students' drawings on arithmetic and geometry teaching were reinforced by the verbal data in the form of an interview (Bland, 2012). Firstly, we can report that the semi-structured interview provided additional information, providing a more comprehensive picture of mathematics lessons. Using the interview guidelines, additional subdimensions were addressed in the domains associated with them. Consequently, through the interview questions, more information was obtained so that more subdimensions with accompanying scales could be identified in the corresponding domains in addition to the illustrated subdimensions. Secondly, it must be noted that there has shown to be some drawbacks in using drawings: some children had difficulties drawing, some did not like drawing, and some aspects could only be expressed in a limited way through drawing. In such cases, additional data sources (i.e., a semi-structured interview) is necessary (Kuzle & Glasnović Gracin, 2020). Thirdly, interpretation of students' drawings has proven to be a challenging task since their analysis should be understood as interpreting the meanings that the students had given to presented situations and objects (Blumer, 1969). Thus, in order to avoid the coder's own interpretation, not only analyst triangulation is needed, but also methodologies, such as participant-produced drawings (Kearney & Hyle, 2004), allowing each child to interpret his or her own drawing, which consequently allows an in-depth understanding of what the child had drawn.

The amount of researcher-imposed structure and its clarity on the drawing process is a determinant in what aspects are portrayed in the drawings. One can distinguish between different types of image representations (e.g., De Beni & Pazzaglia, 1995), such as general, specific, and episodic. An image may be general referring to a concept without any reference to a particular example or to specific characteristics of the item (e.g., a table is described as a surface with four legs). On the other hand, one can have a specific image of a table, such as a classroom table. Thus, a specific image refers to a single, well-defined example of the concept without reference to a specific episode. Lastly, an episodic-autobiographical image refers to the occurrence of a single episode at a particular time and place in the subject's life connected to the concept (e.g., a student draws a successfully completed task during a mathematics lesson in the past). Most students' drawings in our study represented episodic-autobiographical images (De Beni & Pazzaglia, 1995) illustrating the occurrence of a single episode (i.e., a geometry lesson or an arithmetic lesson) at a particular time and place in their mathematics class. Thus, giving students a concrete drawing assignment but with little structure allowed them to illustrate and communicate unique and personally significant experiences and avoided imposing a particular perspective on them.

Participant-produced drawings create a window into students' perceptions of mathematics classroom learning milieus from a social perspective, making them viable tools for teachers. Teachers are the most significant influencing factor in students' learning (Hattie, 2013); their attitude and willingness to teach determine the development of students' content-related and process-related competencies. Trickett and Moos (1973) already emphasized that teachers can learn a lot about their teaching through classroom climate instruments. Drawings offer an even greater potential for teachers to capture children's thoughts and perceptions (Anning, 1997; Anning & Ring, 2004). Their use in the classroom could make students' perceptions and experiences of the teaching process more visible. For instance, the aspects of the classroom social climate model that occur less frequently may have played a subordinate role in classroom instruction. Thus, children's drawings and their interpretations are productive ways of promoting constructive dialogue about teaching and learning between students and their teachers, and in that sense help them plan and implement changes for future lessons (Anning, 1997; Anning & Ring, 2004). This is paramount since characteristics of learning environments are powerful predictors of students' academic success (e.g., Evans et al., 2009).

The modified classroom social climate model emerging from the participant-produced drawings is independent of the mathematical content. The analytical tools that emerged from the participant-produced drawings with respect to geometry and arithmetic lessons showed great similarities. The only differences emerged in the description of some subscales pertaining to the "Interpersonal Relationship" category with the associated subdimensions "Participation" and "Responding" and subscales pertaining to the "Personal Growth" category with the associated subdimension "Teaching materials and tools". With respect to the latter, the differences were bigger since the teaching materials and tools differ greatly depending on the mathematical content area. Nevertheless, the modified model of the classroom social climate model emerging from the participant-produced drawings proved to be independent of the mathematics subject area (i.e., arithmetic and geometry). Thus, it is our opinion that – independent of the subject whose classroom social climate is in focus and independent of the different subject content areas – our classroom social climate model is viable and can be used for diverse qualitative inquiries in education in general.

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Att utveckla elevers förmåga att formulera undersökningsbara frågor i naturvetenskap: Mangling av en didaktisk modell

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[5] Nacka kommun, Sverige

En viktig målsättning för naturvetenskaplig undervisning är att utveckla förmågan att formulera undersökningsbara frågor. Syftet med den här studien är att undersöka hur undervisning som utformats med hjälp av metoden *Question Formulation Technique* (QFT) kan stödja utveckling av elevers förmåga att formulera naturvetenskapligt undersökningsbara frågor. QFT är en modell för att utveckla elevers förmåga att formulera och värdera sina egna frågor i allmänhet. I studien prövas QFT i en svensk skolkontext och inom ramen för naturvetenskaplig undervisning. Studien genomfördes som en interventionsstudie i gymnasieskolan och inom ramen för kursen Gymnasiearbete. I kursen ska eleverna genomföra en egen naturvetenskaplig undersökning. QFT användes för att utforma undervisning som del av introduktionen till kursen. Data består av videoinspelningar av elevsamtal från undervisning som har analyserats utifrån ett pragmatiskt ramverk med organiserande syften och praktisk epistemologisk analys. Resultaten visar vilka närliggande syften som etableras i elevernas samtal om undersökningsbara frågor i undervisningen: (A) att producera så många frågor som möjligt, (B) att bedöma vilka frågor som är mest relevanta, (C) att kategorisera frågor, (D) att hitta och specificera ett undersökningsobjekt och (E) att planera för att genomföra en undersökning. Slutsatsen är att QFT kan fungera som stöd för lärares planering av undervisning om naturvetenskapligt undersökningsbara frågor under förutsättning att läraren aktivt stödjer eleverna i att uppmärksamma centrala kvaliteter avseende undersökningsbarhet och genom att binda samman närliggande syften med det övergripande syftet.

Nyckelord: Systematiskt undersökande, undersökningsbara frågor, Question Formulation Technique (QFT), didaktisk modellering, gymnasiearbete

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1 Bakgrund

Att utveckla elevernas förmåga att genomföra systematiska undersökningar är en viktig del av naturvetenskaplig undervisning på alla nivåer (Hodson, 2014; Hofstein & Lunetta, 2004). I de svenska styrdokumenten för gymnasieskolan (Skolverket, 2011a) ges naturvetenskapliga arbetsmetoder en framträdande plats och målen för undervisning i ämnena kemi, biologi och fysik innefattar specifikt "naturvetenskapliga arbetsmetoder som att formulera och söka svar på frågor" (Skolverket, 2011a). Förmågan att genomföra systematiska undersökningar innefattar hela den undersökande processen: från att formulera undersökningsbara frågor och välja undersökningsmetoder, till att hantera material och utrustning, värdera resultat och slutligen formulera och redovisa slutsatser. I den här artikeln fokuserar vi specifikt på förmågan att formulera undersökningsbara frågor i naturvetenskap.

Tidigare forskning om formulering av undersökningsbara frågor i naturvetenskap

I ett internationellt perspektiv är forskningen om laborationer och undersökande arbete i naturvetenskaplig undervisning mycket omfattande (för en forskningssammanställning av laborationer i naturvetenskaplig undervisning se Skolforskningsinstitutet, 2020). Laborationer och undersökande arbetssätt kan svara mot olika mål i undervisningen. Det kan handla om att eleverna ska lära sig att göra naturvetenskapliga undersökningar (*doing science*), att lära sig om vad som kännetecknar naturvetenskapliga arbetssätt och naturvetenskapens karaktär (*learning about science*) samt att lära sig naturvetenskapliga begrepp, teorier och modeller (*learning science*) (Gyllenpalm, 2010; Hodson, 2014). Tidigare forskning visar att de olika målen med laborativt arbete som undervisningsmetod och som innehåll tenderar att sammanblandas (Abrahams & Millar, 2008; Gyllenpalm, Wickman, & Holmgren, 2010; Hodson, 1996). Lederman (2007) pekar också på att målen setts som olika viktiga. Oftast har målen om att eleverna ska lära sig göra naturvetenskapliga undersökningar, lära sig om naturvetenskapliga arbetssätt och naturvetenskapens karaktär betraktats som mindre viktiga än att eleverna ska lära sig naturvetenskapliga begrepp. Detta gäller även svenska lärare som enligt Högström, Ottander och Benckert (2012) oftast använder laborativa arbetssätt som en metod för att utveckla elevernas förståelse för naturvetenskapliga begrepp och fenomen.

Utrymme för elever att formulera vilka frågor som ska undersökas ges inom den slags undersökande uppgifter som kallas öppna undersökningar. Wolf och Fraser (2008) skiljer på öppna och bekräftande undersökningar (*inquiry* och *non-inquiry laboratory teaching*). I öppna undersökningar kan elever ges möjlighet att ställa egna undersökningsfrågor, välja metod och sätt att sammanställa resultaten från undersökningarna. I en bekräftande laboration styrs däremot såväl frågor som metod och sätt att tolka data av läraren. Principiellt sett kan både öppna och bekräftande undersökningar svara mot de olika målen med laborativt undersökande arbetssätt även om det i praktiken bara är i de allra öppnaste formerna av undersökande arbete som elever brukar uppmuntras till att formulera egna frågor (Bianchi & Bell, 2008; Stokhof m.fl., 2019). För att utveckla elevernas förmåga till systematiskt undersökande och mer specifikt förmågan att formulera undersökningsbara frågor krävs dock ett systematiskt arbete med just formulering och bearbetning av frågor.

Tidigare forskning har visat att lärare sällan låter elever formulera egna undersökningsbara frågor utifrån en övergripande forskningsfråga eller problemsituation. Enligt en svensk studie av Lunde, Rundgren och Chang Rundgren (2015), som undersökte lärares strategier för att involvera elever i naturvetenskapligt undersökande arbete, gav lärarna återkommande uttryck för att det var svårt att planera aktiviteter som kunde ge eleverna större inflytande över de undersökande aktiviteterna än traditionellt styrda laborationer. En strategi som lärarna använde var att “öppna upp” styrda laborationer som de tidigare använt i undervisningen. En annan strategi var att imitera uppgifter i de nationella proven med fokus på systematiskt undersökande. Anpassningen av laborationerna innefattade att lärarna formulerade om styrda laborationer för att skapa större utrymme för eleverna att planera, genomföra och utvärdera dem. Vid imitation av uppgifter i de nationella proven gav lärarna varje elev en undersökningsfråga som eleven sedan fick göra en planering till. I studien saknas dock helt exempel på laborationer där eleverna kunde få möjlighet att formulera egna forskningsfrågor. Den risk som Lunde m.fl. (2015) pekar på är att naturvetenskapligt undersökande framställs som en sluten process som leder mot givna sanningar.

Observationerna i Lunde med fleras studie (2015) är i linje med resultaten från Rothstein och Santanas (2011) studie som visar att undervisning i USA sällan behandlar konsten att ställa egna frågor. Att ställa frågor ses snarare som något som tillhör lärarens domän. Lärare lägger ofta mycket tid på att formulera och bearbeta frågor i syfte att engagera och väcka diskussion och nyfikenhet. I naturvetenskaplig undervisning finns det också en risk att det uppstår konkurrens mellan undervisning om ämneskunskaper och målet om att eleverna ska utveckla en förståelse för systematiskt undersökande. Detta innebär att undervisning för att utveckla förmågan att formulera frågor ofta prioriteras bort (Rothstein, Santana & Minigan, 2015). Rothstein och Santana (2011) menar dessutom att många elever upplever uppgiften att formulera frågor som utmanande och att frågeformulering i helklass kan upplevas hämmande.

Bielik och Yarden (2016) visar, i en fallstudie av en spetsutbildning med inriktning bioteknik, att undervisning med syfte att utveckla elevers förmåga att formulera undersökningsbara frågor behöver kännetecknas av elevcentrering, dialog och nära interaktion mellan lärare och elever. För att utveckla elevers förmåga att formulera frågor som utgångspunkt för naturvetenskapligt undersökande krävs ett målmedvetet och noggrant arbete av lärare. Undervisningen behöver arrangeras så att eleverna både kan få generera och bearbeta sina egna frågor. Möjligheter för eleverna att arbeta med formulering av egna frågor kan också ha potential att bidra till att göra eleverna mer engagerade i det systematiskt undersökande då de känner att de får ”äga” vad undersökandet riktas mot (Andrée & Lager-Nyqvist, 2012; Andrée, 2012; Chaiklin, 1999; Eriksson m.fl., 2018; Kelly & Cunningham, 2019; Rothstein & Santana, 2011). Den tidigare forskningen är dock inte entydig. Wolf och Fraser (2008) rapporterar att de inte ser några skillnader i kunskapsprov mellan de elever som hade fått arbeta med öppna och bekräftande undersökningar och de som inte gjort det, även om de elever som hade fått göra öppna undersökningar formulerade sig mer positivt om undervisningen.

I den här studien undersöker vi om och i så fall hur en metod som kallas *Question Formulation Technique* (QFT) skulle kunna användas för att stödja elevers förmåga att formulera naturvetenskapligt undersökningsbara frågor. Metoden, som utvecklats av Dan Rothstein och hans kollegor (Rothstein et al., 2015; Rothstein & Santana, 2011), kan beskrivas som elevcentrerad eftersom den fokuserar på elevers formulering och bearbetning av frågor i dialog med varandra. Metoden går i korthet ut på att eleverna på ett strukturerat sätt formulerar och bearbetar frågor i relation till ett eller flera påståenden. En huvudsaklig anledning till vårt val att ta utgångspunkt i QFT i den här studie är att QFT utgör en modell som prövats vetenskapligt i en lång rad sammanhang (både i och utanför undervisning). Metoden eller modellen har som syfte just att utveckla människors förmåga att formulera frågor kring olika temata. QFT har dock inte prövats vetenskapligt i en svensk eller nordisk kontext och inte heller i relation till undervisningssyftet att utveckla elevers förståelse för systematiskt undersökande i skolans naturvetenskapliga ämnen.

Question Formulation technique, QFT, som didaktisk modell

QFT är en metod för att lära elever, oavsett utbildningsnivå, att formulera bättre frågor (Rothstein et al., 2015; Rothstein & Santana, 2011). QFT har utvecklats och prövats i en mängd olika amerikanska undervisningssammanhang från förskola till universitet och inom så vitt skilda ämnesområden som afrikansk historia, geometri och plattektonik (Rothstein & Santana, 2011). QFT har använts både för att introducera elever till nya undervisningsmoment och för att utvärdera elevers kunskaper (Rothstein & Santana, 2011). Tidigare forskning om användning av QFT visar att eleverna tenderar att bli mer delaktiga i undervisningen, utvecklar större grad av ägandeskap (Minigan m.fl., 2017; Rothstein, Santana, & Minigan, 2011) och nyfikenhet inför det innehåll som behandlas i undervisningen (Clark m.fl., 2019). QFT har också prövats som metod för att utveckla aktörskap utanför skolan, exempelvis hos patienter inom hälsovård i USA (Deen m.fl., 2011). Det finns dock inga tidigare publicerade studier där QFT prövats som utgångspunkt för systematiskt undersökande inom naturvetenskaplig didaktisk forskning, inte heller av QFT i en svensk skolkontext.

Undervisning som är utformad utifrån QFT följer nedanstående modell, enligt Rothstein och Santana (2011), med följande sex delsteg:

1. Läraren presenterar ett frågeområde. Syftet är att fånga elevernas uppmärksamhet och att stimulera själva frågeformulerandet. Exempel på frågeområden kan vara påståenden, fraser eller visuella resurser. Ett frågeområde bör dock inte ges i form av en eller flera frågor.
2. Eleverna uppmanas att individuellt producera så många frågor som möjligt till det givna frågeområdet. Frågorna ska antecknas exakt som de först formulerades och eventuella påståenden ska göras om till frågor. Frågorna ska i den här delen av processen inte värderas, diskuteras, bedömas eller besvaras.
3. Eleverna arbetar tillsammans med att bearbeta och förbättra sina frågor. Bland annat identifierar, jämför och analyserar eleverna öppna och slutna frågor och omformulerar öppna frågor till slutna och vice versa. I processen med att bearbeta och förbättra frågorna handleder även läraren eleverna i diskussioner om fördelar- och nackdelar med respektive frågetyp.

4. Läraren tillhandahåller kriterier för hur eleverna ska rangordna eller prioritera de bearbetade frågorna. Det kan exempelvis innebära att eleverna väljer ut tre frågor utifrån vad de vill undersöka vidare.
5. Lärare och elever bestämmer tillsammans hur frågorna ska användas.
6. QFT avslutas med en gemensam utvärdering. Genom att tillsammans synliggöra hur eleverna producerat, förbättrat och prioriterat sina frågor skapas möjligheten att tillämpa kunskaperna i nya sammanhang.

Det övergripande syftet med studien är att pröva QFT som en didaktisk modell. Mer specifikt om och hur QFT fungerar som ett verktyg för undervisning som syftar till att utveckla elevernas förmåga att formulera undersökningsbara frågor i naturvetenskap (jmf Dudas m.fl., 2018; Lunde & Sjöström, 2020; Seifeddine Ehdwall, & Wickman, 2018). Enligt Wickman, Hamza och Lundegård (2018) är en didaktisk modell utformad för specifika syften, användare och användningsområden. Didaktiska modeller är med andra ord sällan generiska utan riktar sig mot olika grupper av lärare och/eller elever. Didaktiska modellers överförbarhet och relevans kan dock prövas genom mangling. Manglingen innebär att en modell skapad utifrån analys av situerade undervisningssituationer prövas i nya undervisningssammanhang. En mangling är alltså en kritisk prövning med syfte att skapa insikter om modellens begränsningar och ge möjligheter till en vidareutveckling av modellen (Wickman, Hamza, & Lundegård, 2018). QFT har dock utvecklats med anspråk på att vara en generell modell som kan användas såväl i som utanför skolan.

2 En pragmatisk utgångspunkt

Studien tar sin teoretiska utgångspunkt i ett pragmatiskt perspektiv på undervisning och lärande där kunskap ses som förankrad i sociala och kulturella praktiker. Enligt ett pragmatiskt perspektiv på lärande sker allt meningsskapande genom interaktion med andra människor och omgivningen. Det innebär att lärande är situerat. Centrala begrepp inom pragmatismen är erfarenhet och syften (Dewey, 1938/2015). Erfarenhet skapas i transaktion med omgivningen och är aldrig enbart individuell. Erfarenhet karakteriseras av två grundläggande principer: kontinuitet och interaktion. Kontinuitet innebär att tidigare erfarenheter transformeras i nya sammanhang. I undervisning kan det innebära att elevens tidigare unika erfarenheter samverkar och omvandlas i en undervisningskontext. Kontinuitet i handling innebär att det som eleven lärt sig i en undervisningssituation hjälper denne att förstå och handskas med kommande situationer (Wickman, 2006; 2014). Begreppet interaktion beskriver omgivningens samspel med elevens behov, syften, önskningar och möjligheter. Det möjliggör i sin tur konstruktioner av nya erfarenheter. Interaktionsbegreppet belyser därmed behovet av att läraren arbetar med att binda samman elevernas förmågor, behov och tidigare erfarenheter med undervisningens syften (Dewey, 1938/2015). Utifrån ett pragmatiskt perspektiv väcks frågor kring om och hur QFT kan fungera som redskap för lärare i att stötta elevers utveckling av förmågan att formulera naturvetenskapligt undersökningsbara frågor. Med andra ord undersöks i vilka avseenden QFT kan bidra till att elevernas tidigare erfarenheter av olika naturvetenskapliga fenomen och laborativt arbete aktualiseras, rekonstrueras och transformeras i relation till formulering av naturvetenskapligt undersökningsbara frågor.

Den här studien fokuserar specifikt på hur kontinuitet etableras mellan elevernas arbete med att utveckla frågor genom undervisning som planerats med stöd av QFT och undervisningens syfte att utveckla elevernas förmåga att formulera naturvetenskapligt undersökningsbara frågor. I ett pragmatiskt perspektiv kallas de syften som etableras i elevernas samtal för närliggande syften och de syften som avser det lärande som undervisningen bör leda fram till för övergripande syften. Tillsammans benämns de olika syftena för organiserande syften (Johansson & Wickman, 2011). Det övergripande syftet kan exempelvis vara kursplanens syfte eller de syften som läraren formulerar för en lektion. Det övergripande syftet behöver inte vara klart för eleverna från början. I den här studien var undervisningens övergripande syfte att eleverna skulle utveckla förmågan att formulera naturvetenskapligt undersökningsbara frågor. De närliggande syftena etablerades i elevernas samtal. Johansson och Wickman (2011) menar att möjligheter till meningsskapande skapas när det närliggande och det övergripande syftet blir kontinuerliga med varandra. På så sätt kan en kontinuitet mellan tidigare och nuvarande undervisningssammanhang, både vad gäller naturvetenskapligt innehåll och systematiskt undersökande, etableras (jfr Andrée, Wickman & Lager-Nyqvist, 2017). I planeringen av en lektion kan läraren planera för vissa närliggande syften men huruvida det närliggande syftet blir kontinuerligt med undervisningens övergripande syfte är en empirisk fråga. Det handlar om vilka syften som etableras i de samtal som tar form i undervisningen. I interaktion mellan lärarens instruktioner och elevernas erfarenheter, intressen och behov kan elevernas samtal ta vägar som leder bort från undervisningens planerade övergripande syften. Eleverna kan alltså skapa mening genom arbete med uppgiften och de olika stegen i QFT, även på andra sätt än de avsedda. Huruvida lärarens och elevernas syften blir kontinuerliga blir därmed en viktig del i analysen och något som synliggörs med hjälp av de organiserande syftena.

3 Syfte och forskningsfråga

Syftet med den här studien är att undersöka hur undervisning som utformats med hjälp av QFT kan stödja utveckling av elevers förmåga att formulera naturvetenskapligt undersökningsbara frågor. Målet med studien är att mangla QFT som didaktisk modell i en svensk undervisningskontext och på så vis bidra till utveckling av undervisning som kan stödja utvecklingen av förmågan till systematiskt undersökande.

Då studien utgår från en pragmatisk syn på lärande handlar syftet om i vilka avseenden QFT kan bidra till att skapa kontinuitet mellan de närliggande syften som etableras i undervisningen och det övergripande syftet om naturvetenskaplig undersökningsbarhet. Den forskningsfråga som fokuseras är: *Vilka närliggande syften etableras i elevernas arbete med att formulera undersökningsbara frågor i naturvetenskap, i undervisning som utformats utifrån QFT?*

4 Metod

Studien ingår i ett större design-baserat projekt som genomförts inom ramen för *Stockholm Teaching & Learning Studies*[1]. Design-baserad forskning kännetecknas av ett arbete med interventioner där sätt att utforma undervisning designas, prövas och analyseras i cykler i verkliga klassrum (McKenney & Reeves, 2012; The Design-Based Research Collective, 2003). Den design-baserade forskningsansatsen syftar till samtidig utveckling av sätt att designa undervisning och sätt att förstå undervisning – teori om undervisning. Projektet bygger på ett samarbete mellan lärare och forskare som tillsammans utgjorde en forskargrupp. Projektet omfattar totalt 222 elever, nio lärare, sex forskare och en lärarstudent som genomförde en verksamhetsförlagd del av sin grundläggande lärarutbildning med en av de deltagande lärarna som handledare. Alla forskare har en bakgrund som antingen högstadie- eller

[1] STLS är en plattform för undervisningsutvecklande forskning i samverkan mellan lärare och forskare i stockholmsregionen.

gymnasielärare i naturvetenskapliga ämnen och lärarna undervisade i naturvetenskapliga ämnen på gymnasiet. Några av lärarna har dessutom bakgrund som forskare i naturvetenskapliga ämnen. Projektet genomfördes med interventioner i tre cykler på tre olika gymnasieskolor under två år. De data som genererats genom interventionerna i form av elevsamtal har analyserats och redovisats i en annan studie med fokus på vad som kännetecknar gymnasieelevers förmåga att formulera undersökningsbara frågor i naturvetenskap (Björnhammer m.fl., 2020). I den här studien görs en näranalys av den första interventionen (som genomfördes under den första cykeln på en av de tre skolorna) där QFT användes som utgångspunkt för design av undervisningen.

Planering och genomförande av undervisning utifrån QFT

Då två av de deltagande lärarna skulle starta upp kursen Gymnasiearbete bestämdes att QFT skulle prövas i relation till just den kursen. I interventionen deltog två klasser med tredjeårselever som just skulle påbörja sitt gymnasiearbete. Kursen Gymnasiearbete ingår i alla gymnasieprogram, men är relaterad till examensmålen för respektive nationellt gymnasieprogram och syftar till att visa att eleven är förberedd för yrkesutövning eller fortsatta studier. Vad gäller gymnasiearbetet för det naturvetenskapliga programmet ska det "utföras på ett sådant sätt att eleven formulerar en frågeställning samt planerar, genomför och utvärderar ett större arbete som utgår från centrala kunskapsområden inom programmet" (Skolverket, 2011b, s. 248).

Inför elevernas kursstart arbetade forskningsgruppen med att anpassa QFT så att den skulle kunna fungera som en introduktion till gymnasiearbetet. Flera av de lärare som ingick i forskargruppen hade undervisat eleverna i flera andra naturvetenskapliga kurser och hade därför kunskap om elevgrupperna. Förutom en kort genomgång av de olika delsteg som utgör QFT presenterade lärarna även tre olika frågeområden för eleverna (steg 1 i QFT). De aktuella frågeområdena var ”Ipren och Alvedon har olika maxdosering per dygn” (kemi), ”Honung innehåller antibakteriella ämnen” (biologi) och ”Det finns LED- och halogenlampor” (fysik). Utgångspunkt för forskargruppens val av frågeområden var att gymnasiearbetet ska ta sin utgångspunkt i centrala kunskapsområden för respektive gymnasieprogram. Därför valdes ett frågeområde för vart och ett av skolämnena kemi, biologi och fysik. Som steg 2 instruerades eleverna att individuellt formulera minst tjugo frågor utifrån ett eller flera av de givna frågeområdena. De individuella frågorna skulle formuleras förutsättningslöst, i snabb följd och inte bearbetas. Det tredje steget, med huvudsyftet att frågorna skulle bearbetas och förbättras, genomfördes parvis. Uppgiften att bearbeta frågorna innefattade att eleverna skulle kategorisera frågorna, resonera kring fördelar och nackdelar med varje frågetyp, ändra en öppen fråga till en sluten och vice versa, fundera över om frågan innehåller oberoende/beroende/kontrollvariabler och samtala om för vem frågorna har relevans. Delsteget innebar att eleverna även skulle försöka omformulera frågorna. Då tredjeårseleverna redan läst många av de grundläggande naturvetenskapliga gymnasiekurserna så beskrev introduktionen till delsteget inte begrepp som variabler eller öppna/slutna frågor närmare, utan eleverna förutsattes vara väl förtrogna med begreppen. Efter att eleverna på olika sätt bearbetat frågorna så introducerades delsteg 4, som innebar att eleverna skulle rangordna och prioritera frågorna utifrån undersökningsbarhet och relevans. Eleverna, som i detta steg arbetade i grupper om fyra, skulle också försöka att motivera sina val och välja tre frågor att arbeta vidare med. I steg 5 ombads eleverna att skissa på en design av en naturvetenskaplig undersökning med utgångspunkt i någon av de tre frågor de valt i det föregående delsteget. Att skissera en undersökning var en övningsuppgift inför det kommande gymnasiearbetetsprojektet och det ingick alltså inte i uppgiften att genomföra den skisserade undersökningen praktiskt. Det sjätte och sista steget var en gemensam utvärdering. I detta steg fick eleverna reflektera i helkass över de frågor som de formulerat och bearbetat i relation till design och genomförande av en praktisk undersökning. Då elevernas egenformulerade frågor eller designer av undersökningar inte utgör studiens forskningsobjekt finns de inte redovisade i samband med att studiens resultat presenteras. I studiens resultatdel analyseras istället elevernas samtal om att formulera naturvetenskapligt undersökningsbara frågor.

Sammanfattningsvis syftade den genomförda lektionen till att ge en introduktion till och inspiration inför det egna projekt som eleverna skulle genomföra inom kursen Gymnasiearbete. Efter introduktionen som planerats utifrån QFT gavs eleverna, under samma lektion, även tid till att formulera frågor utifrån egna intresseområden. I gymnasiearbetets fortsatta undervisning delades eleverna in i ämnesgrupper för att handledas av en lärare med kompetens inom det valda naturvetenskapliga området. Denna process ligger dock utanför den här studien.

Datainsamling

I den intervention som ligger till grund för den här studien ingick två klasser med 32 elever vardera. Två av lärarna i forskningsgruppen ansvarade för en lektion om 130 minuter i varsin klass. De båda lektionerna videofilmades i sin helhet och vid gruppdiskussionerna riktades videokameror mot eleverna. Då eleverna både arbetade individuellt, i par och i grupper om fyra är det dock svårt att redogöra för hur många elevgrupper som dokumenterades. Forskningsgruppen arbetade sedan gemensamt med att transkribera hela det inspelade materialet.

Dataanalys

Data har analyserats med hjälp av praktisk epistemologisk analys, PEA. PEA är en metod för att analysera meningsskapande i klassrummet som utvecklats utifrån ett pragmatiskt ramverk (Wickman & Östman, 2002). PEA synliggör lärande som process, det vill säga hur deltagande och interaktion medieras genom språk och artefakter, och blir därmed ett sätt att försöka förstå sambandet mellan vad eleverna gör och säger i relation till lärande. En PEA utgår ifrån de analytiska begreppen: syften, stå fast, relation, möten och mellanrum (Wickman & Östman, 2002). I en undervisningsaktivitet sker möten mellan elever, deras tidigare och nya erfarenheter och omgivningen. Det är också så att viss språkanvändning och handling står fast. Att något står fast innebär att det finns en gemensam förståelse, en gemensam utgångspunkt för eleverna och att de inte behöver utforska detta vidare. Möten kan också skapa mellanrum, vilket betyder att något i situationen är oklart. När mellanrummet sedan fylls rekonstrueras och transformeras tidigare erfarenheter till nya i relation till det som i aktiviteten står fast. Det kan också vara så att ett mellanrum inte fylls utan istället dröjer kvar (Wickman, 2006; Wickman & Östman, 2002).

De elevsamtal som dokumenterades under elevernas arbete med QFT analyserades enligt följande: Vid genomläsning av transkripten identifierades fem närliggande syften i elevernas samtal. I det fortsatta analysarbetet förhandlades och bearbetades sedan de närliggande syftena av flera av forskargruppens deltagare. Det gemensamma arbetet kan ses som en form av validering där olika tolkningar prövas (Bryman, 2011). Därefter har en näranalys genomförts med hjälp av praktisk epistemologisk analys, PEA (Wickman & Östman, 2002). De fem närliggande syftena exemplifieras med excerpt i resultatdelen. Dessa har valts för att så tydligt som möjligt illustrera de identifierade närliggande syftena.

Etiska överväganden

De elever och lärare som på olika sätt varit involverade i studien har informerats muntligt och skriftligt i enlighet med de forskningsetiska principerna om informationskravet, samtyckeskravet, konfidentialitetskravet och nyttjandekravet (Vetenskapsrådet, 2017). Samtliga elever är över 15 år och har även gett sitt skriftliga samtycke till att delta i studien. Av konfidentialitetsskäl anges deltagarna med pseudonymer i de excerpt som redovisas. Även skolans namn har avlägsnats. Transkribering och analys av elevernas samtal har skett efter kursens slut då de deltagande eleverna fått betyg på kursen Gymnasiearbete och därmed avslutat sina gymnasiestudier.

5 Resultat

Med hjälp av organiserande syften identifierades fem närliggande syften i elevernas samtal. De närliggande syften som identifierats är inte direkt relaterade till de olika stegen i QFT även om analysen visar att eleverna i sina samtal har stöd av den struktur som användningen av QFT innebär. De närliggande syften som identifierades var:

A. Att producera så många frågor som möjligt
B. Att bedöma vilka frågor som är mest relevanta
C. Att kategorisera frågor
D. Att hitta och specificera ett undersökningsobjekt
E. Att planera för att genomföra en undersökning

I studien är det framförallt elevernas samtal som är i fokus och lärarens röst återfinns därför inte i excerpten från samtalen. Lärarna är dock hela tiden närvarande i undervisningen och strukturerar och leder elevernas arbete i linje med planeringen.

A. Att producera så många frågor som möjligt

Det första närliggande syftet som etableras i elevernas samtal visar att det i undervisningen blir viktigt för eleverna att producera och redovisa ett visst antal frågor för varandra:

Excerpt 1: "Att pumpa tjugo frågor."

1. Axel: Ja, och det var mina frågor.

2. Theo: Ja, och jag tyckte de var jävligt bra. Det där är bra frågor!
3. Axel: Ja, men jag känner mig ganska nöjd! Jag lyckades pumpa tjugo frågor!
4. Theo: Den här, den här känns lite hög men… ((pekar på en fråga))
5. Axel: ((skratt)) Ja, det kan man ju [ohörbart] säga att det var.
6. Theo: Men du fick fler frågor än vad jag fick.
7. Axel: Det blev lite krystat här på slutet.
8. Theo: Men du fick mer frågor än jag fick. Det är ganska beklagligt.

Att eleverna ska producera många frågor är något som framstår som självklart, det vill säga står fast i samtalet. I samtalet uttrycks det genom att eleverna redovisar och jämför sina egenproducerade frågor med varandra (rad 1, 3, 6). I elevernas samtal uppstår dock ett mellanrum om betydelsen av kvaliteten på de frågor som ska produceras (rad 4, 7). Eleverna försöker helt enkelt att väga kvalitet (rad 4, 5, 7) mot antalet frågor (rad 2, 3, 6, 8). Att producera många frågor utgör ett närliggande syfte, dock inte helt utan reflektion från eleverna. En elev beskriver frågorna som formulerats i slutet av uppgiften som "lite krystat" (rad 7).

I andra elevsamtal återkommer varianter av det närliggande syftet uttryckt som vilket antal frågor som ska rangordnas, väljas, jämföras eller på något annat sätt bearbetas i grupp. Till exempel uttrycker Viggo i en annan elevgrupp: "Nu, nu kommer det. Rangordna frågorna utifrån undersökningsbarhet och relevans, välj tre frågor. Vi har redan tagit fram tre frågor." I detta samtal blir produktion av "rätt antal" frågor ett närliggande syfte. Sammanfattningsvis innebär detta närliggande syfte att eleverna riktar sin uppmärksamhet mot antalet frågor snarare än frågornas kvalitet.

B. Att bedöma vilka frågor som är mest relevanta

Det andra närliggande syftet som etableras i elevernas samtal behandlar frågors relevans. Excerptet nedan kommer från när en elevgrupp om två par jämför och värderar sina frågor (QFT, delsteg 4) inom frågeområdet "Honung innehåller antibakteriella ämnen". I excerptet samtalar Viggos grupp kring vilka frågor de anser mest intressanta:

Excerpt 2: "Vilka frågor tycker vi var mest intressanta?"

1. Viggo: Vilka frågor tyckte vi var mest intressanta?
2. Linnea: Vi hade lite, några som var lite jämna, samma typ.
3. Emil: Den här kan vi stryka. ((stryker i sitt block))
4. Linnea: Någonting med så här… eh…
5. Emil: Vi stryker den här också. Den är lite för B. ((stryker igen i blocket))

6. Linnea: Hade ni någonting om så här “medicin-ish”? Att det händer något i kroppen?
7. Viggo: Nej, egentligen inte! Vi, jo, men vi hade typ så här: “Är det bra att ha det i te?”
8. Linnea: Ja, just det!
9. Viggo: Men annars var det typ så här: “Hur kan vi på ett naturligt sätt förändra innehållet i honung så att det bättre hjälper oss bättre att… [ohörbart]?
10. Linnea: Ja, just det!
11. Emil: När man har det i varmt vatten så tänkte ni att egenskaperna skulle förändras eller göra verkningar på kroppen?
12. Viggo: Ja, vad är upptagningsförmågan? Vad är skillnaden?
13. Oskar: Jag behöver gå på toa! ((reser sig och lämnar gruppen))
14. Linnea: Ja, det är väl lite samma som, lite samma som det här med hals-grejen typ?
15. Emil: Den, den gillar jag. ((pekar på en fråga))
16. Linnea: Vi skulle kunna ta något sånt. Ja?
17. Viggo: Det var en fråga. Nu ska vi bara välja två till.
18. Emil: Jag kan tänka mig en om immunförsvaret.
19. Viggo: Immunförsvaret den har vi också på ((pekar på listan med frågor))! Eller resistens.
20. Emil: Ja!
21. Linnea: Ja!
22. Emil: Den är ju ganska intressant!
23. Viggo: Resistens eller immunförsvaret?

Elevernas samtal handlar om vilka frågor som är intressanta (rad 1, 22). I det här samtalet tolkar vi “intressant” som en fråga om relevans. Att frågorna ska vara intressanta är något som står fast för eleverna. Några frågor anses inte intressanta – till exempel är de “lite för B” (rad 3, 5). En av eleverna, Linnea, föreslår att intressanta frågor skulle kunna vara lite “medicin-ish” och att det ska handla om något i kroppen (rad 6). Samtalet tar därefter en medicinsk riktning och eleverna börjar fylla mellanrummet med frågor om immunförsvaret och resistens (rad 18–19). Att medicinska frågor klassas som intressanta blir något nytt som står fast i elevernas samtal.

Det närliggande syftet om relevans innehåller i andra elevsamtal även diskussioner om angelägenhet, samhälleliga eller naturvetenskapliga aspekter, undersökningens konsekvenser, huruvida frågorna skiljer sig eller liknar varandra men även utifrån icke-mänskliga aspekter, och uppvisar därmed en bredd av olika aspekter.

C. Att kategorisera frågor

I elevernas arbete med att välja några frågor att bearbeta och förbättra etablerades ett tredje närliggande syfte om kategorisering av frågor. I excerptet nedan återvänder vi till det första elevparet, Axel och Theo, när de arbetar med att kategorisera de frågor som de skrivit (QFT, delsteg 3) till frågeområdet ”Ipren och Alvedon har olika maxdosering per dygn”:

Excerpt 3: “Först kategorisera frågorna...”

1. Theo: Förbättra?
2. Axel: Först kategorisera frågorna.
3. Theo: *Well* jag, mina går...
4. Axel: Mina är redan ganska...
5. Theo: ... kategoriserade! Det finns så här varför...
6. Axel: Ja, precis!
7. Theo: Det finns varför-frågor och påstående-frågor som är i princip omgjorda till frågor. Alltså påståenden som är omgjorda till frågor och varför liksom.
8. Axel: Ja, här skulle man, framförallt så tänker jag mig att man skulle kunna... de här liksom mer samhällsperspektiv...
9. Theo: Ja, okej. ((gäspar))
10. Axel: Det är verkligen samhällsperspektiv! Vilken är den vanligaste användningen vid överdoseringen? Hur vanligt är det? Vilken är mest populär?
11. Theo: Och sedan har vi PETA-frågor!
12. Axel: Ja, precis. ((skratt)) Sen har vi mera koppling till andra djur, liksom andra levande organismer. Sen har vi lite sådant här hur fungerar det egentligen...
13. Theo: Ja, exakt!
14. Axel: ... hur är de uppbyggda och vad är det de gör i kroppen?
15. Theo: Ja, det finns ju lite många, lite flera så här, vad ska man säga, kemiska frågor i form av beteckning om du förstår vad jag menar. Inte beteckningen men hur funkar det?
16. Axel: Ja, precis!
17. Theo: Sen finns det så här mera “ish”-sociala om du fattar vad jag menar, i form av att det finns så här yttre och inre. Fattar du vad jag menar? Jag tycker man kan kategorisera det för många av dina frågor är så här “Vad händer?” och sen så är det “Vad egentligen händer i kroppen?”
18. Axel: Ja.
19. Theo: Och det är lite grand det här “Vad händer och varför?” som jag har.
20. Axel: Precis, precis, fast du har mer generellt. Jag har tagit lite mer specifika frågor men egentligen har vi i stort sett samma frågor.

Att eleverna ska kategorisera sina frågor är något som står fast i det här samtalet (rad 2-4). Däremot uppstår ett mellanrum kring utifrån vilka kriterier frågorna ska kategoriseras. En av eleverna väljer att inordna sina frågor i kategorierna “varför-frågor” och “påstående-frågor” (rad 7). Då det uppstår ett mellanrum om vad som menas med dessa kategorier tar samtalet en annan riktning (rad 8). I det fortsatta samtalet kategoriseras frågorna istället som frågor med samhällsperspektiv (rad 8, 10) och som “PETA-frågor” (rad 11). Här har vi tolkat “peta-frågor” som djurrättsfrågor (PETA är en förkortning av *People for the Ethical Treatment of Animals*) vilket stöds av att Axel som i sin utsaga skapar en relation mellan “PETA” och “andra djur, liksom andra levande organismer” (rad 12). En annan kategorisering som Axel föreslår är “hur fungerar det egentligen?” (rad 12), vilket eleverna vidareutvecklar till vad olika kemikalier gör i kroppen och hur ämnen är kemiskt uppbyggda (rad 14-15). Eleverna introducerar slutligen en kategorisering av frågor som sociala (rad 17) innan de avslutar jämförelsen.

I andra elevers samtal förekommer andra kategoriseringar. En återkommande kategorisering är i öppna och slutna frågor. I excerpt 4 diskuterar Axel och Theo vidare kring vad som räknas som en öppen eller sluten fråga samt om en öppen fråga är bättre än en sluten fråga (QFT, delsteg 3).

Excerpt 4: ”Vad är en öppen och sluten fråga?”

1. Axel: Okej, ändra en öppen fråga till en sluten fråga och vice versa. ((läser från instruktionen)) Ska vi bara?
2. Theo: Finns öppen? Vad ska man ta?
3. Axel: Okej, vad räknas som en öppen fråga och en sluten fråga?
4. Theo: En öppen fråga är väl mer generell och en sluten fråga är specificerad till ett särfall. *Right*?
5. Axel: Ja.
6. Theo: Det vi behöver göra till exempel, finns det ett uppenbart bättre? Det är en öppen fråga.
7. Axel: Jag tror att en öppen fråga är typ hur vanligt är det med överdosering? Och en sluten skulle kunna vara: Är det vanligare hos kvinnor än hos män?
8. Theo: Exakt, det är exakt det! Jag är ganska, jag tänker också det.
9. Axel: Jag tycker att det känns ganska, ja.
10. Theo: För det du gör är att du säger okej, är det vanligare med överdosering och sedan går du in på ett särfall.
11. Axel: Ja precis. Och att göra en sluten fråga till en öppen. Har vi några slutna frågor?
12. Theo: Eh, vi har typ inga slutna frågor. Kan växter överdosera är en ganska sluten fråga.
13. Axel: ((skratt))
14. Theo: Ja, men det är ju det för det är samma sak som du sa förut. *Right*. Att du gick in förut på, okej, nu har vi ett särfall här och det är växter. Kan de överdosera? Då ändrar vi det till: Hur vanligt är det generellt att överdosera? Varför blir de här växterna? Varför har just de här växterna annorlunda nummer om de har det?
15. Axel: Ja, det är inte omöjligt att det är något sånt de menar.
16. Theo: Ja, det är ju i princip bara att vi ändrar den här frågan tillbaka det vi sa förut. Vi ändrar kvinnor, det du sa förut med att kvinnor specifikt överdoserar till generella frågor.

17. Axel: Ja, precis.
18. Theo: *Right*!
19. Axel: Ja.

I det fjärde excerptet diskuterar Axel och Theo skillnaden mellan öppna och slutna frågor samt ger exempel på hur frågor kan omformuleras. Att det finns öppna och slutna frågor är något som står fast i elevernas samtal. En öppen fråga formuleras som mer generell medan en sluten fråga formuleras som “specificerad till ett särfall” (rad 4). Eleverna försöker även att beskriva skillnaden mellan öppna respektive slutna frågor genom exempel om överdosering (rad 7, 10, 12, 14, 16). Allteftersom elevernas samtal fortgår uppstår det ett mellanrum om vad som skiljer en öppen från en sluten fråga (rad 3, 6, 11, 12, 16). Eleverna blir mindre och mindre säkra. Trots flera försök att slå fast innebörden av öppna respektive slutna frågor lyckas eleverna inte fylla mellanrummet, utan det dröjer kvar. Eleverna saknar kriterier för att fullt ut kunna genomföra arbetet eftersom vad ett kategoriserande av frågor innebär tagits för givet i instruktionerna till eleverna. Samtidigt öppnar avsaknaden av en entydig definition av öppna och slutna frågor upp för ett utforskande av innebörder.

D. Att hitta och specificera ett undersökningsobjekt

I elevernas diskussioner etablerades även ett fjärde närliggande syfte om vad som ska undersökas och hur det kan förstås, det vill säga hur undersökningsobjektet kan specificeras. I excerpt 5 utforskar Theo och Axel tillsammans med Maja och Sixten ett möjligt undervisningsobjekt i relation till frågeområdet ”Ipren och Alvedon har olika maxdosering per dygn” (QFT, delsteg 4).

Excerpt 5: “Man skulle vilja hitta och specificera någon specifik långvarig konsekvens.”

1. Maja: Vad har regelbunden behandling med Ipren respektive Alvedon för långvariga konsekvenser?
2. Axel: Mmmm.
3. Maja: Då är alltså kontrollvariabeln det du ser, den oberoende variabeln är tid och den beroende variabeln är effekt och [ohörbart]. Nu inser jag att vad som helst kan hända med kontrollvariabeln.

[...]

4. Axel: Läs den ((frågan)) igen lite långsammare.
5. Maja: “Vad har regelbunden behandling med Ipren respektive Alvedon för långvariga konsekvenser?”
6. Theo: [ohörbart]

7. Axel: Man skulle definitivt vilja specificera det rekommenderade intaget och dessutom…
8. Sixten: Kanske långvarigt och hur långvarigt…
9. Axel: Man skulle vilja hitta och specificera någon specifik långvarig konsekvens.
10. Sixten: Men det är det som är frågan: Vad är en långvarig konsekvens?
11. Axel: Ja, precis men man skulle vilja har någonting snarare, vad har det här, vad har det för långvarig påverkan på typ det här enzymet i hjärnan.
12. Theo: Exakt!
13. Maja: Ja, men faktiskt.
14. Theo: Vad finns det för generella [ohörbart].
15. Axel: Ja, snarare än öppet alltså. Vad finns det för långvariga effekter? Hur påverkar det just det här?
16. Theo: Ja.
17. Maja: Okej, vad kan man tänka sig för typ av konsekvens? Smärtgräns?
18. Axel: Ja.
19. Maja: Smärttröskel?
20. Axel: Någon form av typ beroende.
21. Maja: Ja, metabolism alltså vi ska ha mätdata-tänk.
22. Theo: [ohörbart] … vi har dålig uppfattning om vad som är smärtgräns.
23. Axel: Ja, men det bygger på vad är det egentligen de olika ämnena gör i kroppen och hur påverkar då vad …
24. Theo: Om du tar en varje gång du får lite huvudvärk så får du ta den varje gång …
25. Sixten: Men sen så är det ju också så att nu så undersöker vi ju konsekvenser till att vilja höja smärttröskel för att se vad som påverkar. Det är ju den saken som förändrar sig.
26. Axel: Ja, det är ju inte omöjligt att en långvarig medicinering av det skulle liksom…
27. Maja: Höja va? Nej?
28. Axel: Jo, det borde höja smärtgränsen. Nerverna borde domna av typ.
29. Maja: Man vänjer sig.

((Maja och Theo pratar i munnen på varandra.))

30. Maja: Smärttröskeln vi tar det då.
31. Axel: Ja, men det är lite intressant!

Att ett regelbundet intag av värktabletter kan få långvariga konsekvenser för användaren är något som står fast i elevernas samtal (rad 1). Det uppstår trots detta ett mellanrum om vad som menas med långvarighet och konsekvens (rad 9-10) och hur detta ska kunna undersökas (rad 11-13, 23). Eleverna försöker att fylla mellanrummet om vad som ska undersökas utifrån termer om mätbarhet. Termer som smärttröskel/smärtgräns (rad 17, 19, 22, 25, 28, 30), beroende (rad 20) och metabolism (rad 21) diskuteras. Det blir i analysen tydligt att eleverna inte riktigt vet hur de ska kunna undersöka konsekvenserna från långvarig användning av smärtstillande preparat. Mellanrummet dröjer alltså kvar trots att eleverna själva väljer smärtgräns som ett undersökningsobjekt.

E. Att planera för att genomföra en undersökning

Det femte och sista närliggande syftet handlar om hur en undersökning skulle kunna genomföras utifrån de frågor som eleverna formulerat. I excerptet fortsätter vi att följa Maja, Sixten, Theo och Axel när de försöker att skissera en tänkt undersökning som svarar mot frågeområdet ”Ipren och Alvedon har olika maxdosering per dygn”.

Excerpt 6: “Jobbigt att testa själv!”

1. Maja: Ska vi svara på alla eller ska vi välja en?
2. Axel: Ingen aning.
3. Theo: Vi kör en.
4. Axel: Vi börjar med en och vilken börjar vi med?
5. Maja: Vad har en behandling med rekommenderad daglig dos i tre månader med Alvedon respektive Ipren för långvariga konsekvenser för smärttröskel? Man skulle ju...
6. Axel: Den blir jobbigt att testa själv!
7. Maja: Ja.
8. Sixten: Ja, som den är nu.
9. Axel: Då skulle man snarare vilja gå in teoretiskt.
10. Maja: Alltså litteraturstudie?
11. Axel: Snarare teoretiskt vad är det de gör i hjärnan och vad är det de egentligen gör i kroppen? Och gå vidare på: Vad är det som påverkar smärttröskeln och vad finns det för samband mellan de två?
12. Maja: Så då är det teori? Vad är smärttröskeln?
13. Axel: Den är ju ganska svåra att testa. Det krävs en långvarig studie.

I excerpt 6 jämför eleverna de båda kemiska preparaten Ipren och Alvedon. Att en långvarig användning av smärtstillande preparat kan få konsekvenser är något som står fast för eleverna. Det uppstår dock ett mellanrum om hur en undersökning av dessa konsekvenser skulle kunna genomföras (rad 6, 8). Eleverna försöker att fylla mellanrummet om undersökningsupplägget med förslag om en teoretisk studie (rad 9, 10). Enligt eleverna skulle en teoretisk studie kunna reda ut hur de smärtstillande preparaten arbetar för att sänka individens smärttröskel (rad 11). Eleverna lyckas dock inte reda ut hur en empirisk undersökning i ämnet skulle kunna gå till, vilket leder till att mellanrummet dröjer kvar (rad 13).

I excerpt 7 fortsätter eleverna att diskutera hur effekten av de smärtstillande preparaten Alvedon och Ipren kan mätas:

Excerpt 7: ”Att kunna mäta effekten.“

1. Axel: Det första man vill kolla på i så fall på den frågan är ju: Vad är det för ämnen som är verksamma?

2. Maja: Vi har ju redan [ohörbart].
3. Axel: Ja, men precis, men… ja, ja, det… då skulle man behöva kolla på vad är det…
4. Maja: Det skulle man faktiskt kunna göra!
5. Axel: Man skulle kunna kolla vad är det som plockar upp det? Vad har det ämnet för kapacitet? Om det ämnet som plockar upp det där är ett ämne som är ganska likt dem så kan man ju gissa. Det skulle också kunna rinna igenom. Så det är också ganska…
6. Maja: [ohörbart]
7. Axel: Ja, ja.
8. Maja: Men då måste man kunna mäta effekten av det liksom?
9. Axel: Ja, eventuellt så skulle man kunna göra någon form av typ titrering på något vänster för att testa.
10. Maja: Okej, man tar reda på ämnet genom att man titrerar?
11. Axel: Det är ju det! Om du tänker dig att du har det här ämnet upplöst i vatten och du har ämnet som plockar upp det upplöst i vatten. Testa att dränka en [ohörbart, Axel blir avbruten av Maja som ställer en ohörbar fråga]. Ja, i och för sig. Men fortfarande, man skulle kunna göra en faktisk studie på det. Förmodligen.
12. Maja: Ja, men du, det är fan inte så jävla dumt! Sen så skulle man skulle ju kunna testa det, [ohörbart] typ när, var i sin metabolismcykel man är.
13. Axel: Ja.
14. Maja: Och om man har andra saker på sig.
15. Axel: Ja, förmodligen finns det andra hormoner och skit som påverkar. Men det får man ju då väga in i diskussionen.
16. Theo: Exakt!

Den första delen av excerptet innefattar en specificering av undersökningsobjektet genom en diskussion om vilka ämnen som är verksamma i Ipren och Alvedon (rad 1-5). Maja föreslår då att en undersökning måste vara mätbar (rad 8). Undersökningars mätbarhet är något som står fast för eleverna. Däremot skapas ett mellanrum om hur effekten av de smärtstillande preparaten Alvedon och Ipren kan mätas. Eleverna försöker att fylla mellanrummet genom att föreslå metoden titrering då värktabletter kan lösas i vatten (rad 9-11). I det fortsatta samtalet uppstår ett nytt mellanrum om titreringens applicerbarhet/generaliserbarhet på en verklig kropp (12-15). Eleverna försöker att fylla det nya mellanrummet genom att lyfta de hormonella och metaboliska processer som kan inverka på en undersökning (rad 12, 15).

6 Diskussion

Resultaten visar att fem närliggande syften etablerades under elevernas arbete med QFT: (A) att producera så många frågor som möjligt, (B) att bedöma vilka frågor som är mest relevanta, (C) att kategorisera frågor, (D) att hitta och specificera ett undersökningsobjekt och (E) att planera för att genomföra en undersökning. Flera av de närliggande syften som identifierades i elevernas samtal väcker frågor om hur lärarnas instruktioner och arbetsgången för QFT tolkades ur ett elevperspektiv. Innan vi fördjupar oss i analysen av elevernas arbete så vill vi betona att både de instruktioner som eleverna fick och de modifieringar som gjordes av QFT var ett resultat av det gemensamma arbetet i forskningsgruppen. Det vill säga den kritiska blicken och analysen gäller både forskningsgruppens egna planeringsarbete och hur instruktionerna togs emot av eleverna.

Kontinuitet mellan närliggande och övergripande syften

Enligt Johansson och Wickman (2011) synliggörs lärandeprogression genom att det närliggande och övergripande syftet är kontinuerligt med varandra. De närliggande syften som etablerades i elevernas samtal stämmer i huvudsak överens med de olika stegen i QFT. Vi kan se detta som ett uttryck för att den genomförda undervisningen faktiskt strukturerades utifrån QFT samt att eleverna ansträngde sig att följa lärarnas instruktioner och de deluppgifter som de fått.

Det första närliggande syftet om ”att producera så många frågor som möjligt” överensstämmer med QFT, delsteg 2. Delsteget innebar att eleverna skulle producera många frågor utan att värdera eller reflektera närmare över frågornas kvalitet i relation till undersökningsbarhet. Det visade sig att eleverna tog uppgiften om att producera en stor mängd frågor på största allvar, något som kom att påverka deras fortsatta arbete. Då huvudfokuset låg på antalet frågor, snarare än att frågorna skulle vara undersökningsbara skapades en inbyggd svårighet i att senare förbättra dem. Att inte frågornas kvalitet betonas i instruktionen för QFT ser vi som en begränsning, då det i vår studie innebar att frågan om undersökningsbarhet inte sattes i förgrunden av eleverna.

De närliggande syftena om ”att bedöma vilka frågor som är mest relevanta” och ”att kategorisera frågor” kan även de relateras till delsteg och instruktioner för QFT. Enligt delsteg 3 och 4 skulle eleverna förbättra sina frågor genom att bland annat kategorisera dem och fundera på skillnaden mellan öppna och slutna frågor. Eleverna skulle även rangordna frågorna utifrån relevans. Vad som menas med kategorisering eller relevans är dock inte något som förklaras närmare i lärarnas instruktioner. Det visade sig ge upphov till en hel del förvirring bland eleverna. Som vi tidigare sett exempel på så famlar eleverna i kategoriseringen av frågorna och väljer att dela in dem i kategorier som “varför-frågor”, “påstående-frågor”, “peta-frågor”, “hur fungerar det egentligen?” eller “frågor med samhällsperspektiv”. Eleverna återkommer i samtalen också till distinktionen öppna och slutna frågor. Distinktionen introducerades av den undervisande läraren i inledningen av lektionen men innebörderna av den utvecklades inte. Eleverna förutsattes veta skillnaden. I samtalen framstår eleverna som väl införstådda i att frågor kan ses som öppna eller slutna. Att kategorisera egna redan formulerade frågor var dock lättare sagt än gjort. Det är inte självklart att en mer specificerad instruktion och genomgång av öppna och slutna frågor skulle ha bidragit till en högre grad av kontinuitet mellan det närliggande syftet att kategorisera frågor och det övergripande syftet att utveckla elevernas förmåga att formulera undersökningsbara frågor i naturvetenskap. Detta är något som skulle behöva undersökas vidare. Vad gäller det närliggande syftet om relevans så framgår att eleverna inte riktigt vet hur de ska ta sig an frågan om relevans – de saknar en gemensam förståelse av utifrån vilka kriterier de ska bedöma relevans. Relevans blir istället något som eleverna förhandlar om. I inledningen av elevexcerpt 2 så uttrycker eleverna att frågor som är lite “medicin-ish” ses som relevanta. I elevsamtalen om relevans ingår även diskussioner om angelägenhet, samhälleliga eller naturvetenskapliga aspekter, undersökningens konsekvenser, huruvida frågorna skiljer sig eller liknar varandra men även utifrån icke-mänskliga aspekter. Det närliggande syftet innefattar därmed en bredd av olika aspekter. Sammanfattningsvis visar analysen att de närliggande syftena ”att bedöma vilka frågor som är mest relevanta” och ”att kategorisera frågor” etableras då eleverna försöker greppa vad relevans och kategorisering innebär. Dessa närliggande syften blir därmed inte direkt kontinuerliga med undervisningens och studiens övergripande syfte att utveckla elevernas förståelse av undersökningsbarhet.

Det närliggande syftet om ”att planera för att genomföra en undersökning”, är ett syfte som också är relaterat till lärarnas instruktioner och QFT. I syftet diskuterar eleverna ett praktiskt genomförande av de frågor de formulerat utifrån parametrar som mätbarhet, observationer, laborationsutrustning och metoder, felkällor och litteraturstudier. De samtalar även om olika etiska, samhällsvetenskapliga och miljövetenskapliga aspekter av själva genomförandet av undersökningen. Att designa en undersökning var en uttolkning av QFT:s delsteg 5 där lärare och elever tillsammans skulle bestämma hur de tidigare frågorna ska användas. Att eleverna skulle pröva att designa en undersökning var en anpassning till att lektionen var en introduktion till kursen Gymnasiearbete. Elevernas samtal visar dock att de har svårt att konstruera undersökningar som är genomförbara i en gymnasieskolas laborationssalar utifrån de frågor de formulerat. Deras samtal blir mer som teoretiska antaganden om vad som kan tänkas vara möjligt att undersöka. Deras mer eller mindre kvalificerade antaganden kan tänkas både vara en konsekvens av att de tre frågeområdena är för långt ifrån elevernas egna laborativa kunskaper men också en konsekvens av att instruktionerna inte betonat att undersökningarna ska genomföras i en gymnasiekontext. Då det närliggande syftet väcker frågan om undersökningsbarhet så blir detta närliggande syfte kontinuerligt med studiens övergripande syfte.

Det närliggande syftet ”att hitta och specificera ett undersökningsobjekt” visar att förhandla, diskutera, värdera och samtala om *vad* som kan bli intressanta undersökningsobjektet var en viktig del i elevernas arbete med att bearbeta sina frågor. Att förhandla om, diskutera eller värdera undersökningsobjektet, och på så vis forskningsfrågan, var dock inte en i förväg planerad aktivitet och ingår inte i modellen för QFT. Förhandlingen om undersökningsobjektets specificering uppstod istället spontant i elevernas samtal. Det närliggande syftet om specificering av undersökningsobjektet bidrar dock till att skapa kontinuitet med det övergripande syftet att utveckla elevernas förmåga att formulera naturvetenskapligt undersökningsbara frågor. Utan ett arbete med att precisera undersökningsobjektet blir det svårt att formulera naturvetenskapligt undersökningsbara frågor.

QFT som modell för att formulera undersökningsbara frågor i naturvetenskap

Didaktiska modeller genereras för specifika syften, användare och användningsområden och är därmed sällan generiska (Wickman m.fl., 2018). Didaktiska modeller kan dock prövas i nya undervisningssammanhang genom en så kallad mangling. I den här studien har vi valt att mangla QFT i ett svenskt skolsammanhang och i relation till naturvetenskaplig undervisning med fokus på systematiskt undersökande. Resultaten pekar på att QFT tenderade rikta eleverna mot formuleringar av frågor mer allmänt och till att följa de givna instruktionerna. QFT visade sig dock vara användbar ur forskningssynpunkt i och med att modellen bidrog till att synliggöra en diskrepans mellan våra förväntningar på elevernas förmåga att formulera naturvetenskapliga frågor och deras kunskaper om vad ett naturvetenskapligt frågeformulerande kan innebära. Tidigare forskning om öppna undersökningar pekar också på att det är viktigt att elever ges stöd i genomförandet av öppna undersökningar initialt (Bianchi & Bell, 2008; Stokhof m.fl., 2019). Mot bakgrund av att förekomsten av öppna undersökningar med möjligheter för elever att formulera egna frågeställningar är mycket begränsade i svensk skola (Lunde m.fl., 2015) blir vår slutsats att detta är något som den naturvetenskapliga undervisningen behöver adressera oftare och återkommande.

För att QFT ska kunna fungera som en utgångspunkt för att utveckla elevers förmåga att formulera naturvetenskapligt undersökningsbara frågor behöver dock läraren handleda eleverna i diskussioner om fördelar- och nackdelar med respektive frågetyp samt bidra med redskap för att värdera och specificera frågor. Distinktionen öppna-slutna frågor som användes i undervisningen visade sig inte fungera som redskap för specificering. Eleverna kunde använda distinktionen för att klassificera frågor som de formulerat men inte för att utveckla frågorna så att de blev mer undersökningsbara. Däremot bidrog avsaknaden av en entydig definition till att eleverna riktade sin uppmärksamhet mot vad som kännetecknar de två frågetyperna. En tänkbar utveckling av modellen skulle kunna vara att introducera begreppsliga redskap som ligger närmare utformningen av naturvetenskapliga undersökningar och avsätta tid för gemensam uppföljning där olika frågetyper kan jämföras. En möjlighet skulle kunna vara att pröva fruktbarheten i att använda exempelvis Schwabs (1978) begrepp för olika kunskapsintressen inom naturvetenskap: taxonomiskt intresse (att beskriva, sortera, benämna), sambandsintresse (att förutsäga nya observationer ur redan gjorda observationer) och förklaringsintresse (att förklara varför något observeras). Eleverna skulle kunna få i uppgift att kategorisera sina frågor med hjälp av kunskapsintressen, men också få pröva att omformulera frågor från ett kunskapsintresse till ett annat.

Vår studie indikerar även att en växelverkan mellan teori och praktik, det vill säga ett frågeformulerande i kombination med ett laborativt undersökande hade varit gynnsamt för att utveckla förmågan att formulera naturvetenskapligt undersökningsbara frågor. I en artikel från 2017, publicerad efter att vi genomfört vår studie, betonar Minigan och Beer (2017) vikten av att frågeområdet fångar elevernas uppmärksamhet och att det stimulerar själva frågeformulerandet. Att hitta ett intressant frågeområde kan enligt Minigan & Beer (2017) vara en av de största utmaningarna med metoden. Vår analys av elevernas samtal visar att de frågeområden som vi formulerat utgjorde en begränsning för elevernas arbete med att formulera frågor. En möjlig tolkning är att eleverna hade för liten teoretisk och laborativ kunskap om de olika frågeområdena för att kunna formulera kvalitativa frågor och designa genomförbara undersökningar. En annan möjlig tolkning är att eleverna saknade förmåga att knyta an till tidigare erfarenheter av systematiskt undersökande i naturvetenskapliga ämnen. För att stötta eleverna skulle läraren kunna lägga mer vikt vid det kollektiva samtalet i undervisningen. På så sätt skulle en kontinuitet mellan tidigare undervisningssammanhang och det pågående arbetet med formulering och bearbetning av undersökningsbara frågor kunna skapas (jfr Andrée, Wickman & Lager-Nyqvist, 2017). Stokhofs m.fl. (2019) forskning kring användning av tankekartor för formulering av frågor stödjer den senare tolkningen. De visade att lärare genom att etablera ett delat ansvar och kollektivt ägarskap för arbetet med tankekartor i klassrummet kunde involvera eleverna i ett mer intensivt kollektivt kunskapsbyggande. På motsvarande sätt är det möjligt att förberedande, eller mer omfattande, inslag i undervisningen av kollektiv formulering och bearbetning av frågor kan skapa bättre förutsättningar för utveckling av elevernas förmåga att formulera undersökningsbara frågor.

En slutsats är att QFT som modell är för generell för att fungera i naturvetenskaplig undervisning och att undervisningen måste planeras i närmare växelverkan mellan styrdokument, lärarens ämneskompetens och elevgruppens behov. Denna slutsats ligger i linje med Minigan och Beers (2017) slutsats att även om QFT presenteras som en steg för steg-process så finns det fortfarande utrymme för läraren att skräddarsy och förändra metoden utifrån sin kunskap om de aktuella eleverna samt egna insikter om vilka praktiker som bäst passar det egna klassrummet. QFT bör snarare tillskrivas ett dynamiskt förhållningssätt än ses som en statisk strategi. Efter vår egen mangling av QFT kan vi inte annat än hålla med Minigan och Beer (2017) i att QFT behöver tillskrivas ett dynamiskt förhållningssätt och anpassas till rådande undervisningskontext.

Behov av fortsatt mangling

Om QFT ska kunna bli användbar som didaktisk modell för undervisning rörande formulering av undersökningsbara frågor i naturvetenskap krävs fortsatt mangling av QFT utifrån såväl tidigare forskning om undersökande arbete i naturvetenskap och naturvetenskapens karaktär som undervisning i naturvetenskap inom andra ämnesområden och sammanhang.

Ett motiv för genomförandet av den här studien var att pröva QFT som didaktisk modell för att utveckla undervisning med fokus på formulering av naturvetenskapligt undersökningsbara frågor. Bakgrunden till detta var problematiken med att eleverna själva sällan får möjlighet att träna förmågan att formulera undersökningsbara frågor utifrån en övergripande forskningsfråga eller problemsituation (Lunde m.fl., 2015;

Rothstein & Santana, 2011). Flera av de närliggande syften som etablerades i elevernas samtal visar också på en ovana hos eleverna att formulera egna undersökningsbara frågor. En tolkning är att denna ovana kan förstås i sken av en avsaknad av explicit undervisning om formulering av undersökningsbara frågor i naturvetenskap (jfr Lunde m.fl., 2015; Stokhof m.fl., 2019). Ytterligare en möjlighet är att eleverna, i likhet med de elever som fick göra öppna undersökningar i Wolf och Frasers (2008) studie, behövde mer stöd och vägledning från lärare i början för att kunna bli alltmer självständiga efterhand.

I parallell med Ledermans (2007) argument om att utveckling av elevers förståelse för naturvetenskapens karaktär (*nature of science*) kräver undervisning som explicit behandlar dessa frågor, så kräver också utveckling av elevers förmåga att formulera naturvetenskapligt undersökningsbara frågor en undervisning som explicit fokuserar frågeformulering. Resultaten från den här studien pekar också på nödvändigheten av att situera arbetet med formulering av frågor i en naturvetenskaplig kontext där eleverna får möjlighet att uppmärksamma olika kvaliteter av undersökningsbarhet inom olika naturvetenskapliga ämnesområden.

Våra resultat ger exempel på hur eleverna försöker reda i vad som kan utgöra ett undersökningsobjekt: Vad kan det innebära ”att hitta och specificera ett undersökningsobjekt”? Eleverna bearbetning av undersökningsobjektet tenderar dock bli abstrakt vilket kan förstås i skenet av att de givna frågeområdena kan ha varit alltför generella och långt ifrån elevernas erfarenheter. Vi har i en tidigare studie (Björnhammer m.fl., 2020) visat på behovet av att skapa möjligheter för elever att aktivt arbeta med att precisera eller specificera själva undersökningsobjektet, det vill säga att låta eleverna arbeta med frågan om *vad* de avser att undersöka. I den tidigare studien identifierade vi även att elevers förmåga att formulera undersökningsbara frågor i naturvetenskap innefattar en dimension av operationalisering av undersökningsobjektet. Också den här studien understryker behovet av att arbeta med hur frågor kan göras undersökningsbara. Vi menar även att vidare forskning behövs för att studera hur detta kan genomföras: i vilken utsträckning formulering av undersökningsbara frågor bör eller måste utgöra del av en undersökningspraktik, vilken betydelse elevernas tidigare erfarenheter av undersökningsmetoder och kunskaper om det ämnesområde som ska undersökas har, samt betydelsen av begreppsliga redskap för att göra frågan om centrala aspekter av undersökningsbarhet explicita (jfr Lederman, 2007 om nödvändigheten av att hantera frågor om naturvetenskapens karaktär som ett explicit innehåll i undervisningen).

Överförbarhet och begränsningar

I den här studien har QFT manglats som didaktisk modell i ett svenskt skolsammanhang och i relation till naturvetenskaplig undervisning med fokus på systematiskt undersökande. I detta arbete tolkades och omsattes QFT till undervisning i ett specifikt sammanhang. Undervisningen planerades för en specifik grupp elever vid en viss tidpunkt och på en viss plats av en särskild grupp forskande lärare med intresse för såväl den specifika undervisningssituationen som för hur den planerade och genomförda undervisningen kan ta fram ny kunskap om undervisning. En grundläggande fråga att ställa – givet studiens utgångspunkt i och intresse för det partikulära sammanhanget – är vilka möjligheter som finns att ta fram resultat som blir generaliserbara och överförbara till andra sammanhang.

Larsson (2009) argumenterar för en pluralistisk förståelse för generaliserbarhet i kvalitativ forskning som innebär tre olika sätt att resonera: (I) skapa förutsättningar för generalisering genom att maximera variation, (II) generalisering genom kontextlikhet och (III) generalisering genom mönster-igenkänning. I den här studien har förutsättningar för generalisering genom variation framförallt skapats genom att analysen designades för att fånga den variation som uppstår i undervisningen avseende vilka närliggande syften som etableras i de deltagande elevgrupperna. Genom att synliggöra den bredd av variation som uppstår i undervisningen skapas rikare möjligheter till generaliserbarhet. Variationen är dock begränsad till de specifika klasser, elever och lärare som ingår i studien.

Det andra alternativet till generalisering, generalisering genom kontextlikhet, synliggör vissa begränsningar avseende möjligheterna att generalisera resultaten. Studien är empiriskt begränsad till kursen Gymnasiearbete vid naturvetenskapligt program. Detta innebär att de elever som deltar i studien går sista året av tre på naturvetenskapligt program och har därmed merparten av de naturvetenskapliga studierna i gymnasieskolan bakom sig. Studien är också begränsad till en skola i stockholmsregionen med resursstarka elever (sett till meritvärden för antagning till årskurs 1 och bakgrundsfaktorer i form av föräldrarnas genomsnittliga utbildningsnivå och andel nyinvandrade elever jämfört med övriga skolor i Sverige). Sammantaget innebär de empiriska förutsättningarna att QFT har prövats i ett sammanhang med mycket kunniga elever i slutet av sina gymnasiestudier. Det är rimligt att anta att elever i gymnasieskolans tidigare årskurser eller grundskolan kan etablera andra närliggande syften än de som etablerades i denna studie. Vi skulle exempelvis kunna förvänta oss att elever i andra sammanhang i mindre grad utgår från kvaliteter avseende frågors undersökningsbarhet eller att de synliggör andra dimensioner av relevans och genomförbarhet.

Avseende generalisering genom mönsterigenkänning innebär detta att en del av ansvaret för att pröva resultatens generaliserbarhet förläggs till läsaren. Det handlar om att den lärare som tar del av resultaten behöver pröva i vilka avseenden de närliggande syften som identifierats i studien fungerar som sätt att beskriva och tolka utmaningar som uppstår i undervisning som syftar till att utveckla elevers förmåga att formulera och bearbeta undersökningsbara frågor i nya sammanhang. I avsnitten ”QFT som modell för att formulera undersökningsbara frågor i naturvetenskap” och ”Behov av fortsatt mangling” pekar vi på några av de resultat som vi ser har tydligast bäring på naturvetenskaplig undervisning i andra sammanhang. I viss mån kan möjligheterna att generalisera resultaten också anses ha prövats gentemot nya sammanhang mot bakgrund av att alla lärare som deltagit i forskargruppen representerar erfarenheter från skolor med olika förutsättningar från såväl gymnasiet som högstadiet.

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Matematiikan parhaat osaajat lukion lopussa ja heidän matematiikka-asenteissaan tapahtuneet muutokset

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Tutkimus perustuu Opetushallituksen ja Kansallisen koulutuksen arviointikeskuksen keräämään pitkittäisaineistoon. Samaan ikäluokkaan kuuluvat oppilaat ovat osallistuneet kansallisiin matematiikan kokeisiin ja matematiikka-asenteita kartoittaviin kyselyihin vuosien 2005–2015 aikana neljällä eri mittauskerralla perusopetuksen kolmannelta vuosiluokalta toisen asteen loppuun. Tutkimusaineiston kokonaisotos käsittää yhteensä 3896 oppilasta. Tutkimuksessa keskitytään tarkastelemaan matematiikassa parhaiten menestyneitä opiskelijoita. Matematiikan parhaiksi osaajiksi määritetään kansalliseen matematiikan kokeeseen osallistuneet lukiolaiset, jotka saivat pitkän matematiikan ylioppilaskokeesta arvosanan laudatur tai eximia cum laude approbatur (n = 146). Ensin tutkimuksessa selvitetään, miten parhaiden osaajien matematiikka-asenteet muuttuivat perusopetuksesta lukion loppuun ja toiseksi, miten opetuksen pedagogiset ratkaisut yläkoulussa ja lukiossa selittävät osaamiseltaan parhaiden tyttöjen ja poikien asenteissa tapahtuneita muutoksia. Selittävien tekijöiden analyysissa käytetään päätöspuuanalyysia (DTA) ja lineaarista regressioanalyysia. Matematiikan parhaiden osaajien matematiikasta pitäminen kasvoi lukio-opintojen aikana, mutta minäkäsitys ja kokemus matematiikan hyödyllisyydestä laskivat. Matematiikassa parhaiten menestyneiden tyttöjen asenteissa tapahtuneet muutokset poikkesivat asenteiden yleisestä muutossuunnasta. Parhaiden tyttöjen minäkäsitys kasvoi yläkoulun ja lukion aikana lähes parhaiten menestyneiden poikien tasolle ja tytöt pitivät matematiikasta lukion lopussa poikia enemmän. Matematiikassa parhaiten menestyneiden tyttöjen ja poikien asenteiden kehittymistä selittivät erilaiset opetuksen pedagogiset ratkaisut. Molemmilla myönteisiä asenteita vahvistivat yleisesti oppilaskeskeisyyteen, yhteistoiminnallisuuteen ja oppijoiden tarpeiden huomioimiseen liittyvät pedagogiset ratkaisut.

AVAINSANAT: matematiikan parhaat osaajat, pitkittäistutkimus, kansallinen arviointi, matematiikka-asenteet, toisen asteen koulutus

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1 Johdanto

Matematiikan opetuksen tavoitteena perusopetuksessa ja lukiossa on pitää yllä oppilaan innostusta ja kiinnostusta matematiikkaa kohtaan sekä tukea oppilaan myönteistä minäkuvaa ja itseluottamusta (Opetushallitus, 2004; 2014; 2015; 2019). Tämä ei kuitenkaan näyttäisi toteutuvan, sillä matematiikkaan liittyvät asenteet heikkenevät yleisesti perusopetuksen aikana. Tämä saattaa vaikuttaa oppilaiden toista astetta koskeviin koulutusvalintoihin ja myös tulevissa opinnoissa ja elämässä menestymiseen (Metsämuuronen, 2013). On esitetty myös huoli tyttöjen matematiikkaan liittyvästä itseluottamuksesta, joka laskee selvästi poikien minäpystyvyyttä alhaisemmaksi jo peruskoulun aikana (Tuohilampi & Hannula, 2013; Metsämuuronen, 2017). Tässä tutkimuksessa on tarkoitus selvittää, miten matematiikkaan liittyvät asenteet kehittyvät perusopetuksesta lukion loppuun, kun tutkimuskohteena ovat matematiikassa parhaiten menestyneet opiskelijat.

Tutkimus perustuu Opetushallituksen ja Kansallisen koulutuksen arviointikeskuksen keräämään matematiikan oppimistuloksia käsittelevään pitkittäisaineistoon, jossa samaan ikäluokkaan kuuluvia oppilaita on seurattu perusopetuksen kolmannelta vuosiluokalta toisen asteen koulutuksen loppuun vuosien 2005–2015 aikana. Matematiikan parhaiksi osaajiksi määritetään lukio-opiskelijat, jotka ovat menestyneet parhaiten pitkän matematiikan ylioppilaskokeessa. Aikaisempiin tutkimustuloksiin (Julin & Rautopuro, 2016; Niemi ym., 2021) perustuen tiedetään, että opiskelijat, jotka suuntautuivat lukiossa pitkän matematiikan opiskeluun, menestyivät keskimäärin jo yhdeksännellä luokalla selvästi muita paremmin kaikilla matematiikan osa-alueilla ja suurin osa yhdeksännen vuosiluokan kokeessa parhaiten menestyneistä osaajista oli parhaita myös toisen asteen lopussa pidetyssä kokeessa. Toisaalta samaan tutkimusaineistoon perustuvien aikaisempien analyysien mukaan tiedetään, että erinomaiseen menestymiseen matematiikassa voi yltää myös keskitasoa heikommastakin lähtötasosta (Niemi ym., 2020). Emme ajattelekaan menestymistä matematiikassa synnynnäisenä ja muuttumattomana lahjakkuutena (ks. mm. Sternberg & Davidson, 2005) vaan näemme matemaattisten taitojen olevan kehitettävissä (ks. mm. Leikin, 2014).

Opettajalla on tärkeä rooli oppilaiden myönteisten asenteiden edistämisessä. Esimerkiksi yhteistoiminnallisten opetusmenetelmien käytön on havaittu parantavan keskitasoa parempien osaajien matematiikkaan liittyviä asenteita ja näiden menetelmien käyttö on vaikuttanut erityisesti poikien myönteiseen asennekehitykseen (Hannula & Oksanen, 2013). Tutkimuksen toisena tavoitteena onkin selvittää, millaiset opetuksen pedagogiset ratkaisut vahvistavat matematiikassa parhaiten menestyneiden opiskelijoiden matematiikka-asenteita. Matematiikan parhaiden osaajien asenteita koskeva tutkimus on vähäistä ja tällä tutkimuksella saadaan tietoa siitä, millaiset pedagogiset ratkaisut yläkoulussa ja lukiossa ovat tärkeimpiä, kun halutaan tukea matematiikassa parhaiten menestyneiden tyttöjen ja poikien myönteistä suhtautumista matematiikkaa kohtaan.

2 Matematiikkaan liittyvät asenteet

Matematiikkaan liittyviä asenteita ja niiden suhdetta osaamiseen voidaan tarkastella monesta eri näkökulmasta. Tämä tutkimus perustuu kansalliseen matematiikan oppimistulosten arviointiin, jossa oppilaiden matematiikka-asenteita kartoitettiin kolmella ulottuvuudella: matematiikasta pitäminen, käsitys itsestä matematiikan osaajana ja kokemus matematiikan hyödyllisyydestä (ks. Metsämuuronen, 2009).

Lukuisat tutkimukset ovat osoittaneet matematiikkaan liittyvien asenteiden ja matematiikan osaamisen välillä olevan positiivinen korrelaatio (mm. Bandura, 1986 lähtien; Hannula & Laakso, 2011; Ma & Kishor, 1997; Roesken ym., 2011). Vaikka asenteiden ja saavutusten välistä kausaliteettisuhdetta on tutkittu yhä enemmän, vuorovaikutuksen suunnasta on saatu ristiriitaisia tuloksia. PISA-aineistosta tehdyn analyysin perusteella suomalaisoppilailla osaamisen vaikutus minäpystyvyyteen on yksi suurimmista (Williams & Williams, 2010). Suomalaisessa pitkittäistutkimuksessa oppilaiden matematiikan osaamisen on havaittu vaikuttavan minäpystyvyyden kehittymiseen koko peruskoulun ajan, mutta vastakkaissuuntainen vaikutus voimistuu vähitellen ollen peruskoulun loppuvaiheessa sen kanssa samalla tasolla (Hannula ym., 2014). Matematiikasta pitämisen tai matematiikan hyödyllisyyden kokemisen yhteyttä osaamiseen on tutkittu vähemmän. Ma ja Kishor (1997) saivat meta-analyysissaan tulokseksi, että matematiikasta pitäminen vaikuttaa osaamiseen, mutta vaikutus heikkenee opiskelijoiden iän myötä. Meta-analyysissa oli mukana 113 tutkimusta koskien perusopetuksen ja toisen asteen opintoja. Ma ja Xu (2004) puolestaan osoittivat, että toisen asteen opinnoissa matematiikan osaaminen selittää, miten hyödylliseksi oppilas kokee matematiikan.

Matematiikkaan liittyvien asenteiden on havaittu Suomessa heikkenevän kouluvuosien aikana. Aluksi heikkenee ensisijaisesti matematiikasta pitäminen alakoulun aikana ja sen jälkeen yläkoulussa minäpystyvyys (Metsämuuronen, 2013). Pitkittäistutkimuksessa oppilaat pitivät matematiikasta eniten ensimmäisellä mittauskerralla, kolmannen luokan alussa. Pitäminen heikkeni huomattavasti kolmannen ja kuudennen luokan välillä. Minäpystyvyys ei juurikaan muuttunut vielä kolmannelta luokalta kuudennelle, mutta se heikkeni huomattavasti yläkoulun aikana. Oppilaat kokivat matematiikan hyödyllisyyden korkeaksi kuudennella luokalla, mutta koettu hyödyllisyys heikkeni samanaikaisesti minäpystyvyyden tunteen kanssa yläkoulun aikana. Oppilaat alkavat iän myötä myös arvioida osaamistaan enemmän suhteessa muihin samassa ryhmässä opiskeleviin oppilaisiin (mm. Tuohilampi & Hannula, 2013). *Big fish, little pond* -efekti selittää oppilaan minäpystyvyyden kokemisessa tapahtuneita muutoksia, kun hän vertaa omaa osaamistaan ryhmän keskimääräiseen osaamiseen. Jos oppilaan taidot ovat keskivertoa paremmat, hänen käsityksensä itsestä matematiikan osaajana paranee ja vastaavasti oppilaan, joka kokee taitonsa keskitasoa heikommaksi, käsitys itsestä osaajana heikkenee (Marsh ym., 2019; Holm ym., 2020).

Kansallisissa oppimistulosarvioinneissa asenteilla on havaittu olevan suuri merkitys opiskeluun toisen asteen koulutuksessa. Yhdeksännen luokan kokonaisasenne ja kokonaisosaaminen selittävät sekä lukioon hakeutumista että matematiikan kurssien määrää toisella asteella. Mitä parempaa opiskelijan osaaminen on 9. luokalla, sitä positiivisempi on hänen käsityksensä matematiikasta oppiaineena ja sitä todennäköisemmin hän valitsee lukio-opinnot ja pitkän matematiikan (Metsämuuronen, 2017).

Matematiikkaan liittyy vahvoja sukupuolittuneita stereotypioita, jotka määrittävät tyttöjen ja poikien käsityksiä itsestään matematiikan oppijoina jo varhain. Jo koulun alussa tytöt arvioivat olevansa poikia heikompia matematiikassa, vaikka tyttöjen ja poikien matematiikan taidoissa ei olekaan eroa (Cvencek ym., 2011; Lindberg ym., 2013). Myös Oppermann ja kanssakirjoittajat (2021) osoittivat tutkimuksessaan, että sukupuoli vaikuttaa matematiikan opiskeluintoon jo peruskoulun toisella ja kolmannella luokalla. Tutkimuksessa löydettiin kolme erilaista ryhmää, joissa poikia oli eniten ryhmissä, joissa matematiikka koettiin innostavana ja tyttöjä enemmän ryhmässä, jossa oppilaiden suhtautuminen opiskeluun oli korkea kaikissa oppiaineissa. Tyttöjen käsitykset omasta matematiikan osaamisesta heikkenevät poikia voimakkaammin kouluvuosien edetessä ja sukupuolten välinen ero kasvaa (Lindberg ym., 2013). Sukupuolierot matematiikkaan liittyvässä itseluottamuksessa olivat Suomessa suuremmat kuin monessa muussa PISA-tutkimusmaassa (Williams & Williams, 2010). Tyttöjen kiinnostuksen matematiikkaan ja luonnontieteisiin on nähty viime vuosina kuitenkin lisääntyvän. Korkeakoulujen matematiikkaa painottava todistusvalinta on saanut enemmän naisia suorittamaan pitkän matematiikan ylioppilaskokeen. Vuonna 2020 jo yli puolet pitkän matematiikan hyväksytysti suorittaneista opiskelijoista oli naisia (Ylioppilastutkintolautakunta, 2021).

Lahjakkaiden oppilaiden asenteita on tutkittu melko vähän. Erdogan ja Yemenli (2019) selvittivät matemaattisesti lahjakkaiden viidesluokkalaisten asenteita matematiikkaa kohtaan oppilaiden kertomusten ja haastattelujen avulla. Tulosten mukaan enemmistöllä oppilaista oli matematiikkaan positiivinen ja melko vakaa asenne, joka kehittyi varhaisessa iässä. Salmela (2016) tutki laudaturylioppilaiden vahvuuksia selvittääkseen, miten vahvuuksia voidaan tukea. Hänen tuloksensa osoittivat, että lahjakkaita oppilaita yhdisti vahva koulumyönteisyys ja sitoutuneisuus. Salmelan tutkimat laudaturylioppilaat menestyivät hyvin koulussa alaluokilta ylioppilastutkintoon ja heidän opiskeluaan kuvasi sinnikkyys, oma-aloitteisuus ja itseohjautuvuus.

3 Opetuksellisten tekijöiden yhteys opiskeluasenteisiin

Opettajan toiminnalla, opetuskäytännöillä ja vuorovaikutuksella on keskeinen merkitys oppilaiden opiskeluasenteiden kehittymiselle ja ylläpitämiselle (Salmela-Aro, 2018). Perusopetuksen ja lukion opetussuunnitelmien (Opetushallitus, 2004; 2014; 2015; 2019) mukaan matematiikan opetuksen tulee tukea oppilaiden myönteistä asennetta matematiikkaa kohtaan ja myönteistä minäkuvaa matematiikan oppijoina. Opetuksen tulisi myös ohjata oppilaita ymmärtämään matematiikan hyödyllisyys omassa elämässään ja laajemmin yhteiskunnassa. Lisäksi yläkoulussa oppilasta tulisi ohjata kehittämään oppimaan oppimisen taitoja ja opiskeluvalmiuksia tulevia jatko-opintoja varten (Opetushallitus, 2004; 2014). Tutkijat (mm. Hannula & Oksanen, 2013; Koskinen, 2016; Metsämuuronen, 2017) ovat tunnistaneet erilaisia tapoja toteuttaa näitä tavoitteita. Keskeisiä ulottuvuuksia asenteisiin vaikuttavassa pedagogiikassa ovat oppilaskeskeisyys, monipuolisten opetusmenetelmien käyttö ja oppijoiden tarpeiden huomioiminen opetuksessa.

Oppilaat suhtautuvat opiskeluun myönteisemmin, kun opetuksessa toteutetaan oppilaskeskeisyyttä eli oppilaat voivat itse vaikuttaa ja päättää tekemisestään ja tuntea yhteenkuuluvuutta muiden oppilaiden kanssa (Salmela-Aro, 2018; Ryan & Deci, 2017). Yhteistoiminnallisten opetusmenetelmien on havaittu tuottavan kansallisessa oppimistulosarvioinnissa hyviä tuloksia sekä osaamisen että asenteiden suhteen (Hannula & Oksanen, 2013). Se, että oppilaat neuvoivat toisiaan, paransi osaamiseltaan keskitasoa parempien oppilaiden asenteita. Lisäksi yhteistoiminnallisten menetelmien käyttö ja opettajan hyvä oppilaiden käyttäytymisen hallinta vaikuttivat voimakkaammin poikien myönteisempään asennekehitykseen. Myös laudaturylioppilaita tutkineen Salmelan (2016) mukaan opiskelijoiden vahvuuksia ja opintomenestystä tukee parhaiten opetus, jota ilmentävät muun muassa opiskelijakeskeisyys ja oppimisen yhteisöllisyys sekä palautteen antaminen ja kannustaminen. Yhteistoiminnalliset menetelmät noudattavat sosiokonstruktivistista oppimiskäsitystä, jolle myös perusopetuksen opetussuunnitelman perusteet (Opetushallitus, 2004; 2014) pohjautuvat.

Monipuolisten opetusmenetelmien käyttö tarkoittaa, että opiskelussa käytetään vaihtelevia työtapoja. Perusopetuksen opetussuunnitelman perusteissa (Opetushallitus, 2014) ohjataan luomaan oppimisympäristö, jossa konkretia ja välineet ovat osana matematiikan opiskelua. Opiskelua on itsenäisesti ja yhdessä, hyödynnetään oppimispelejä ja käytetään tieto- ja viestintäteknologiaa. Yhteistoiminnallisuuden lisäksi oppimisen tulisi siis olla mielekästä, jossa konkreettisuus ja kontekstuaalisuus tukevat ymmärtämistä (Koskinen, 2016). Innovatiivisten opetusmenetelmien käytön on havaittu tukevan perinteisiä opetusmenetelmiä paremmin oppilaiden myönteisten asenteiden kehittymistä (mm. Ogbuehi & Fraser, 2007).

Oppijoiden tarpeiden ja yksilöllisten erojen huomioiminen opetuksessa on keskeistä positiivisen asennoitumisen vahvistamiseksi (mm. Bloom, 1984). Opetuksessa tulisi huomioida oppijoiden yksilölliset erot ja tarjota taitotason mukaisia haasteita. Vanttajan (2002) tutkimuksessa osa laudaturylioppilaista koki opiskelun liian helpoksi eikä opetus ja oppilaan taitotaso kohdanneet. Perusopetuksen opetussuunnitelman perusteiden (Opetushallitus, 2014) mukaan kaiken opetuksen pedagogisena lähtökohtana tulisi olla eriyttäminen, joka perustuu oppilaiden tarpeille ja mahdollisuuksille muun muassa edetä yksilöllisesti. Eriyttämiseen liittyy myös mahdollisuudet suunnitella itse opiskelua ja valita erilaisia työtapoja. Eriyttämällä tuetaan oppilaan itsetuntoa, motivaatiota ja turvataan oppimisen rauhaa (mts. 30). Mielekäs eriyttäminen taitotason mukaisesti parantaa asenteiden lisäksi myös oppilaan osaamistasoa (Metsämuuronen, 2017).

4 Tutkimuskysymykset

Tutkimuksessa on tarkoitus selvittää, miten matematiikan parhaiden osaajien matematiikkaan liittyvät asenteet muuttuvat perusopetuksesta lukion loppuun ja millaiset opetuksen pedagogiset ratkaisut selittävät asenteissa tapahtuneita muutoksia.

Tutkimuskysymykset ja niille asetut hypoteesit ovat seuraavat.

1. Miten matematiikan parhaiden osaajien matematiikkaan liittyvät asenteet muuttuvat perusopetuksesta lukion loppuun ja millaisia eroja tässä kehityksessä on tyttöjen ja poikien välillä?

Hypoteesi: Matematiikka-asenteiden tiedetään heikkenevän kouluvuosien myötä (mm. Metsämuuronen, 2013). Matematiikasta pitäminen laskee jo alakoulun aikana ja minäpystyvyys ja hyödyllisyyden kokeminen yläkoulun aikana (Tuohilampi & Hannula, 2013). Oletamme näin tapahtuvan myös parhailla osaajilla. *Big fish, little pond* -efektin (Marsh ym., 2019; Holm ym., 2020) mukaan esitämme, että parhailla osaajilla minäpystyvyys keskimäärin heikkenee. Tyttöjen käsitykset omasta matematiikan osaamisesta heikkenevät poikia voimakkaammin (mm. Lindberg ym., 2013).

2. Millaiset opiskelijoiden raportoimat yläkoulun ja lukion aikaiset opetuksen pedagogiset ratkaisut ovat yhteydessä matematiikassa parhaiten menestyneiden tyttöjen ja poikien asenteissa tapahtuneisiin muutoksiin yhdeksänneltä luokalta lukion loppuun?

Hypoteesi: Yleisesti myönteisiä asenteita voidaan vahvistaa pedagogisilla ratkaisuilla, joissa keskitytään oppilaskeskeisyyteen ja yhteistoiminnallisuuteen, matematiikan käytännönläheisyyteen sekä oppijoiden yksilöllisiä tarpeita huomioiviin toimintamalleihin (Ryan & Deci, 2017; Salmela-Aro, 2018; Hannula & Oksanen, 2013; Salmela, 2016; Koskinen, 2016; Ogbuehi & Fraser, 2007; Bloom, 1984; Vanttaja, 2002). Yhteistoiminnalliset menetelmät opetuksessa selittävät erityisesti pojilla asenteiden myönteistä kehittymistä (Hannula & Oksanen, 2013). Oletamme näiden tekijöiden vahvistavan myös parhaiden osaajien matematiikka-asenteita.

5 Tutkimuskohde

Tutkimuksessa käytetään Opetushallituksen ja Kansallisen koulutuksen arviointikeskuksen keräämää aineistoa, jossa samaan ikäluokkaan kuuluvia oppilaita on seurattu vuosien 2005–2015 välisenä aikana perusopetuksen kolmannelta vuosiluokalta toisen asteen koulutuksen loppuun. Oppilaat ovat tehneet matematiikan osaamista kartoittavat kokeet 3. ja 6. luokan alussa, 9. luokan lopussa sekä ammatillisen koulutuksen tai lukiokoulutuksen lopussa. Lisäksi oppilaat ovat vastanneet matematiikan asenteita kartoittavaan kyselyyn ja heiltä on kerätty myös erilaista taustatietoa demografisiin tietoihin, yksilöön, kouluun ja kotitaustaan liittyen. Tutkimusaineisto on laaja ja kansallisesti edustava. Kokonaisaineisto käsittää yhteensä 3896 opiskelijaa puuttuvien havaintojen mallintamisen jälkeen (ks. Niemi ym. 2021, s. 20–21). Asennemuuttujien osalta puuttuvia arvoja ei korvattu. Näiltä osin aineisto käsittää 2048 opiskelijaa.

Tutkimuskohteena ovat matematiikan parhaat osaajat. Matematiikan parhaiksi osaajiksi määritetään tässä tutkimuksessa kansalliseen matematiikan kokeeseen toisen asteen lopussa osallistuneet lukiolaiset, jotka saivat pitkän matematiikan ylioppilaskokeesta arvosanan laudatur tai eximia cum laude approbatur (jatkossa lyhyemmin eximia). Parhaat osaajat olisi voitu valita myös kansallisen kokeen perusteella. Valtaosa kansallisen kokeen parhaista osaajista kirjoitti pitkän matematiikan ja menestyi erinomaisesti ylioppilaskokeessa. Kokonaisuudessaan arvioimme ylioppilastutkinnon antavan kansallista koetta todenmukaisemman kuvan opiskelijoiden osaamisen tasosta toisen asteen lopussa. Ylioppilastutkinnolla on suuri painoarvo jatko-opintojen kannalta ja opiskelijat panostavat siihen kansallista koetta enemmän. Tämä rajaus myös mahdollistaa paremmin opetuksellisten tekijöiden merkityksen selvittämisen, sillä heidän opiskelukokemuksensa toisella asteella ovat keskenään samankaltaisemmat kuin jos mukaan olisi otettu myös lyhyen matematiikan ja ammatillisen koulutuksen suorittajat.

Kaikkiaan kansalliseen kokeeseen osallistuneista 54 prosenttia oli lukiolaisia ja lukiolaisista pitkän matematiikan ylioppilaskokeen suoritti yhteensä 490 opiskelijaa (12,6 % koko aineistosta ja 23,3 % lukiolaisista.) Myös kaikkiaan keväällä 2015 ylioppilastutkintoon ilmoittautuneista opiskelijoista noin 23 prosenttia ilmoittautui pitkän matematiikan ylioppilaskokeeseen (Ylioppilastutkintolautakunta, 2021). Tutkimusaineiston keruussa toisen asteen osalta on jonkin verran katoa, mutta ikäluokasta on tavoitettu kuitenkin riittävän kattava määrä opiskelijoita maan eri osista, kuntatyypeistä ja kieliryhmistä (Metsämuuronen, 2017). Laudaturin tai eximian kirjoittaneita opiskelijoita oli yhteensä 146 (3,8 % koko aineistosta ja 30,0 % pitkän matematiikan kirjoittajista). Parhaista osaajista poikia oli 85 (58,2 %) ja tyttöjä 61 (41,8 %).

Parhaiden osaajien vertailuryhmänä pidetään niitä lukiolaisia, jotka saivat pitkän matematiikan ylioppilaskokeessa muun arvosanan kuin eximia tai laudatur (jatkossa keskitason osaajat, n = 344). Taulukossa 1 nähdään parhaiden ja keskitason osaajien kokonaispistemäärien eroja lukion lopussa pidetyssä kokeessa. Osaamista kuvaavat pistemäärät esitetään samalla asteikolla kuin PISA- ja TIMSS-tutkimuksissa. Tällä asteikolla osaamiseltaan keskitason oppilas saa noin 500 pistettä ja keskihajonta on 100 pistettä (ks. Metsämuuronen, 2017, s. 214–215).

Taulukko 1. Kansallisen kokeen kokonaispisteet toisen asteen lopussa

Koepisteet	Yo-kokeen parhaat osaajat (n = 146)	Yo-kokeen keskitason osaajat (n = 344)	Koko otos (n = 3896)
Keskiarvo	772,3	672,9	542,7
Minimi	593,6	439,5	192,6
Maksimi	934,6	832,9	934,6
Keskihajonta	59,4	64,7	124,7

6 Tutkimuksessa käytettävät mittarit

Opiskelijoiden osaamista ja asenteita kartoitettiin neljällä eri mittauskerralla heidän ollessaan 3, 6. ja 9. vuosiluokalla sekä kolme vuotta myöhemmin, jolloin tavoitetut opiskelijat olivat toiseen asteen opintojen loppuvaiheessa ammatillisessa koulutuksessa tai lukiossa. Lisäksi opiskelijoilta kerättiin mittauskerroilla erilaisia taustatietoja, joista tässä tutkimuksessa keskitytään tarkastelemaan opiskelijoiden raportoimia opetuksen pedagogisia ratkaisuja yläkoulussa ja lukiossa.

Koetehtävät

Osaamista kartoittavat kansallisten kokeiden tehtävät, arvosteluperusteet ja pisteytysohjeet on laadittu asiantuntijaryhmissä ja tehtäväsarjat on esitestattu. Tehtäväsarjoissa oli vaikeustasoltaan helppoja, keskivaikeita ja vaikeita osioita (ks. tarkemmat tiedot Metsämuuronen, 2009). Kokeiden tehtävät pohjautuvat perusopetuksen opetussuunnitelman perusteissa (Opetushallitus, 2004) sekä lukion opetussuunnitelman perusteissa (Opetushallitus, 2003) ja ammatillisen koulutuksen tutkintoperusteissa (Opetushallitus, mm. 2009) määriteltyihin matematiikan tavoitteisiin ja sisältöihin.

Jotta eri koeversioiden tuloksia voidaan vertailla keskenään, kokeiden pistemäärät on vertaistettu eli saatettu yhteismitallisiksi osiovasteteoriaan (*Item Response Theory*) perustuvan IRT-mallituksen avulla (Rasch, 1960; Lord & Novick, 1968). Vertaistamisessa on käytetty linkkitehtäviä, joiden avulla osioiden vaikeustasoa voidaan arvioida. Lukion ja ammatillisen koulutuksen kokeissa 78 prosenttia tehtävistä oli suoraan yhdeksännen luokan kokeesta ja osa tehtävistä oli mukana jo kuudennen ja kolmannen luokan kokeissa (ks. tarkemmin mm. Metsämuuronen, 2017, s. 213–214).

Asennemittarit

Asenteita kartoitettiin 15 osion Likert-asteikollisella mittarilla, joka pohjautuu laajalti käytettyyn Fenneman ja Shermanin (1976) matematiikka-asennemittariin. Käytetty asteikko oli viisiportainen kolmannen luokan versiota lukuun ottamatta. Asenteita kartoitettiin kolmatta luokkaa lukuun ottamatta kaikilla muilla luokka-asteilla kolmesta näkökulmasta: matematiikasta pitäminen, käsitys itsestä matematiikan osaajana ja matematiikan hyödyllisyyden kokeminen. Osiota matematiikan hyödyllisyydestä ei kysytty kolmannella luokalla. Osioista *matematiikasta pitäminen*, *käsitys itsestä matematiikan osaajana* ja *matematiikan hyödyllisyys* koottiin luokka-asteen kokonaisasennetta kuvaava summamuuttuja. Lopullisten summien asteikko skaalattiin uudelle asteikolle, jonka arvot vaihtelivat välillä 0–4 (0–1 = negatiivinen asenne, 2 = neutraali asenne, 3–4 = positiivinen asenne) (Metsämuuronen, 2017, s. 37).

Yhdeksännellä luokalla ja toisella asteella kerättiin asenteisiin liittyvää tietoa myös oppilaiden kokemasta matematiikka-ahdistuksesta ja toisella asteella matematiikan opiskeluun liittyvistä tunnetiloista (Metsämuuronen & Tuohilampi, 2017). Tässä tutkimuksessa keskitytään matematiikan minäkäsityksen, matematiikasta pitämisen ja matematiikan hyödyllisyyden kokemisen osa-alueisiin, koska halutaan analysoida asenteissa tapahtuneita muutoksia ja näistä kolmesta löytyy tietoa kaikilta tutkituilta kouluasteilta. Eri vuosiluokkien asenteita vertailtaessa käytetään prosenttiosuuksia käytettyjen asteikkojen maksimipistemääristä.

Opetuksen pedagogiset ratkaisut

Opetuksen pedagogisia ratkaisuja on kysytty opiskelijoilta yhdeksännellä vuosiluokalla ja toisen asteen lopussa. Opiskelijat ovat arvioineet pedagogisten ratkaisujen toistuvuuden toteutumista viisiportaisella Likert-asteikolla (1= *ei* *lainkaan*, 2= *harvoin*, 3= *joskus*, 4 = *usein*, 5= *lähes aina*).

Opetuksen pedagogisten ratkaisujen ryhmittelyyn käytettiin eksploratiivista faktorianalyysia. Siinä tutkitaan korrelaatiomatriisien rakennetta eli etsitään muuttujien kombinaatioista sellaista mallia, joka selittää parhaiten muuttujien välistä vaihtelua (Metsämuuronen 2011, s. 667). Lataukset estimoitiin suurimman uskottavuuden menetelmällä (*Maximum likelihood)* ja rotaatiomenetelmistä käytettiin vinokulmaista *Oblimin*-rotaatiomenetelmää, jossa sallitaan, että faktorit voivat korreloida keskenään (Jokivuori & Hietala, 2007). Kun analyysin avulla löydettiin sopivat faktorit, ne nimettiin ja niiden perusteella muodostettiin summamuuttujat, joita käytetään myöhemmin tulosten analyyseissa. Tässä tutkimuksessa opetuksellisia tekijöitä haluttiin ryhmitellä paremmin ymmärrettäviksi kokonaisuuksiksi. Tulosten analyyseissa käytetään sekä näitä jäsennettyjä kokonaisuuksia että alkuperäisiä yksittäisiä muuttujia.

Yksittäisistä muuttujista muodostettiin eksploratiivisen faktorianalyysin avulla ryhmiä, jotta tunnistettaisiin yksittäisiä opetusmenetelmiä laajempia pedagogisia lähestymistapoja. Faktorianalyysi tehtiin tutkimusaineistolle, joka käsittää yhteensä 3896 opiskelijaa. Yhdeksännen luokan opetuksen pedagogisten ratkaisujen faktorianalyysi tehtiin 3455 opiskelijan vastausten perusteella ja toisen asteen lopun pedagogisten ratkaisujen faktorianalyysi 1934 opiskelijan vastausten perusteella.

Rotatoidussa faktorimatriisissa (taulukko 2) näkyvät yläkoulun tekijöihin liittyvät faktorit ja niille latautuneet muuttujat.

Taulukko 2. Rotatoitu faktorimatriisi yläkoulun aikaisista opetuksen pedagogisista ratkaisuista

	1	2	3
Oppilaat selittävät muille, miten ovat tehtävänsä ratkaisseet.	**,857**	-,055	-,154
Pohditaan, onko tehtävän vastaus järkevä.	**,530**	,095	,105
Oppilaat neuvovat toisiaan.	**,519**	-,070	,093
Oppilaat asettavat itselleen tavoitteita ja arvioivat edistymistään.	**,372**	,258	,086
Tehdään projektitöitä.	-,036	**,708**	-,165
Oppilaat käyttävät tietokonetta.	-,074	**,565**	-,062
Opitaan mittaamalla, rakentelemalla ja muulla tavoin tekemällä.	,056	**,550**	,118
Opiskellaan ryhmissä tai pareittain.	,077	**,386**	,073
Sovelletaan matematiikan taitoja arkielämän tilanteisiin.	,176	**,333**	,218
On yhteistä opetusta opettajan johdolla.	,030	-,097	**,525**
Opettaja ottaa huomioon opetukseen liittyvät oppilaiden ideat ja toiveet.	,029	,298	**,524**
Annetut kotitehtävät olen tehnyt sovitulla tavalla.	-,002	-,019	**,359**

Yläkoulun 12 tekijää muodostivat kolme faktoria, jotka nimettiin seuraavasti: Oppilaskeskeisyys (faktori 1), Monipuoliset opetusmenetelmät (faktori 2) ja Opettajajohtoisuus (faktori 3). Analyysista poistettiin kolme muuttujaa (*Harjoitellaan päässälaskuja*, *Pidetään testejä ja kokeita*, *Kukin ratkaisee itselleen sopivan vaikeita tehtäviä*), joiden lataus kaikille alkuperäisille faktoreille oli alle 0,30. Analyysin yhteydessä varmistettiin, että korrelaatiomatriisit soveltuvat faktorianalyysiin (Kaiserin testitulos 0,832 ja Barlettin sväärisyystesti: $\chi^2(66) = 7023{,}041$; $p < 0{,}001$).

Lukion aikaisiin tekijöihin liittyvät faktorit ja niille latautuneet muuttujat näkyvät alla rotatoidussa faktorimatriisissa (taulukko 3).

Taulukko 3. Rotatoitu faktorimatriisi lukion aikaisista opetuksen pedagogisista ratkaisuista

	1	2	3	4
Opetus on sidottu käytännön tilanteisiin.	**,808**	,061	,087	-,128
Sovelletaan matematiikan taitoja arkielämän tilanteisiin.	**,479**	,129	,196	-,136
Opiskeltavat asiat tulevat selväksi.	,050	**,694**	-,023	,032
Opettaja ottaa huomioon opetukseen liittyvät oppilaiden ideat ja toiveet.	-,031	**,589**	,073	-,090
Kukin ratkaisee itselleen sopivan vaikeita tehtäviä.	-,145	**,476**	,159	-,110
On yhteistä opetusta opettajan johdolla.	-,046	**,428**	-,277	-,112
Annetut kotitehtävät olen tehnyt sovitulla tavalla.	,130	**,381**	-,022	,071
Opiskelijat etenevät omassa tahdissaan.	,096	**,331**	,163	-,029
Tehdään projektitöitä.	,058	-,026	**,748**	,058
Opitaan mittaamalla, rakentelemalla tai muulla tavoin tekemällä.	,139	-,001	**,655**	-,045
Opiskelijat käyttävät tietokonetta.	,063	-,020	**,598**	,050
Opiskellaan ryhmissä tai pareittain.	-,103	,119	**,433**	-,176
Opiskelijat selittävät muille, miten ovat tehtävänsä ratkaisseet.	,026	-,130	-,013	**-,834**
Pohditaan, onko tehtävän vastaus järkevä.	,163	,082	-,042	**-,581**
Opiskelijat asettavat itselleen tavoitteita ja arvioivat edistymistään.	,144	,086	,243	**-,356**
Opiskelijat neuvovat toisiaan.	-,132	,236	,023	**-,356**

Lukion 16 tekijästä muodostui neljä faktoria: Matematiikan yhteys käytäntöön (faktori 1), Oppijoiden tarpeiden huomioiminen (faktori 2), Monipuoliset opetusmenetelmät (faktori 3) ja Oppilaskeskeisyys (faktori 4). Analyysista poistetiin kaksi muuttujaa (*Harjoitellaan päässälaskuja, Pidetään testejä ja kokeita*), joiden lataus kaikille alkuperäisille faktoreille oli alle 0,30. Kaiserin testin tulos (0,835) ja Barlettin sväärisyystesti (χ^2 (120) = 7470,293; p < 0,001) osoittivat, että korrelaatiomatriisi soveltuu faktorianalyysiin.

Faktoreiden mukaan muodostettiin summamuuttujat, joiden reliabiliteetit näkyvät taulukoissa 4 ja 5.

Taulukko 4. Summamuuttujat yläkoulun opetuksen pedagogisista ratkaisuista ja niiden reliabiliteetit

	Muuttujien määrä	Reliabiliteetti (α)
1 Oppilaskeskeisyys	4	0,688
2 Monipuoliset opetusmenetelmät	5	0,634
3 Opettajajohtoisuus	3	0,473

Taulukko 5. Summamuuttujat lukion opetuksen pedagogisista ratkaisuista ja niiden reliabiliteetit

	Muuttujien määrä	Reliabiliteetti (α)
1 Matematiikan yhteys käytäntöön	2	0,728
2 Oppijoiden tarpeiden huomioiminen	6	0,653
3 Monipuoliset opetusmenetelmät	4	0,708
4 Oppilaskeskeisyys	4	0,668

Koska yläkoulun opettajajohtoisuuteen liittyvän summamuuttujan reliabiliteettiarvo jäi heikoksi (alle 0,6), jätettiin kyseinen summamuuttuja analyyseista pois ja tuloksia analysoidaan yksittäisten muuttujien osalta.

7 Tulosten analysointi

Tulosten kuvailussa käytetään perustunnuslukuja kuten frekvenssi- ja prosenttijakaumia sekä keskiarvo- ja keskihajontalukuja. Ryhmien välisiä eroja analysoidaan parametrisin testein kuten t-testillä ja yksisuuntaisella varianssianalyysilla. Efektikoon mittana käytetään Cohenin *f*-arvoa, joka ilmaisee varianssianalyysin yhteydessä havaittujen keskiarvojen välisen eron suuruuden (efektikoko on suuri, kun Cohenin $f > 0{,}40$) (Cohen, 1988; Metsämuuronen, 2011).

Opetuksellisten tekijöiden yhteyttä asenteisiin analysoidaan monimuuttujamenetelmin. Analysoinnissa käytetään päätöspuuanalyysia (*decision tree analysis*, DTA) ja lineaarista regressioanalyysia. DTA-analyysilla tutkitaan laajaa aineistoa ja etsitään muuttujia, jotka erottelevat ja luokittelevat selitettävää muuttujaa. DTA-analyysin avulla saadaan tunnistettua ei-lineaarisia ilmiöitä.

Analyysissa käytetään CHAID-algoritmia, joka etsii tilastollisesti samankaltaisia arvoja selittävän ja selitettävän muuttujan välillä. Algoritmi etsii ja luokittelee ryhmiä, joiden välinen ero on mahdollisimman suuri vertaamalla testien p-arvoja. Tässä tutkimuksessa p-arvoja etsitään F-testin avulla, kun selitettävät muuttujat ovat jatkuvia (Kass, 1980; Metsämuuronen, 2011).

Asiayhteyksien mallintamista ja asenteiden muutosta ennustavien tekijöiden analysoinnissa hyödynnetään lineaarista regressioanalyysia. Sen avulla muuttujien selitysosuudet saadaan paremmin näkyviin. Analyyseissa käytetään askeltavaa menettelyä, jossa yhtälöön lisätään riippumattomia muuttujia yksi kerrallaan ja samalla testataan, miten kunkin lisätyn muuttujan poistaminen vaikuttaa mallin selitysasteeseen (Metsämuuronen, 2011, s. 724). Lopulliseen malliin jää selitysvoimaltaan tilastollisesti merkitsevät muuttujat. Tabachnick & Fidell (2007) kutsuvat menettelyä tilastolliseksi menettelyksi (*statistical regression*), koska selittävät muuttujat valitaan malliin vain tilastollisin perustein. Regressioanalyysin tulokset esitetään niin, että muuttujat ovat analyysin esittämässä järjestyksessä. Lineaarisen regressioanalyysin käytössä tulee ottaa huomioon, että muuttujien yhteydet eivät ole puhtaan lineaarisia. Havaittu vaikutus saattaa syntyä esimerkiksi muuttujan toisen ääripään voimakkaasta vaihtelusta (Metsämuuronen, 2009, s. 49).

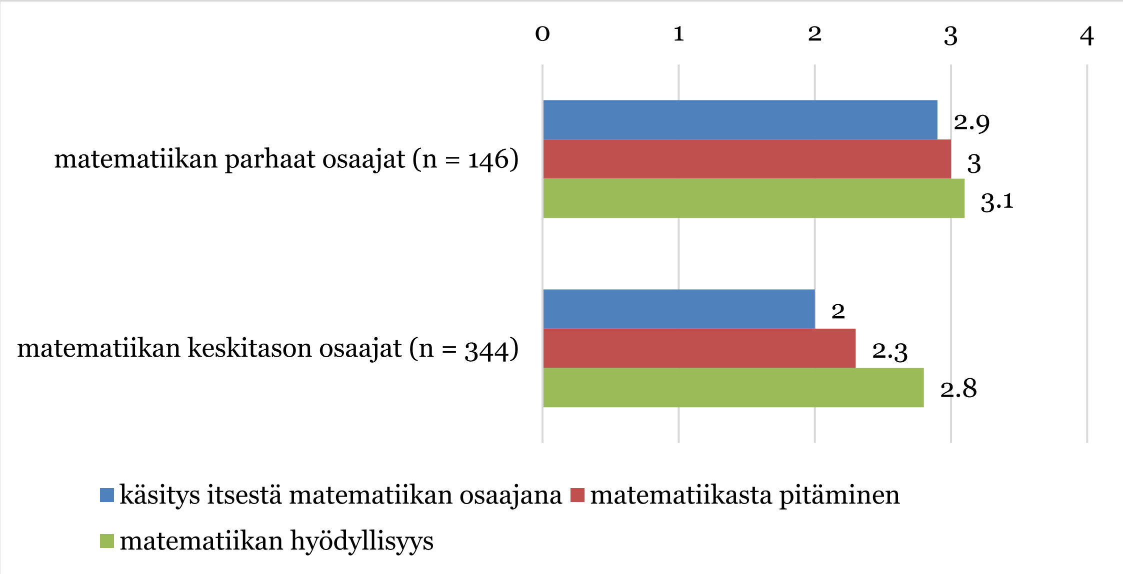
DTA-analyysissa ja regressioanalyysissa analysoidaan erikseen yläkoulun ja lukion aikaiset opetukseen liittyvät tekijät. Ensin pyritään selvittämään, mitkä summamuuttujat selittävät parhaiten asenteen muutoksen vaihtelua ja sen jälkeen, mikä on yksittäisten tekijöiden selitysosuus vaihtelusta. Analyyseissa kontrolloidaan kansallisella kokeella mitattu osaamistaso ottamalla se analyyseihin mukaan. Jos sillä on tilastollisesti merkitsevä yhteys malliin, se esitetään tuloksissa. DTA-analyysi ja lineaarinen regressioanalyysi täydentävät toisiaan. DTA-analyysin avulla voidaan löytää tekijöitä, joita regressioanalyysi ei löydä, sillä DTA-analyysi tunnistaa myös muuttujien väliset epälineaariset ja -hierarkkiset yhteydet.

8 Tulokset

Raportoimme ensin, miten matematiikan parhaiden osaajien matematiikkaan liittyvät asenteet kehittyvät perusopetuksesta lukion loppuun ja millaisia eroja tyttöjen ja poikien välillä on minäkäsityksessä, matematiikasta pitämisessä ja matematiikan hyödyllisyyden kokemisessa tapahtuneissa muutoksissa. Sen jälkeen selvitämme, millaiset opetuksen pedagogiset ratkaisut yläkoulussa ja lukiossa selittävät tyttöjen ja poikien asenteissa tapahtuneita muutoksia.

Matematiikkaan liittyvien asenteiden kehitys perusopetuksesta lukion loppuun

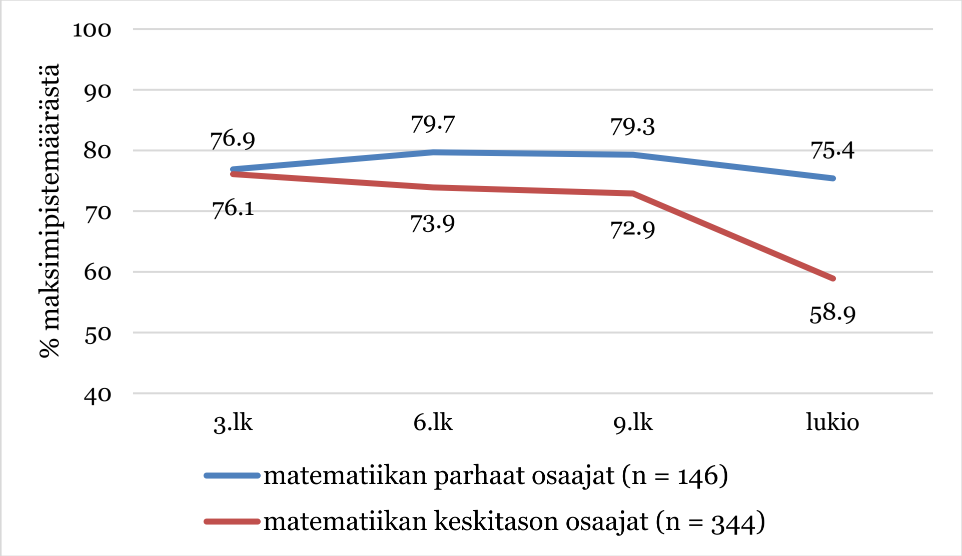
Kuviossa 1 esitetään, millä tasolla matematiikan parhaiden ja keskitason osaajien asenteet olivat lukion lopussa.



Kuvio 1. Matematiikan parhaiden ja keskitason osaajien matematiikka-asenteiden taso lukion lopussa

Yleisesti parhaiden osaajien asenteet olivat keskitason osaajien asenteita positiivisempia. Tämä näkyy erityisesti käsityksessä itsestä matematiikan osaajana ja matematiikasta pitämisessä. Molemmat ryhmät pitivät matematiikkaa lähes yhtä hyödyllisenä.

Kuviossa 2 esitetään, millaisia muutoksia parhaiden ja keskitason osaajien kokonaisasenteessa tapahtui peruskoulun ja lukion aikana.

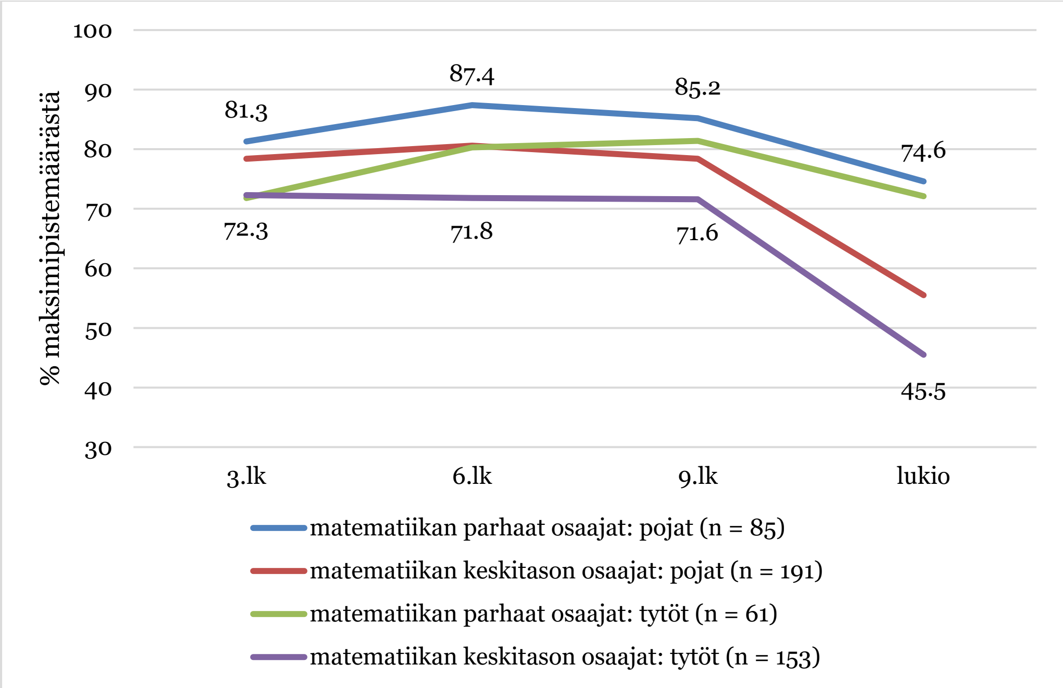


Kuvio 2. Matematiikan parhaiden ja keskitason osaajien kokonaisasenteessa tapahtuneet muutokset perusopetuksesta lukion loppuun

Matematiikan parhaiden ja keskitason osaajien kokonaisasenteen lähtötaso oli lähes sama kolmannen luokan alussa. Parhaiden osaajien asennoituminen pysyi melko vakaana koko perusopetuksen ja lukion ajan. Asennoituminen jopa hieman kasvoi perusopetuksen aikana. Sen sijaan keskitason osaajilla asennoituminen matematiikkaan alkoi heikentyä jo perusopetuksen aikana ja oli lukion lopussa lähes 20 prosenttiyksikköä heikompi kuin parhailla osaajilla.

Käsitys itsestä matematiikan osaajana

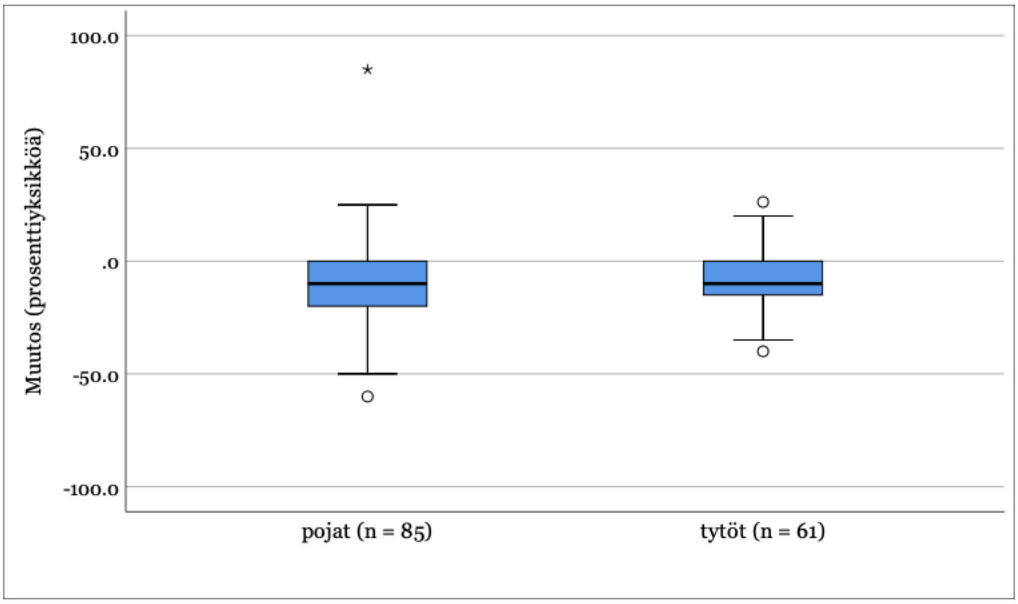
Kuviossa 3 kuvataan, millaisia keskiarvoihin perustuvia muutoksia matematiikassa parhaiten menestyneillä pojilla ja tytöillä tapahtui käsityksessä itsestä matematiikan osaajana perusopetuksesta lukion loppuun. Vertailukohtana ovat keskitason osaajat.



Kuvio 3. Minäkäsityksessä tapahtuneet muutokset perusopetuksesta lukion loppuun osaajaryhmien ja sukupuolen mukaan

Parhaiden poikien käsitys omasta osaamisesta pysyi koko ajan muita korkeammalla tasolla. Erot tyttöjen ja poikien minäkäsityksessä näkyvät selkeästi koko alakouluajan poikien hyväksi, mutta tytöt kuroivat poikien tasoa kiinni yläkoulun aikana ja sen jälkeen. Tyttöjen ja poikien minäkäsityksessä oli tilastollisesti merkitsevä ero vain kolmannella ($t(133) = 2{,}78$; $p = 0{,}006$) ja kuudennella luokalla ($t(144) = 3{,}02$; $p = 0{,}003$). Kolmannella luokalla varianssien ero oli 3,7 prosenttiyksikköä ($F(101{,}59) = 1{,}37$, $p = 0{,}009$, $\eta^2 = 0{,}055$, Cohenin $f = 0{,}23$) ja kuudennella 4,2 prosenttiyksikköä ($F(105{,}07) = 12{,}05$, $p = 0{,}005$, $\eta^2 = 0{,}060$, Cohenin $f = 0{,}24$). Keskitason osaajilla sukupuolten välinen ero säilyi lukion loppuun asti. Kaikilla käsitys omasta osaamisesta heikkeni yläkoulusta lukioon siirtyessä, vaikka parhailla osaajilla heikentyminen ei ollut niin suurta kuin keskitason osaajilla.

Suurin muutos parhaiden osaajien minäkäsityksessä tapahtui siirtymävaiheessa yhdeksänneltä luokalta lukioon. Parhaiden tyttöjen ja poikien välisiä eroja minäkäsityksen muutoksessa havainnollistetaan kuvion 4 laatikko-janakuviolla (*Box Plot*).

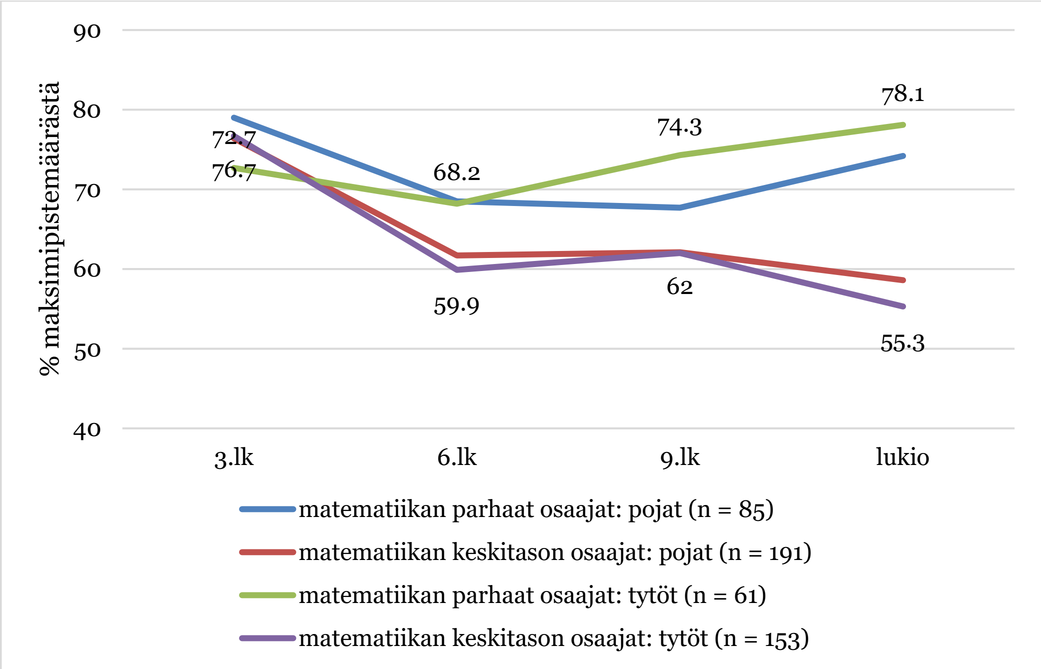


Kuvio 4. Matematiikan parhaiden osaajien minäkäsityksessä yhdeksänneltä luokalta lukion loppuun tapahtuneen muutoksen jakauma sukupuolittain

Minäkäsityksessä tapahtuneet muutokset jakautuvat samankaltaisesti pojilla ja tytöillä. Molemmissa ryhmissä minäkäsitys heikentyi mediaanilla mitattuna keskimäärin 10 prosenttiyksikköä. Pojilla muutoksen keskihajonta oli 19,1 prosenttiyksikköä ja tytöillä 13,1. Pojista neljänneksellä minäkäsitys heikkeni -20 prosenttiyksikköä tai enemmän ja tytöillä -15 prosenttiyksikköä tai enemmän. Neljänneksellä oppilaista minäkäsitys säilyi lukiossa vähintään samalla tasolla kuin se oli ollut yhdeksännellä luokalla tai se vahvistui lukion aikana.

Matematiikasta pitäminen

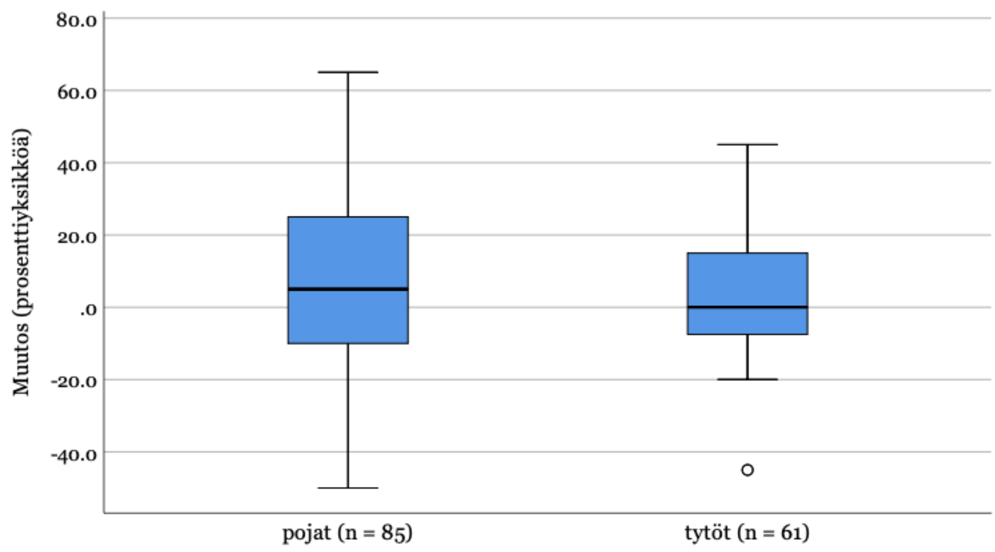
Matematiikasta pitämisessä tapahtuneet keksimääräiset muutokset esitetään kuviossa 5.



Kuvio 5. Matematiikasta pitämisen muutokset perusopetuksesta lukion loppuun osaajaryhmien ja sukupuolen mukaan

Keskitason osaajilla nähdään yleinen muutossuunta, jonka mukaan matematiikasta pitäminen heikkeni koko matkan perusopetuksesta lukion loppuun. Sen sijaan parhailla osaajilla matematiikasta pitäminen vahvistui entisestään lukion aikana. On huomioitavaa, että parhaiden tyttöjen muutossuunta erottuu muista tytöistä sekä parhaista pojista. Parhaat tytöt pitivät matematiikasta kaikkein vähiten kolmannella luokalla. Matematiikasta pitäminen heikkeni tytöillä hieman vielä alakoulun aikana kunnes se lähti jyrkkään kasvuun yläkoulun aikana. Parhaat tytöt pitivät lukion lopussa matematiikasta kaikkein eniten ja saavuttivat parhaiden poikien kolmannen luokan lähtötason. Parhaiden tyttöjen ja poikien väliset erot matematiikasta pitämisessä eivät olleet millään luokka-asteella tilastollisesti merkitseviä.

Matematiikasta pitämisen muutoksissa yhdeksänneltä luokalta lukion kolmannelle vuodelle oli suurta vaihtelua (kuvio 6). Erityisesti pojilla muutoksen vaihteluväli oli tyttöjä suurempi.

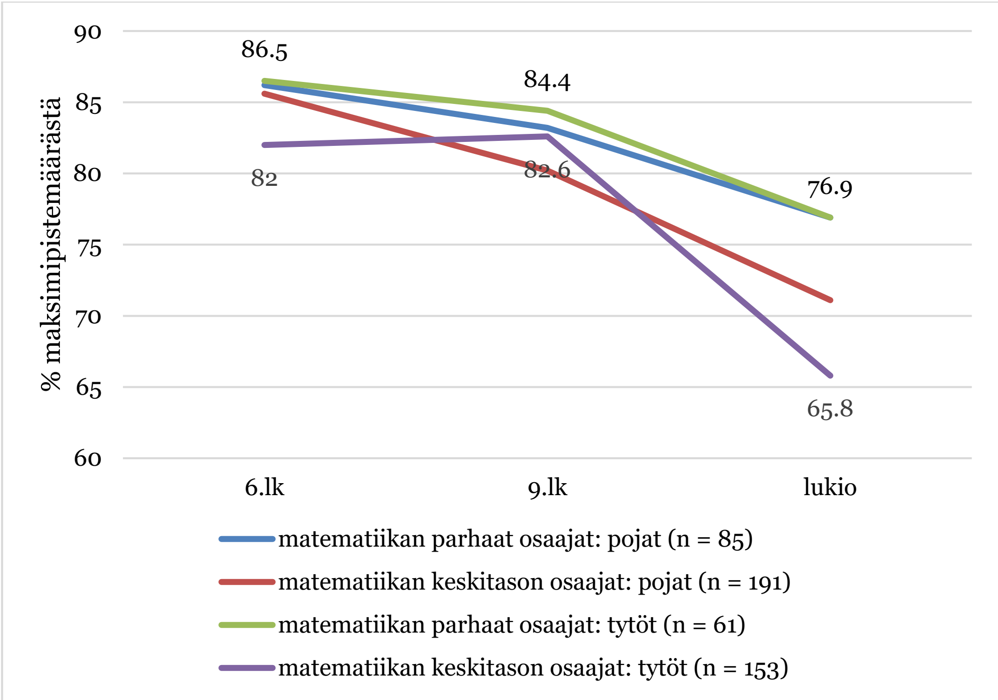


Kuvio 6. Matematiikan parhaiden osaajien matematiikasta pitämisessä yhdeksänneltä luokalta lukion loppuun tapahtuneen muutoksen jakauma sukupuolittain

Muutoksen keskihajonta oli pojilla 24,1 prosenttiyksikköä ja tytöillä 18,8. Mediaanilla mitattuna matematiikasta pitäminen pysyi molemmissa ryhmissä samalla tasolla lukion lopussa kuin yhdeksännellä luokalla. Sekä tytöistä että pojista neljänneksellä matematiikasta pitäminen laski vähintään 10 prosenttiyksikköä. Toisaalta matematiikasta pitäminen kasvoi neljänneksellä pojista vähintään 25 prosenttiyksikköä ja neljänneksellä tytöistä vähintään 15 prosenttiyksikköä.

Matematiikan hyödyllisyys

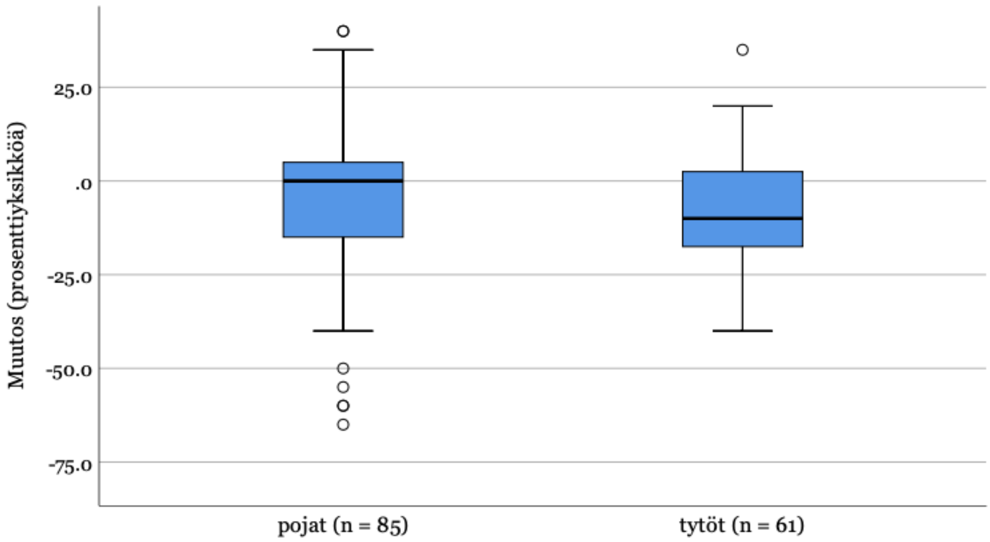
Opiskelijoiden käsitys matematiikan hyödyllisyydestä pääsääntöisesti laski kuudennelta luokalta lukioon ja erityisesti lukiossa (kuvio 7).



Kuvio 7. Matematiikan hyödyllisyyden kokemisen muutokset perusopetuksesta lukion loppuun osaajaryhmien ja sukupuolen mukaan

Parhaiden osaajien kokema matematiikan hyödyllisyys pysyi koko ajan keskitason osaajia korkeammalla tasolla eikä poikien ja tyttöjen välillä ollut havaittavissa juurikaan eroja keskiarvojen suhteen eivätkä erot olleet tilastollisesti merkitseviä. Osaamiseltaan keskitasoa olevien poikien kokema hyödyllisyys oli kuudennella luokalla lähes yhtä korkealla tasolla kuin osaamiseltaan parhaiden poikien kanssa, mutta asennoituminen lähti jyrkkään laskuun yläkoulun aikana ja heikkeni aina lukion loppuun asti. Osaamiseltaan keskitasoa olevien tyttöjen kokemus matematiikan hyödyllisyydestä oli muita hieman alhaisempi kuudennella luokalla, mutta se kehittyi yläkoulun aikana osaamiseltaan parhaiden tyttöjen tasolle. Lukion aikana koettu hyödyllisyys kuitenkin heikkeni lähes 20 prosenttiyksikköä.

Parhaiden osaajien kokemus matematiikan hyödyllisyydestä muuttui voimakkaimmin yhdeksänneltä luokalta lukion loppuun. Parhaiden tyttöjen ja poikien käsitykset hyödyllisyydestä kehittyivät jälleen erisuuntaisesti (kuvio 8). Tytöillä käsitys hyödyllisyydestä keskimäärin laski lukioaikana, kun taas pojilla se pysyi keskimäärin ennallaan. Poikien muutoksessa oli suurempi hajonta kuin tytöillä.



Kuvio 8. Matematiikan parhaiden osaajien matematiikan hyödyllisyyden kokemisessa yhdeksänneltä luokalta lukion loppuun tapahtuneen muutoksen jakauma sukupuolittain

Pojilla muutoksen keskihajonta oli 21,4 prosenttiyksikköä ja tytöillä 15,5 prosenttiyksikköä. Pojilla kokemus matematiikan hyödyllisyydestä mediaanilla mitattuna pysyi lukiossa keskimäärin samalla tasolla kuin yhdeksännellä luokalla ja tytöillä koettu hyödyllisyys heikkeni keskimäärin 10 prosenttiyksikköä. Pojista vähintään neljänneksellä koettu hyödyllisyys heikkeni 15 prosenttiyksikköä tai enemmän ja tytöistä neljänneksellä 20 prosenttiyksikköä tai enemmän. Sekä pojista että tytöistä neljänneksellä matematiikan hyödyllisyyden kokeminen kasvoi 5 prosenttiyksikköä tai enemmän. Pojilla tapahtui tyttöjä enemmän keskiarvoa heikentäviä muutoksia toisessa ääripäässä.

Opetustekijöiden yhteys parhaiden osaajien asennemuutoksiin yhdeksänneltä luokalta lukion loppuun

Seuraavaksi selvitetään, miten opetuksen pedagogiset ratkaisut opiskelijoiden arvioimana yläkoulussa ja lukiossa selittävät matematiikan parhaiden osaajien asenteissa tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun. Analyysit tehdään tytöille ja pojille erikseen ja pyritään selvittämään, millaisia eroja löytyy sukupuolten välillä.

Käsitys itsestä matematiikan osaajana

Parhaiden tyttöjen minäkäsityksen muutosta selittävät DTA-analyysin tulokset esitetään taulukossa 6. Yläkoulun aikaisista tekijöistä ei löytynyt analyysissa muutoksen vaihtelua selittävää muuttujaryhmää tai yksittäistä tekijää vaan tulokset koskevat lukion aikaisia pedagogisia ratkaisuja.

Taulukko 6. Tyttöjen minäkäsityksen muutosta yhdeksänneltä luokalta lukion loppuun erottelevat tekijät DTA-analyysin mukaan (tähdellä * merkitty muuttujien välinen yhteisvaikutus)

	prosenttiosuus parhaista tytöistä (n = 55)	**toistuvuus (asteikolla 1–5)**	**minäkäsityksen muutos (prosenttiyksikköä)**
Opetuksen pedagogiset ratkaisut lukiossa			
Pidetään testejä ja kokeita.	10,9	> 4,0	+6,0
	36,4	(3,0, 4,0]	-15,3
	52,7	≤ 3,0*	-8,3
*Opiskelijat asettavat itselleen tavoitteita ja arvioivat edistymistään.	41,8	> 1,0	-6,0
	10,9	1,0	-17,3

Lukio-opetuksen yksittäisistä tekijöistä minäkäsityksen muutosta erotteli parhaiten se, että matematiikan tunneilla pidetään testejä ja kokeita ($F(2, 52) = 7{,}886$; $p = 0{,}022$). Tytöillä, jotka arvioivat matematiikan testejä ja kokeita lukiossa pidetyn enemmän kuin usein, käsitys omasta osaamisesta kasvoi lukion aikana 6 prosenttiyksikköä. Jos kokeita pidettiin joskus tai tätä harvemmin, tyttöjen minäkäsitys heikkeni noin 8 prosenttiyksikköä. Jos kuitenkin tytöt arvioivat toteuttaneensa itsearviointia useammin kuin harvoin, minäkäsitys heikkeni enää 6 prosenttiyksikköä ($F(1, 27) = 7{,}215$; $p = 0{,}037$). Jos osaamista ei arvioitu millään tavalla, tyttöjen käsitys omasta osaamisesta heikkeni noin 17 prosenttiyksikköä.

Taulukossa 7 esitetään tyttöjen minäkäsityksen muutosta selittävän askeltavan regressioanalyysin tulokset, joissa löytyi vain yksi selittävä muuttuja yläkoulun osalta.

Taulukko 7. Tyttöjen minäkäsityksen muutosta yhdeksänneltä luokalta lukion loppuun selittävät tekijät askeltavan regressioanalyysin mukaan

	Muutoksen vaihtelu minäkäsityksessä, tytöt			
	B	*SE*	*beta*	*p*
Malli 1 – Opetuksen pedagogiset ratkaisut yläkoulussa				
Vakio	15,368	10,417		0,146
Annetut kotitehtävät olen tehnyt sovitulla tavalla.	-5,386	2,255	-0,314	0,021
	$F(1, 52) = 5{,}702$; $p = 0{,}021$; $R = 0{,}314$; $R^2 = 0{,}099$			

Tytöillä minäkäsityksen vaihtelua selitti parhaiten yksittäinen tekijä, jonka mukaan oppilas tekee annetut kotitehtävät sovitulla tavalla. Vaikutus muutokseen on negatiivinen eli minäkäsitys heikentyi lukiossa. Tällaisella oppilaalla minäkäsityksen lähtötaso oli korkea yhdeksännellä luokalla.

Poikien minäkäsityksen muutosta selittäviä tekijöitä etsivän DTA-analyysin tulokset näkyvät taulukossa 8. Yläkoulun aikaisista tekijöistä ei löytynyt muutoksen vaihtelua selittävää tekijää, joten tulokset koskevat vain pedagogisia ratkaisuja lukiossa.

Taulukko 8. Poikien minäkäsityksen muutosta yhdeksänneltä luokalta lukion loppuun erottelevat tekijät DTA-analyysin mukaan (tähdellä * merkitty muuttujien välinen yhteisvaikutus)

	prosenttiosuus parhaista pojista (n = 74)	**toistuvuus (asteikolla 1–5)**	**minäkäsityksen muutos (prosenttiyksikköä)**
Opetuksen pedagogiset ratkaisut lukiossa			
Opiskelijat selittävät muille, miten ovat tehtävänsä ratkaisseet.	13,5	> 4,0	+7,5
	86,5	≤ 4,0*	-13,8
*Opiskellaan ryhmissä tai pareittain.	10,8	> 3,0	-21,9
	60,8	(1,0; 3,0]	-10,2
	14,9	1,0	-22,3

Yksittäisistä lukioon liittyvistä tekijöistä poikien käsitystä omasta osaamisesta vahvisti parhaiten se, että matematiikan tunneilla opiskelijat selittävät muille, miten ovat tehtävänsä ratkaisseet ($F(1, 72) = 12{,}353$; $p = 0{,}003$). Pojista niillä, jotka arvioivat selittäneensä muille tehtäviensä ratkaisuja enemmän kuin usein, arvio omasta osaamisesta kasvoi lähes 8 prosenttiyksikköä. Ne, jotka arvioivat ratkaisujen selittämistä muille tapahtuneen enintään usein, minäkäsitys heikkeni noin 14 prosenttiyksikköä. Se, että opiskelijat eivät selittäneet omia ratkaisujaan toisille kovin usein, oli poikien minäkäsitykselle haitallisinta silloin, kun opiskelijat eivät opiskelleet pareittain tai ryhmissä lainkaan tai pari- tai ryhmätyöskentelyä oli usein.

Taulukossa 9 esitetään askeltavan regressioanalyysin tulokset poikien minäkäsityksen muutoksen selittämisestä. Tilastollisesti merkitsevä muuttuja löytyi vain lukio-opetukseen liittyvistä yksittäisistä tekijöistä.

Taulukko 9. Poikien minäkäsityksen muutosta yhdeksänneltä luokalta lukion loppuun selittävät tekijät askeltavan regressioanalyysin mukaan

	Muutoksen vaihtelu minäkäsityksessä, pojat			
	B	*SE*	*beta*	*p*
Malli 1 – Opetuksen pedagogiset ratkaisut lukiossa				
Vakio	-26,720	7,312		0,001
Opiskelijat selittävät muille, miten ovat tehtävänsä ratkaisseet.	4,866	2,050	0,279	0,020
	F(1, 67) = 5,635; *p* = 0,020; *R* = 0,279; R^2 = 0,078			

Tulos vahvistaa DTA-analyysin tulosta, jonka mukaan parhaiden poikien minäkäsitys vahvistui, kun he selittivät muille, miten ovat tehtävänsä ratkaisseet. Yhteenvetona voidaan todeta, että parhailla tytöillä käsitystä omasta osaamisesta näyttäisi vahvistaneen se, että he saivat osaamisestaan palautetta (kokeet ja itsearviointi) ja opiskelu tapahtui opettajan antamien tehtävien mukaisesti. Parhaiden poikien minäkäsitys vahvistui, kun he saivat osoittaa osaamistaan muille.

Matematiikasta pitäminen

Tyttöjen matematiikasta pitämisessä tapahtuneita muutoksia selittävät tekijät DTA-analyysilla mitattuna näkyvät taulukossa 10. Selittäviä tekijöitä löytyi sekä yläkoulun että lukion pedagogisista ratkaisuista.

Taulukko 10. Tyttöjen matematiikasta pitämisessä tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun erottelevat tekijät DTA-analyysin mukaan (tähdillä * ja ** merkitty muuttujien väliset yhteisvaikutukset)

	prosenttiosuus parhaista tytöistä (n = 55)	**toistuvuus (asteikolla 1–5)**	**muutos matematiikasta pitämisessä (prosenttiyksikköä)**
Opetuksen pedagogiset ratkaisut yläkoulussa			
Monipuoliset opetusmenetelmät (faktori 2)	32,7	> 2,0	-5,6
	12,7	(1,8; 2,0]	16,5
	30,9	(1,4; 1,8]	-2,4
	23,6	≤ 1,4	17,1
Sovelletaan matematiikan taitoja arkielämän tilanteisiin.	50,9	> 2,0*	-3,0
	49,1	≤ 2,0**	10,5
Annetut kotitehtävät olen tehnyt sovitulla tavalla.*	34,5	> 4,0	-8,4
	16,4	≤ 4,0	8,3
Opitaan mittaamalla, rakentelemalla tai muulla tavoin tekemällä.**	30,9	> 1,0	2,1
	18,2	1,0	24,8
Opetuksen pedagogiset ratkaisut lukiossa			
Monipuoliset opetusmenetelmät (faktori 3)	63,6	> 1,3	-1,5
	36,4	≤ 1,3	12,5
Tehdään projektitöitä.	21,8	> 1,0*	-7,5
	78,2	1,0	6,7
Sovelletaan matematiikan taitoja arkielämän tilanteisiin.*	9,1	> 2,0	3,0
	12,7	≤ 2,0	-15,0

Monipuolisten opetusmenetelmien käyttö oli selittävänä muuttujana sekä yläkoulussa että lukiossa. Tällaisten opetusmenetelmien runsas käyttö näyttäisi kuitenkin selittäneen parhaiden tyttöjen matematiikasta pitämistä eri tavalla yläkoulussa ja lukiossa. Yläkoulussa pedagogisten ratkaisujen toistuvuus selitti asenteessa tapahtuneen muutoksen lähtötasoa. Parhaiden tyttöjen matematiikasta pitäminen parani keskimäärin 10 prosenttiyksikköä (muutosten keskiarvo, kun toistuvuus ≤ 2,0), jos monipuolisia opetusmenetelmiä toteutettiin yläkoulussa harvoin tai ei lainkaan ja heikkeni lähes 6 prosenttiyksikköä, jos näitä opetusmenetelmiä toteutettiin ainakin joskus ($F(3, 51) = 7{,}121$; $p = 0{,}015$). Jos yläkoulussa toteutettiin monipuolisia opetusmenetelmiä keskimääräistä enemmän, parhaiden tyttöjen matematiikasta pitäminen oli yläkoulussa korkealla tasolla ja heikkeni tästä lukion aikana. Jos näitä opetusmenetelmiä toteutettiin keskimääräistä vähemmän, matematiikasta pitämisen lähtötaso oli alhaisempi ja matematiikasta pitäminen parani lukion aikana.

Yläkoulun yksittäisistä tekijöistä muutoksen vaihtelua erotteli se, kuinka usein matematiikan taitoja sovelletaan arkielämän tilanteisiin ($F(1, 53) = 8{,}000$; $p = 0{,}020$). Tytöistä puolet arvioi, että soveltamista tapahtui useammin kuin harvoin ja heillä matematiikasta pitäminen heikkeni 3 prosenttiyksikköä. Toisin sanoen tytöt pitivät matematiikasta enemmän yläkoulussa kuin lukiossa, jos taitoja sovellettiin yläkoulussa arkielämän tilanteisiin. Lisäksi, jos oppilas arvioi tehneensä annetut kotitehtävät sovitulla tavalla enemmän kuin usein, heikkeni matematiikasta pitäminen noin 8 prosenttiyksikköä ($F(1, 26) = 7{,}072$; $p = 0{,}026$). Parhaiden tyttöjen matematiikasta pitäminen oli yläkoulussa todennäköisesti sitä korkeammalla tasolla, mitä enemmän taitoja sovellettiin arkielämän tilanteisiin ja mitä useammin oppilaat tekivät kotitehtäviä. Jos matematiikan taitoja sovellettiin harvoin tai ei lainkaan, tyttöjen matematiikasta pitäminen parani lukion aikana noin 11 prosenttiyksikköä. Jos tähän kuitenkin yhdistyi mittaamalla, rakentelemalla tai muulla tavoin tekemällä oppimista vähintään harvoin, matematiikasta pitäminen kasvoi enää noin 2 prosenttiyksikköä. Jos tekemällä oppimista ei ollut lainkaan, matematiikasta pitäminen kasvoi jopa noin 25 prosenttiyksikköä ($F(1, 25) = 15{,}196$; $p = 0{,}001$). Tekemällä oppiminen yläkoulussa paransi parhaiden tyttöjen matematiikasta pitämisen lähtötasoa.

Lukiossa monipuolisten opetusmenetelmien toistuva käyttö näyttäisi heikentäneen parhaiden tyttöjen matematiikasta pitämistä ($F(1, 53) = 7{,}963$; $p = 0{,}034$). Yksittäisistä tekijöistä se, että tunneilla tehdään projektitöitä enemmän kuin harvoin, heikensi tyttöjen matematiikasta pitämistä lähes 8 prosenttiyksikköä ($F(1,53) = 5{,}815$; $p = 0{,}039$). Jos projektitöiden tekemiseen yhdistyi matematiikan taitojen soveltaminen arkielämän tilanteisiin, matematiikasta pitämisen muutos olikin positiivinen (+3,0 prosenttiyksikköä), mutta heikkeni entisestään (muutos -15,0 prosenttiyksikköä), jos soveltamista oli harvoin tai ei lainkaan ($F(1, 10) = 6{,}176$; $p = 0{,}032$). Eniten matematiikasta pitäminen kasvoi, jos projektitöitä ei toteutettu lukiossa opiskelijoiden arvioimana lainkaan (muutos +6,7 prosenttiyksikköä).

Taulukossa 11 on askeltavan regressioanalyysin tulokset tyttöjen matematiikasta pitämisessä tapahtuneiden muutosten selityksiin.

Taulukko 11. Tyttöjen matematiikasta pitämisessä tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun selittävät tekijät askeltavan regressioanalyysin mukaan

	Muutoksen vaihtelu matematiikasta pitämisessä, tytöt			
	B	*SE*	*beta*	*p*
Malli 1 – Opetuksen pedagogiset ratkaisut yläkoulussa				
Vakio	25,634	9,453		0,009
Monipuoliset opetusmenetelmät (faktori 2)	-11,646	4,826	-0,315	0,019
	F(1, 53) = 5,823; p = 0,019; R = 0,315; R^2 = 0,099			
Vakio	20,911	6,994		0,004
Sovelletaan matematiikan taitoja arkielämän tilanteisiin.	-6,595	2,548	-0,338	0,012
	F(1, 52) = 6,701; p = 0,012; R = 0,338; R^2 = 0,114			
Malli 2 – Opetuksen pedagogiset ratkaisut lukiossa				
Vakio	26,121	9,036		0,006
Monipuoliset opetusmenetelmät (faktori 3)	-13,884	5,366	-0,335	0,012
	F(1, 53) = 6,694; p = 0,012; R = 0,335; R^2 = 0,112			
Vakio	19,149	6,945		0,008
Matematiikan tunneilla tehdään projektitöitä.	-12,819	5,193	-0,330	0,017
	F(1, 50) = 6,094; p = 0,017; R = 0,330; R^2 = 0,109			

Tulokset vahvistavat DTA-analyysin tuloksia. Parhaiden tyttöjen matematiikasta pitäminen laski todennäköisesti lukion aikana sitä enemmän, mitä useammin yläkoulun opetuksessa toteutettiin monipuolisia opetusmenetelmiä ja asenteen lähtötaso oli korkealla. Lukiossa monipuolisten opetusmenetelmien toistuvuus sen sijaan heikensi parhaiden tyttöjen matematiikasta pitämistä.

Poikien matematiikasta pitämisessä tapahtuneita muutoksia selittävät DTA-analyysin tulokset ovat taulukossa 12. Yläkoulun tekijöistä ei löytynyt selittäviä tekijöitä, joten tulokset on esitetty vain lukioon liittyvien tekijöiden osalta.

Taulukko 12. Poikien matematiikasta pitämisessä tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun erottelevat tekijät DTA-analyysin mukaan

	prosenttiosuus parhaista pojista (n = 74)	**toistuvuus (asteikolla 1–5)**	**muutos matematiikasta pitämisessä (prosenttiyksikköä)**
Opetuksen pedagogiset ratkaisut lukiossa			
Monipuoliset opetusmenetelmät (faktori 3)	58,1	> 1,5	+0,5
	41,9	≤ 1,5	+16,0
Opiskeltavat asiat tulevat selväksi.	87,8	> 3,0	+10,7
	12,2	≤ 3,0	-19,4

Monipuolisiin opetusmenetelmiin liittyvät tekijät erottelivat parhaiten myös poikien matematiikasta pitämisessä tapahtuneita muutoksia ($F(1, 72) = 8{,}253$; $p = 0{,}032$). Parhaista pojista noin 40 prosentilla matematiikasta pitäminen lisääntyi 16 prosenttiyksikköä, kun lukiossa toteutettiin harvoin tai ei lainkaan monipuolisia opetusmenetelmiä. Jos monipuolisia opetusmenetelmiä toteutettiin harvoin tai tätä useammin, parhaiden poikien matematiikasta pitäminen pysyi samalla tasolla kuin se oli yhdeksännellä luokalla. Yksittäisistä lukioon liittyvistä tekijöistä se, kuinka usein opiskeltavat asiat tulevat opiskelijoiden arvioimana selväksi, erotteli parhaiten poikien matematiikasta pitämisessä tapahtuneita muutoksia ($F(1, 72) = 14{,}576$; $p = 0{,}001$). Erot muutoksessa kahden ryhmän välillä olivat noin 30 prosenttiyksikköä. Suurin osa pojista (noin 88 prosenttia) arvioi, että opiskeltavat asiat tulivat selväksi useammin kuin joskus ja heillä matematiikasta pitäminen kasvoi noin 11 prosenttiyksikköä.

Taulukossa 13 esitetään askeltavan regressioanalyysin tulokset parhaiden poikien matematiikasta pitämisessä tapahtuneisiin muutoksiin. Selittäviä tekijöitä löytyi sekä yläkoulun että lukion osalta.

Taulukko 13. Poikien matematiikan pitämisessä tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun selittävät tekijät askeltavan regressioanalyysin mukaan

	Muutoksen vaihtelu matematiikasta pitämisessä, pojat			
	B	*SE*	*beta*	*p*
Malli 1 – Opetuksen pedagogiset ratkaisut yläkoulussa				
Vakio	37,157	15,072		0,016
Oppilaskeskeisyys (faktori 1)	-8,887	4,378	-0,234	0,046
	F(1, 71) = 4,120; p = 0,046; R = 0,234; R^2 = 0,055			
Vakio	26,710	9,319		0,005
Opettaja ottaa huomioon opetukseen liittyvät oppilaiden ideat ja toiveet.	-6,138	2,794	-0,254	0,031
	F(1, 70) = 4,826; p = 0,031; R = 0,254; R^2 = 0,064			
Malli 2 – Opetuksen pedagogiset ratkaisut lukiossa				
Vakio	-37,763	19,675		0,059
Oppijoiden tarpeiden huomioiminen (faktori 2)	11,941	5,198	0,261	0,025
	F(1, 72) = 5,276; p = 0,025; R = 0,261; R^2 = 0,068			
Vakio	-25,631	13,665		0,065
Kukin ratkaisee itselleen sopivan vaikeita tehtäviä.	8,460	3,559	0,279	0,020
	F(1, 67) = 5,649; p = 0,020; R = 0,279; R^2 = 0,078			

Tulosten mukaan yläkoulussa oppilaskeskeisyyteen liittyvät tekijät näyttivät heikentäneen pitämisessä tapahtuneita muutoksia samoin kuin se yksittäinen tekijä, jonka mukaan opettaja ottaa huomioon opetukseen liittyvät oppilaiden ideat ja toiveet. Koska näihin liittyvät muuttujat eivät vaikuttaneet malliin tilastollisesti merkitsevästi enää lukiossa, on todennäköistä, että parhaiden poikien matematiikasta pitäminen oli yhdeksännellä luokalla korkealla tasolla näiden tekijöiden selittäessä pitämistä. Kun näitä ratkaisuja ei toteutettu toistuvasti enää lukiossa, poikien matematiikasta pitäminen heikentyi.

Lukiossa parhaiden poikien matematiikasta pitäminen vahvistui, kun opetuksessa huomioitiin oppijoiden tarpeet. Se, että parhaat pojat ratkaisivat itselleen sopivan vaikeita tehtäviä, lisäsi heidän matematiikasta pitämistään.

Yhteenvetona voidaan todeta, että parhailla tytöillä monipuolisten opetusmenetelmien toistuvuus selitti matematiikasta pitämistä eri tavalla yläkoulussa ja lukiossa. Monipuolisten opetusmenetelmien toistuvuus yläkoulussa selitti korkeampaa matematiikasta pitämisen tasoa yläkoulun lopussa, mutta heikensi matematiikasta pitämistä lukiossa. Parhailla pojilla monipuolisten opetusmenetelmien lisääminen lukiossa ei näyttänyt vahvistaneen matematiikasta pitämistä. Poikien matematiikasta pitäminen kasvoi, kun opettaja huomioi opetuksessaan opiskelijoiden toiveita ja pojat saivat ratkaista itselleen sopivan vaikeita tehtäviä.

Matematiikan hyödyllisyys

Parhaiden tyttöjen matematiikan hyödyllisyyden kokemisen muutoksia selittäviä tekijöitä löytyi DTA-analyysilla sekä yläkoulun että lukion aikaisista tekijöistä (taulukko 14).

Taulukko 14. Tyttöjen matematiikan hyödyllisyyden kokemisessa tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun erottelevat tekijät DTA-analyysin mukaan (tähdellä * merkitty muuttujien välinen yhteisvaikutus)

	prosenttiosuus parhaista tytöistä (n = 55)	**toistuvuus (asteikolla 1–5)**	**muutos koetussa matematiikan hyödyllisyydessä (prosenttiyksikköä)**
Opetuksen pedagogiset ratkaisut yläkoulussa			
On yhteistä opetusta opettajan johdolla.	87,3 12,7	> 3,0* ≤ 3,0	-5,9 -22,1
Annetut kotitehtävät olen tehnyt sovitulla tavalla.*	63,6 23,6	> 4,0 ≤ 4,0	-9,1 +2,7
Opetuksen pedagogiset ratkaisut lukiossa			
Matematiikan yhteys käytäntöön (faktori 1)	69,1 30,9	> 1,5 ≤ 1,5	-4,2 -16,5
Opiskellaan ryhmissä tai pareittain.	12,7 87,3	> 3,0 ≤ 3,0	+6,4 -10,1

Yläkoulun aikaisista yksittäisistä tekijöistä tytöillä muutosta hyödyllisyyden kokemisessa erotteli parhaiten se, että tunneilla on yhteistä opetusta opettajan johdolla ($F(1, 53) = 7{,}508$; $p = 0{,}017$). Tytöistä noin 87 prosenttia arvioi yhteistä opetusta olleen useammin kuin joskus ja heillä koettu hyödyllisyys heikkeni noin 6 prosenttiyksikköä. Tytöistä niillä, jotka arvioivat yhteistä opetusta opettajan johdolla olleen joskus tai harvemmin, hyödyllisyyden kokeminen heikkeni noin 22 prosenttiyksikköä. Matematiikan hyödyllisyyden kokeminen kasvoi noin 3 prosenttiyksikköä, jos joskus tai useammin toistuvaan opettajajohtoisuuteen yhdistyi se, että oppilas teki annetut kotitehtävät sovitulla tavalla usein tai sitä harvemmin

(F(1, 46) = 7,016; *p* = 0,022). Sen sijaan jatkuva kotitehtävien tekeminen heikensi hyödyllisyyden kokemusta (muutos -9,1 prosenttiyksikköä).

Lukion aikaisista tekijöistä tyttöjen matematiikan hyödyllisyyden kokemisessa tapahtunutta muutosta erotteli parhaiten pedagogiset ratkaisut, joissa matematiikka yhdistyy käytäntöön (F(1,53) = 8,381; *p* = 0,027). Muutos oli vähemmän negatiivinen, jos matematiikkaa yhdistettiin käytäntöön ainakin harvoin. Ero ryhmään, jossa käytännön yhteyttä ei ollut lainkaan, oli noin 12 prosenttiyksikköä. Yksittäisistä tekijöistä hyödyllisyyden kokemisessa tapahtunutta muutosta erotteli opiskelu ryhmissä tai pareittain (F(1, 53) = 7,860; *p* = 0,021). Noin 13 prosentilla tytöistä hyödyllisyyden kokemus kasvoi keskimäärin 6,4 prosenttiyksikköä, jos tunneilla opiskeltiin ryhmissä tai pareittain enemmän kuin joskus. Jos pari- ja ryhmätyöskentelyä oli joskus tai tätä harvemmin, kokemus matematiikan hyödyllisyydestä heikkeni noin 10 prosenttiyksikköä.

Taulukossa 15 ovat askeltavan regressioanalyysin tulokset, joista malliin jäivät vain lukio-opetukseen liittyvät pedagogiset ratkaisut.

Taulukko 15. Tyttöjen matematiikan hyödyllisyyden kokemisessa tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun selittävät tekijät askeltavan regressioanalyysin mukaan

	Muutoksen vaihtelu matematiikan hyödyllisyyden kokemisessa, tytöt			
	B	*SE*	*beta*	*p*
Malli 2 – Opetuksen pedagogiset ratkaisut lukiossa				
Vakio	-26,667	7,020		< 0,001
Matematiikan yhteys käytäntöön (faktori 1)	8,889	3,209	0,356	0,008
	F(1, 53) = 7,675; p = 0,008; R = 0,356; R^2 = 0,126			
Vakio	-39,238	9,082		< 0,001
Sovelletaan matematiikan taitoja arkielämän tilanteisiin.	9,153	2,701	0,406	0,001
Opiskelijat selittävät muille, miten ovat tehtävänsä ratkaisseet.	4,028	1,539	0,302	0,012
Opiskelijat asettavat itselleen tavoitteita ja arvioivat edistymistään.	-4,720	1,684	-0,324	0,007
Opiskellaan ryhmissä tai pareittain.	4,191	2,037	0,246	0,045
	F(4, 47) = 7,487; p < 0,001; R = 0,624; R^2 = 0,389; R^2_{Adj} = 0,337			

Tulosten mukaan tyttöjen kokemus matematiikan hyödyllisyydestä vahvistui lukiossa, kun opetuksessa huomioitiin matematiikan yhteys käytäntöön. Tämä vahvistaa DTA-analyysin tuloksia. Yksittäisistä tekijöistä hyödyllisyyden kokemusta vahvisti eniten se, että matematiikan taitoja sovelletaan arkielämän tilanteisiin. Sen jälkeen hyödyllisyyden kokemisen kasvua selitti se, että opiskelijat selittävät muille, miten ovat tehtävänsä ratkaisseet ja opiskellaan ryhmissä tai pareittain. Omien tavoitteiden ja edistymisen arviointi näyttäisi heikentäneen hyödyllisyyden kokemusta.

Parhailla pojilla hyödyllisyyden kokemuksen muutoksia selittäviä tekijöitä löytyi vain askeltavassa regressioanalyysissa (taulukko 16).

Taulukko 16. Poikien matematiikan hyödyllisyyden kokemisessa tapahtuneita muutoksia yhdeksänneltä luokalta lukion loppuun selittävät tekijät askeltavan regressioanalyysin mukaan

	Muutoksen vaihtelu matematiikan hyödyllisyyden kokemisessa, pojat			
	B	SE	beta	p
Malli 1 – Opetuksen pedagogiset ratkaisut yläkoulussa				
Vakio	-53,539	20,559		0,011
Menestyminen kansallisessa kokeessa (9.lk)	0,067	0,029	0,264	0,024
	$F(1, 71) = 5{,}309$; $p = 0{,}024$; $R = 0{,}264$; $R^2 = 0{,}070$			
Malli 2 – Opetuksen pedagogiset ratkaisut lukiossa				
Vakio	28,022	15,510		0,075
Opiskelijat neuvovat toisiaan.	-8,684	3,695	-0,276	0,022
	$F(1, 67) = 5{,}524$; $p = 0{,}022$; $R = 0{,}276$; $R^2 = 0{,}076$			

Pojilla matematiikan hyödyllisyyden kokemisessa tapahtunutta muutosta näyttäisi selittäneen parhaiten osaamisessa tapahtuneet muutokset. Lukiossa se, että opiskelijat neuvovat toisiaan, näyttäisi heikentäneen opiskelijoiden kokemuksia matematiikan hyödyllisyydestä.

Yhteenvetona voidaan todeta, että parhailla tytöillä muutoksen vaihtelua matematiikan hyödyllisyyden kokemisessa selitti parhaiten opetuksen pedagogiset ratkaisut, joissa matematiikan taitoja sovellettiin arkielämän tilanteisiin ja opiskelijat työskentelivät pareittain tai ryhmissä. Pojilla matematiikan hyödyllisyyden kokemisessa tapahtuneita muutoksia selittäviä tekijöitä ei juurikaan löytynyt. Heillä toisten neuvominen heikensi hyödyllisyyden kokemista.

9 Yhteenveto ja pohdintaa

Tutkimuksessa selvitettiin ensin, miten matematiikan parhaiden osaajien matematiikkaan liittyvät asenteet muuttuvat perusopetuksesta lukion loppuun. Tulokset vahvistivat hypoteeseja. Ensinnäkin parhaiden osaajien matematiikasta pitäminen näytti heikkenevän jo kolmannelta luokalta kuudennelle kuten yleisesti muillakin oppilailla (Metsämuuronen, 2013). Parhailla osaajilla matematiikasta pitäminen pysyi kuitenkin yläkoululuokilla vakaana ja nivelvaiheen jälkeen yläkoulusta lukioon matematiikasta pitäminen kasvoi entisestään. Toisena tarkasteltavana asennetekijänä minäkäsitys pysyi parhailla osaajilla yläkoulun aikana vakaana ja korkealla tasolla, kun yleisesti oppilaiden minäkäsitys lähtee voimakkaaseen laskuun yläkoululuokkien aikana (Metsämuuronen, 2013). Kuitenkin myös parhailla osaajilla minäkäsityksessä tapahtui heikkenemistä lukiovuosien aikana. *Big fish, little pond* -efektin (Marsh ym., 2019; Holm ym., 2020) voidaan nähdä selittävän tätä ilmiötä. Parhaat osaajat kokevat olevansa parhaita yläkoulussa, kun vertailuryhmänä on koko ikäryhmään kuuluvat oppilaat. Opetusryhmä, johon oppilas kuuluu, on hyvin heterogeeninen. Vertailuryhmä kuitenkin muuttuu parhaiden osaajien siirtyessä toiselle asteelle. Lukioon hakeutuu koko ikäryhmästä reilu puolet ja pitkän matematiikan opinnoissa vertailuryhmä muuttuu yhä homogeenisemmäksi. Oppilas ei välttämättä koekaan olevansa enää ryhmän paras, kun ryhmän muutkin opiskelijat ovat hänen kanssaan samalla osaamistasolla. Toki minäkäsityksen heikkenemistä selittää varmasti myös se, että vaatimustaso lukio-opinnoissa kasvaa ja myös parhaat osaajat joutuvat tekemään enemmän töitä osaamisensa eteen. Kolmantena asennetekijänä kokemus matematiikan hyödyllisyydestä heikkeni samansuuntaisesti kuin minäkäsitys, mutta heikkeneminen alkoi jo yläkoulun aikana. Vaikka kokemus matematiikan hyödyllisyydestä heikkeni, parhaat osaajat kokivat matematiikan hieman hyödyllisempänä kuin muut pitkän matematiikan kirjoittaneet opiskelijat. Menestys opinnoissa lisää varmastikin opiskelijoiden kokemusta matematiikan hyödyllisyydestä (Ma & Xu, 2004) ja monella opinnoissaan erinomaisesti menestyvällä opiskelijalla on tavoitteena hakeutua alalle, jossa matematiikan osaamisella on keskeinen merkitys.

Parhaiden osaajien asenteita tarkastellessa on huomioitava, että matematiikassa parhaiten menestyneiden tyttöjen asenteissa tapahtuneet kehityssuunnat eroavat yleisestä kehityssunnasta. Yleisesti tytöt arvioivat olevansa poikia heikompia matematiikassa, vaikka matematiikan taidoissa ei olisikaan eroa (Cvencek ym., 2011;

Lindberg ym., 2013). Käsitykset matematiikan osaamisesta laskevat tytöillä poikia voimakkaammin kouluvuosien edetessä ja sukupuolten välinen ero kasvaa (Lindberg ym., 2013). Tässä aineistossa matematiikassa parhaiten menestyneillä tytöillä käsitys itsestä matematiikan osaajana oli lähtötasoltaan kolmannella luokalla parhaiden poikien käsitystä selvästi heikommalla tasolla, mutta lähti kasvuun heti kolmannen luokan jälkeen ja ero parhaisiin poikiin kaventui yläkoulun aikana. Parhaiden tyttöjen ja poikien minäkäsityksessä ei ollut juurikaan eroa enää lukion lopussa. Muilla pitkän matematiikan kirjoittaneilla tytöillä minäkäsityksen ero poikiin pysyi yhtä suurena lukion loppuun asti. Myös matematiikasta pitämisessä tapahtuneet muutokset ovat kiinnostavia matematiikassa parhaiten menestyneiden tyttöjen osalta. Sekä parhailla tytöillä että pojilla matematiikasta pitäminen heikentyi kolmannelta luokalta kuudennelle. Tämän jälkeen pojilla matematiikasta pitäminen heikkeni vielä hieman yläkoulun aikana ennen kuin se lähti kasvuun lukio-opintojen aikana. Parhaat tytöt alkoivat pitää matematiikasta poikia enemmän kuudennen luokan jälkeen ja matematiikasta pitäminen kasvoi jyrkästi koko yläkoulun ja lukio-opintojen ajan eivätkä parhaat pojat saavuttaneet tyttöjen tasoa edes lukion lopussa.

Toiseksi tutkimuksessa selvitettiin, millaiset opetuksen pedagogiset ratkaisut selittävät tyttöjen ja poikien asenteissa tapahtuneita muutoksia. Selittäviä tekijöitä etsittiin regressioanalyysin ja DTA-analyysin avulla. Analyysimenetelmät täydentävät toisiaan, sillä regressioanalyysin avulla löydetään muuttujien välisiä lineaarisia yhteyksiä ja DTA-analyysi tunnistaa myös muuttujien väliset epälineaariset yhteydet. Tulosten analysoinnissa on kuitenkin huomioitava, että DTA-analyysin tekemät luokittelut eivät ole aina kovin vahvoja, koska löydetyt yhteydet saattavat näkyä vain pienellä osalla tutkimusjoukkoa. Asennemuutoksia selittäviä opetuksen pedagogisia ratkaisuja etsittiin sekä yläkoulun että lukion ajalta. Näiden vaikutukset asenteissa tapahtuneisiin muutoksiin olivat erilaiset. Yläkoulun aikaiset tekijät vaikuttivat oppilaiden asenteisiin yhdeksännen luokan lopussa eli muutoksen lähtötasoon ja lukion aikaiset tekijät oppilaiden asenteisiin lukion lopussa.

Tyttöjen ja poikien asenteiden kehittymistä selittävät erilaiset opetuksen pedagogiset ratkaisut. Molemmilla yleisesti myönteisiä asenteita vahvistivat oppilaskeskeisyyteen, yhteistoiminnallisuuteen ja oppijoiden tarpeiden huomioimiseen liittyvät tekijät aikaisempien tutkimustulosten (mm. Salmela-Aro, 2018; Hannula & Oksanen, 2013; Salmela, 2016; Koskinen, 2016; Vanttaja, 2002) ja niihin perustuvan hypoteesin suuntaisesti. Sellaisten monipuolisten opetusmenetelmien käyttö, jossa yhdistyy muun muassa projektitöiden tekeminen tai konkreettisten välineidän käyttö, ei näytä edistävän myönteisten asenteiden kehittymistä.

Tulosten mukaan tytöillä minäkäsitystä vahvisti parhaiten palautteen saaminen omasta osaamisesta (testit ja kokeet sekä itsearviointi) ja kotitehtävien säännöllinen tekeminen. Tämä noudattaa Salmelan (2016) tuloksia laudaturylioppilaista, joiden opintomenestystä tuki opetus, jossa ilmeni palautteen antaminen ja kannustaminen. Pojilla minäkäsitystä vahvisti se, että he pääsivät selittämään muille, miten ovat tehtävänsä ratkaisseet ja se, että opiskeltiin pareittain tai ryhmissä. Tämä vastaa Hannulan ja Oksasen (2013) yläkoulua koskevia tutkimustuloksia yhteistoiminnallisten opetusmenetelmien suhteesta parempiin oppimistuloksiin. Tytöt tarvitsevat kannustusta ja vahvistusta uskoakseen itseensä ja poikien itsetunto kasvaa, kun he saavat näyttää osaamistaan muille.

Matematiikasta pitämisessä tapahtuneita muutoksia selitti osin samat tekijät sekä tytöillä että pojilla. Tytöillä monipuolisten opetusmenetelmien toistuvuus lukiossa heikensi matematiikasta pitämistä, mutta oli yläkoulussa yhteydessä parempaan asennoitumisen lähtötasoon. Lukiossa erityisesti projektitöiden tekeminen näyttäisi heikentäneen tyttöjen matematiikasta pitämistä. Myöskään pojilla monipuolisten opetusmenetelmien lisääminen lukiossa ei lisännyt matematiikasta pitämistä. Lukiossa opiskelun aikataulu on usein tiukka ja parhaat osaajat saattavat kokea projektitöiden tekemisen työlääksi ja aikaa vieväksi. Voi olla, että projektitöiden tekeminen aiheuttaa opiskelijoissa stressiä, jos he asettavat erityisen korkeat tavoitteet itselleen. Pojilla matematiikasta pitämistä näytti lisäävän oppilaskeskeisyyteen ja oppijoiden tarpeiden huomioimiseen liittyvät tekijät. Pojille oli tärkeää, että opiskeltavat asiat tulevat selväksi ja he saavat tehdä oman taitotasonsa mukaisia tehtäviä. Myös Salmelan (2016) mukaan opiskelijakeskeisyys oli yksi tekijä, joka ilmeni laudaturylioppilaiden opintomenestystä ja vahvuuksia tukevana opetuksen piirteenä.

Tytöillä matematiikan hyödyllisyyden kokemusta lisäsi opetuksen pedagogiset ratkaisut, joissa matematiikan taitoja sovellettiin arkielämän tilanteisiin. Myös yhteistoiminnalliset ratkaisut matematiikan soveltamisen ohella vahvistivat tyttöjen kokemusta matematiikan hyödyllisyydestä. Yhteistoiminnallisuus yhdessä käytäntöön liittyvän matematiikan kanssa tukee työelämässä tarvittavia valmiuksia. Pojilla ei löytynyt selkeitä opetukseen liittyviä ratkaisuja, jotka olisivat yhteydessä lisääntyvään kokemukseen matematiikan hyödyllisyydestä.

Matematiikkaan liittyvien asenteiden vahvistaminen yläkoulussa ja lukiossa on tärkeää, jotta parhailla osaajilla kehittyy ja säilyy mielenkiinto matemaattisia aineita ja aloja kohtaan. Vahvaa matematiikan osaamista tarvitaan yhteiskunnan eri aloilla ja erityisesti tyttöjen mielenkiintoa matematiikkaa kohtaan tulee vahvistaa. Kuten tuloksissa havaittiin, matematiikan parhailla osaajilla ei ole lukion lopussa minäkäsitykseen liittyviä sukupuolieroja ja parhaat tytöt pitävät matematiikasta jo yläkoulussa parhaita poikia enemmän. Jotta asenteet matematiikkaa kohtaan pysyvät korkealla tasolla osaamisen kanssa, tarvitaan kannustamista, yhteistoiminnallisuutta ja oppijoiden yksilöllisiä tarpeita huomioivia opetuksellisia ratkaisuja. Projektitöiden teettämistä, tekemällä oppimista ja vastaavien opetusmenetelmien tarkoituksenmukaisuutta lukiossa tulee pohtia. Lisäksi jo yläkoulussa tulisi lisätä oppilaiden tietoisuutta matematiikan hyödyntämisestä yhteiskunnan eri alueilla ja ammateissa, jotta erityisesti tyttöjä saataisiin valitsemaan enemmän matemaattisia aineita jatko-opinnoissaan. Todennäköisesti jo korkeakoulujen matematiikkaa painottavalla todistusvalinnalla on ollut merkitystä sille, että tytöt ovat olleet motivoituneempia opiskelemaan matematiikkaa toisella asteella. Pitkän matematiikan kirjoittaneista suurin osa on jo toista vuotta peräkkäin tyttöjä (Ylioppilastutkintolautakunta, 2021).

On syytä ottaa huomioon, että tutkimus koskee vain matematiikassa parhaiten menestyneitä opiskelijoita, jotka ovat suorittaneet pitkän matematiikan ylioppilaskokeen ja tuloksia tulee tulkita heidän näkökulmastaan. On kuitenkin tärkeä muistaa, että myös muita kuin vain erinomaisesti matematiikassa menestyneitä osaajia tarvitaan laajasti yhteiskunnan eri aloilla ja tietoisuutta tästä tulisi lisätä myös lukio-opintojen aikana. Jatkossa olisi kiinnostavaa tutkia, voidaanko matematiikka-asenteita vahvistamalla lisätä myös keskitason osaajien halua opiskella enemmän matemaattisia aineita ja kykyä kasvattaa heidän omaa suoritustasoaan.

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Drawing out emotions in primary grade geometry: An analysis of participant-produced drawings of Grade 3–6 students

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Research on psychosocial classroom learning environments has a strong tradition due to the early discovery of a relationship between positive classroom climate and academic performance and motivation, engagement, participation, and attitude towards school and teaching. Yet, more research is needed in this area due to the rich concept of classroom climate. In this paper, I focus on the emotional classroom climate in specific mathematics lessons, namely geometry lessons. The goals of this paper are threefold: (a) to present an analytical tool to determine the emotional classroom climate in geometry lessons using participant-produced drawings, (b) to provide insight into the emotional classroom climate in primary grade geometry lessons (Grades 3-6), and (c) to report on the differences and similarities between the grade levels regarding the emotional classroom climate. In total, 114 German primary grade students participated in the study. The emotional classroom climate was analyzed using participant-produced drawings. The results showed that the emotional classroom climate in all grades could be described as positive and relatively stable. However, positive emotional classroom climate dominated in Grade 3 geometry lessons only. Negative classroom climate was elicited in very few cases, if at all. Still, an ambivalent classroom climate (both positive and negative emotions) increased from the lower to the higher grades. Lastly, versatile implications for theory and practice are discussed regarding the methodology as well as possible future directions.

KEYWORDS: Emotional classroom climate, geometry lessons, mathematics-related affect, participant-produced drawings, primary education

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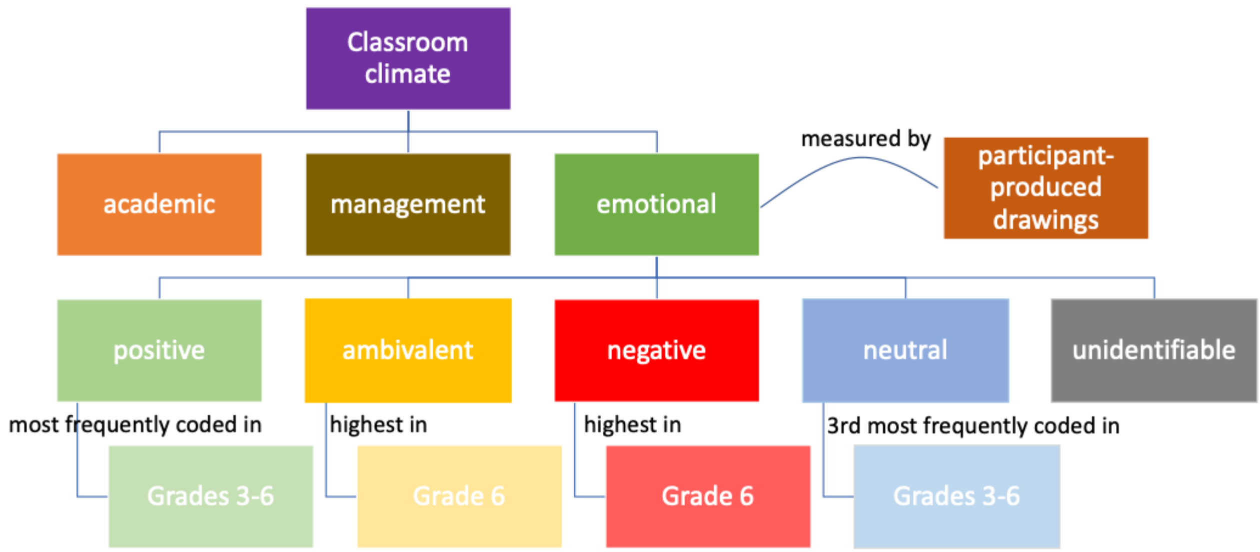

1 Introduction

Over the course of their school years, many students experience both positive and negative emotions in various subjects (Reindl & Hascher, 2013; vom Hofe et al., 2002). Among other things, emotions determine the behavior of those involved in teaching (Evans et al., 2009) as well as the willingness to learn and to perform, which are important components for school well-being (Schiepe-Tiska & Schmidtner, 2012). Furthermore, a positive teacher-child relation advances both students' social accommodation and their orientation to school, and thus is an important foundation for their academic career in the future (Harrison et al., 2007). In recent decades, the study of emotions has gained greater prominence in educational research (Hascher & Edlinger, 2009), often with a focus on specific emotions, such as the joy of learning (Schmude, 2005). The international comparative study PISA 2012 also analyzed, among other things, emotional orientation in mathematics (Schiepe-Tiska & Schmidtner, 2012). In PISA 2012, Germany performed slightly below the OECD average in terms of the emotional orientation of enjoyment in mathematics (Schiepe-Tiska & Schmidtner, 2012). Overall, only 39% of 15-year-old students reported liking mathematics and engaging in mathematics because they enjoyed it (Schiepe-Tiska & Schmidtner, 2012).

In mathematics education, the topic of emotions already has its field of research (e.g., Dahlgren Johansson & Sumpter, 2010; Laine et al., 2013, 2015; Reindl & Hascher, 2013; Tuohilampi et al., 2016; vom Hofe et al., 2002). For example, in the PALMA study ("Project for the Analysis of Learning and Achievement in Mathematics"), it has been confirmed that emotions have a strong influence on the mathematical competence growth of Grade 5-10 students (vom Hofe et al., 2002). Here, the joy of learning correlated positively with interest and motivation to learn, among other things, whereas anxiety and boredom were recognized as negative influencing factors (vom Hofe et al., 2002). Similarly, Frenzel and Stephens (2007) in their study with Grades 5-10 students reported on a close connection between the classroom climate and learning achievements as well as emotional and social experiences. These results make clear what significance both positive and negative emotions may have for mathematics development at primary school age.

Another area of research in the field of emotions focuses on the development of emotions over the school years. In a study by Reindl and Hascher (2013), a decrease in positive emotions was recorded in mathematics classes, which negatively affected attitudes toward mathematics as a subject. Hascher et al. (2011) also pointed out that positive emotions decrease over the elementary school years. Previous research on emotions focused mainly on mathematics education in general, but not specifically on different mathematical content, such as arithmetic (Reindl & Hascher, 2013; Schmude, 2005), geometry (Glasnović Gracin & Kuzle, 2018), or how these develop during the elementary school years. School geometry is a subject area of mathematics that is often disregarded and referred to as the "stepchild" of mathematics education (Backe-Neuwald, 2000; Eichler, 2005). However, geometry didactics (e.g., Krauthausen, 2018) especially emphasizes that geometry instruction may help students develop a positive attitude towards mathematics due to its motivating effect on students through alternative instructional concepts (e.g., action-oriented instruction, discovery learning) and a sense of achievement by experiencing success. This led to an increased interest in how elementary school children feel during this large laissez-faire subfield of mathematics lessons. More specifically, the main goal of the inquiry presented in this paper was to provide insight into the emotional classroom climate in primary grade geometry lessons[1] and to find out if previously reported trends also apply to them. For this purpose, participant-produced drawings were used, which allow a constructive process of thinking in action, rather than seeing drawings as simple representations of the participant's worldviews (Kearney & Hyle, 2004).

2 Theoretical foundation

In this section, I present the constructs of classroom climate and emotional classroom climate and place them in the context of this work (i.e., drawings as a research method). Emotional classroom climate research in mathematics using drawings is then presented. The section ends with the two research questions that guided the study.

2.1 Emotional classroom climate

The classroom is a significant environment in the development of children which with time develops a distinct climate or feel (Ashkanasy, 2003). It shapes students' essential perceptions, and it allows each child to acquire new concepts and

[1] In the federal states of Berlin and Brandenburg (Germany) where the study was conducted, primary education covers Grades 1 to 6.

procedures, which are supported by the teacher and the teacher's choice of activities (Ahtee et al., 2016). According to researchers (Evans et al., 2009; Hannula, 2012), the classroom climate refers to a shared subjective representation of important characteristics of the classroom which, however, differ in their climate-creating determinants. Götz et al. (2011) presented a general statement for the "emotional aspects" of climate. They refer to both positive and negative emotions of a group. In addition, "perceived affective attitudes" (Götz et al., 2011, p. 506) related to the school, people who are associated with the school, areas of specialization, and subjects taught are, among others, the emotional aspects of classroom climate. On the other hand, Evans et al. (2009) divided the notion of classroom climate into three complementary components: *academic*, referring to pedagogical and curricular elements of the learning environment; *management*, referring to discipline styles for maintaining order; and *emotional*, referring to affective interactions within the classroom. Furthermore, Evans et al. (2009) argued for the importance of treating emotional climate as a distinct aspect of classroom climate as it is "superordinate to other classroom climate domains since it interfaces with the conventional academic and management elements of effective learning environments" (p. 131). In this study, I concentrate on the last component, namely the emotional classroom climate, which can be described as an emotional relationship between the students and the teacher.

According to Hannula (2012), the emotional climate in the classroom can be regarded from a psychological and social point of view (see Table 1). The *psychological dimension* refers to the level of an individual and involves affective conditions (i.e., emotions and emotional reactions, thoughts, meanings, and goals), and affective properties (i.e., attitudes, beliefs, values, and motivational orientations). The *social dimension* refers to the classroom community. Its affective conditions refer to social interaction, communication, and the atmosphere in a classroom (momentarily), whereas affective properties refer to norms, social structures, and the atmosphere in the classroom. Here, the focus is on the former.

Table 1. Dimensions of the emotional climate in a classroom

	Psychological dimension or the level of the individual	**Social dimension or the level of the community (classroom)**
Affective condition (state)	Emotions and emotional reactions Thoughts Meanings Goals	Social interaction Communication Atmosphere in the classroom (momentarily)
Affective property (trait)	Attitudes Beliefs Values Motivational orientations	Norms Social structures Atmosphere in the classroom

In both dimensions, one can distinguish between two temporal aspects of affect, namely state and trait. *State* (affective condition) refers to the emotional atmosphere at a specific moment in the classroom, such as different emotions and emotional reactions (e.g., fear and joy), thoughts (e.g., "This is difficult."), meanings (e.g., "I could do it."), and aims (e.g., "I want to solve this task.") (Laine et al., 2013). They influence critical decisions and determine whether a problem is solved by an individual or not and change rapidly. *Trait* (affective property) refers to more stable conditions or properties, such as attitudes (e.g., "I like math."), beliefs (e.g., "Math is difficult."), values (e.g., "Math is important."), and motivational orientations (e.g., "I want to understand.") (Laine et al., 2013, 2015). They provide a consistent pattern of how an individual thinks or feels in a situation (Hannula, 2012). Both temporal aspects of affect can be applied to the context of the school. For instance, in situations of a similar nature that occur repeatedly in the classroom (i.e., checking of homework, discussion of assignments), students develop affective characteristics (traits) typical of that situation. Social norms, social structures, and the prevailing atmosphere in the classroom are described as such traits (Hannula 2012; Laine et al. 2013). Given the teacher's central role in constructing the emotional climate or being the emotional force in day-to-day school lessons (Evans et al., 2009), recurring situations may have an influence on students developing more stable affective traits typical to a certain classroom (Laine et al., 2013).

2.2 Drawings as a research method

In recent decades childhood research has experienced a paradigm shift that has had a comprehensive impact on research design and methods. While it was common to view children as objects, the shift has led to children being increasingly viewed as subjects in the research process by using methodologies such as observations or test procedures (Hill, 1997). Among other things, this shift has led to increased use of participatory and visual methods and processes in childhood research. With respect to visual research, drawings, videos, and photographs have been recognized as one of the crucial methods (Einarsdóttir, 2007). Visual methods are not only effective because of the richness of produced data, but also because of the quality of the data providing insights into children's everyday lives (Einarsdóttir, 2007). Drawing is a creative method based on inventive and imaginative processes with drawings as a research tool having the function of capturing children's individual experiences (Veale, 2005). Children perceive drawing as a way of expressing themselves (Laine et al., 2015). Their drawings are shaped by "perception, emotions and motivations, cognitions, and skills and abilities" (Gramel, 2008, p. 34). Guided by emotion, children communicate through drawings what occupies them, what is important to them, and what they experience (Gramel, 2008), and thus provide a holistic insight into their emotional lifeworld (Einarsdóttir, 2007; Kearney & Hyle, 2004; Veale, 2005). For children, drawing is much more than a simple representation of what they see before them; rather it can be understood as one way in which they are making sense of their experiences (Anning & Ring, 2004).

Drawings are considered a useful research method when subjects cannot adequately express or verbalize content in response to research questions, as they require little or no language mediation (Thomson, 2008; Weber & Mitchell, 1996). Another advantage of using drawings as a research object is that the interviewees, through the support of the drawings, answer honestly and reduce their answers to the essentials (Nossiter & Biberman, 1990). Moreover, children participate more in research when it is fun for them and when they can express their creativity (Punch, 2002). Through colors, in particular, children can highlight emotions and create an effect in the viewer and the drawer themselves (Neuß, 2014). According to Harrison et al. (2007), emotions that are felt particularly negatively can be better expressed in drawings than through language. Furthermore, the drawers can be observed while drawing (Clark, 2005; Einarsdóttir, 2007; Punch, 2002), and thus the interpretations of the drawers themselves, or explanations about the drawn image can be experienced. Additionally, Kearney and Hyle (2004) found that using participant-produced drawings is more likely to accurately represent participants' experiences and emotions. Participant-produced drawings function as a catalyst, helping participants to articulate their feelings, emotions, and lived experiences. In that manner, the participant approach allows for depth of discussion, participant's shaping of agenda, and encourages collaborative meaning-making as well as reliable and trustworthy data (Kearney & Hyle, 2004).

2.3 Emotional classroom climate research

Current research on emotions and their development in mathematics education focused mainly on the secondary and less on the primary level (Reindl & Hascher, 2013). Yet, research across disciplines shows there is a decline in enthusiasm for learning and school over the first years of education, and everyday school life is increasingly accompanied by negative emotions (e.g., Helmke, 1993; Reindl & Hascher, 2013). Here, especially negatively experienced emotions, such as boredom, are the main accompanying symptoms of school experience (Eder, 1995). For instance, Reindl and Hascher (2013) investigated the emotional feelings (i.e., joy, interest, anger, fear, and boredom) of 165 Austrian elementary school students[2] in Grades 1 to 4 during mathematics lessons using a questionnaire at different points in time over a period of two school years. In particular, they were interested in whether a decrease in the positively experienced emotions of joy and interest had an effect on the negatively experienced anger, fear, and boredom. They reported that positive emotions were more pronounced than negative emotions in each of the grade levels studied, with Grade 1 students showing the most positive emotions, meaning that "children [...] experience more positive and fewer negative emotions toward mathematics" (Reindl & Hascher, 2013, p. 283) at the beginning of their schooling. Over the course of the first school year, the positive emotions decreased. In the survey, positive emotions increased in Grade 2 and remained relatively stable in Grades 3 and 4 (Reindl & Hascher, 2013). Specifically, the students experienced overall the emotion of joy at a relatively high level. Within the respective school years, a decrease in negatively experienced emotions was always ascertainable (Reindl & Hascher, 2013). Although negative emotions are subject to a slight recovery effect during the transition from primary to secondary school (van Ophuysen, 2008), a stronger focus on primary

[2] The sample was reduced to 121 students in the course of the study.

grades is important and necessary.

Although numerous studies researching emotions employed methods such as interviews, observations, and questionnaires (e.g., Reindl & Hascher, 2013; Schmude, 2005), in the last decade, there has been an increase in studies on emotions and emotional classroom climate using visual research methods, such as drawings (e.g., Dahlgren Johansson & Sumpter, 2010; Glasnović Gracin & Kuzle, 2018; Laine et al., 2013, 2015, 2020; Tuohilampi et al., 2016). These studies have shown that children's drawings have great potential to provide a thorough insight into different aspects of classroom climate in school mathematics. For instance, Laine et al. (2013) investigated the emotional atmosphere of 133 Finnish Grade 3 students from a total of nine mathematics classes, using students' drawings only. The emotional classroom climate was classified into five categories (i.e., positive, ambivalent, negative, neutral, unidentifiable) based on the students' and teachers' mode (i.e., facial expressions) as well as on their speech and thought bubbles illustrated in the drawings. Overall, 38% of the drawings showed a positive emotional atmosphere in mathematics class. In addition, 33% of the drawings were rated as ambivalent, 15% as neutral, 5% as unidentifiable, and 10% with a negative emotional atmosphere (Laine et al., 2013). Since the difference between positive and ambivalent categories was not that big, Laine et al. (2013) concluded that the emotional atmosphere in Grade 3 mathematics classes was mainly positive. In a further study, Laine et al. (2015) researched the emotional atmosphere of 136 Finnish Grade 5 students from a total of eight mathematics classes. The research design and the evaluation of the children's drawings followed the same criteria as in the study of third graders (Laine et al., 2013). Overall, 36% of the drawings showed a positive classroom climate, 34% ambivalent, 13% neutral, 14% negative, and 3% were not identifiable. Since both reported studies (Laine et al., 2013, 2015) addressed the same research questions, a comparison of the results between Grade 3 and Grade 5 students is possible. Over the course of the school years, the proportions of atmospheres drawn that were rated as positive as well as those rated as negative changed between the two grade levels. The assessment of drawings depicting negative emotional atmospheres in mathematics lessons noticeably increased. There was an increase of 4% compared to the third graders from the study two years earlier. In total, 14% of the drawings were assessed as negative, whereas the percentage of positively rated drawings (38%) decreased by 2% to 36% (Laine et al., 2015). Thus, the results of both studies suggested there was a negative trend regarding emotional atmosphere in mathematics classrooms over the school years (Laine et al., 2020).

Similarly, Dahlgren Johansson and Sumpter (2010) presented a comparative analysis of children's conceptions of mathematics and mathematics education in Grades 2 (N = 19) and 5 (N = 11) using drawings. The results showed that there was a significant decrease in positively perceived attitudes toward the subject of mathematics from the study group of Grade 2 to the study group of Grade 5 students. Overall, only five Grade 5 students rated their attitude toward mathematics as positive, in contrast to 17 positive ratings from Grade 2 students. The most frequently displayed emotion of Grade 2 students was happiness, sometimes combined with a quirky thoughtfulness, whereas Grade 5 students most frequently displayed calmness and frustration (Dahlgren Johansson & Sumpter, 2010).

In comparison to the above-mentioned studies, Glasnović Gracin and Kuzle (2018) analyzed the emotional climate in school mathematics in the context of geometry lessons using participant-produced drawings. Here, a multiple case study with four high-achieving Croatian students from Grades 2 to 5 was conducted. The drawings were analyzed based on facial features, and thought and speech bubbles as suggested by Laine et al. (2013, 2015), and Zambo (2006), but expanded by also looking at body language. The results of the study were aligned with those of Laine et al. (2013) with the emotional climate in geometry lessons on the level of the individual being positive (Grades 2-3), unidentifiable (Grade 5) or ambivalent (Grade 4), but in no case dominantly negative. Since a multiple case study was conducted, Glasnović Gracin and Kuzle (2018) could not portray a comprehensive picture of the emotional climate in geometry lessons, but rather case-based results. For that reason, the results were neither representative of a broader population nor generalizable.

2.4 Research questions

In this study, I aim to contribute to the research of the mathematics-related affect by presenting a detailed inventory to determine the emotional classroom climate in geometry lessons using participant-produced drawings, by providing insight into the emotional classroom climate in primary grade geometry lessons, and by examining the grade level's effect on that. The exact research questions are:

1. What kind of emotional classroom climate in geometry lessons can be seen in primary Grade 3-6 participant-produced drawings?

2. What similarities and differences in participant-produced drawings exist among elementary Grades 3–6 from the perspective of the emotional classroom climate in geometry lessons?

3 Research process

3.1 Research design and subjects

For this study, an explorative cross-sectional qualitative research design (Patton, 2002) using participant-produced drawings (Kearney & Hyle, 2004) was chosen. The study participants were 114 elementary school students (Grades 3 to 6). This age group was optimal as they had already gathered enough experience in school mathematics. Furthermore, according to Lucquet's developmental-stage theory (1913, 1923, in Anning & Ring, 2004), the children are either at a schematic stage (ages 7 to 9 years) or visually unrealistic stage (ages 9 to 11 years) of drawing. Thus, the quality of drawings is already solid to high enough to allow rich insights into the emotional classroom climate. The distribution of students was as follows: 25 students from Grade 3, 33 students from Grade 4, 28 students from Grade 5, and 28 students from Grade 6. In the study, multiple urban schools from two federal states in Germany (i.e., Berlin and Brandenburg) participated in the project. Here, elementary schools were approached that fit the profile (i.e., not high- or low-ranked schools, but average urban schools). From the schools that agreed to participate, classes were selected that had at least one geometry lesson per week and, according to the teachers, may reflect the variety of emotional states in geometry instruction. From the same school, a maximum of two average students were randomly selected. Typical case sampling as a type of purposive sampling was utilized as a way of collecting rich and in-depth data and to allow for a comparison between other similar samples (Patton, 2002).

3.2 Data collection instruments and procedure

The main source of data was student work, namely student drawings, and a semi-structured interview. Student work was based on an adaptation of the instrument from the work of Ahtee et al. (2016), and Laine et al. (2013, 2015). The research data were collected in a one-to-one setting between a student and the author of the paper. For the drawing, the students received instructions in the form of an Anna letter (Dohrmann & Kuzle, 2014). Each student was given a piece of A4-paper with an assignment given by a fictional bright 12-year old girl by the name of Anna: “Dear __________, I am Anna and new to your class. I would like to get to know your class better. Draw two pictures of your mathematics lessons. The first drawing should show what your arithmetic lessons are like and how you view them. The second drawing should show what your geometry lessons are like and how you view them. In each drawing, include your teaching group, the teacher, and the pupils. Use speech bubbles and thought bubbles to describe conversation and thoughts. Mark the pupil that represents you in the drawing by writing “ME”. Thank you and see you soon! Yours Anna.” Here, only the second drawing is of relevance. After the students had finished drawing, the drawings were used as a catalyst for a semi-structured interview, as suggested by Kearney and Hyle (2004). During the interview, both a free description of the drawing on the part of the child were given (e.g., “Describe your picture to me.”) and specific questions based on the child’s description were posed (e.g., “How does the child 1, 2, etc. feel in the second drawing?”, “What is the reason for that?”).

3.3 Data analysis

The analysis of drawings was based on the holistic evaluation of the emotional classroom climate as suggested by Laine et al. (2013, 2015). Here each drawing was analyzed one content category at a time. Specifically, the evaluation was based on both the students’ and the teacher’s moods as well as on their speech and thought bubbles illustrated in the drawings. According to Koike (1997, cited in Gramel, 2008, p. 36) feelings can be divided into five categories of expression in drawings, namely facial expression, gestures, the facial schema, the representation of situations triggering emotions, and symbols. In the study, different facial features, and speech and thought bubbles were analyzed based on the coding manual developed by Zambo (2006), which was expanded with physical body features (i.e., body posture, arm position) as suggested by Glasnović Gracin and Kuzle (2018), to achieve a more accurate representation (see Table 2).

Table 2. An excerpt from the coding manual

Feature and thoughts	Nature and ranking	Clues
Physical facial features		
eyes	positive (+1)	wide open
mouth	neutral (0)	drawn as a straight line
symbols drawn on face	negative (-1)	tears, tongue stuck out, teeth in a growl
Physical body features		
arms	positive (+1)	arms in the air, open arms
arms	neutral (0)	arms on the table
arms	negative (-1)	crossed arms
Thoughts		
symbols, signs, words, emotional words	positive (+1)	hearts, peace signs, thumbs up, “easy”, “fun”, “I like”
symbols, signs, words	neutral (0)	no expression
symbols, signs, words, emotional words	negative (-1)	dark scribbles, sad, “blah, blah”, “hate”, “too hard”

Concretely, in each drawing, both the depicted students as well as the teacher were examined according to the developed inventory. The analysis of the latter was necessary because the teacher influences the affective climate of the class (e.g., Harrison et al., 2007). I use Figure 1 for the purpose of explaining the coding process. The drawing does not represent a prototypical drawing, but rather has been selected on the basis of the scan quality. As shown in Tables 2 and 3, if a child’s rating of a category was emotionally positive, a counter (+1) was noted. If the assessment was negative, a negative counter (-1) was noted, and if the assessment was neutral, the symbol 0 was noted (Zambo, 2006). If none of the categories was drawn, it was classified as unidentifiable and received a dash (-). After rating each feature, the “counters” were balanced against each other. If the score was 0, the emotional state of the respective child was rated as neutral; if the score was positive, it was rated as positive; and if the score was negative, it was rated as negative. If an individual contained both positive and negative characteristics, it was coded as ambivalent. Thus, Kevin, depicted in Figure 1, was assigned +2 counters, his emotional feeling was coded as positive. The same procedure was then used for all the other protagonists in the drawing (i.e., Jessica, Lucas, Leonie, the teacher).

Figure 1. Exemplary coding of the emotional feeling of the drawn children

Following the rating of the children drawn, the holistic evaluation of the emotional climate in a geometry lesson was assessed. This was based on five categories reported in Laine et al. (2013, 2015) which was slightly adapted. These include the following emotional classroom climate categories: positive (i.e., the majority of persons smile, or think or behave positively, although some of the expressions can be neutral), ambivalent (i.e., there are both positive and negative facial/body language expressions or thoughts), negative (i.e., the majority of persons are sad or angry or think/behave negatively, although some of the expressions can be neutral), neutral (i.e., all facial/body language expressions or other thoughts are neutral, although some of the expressions can be either positive or negative), and unidentifiable (i.e., no facial/body language expressions or thoughts). If identifiable and non-identifiable persons were illustrated, only the non-identifiable ones were identified in the overall image analysis but were scored as neutral. To facilitate the interpretation of the children's drawings, the semi-structured interviews were transcribed. Multiple data sources (i.e., data triangulation) were used to assess the consistency of the results, and to increase the validity of the results (Kuzle & Glasnović Gracin, 2020; Patton, 2002). Furthermore, they gave precise indications of the two temporal aspects of affect (i.e., state, trait). Going back to Figure 1, the interview confirmed that Kevin was in a positive mood and that he enjoyed geometry lessons very much. In addition, it provided information about the emotional state of two students, namely Lucas and

Leonie, who were drawn from the back. In the drawing, only arm posture is visible (i.e., arms open downward) which would imply neutral counters in each case, and hence neutral emotional feeling. The interview, however, revealed that both Lucas and Leonie disliked geometry lessons, were not happy, and for that reason the drawer did not want to draw their faces. In this case, the interview added new information – or even gave a completely different picture of the emotional state of both children than one would take from the drawing itself. Thus, both children did not reflect a neutral emotional state but rather a negative one.

Table 3. Exemplary coding of the emotional feeling of the drawn child

Child	Physical and speech/thought bubble features	Feature clues	Explanation	Score
Kevin	Facial features	mouth	smiling	+1
		eyes/eyebrows	wide open	+1
		face drawn symbols	-	-
	Total: Physical face features			+2
	Body features	arm position	downward	0
	Speech/thought bubble features	symbols	-	-
		signs	-	-
		words	-	-
	Total: Speech/thought bubble features			-
Total: Kevin				+2

The two researchers coded the students' data separately from one another. The interrater reliability was high (90% agreement). Nevertheless, we discussed the differences in coding taking into consideration both students' products, and refined the coding manual. It was agreed that the final decision about the nature of a counter assigned to a particular physical feature would be based on the interview data, as was exemplarily elaborated earlier when describing the process of data analysis. This decision mainly related to the drawings in which the protagonists were depicted from behind or in an extremely simplified or generic manner. Furthermore, there were a few disagreements regarding the nature of individual thought features, such as "good", "okay" which were then discussed. Adjustments were subsequently made to our coding, after which the interrater reliability was 100%.

4 Results

This section is divided into two parts. The first part focuses on the emotional classroom climate in Grades 3-6 geometry lessons. The second part focuses on the differences and similarities in the emotional classroom climate across four grade levels.

4.1 Emotional classroom climate in geometry lessons through the lens of Grades 3-6 participant-produced drawings

After analyzing the physical features (i.e., face and body), and the speech and thought bubbles of the children and the teacher depicted in the drawings, they were classified into five categories (i.e., positive, negative, ambivalent, neutral, unidentifiable). In Table 4 the results regarding the emotional classroom climate in Grades 3-6 geometry lessons are presented.

Table 4. Emotional classroom climate in primary grade geometry: absolute and relative frequencies

		Emotional classroom climate categories				
Grade	N	positive	ambivalent	negative	neutral	unidentifiable
3	25	15 (60%)	6 (24%)	1 (4%)	2 (8%)	1 (4%)
4	33	14 (42%)	9 (27%)	3 (9%)	5 (15%)	2 (6%)
5	28	13 (46%)	10 (36%)	2 (7%)	2 (7%)	1 (4%)
6	28	13 (46%)	12 (43%)	0 (0%)	2 (7%)	1 (4%)
Total	114	55 (48%)	37 (33%)	6 (5%)	11 (10%)	5 (4%)

4.1.1: Emotional classroom climate in Grade 3. In total, 15 drawings (60%) by Grade 3 students represented the emotional climate in the geometry classroom as positive. The drawing shown in Figure 2 is an example of a drawing that was rated as positive, as both the children and the teacher are smiling. The eyes of all three protagonists are typical with no expression (i.e., round eyes), which were coded as neutral. The arm posture of child 1 is neutral (i.e., arms closed next to the body holding two objects), whereas that of child 2 and of the teacher is positive as their arms are open upwards. The arm posture of child 2 signalizes that she raised her hand to answer the teacher's question, and in that manner is engaging in conversation, though the drawer did not have any speech bubbles. The interview revealed that this drawing illustrates an everyday teaching situation in geometry lessons. The teacher is often in a good mood and the students like geometry. Thus, these are permanently valid characteristics of geometry lessons (trait). The positive emotional climate was in almost all cases described as a trait.

Figure 2. A Grade 3 student's drawing of a positive emotional classroom climate.

In total, six drawings (24%) by Grade 3 students represented the emotional climate in the geometry classroom as ambivalent since both positive (e.g., protagonists smiling, arms open upwards, "I am in a good mood.", "I like geometry."), as well as negative features (e.g., mouth turned downward, closed eyebrows, mouth open in a scream, "I find geometry hard.", "boring"), were illustrated. The interviews revealed that in half of the drawings this was a permanent characteristic of geometry lessons (trait), whereas in the rest of the drawings it was a temporal characteristic of geometry lessons (state). Only one Grade 3 student (4% of drawings) represented the emotional climate in the geometry classroom as negative with almost only negative features (e.g., mouth turned downward, tears on the face, eyebrows slanted inward and contracted, tears, mouth open in a scream, "!") and some neutral ones (e.g., arms closed downwards, eyes typical with no expression) were illustrated. However, the drawer elaborated in the interview that the mood was determined by a quarrel between the students, and that this situation did not reflect a permanent condition in geometry lessons, but a condition of the emotional classroom climate at a specific moment (state).

In total, two Grade 3 students (8% of drawings) represented the emotional climate in the geometry classroom as neutral. This can be recognized by neutral body language (e.g., hands placed on the desks) and neutral thought and speech bubbles (e.g., "Open your books at pages 16-17. We are doing these pages now.", "OK."). Whereas the interview revealed that the first drawing represented a stable condition (trait), the second drawing represented a specific moment in the geometry class (state). Only one Grade 3 student's drawing was rated as unidentifiable. In this drawing, there were no facial or body expressions, and speech and thought bubbles could not be identified. Children's names are written on drawn rectangles, which most likely represented desks. Likewise, the interview did not provide any further information.

To summarize, in the sample of Grade 3 students' participant-produced drawings, the emotional classroom climate in geometry was predominantly (more than half) positive and a rather stable condition (trait). Only one drawing showed a completely negative emotional classroom climate which, however, reflected a temporary situation of geometry teaching (state). The few cases of ambivalent classroom climate had more positive emotional features than negative ones.

4.1.2: Emotional classroom climate in Grade 4. In total, 14 drawings (42%) by Grade 4 students represented the emotional climate in the geometry classroom as positive, depicting only positive features (e.g., full smile, arms open upwards, wide-open eyes, "That is easy.") with occasionally also neutral ones (e.g., eyes with no expression, mouth drawn as a straight line, a mathematical expression such as "a circle") were illustrated. Furthermore, nine drawings (27%) by Grade 4 students represented the emotional climate in the geometry classroom as ambivalent since both positive (e.g., protagonists smiling, wide open eyes, arms open upwards, "I like it.", "I can do geometry."), as well as negative features (e.g., mouth turned downward, tears on the face, eyes in a downward slant, "boring", "oh no, not today", "zzz"), were illustrated. Only three students (9% of drawings) represented the emotional climate in the geometry classroom as negative since only negative features (e.g., mouth turned downward, eyebrows slanted inward, "not again", "Finally, we get to go home.") with some neutral ones (e.g., arms closed downwards) were illustrated. The drawing shown in Figure 3 is an example of a drawing that was rated as negative. The arm posture of all three protagonists is neutral (i.e., arms in action). The eyes of child 2 are typical with no expression (i.e., round eyes) which were coded as neutral. However, a speech bubble contains the word "aua" which reveals a negative state of mind (i.e., "uoch"). The teacher's thought bubble "Finally home." also reflects a negative condition. The interview revealed that the drawer provided an illustration of an everyday situation: some children are bored and his math teacher is happy that it is the end of the lesson. Thus, these are permanently valid characteristics of geometry lessons (trait).

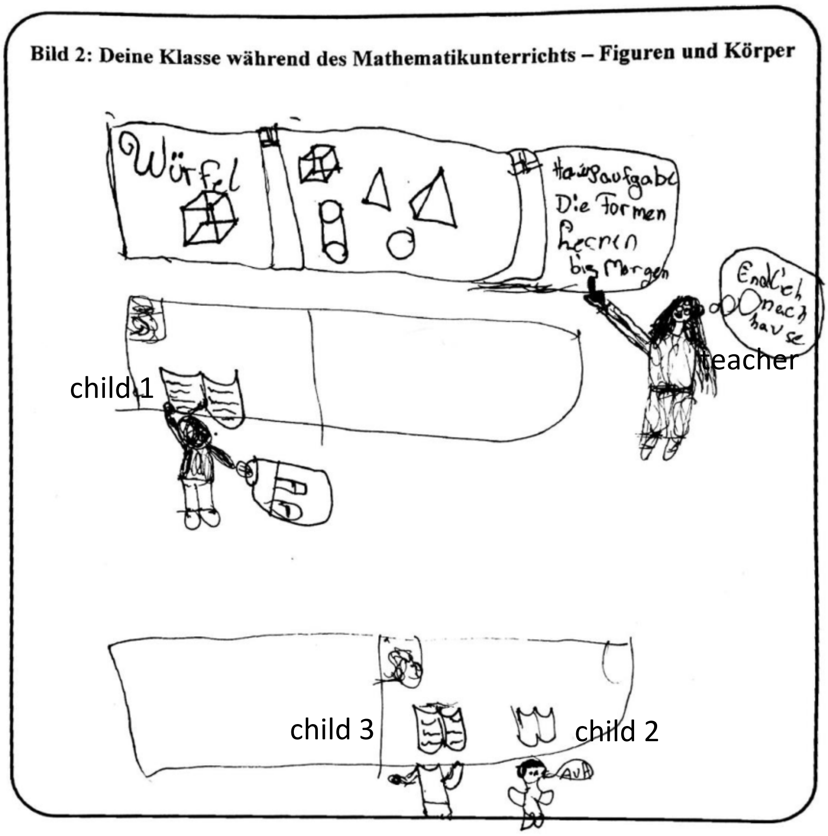

Figure 3. A Grade 4 student's drawing of a negative emotional classroom climate.

In total, five Grade 4 students (15% of drawings) represented the emotional climate in the geometry classroom as neutral since only neutral features were illustrated or mentioned (e.g., hands on the table, eyes with no expression, mouth drawn as a straight line, mathematical statements such as "A circle."). Two drawings were rated as unidentifiable (6% of drawings). In these drawings, there were no facial or body expressions, and speech and thought bubbles could not be identified. Likewise, the interview did not provide any further information.

To summarize, in the sample of Grade 4 students' participant-produced drawings, the drawings portraying a positive classroom climate had the highest frequency, but they did not predominate. However, in all but one drawing a positive emotional climate was a stable condition (trait). Only three drawings showed a completely negative emotional classroom climate which, on the downside, reflected a permanent condition (trait). The cases of drawings reflecting an ambivalent classroom climate were dominated by positive emotional features with a few negative ones.

4.1.3: Emotional classroom climate in Grade 5. Almost half of the drawings (46%, n = 13) by Grade 5 students represented the emotional climate in the geometry classroom as positive since often only positive features (e.g., full smile, arms open upwards, wide open eyes) with occasionally also neutral ones (e.g., eyes with no expression) were depicted. The interviews revealed that in all cases but two, the drawings represented a stable condition in the geometry lesson (state). More than one-third of the drawings (36%, n = 10) were rated as ambivalent with both positive (e.g., arms open upwards, full smile, "I understand it.") and negative features (e.g., mouth turned downward, "boring", "wake up") being illustrated or mentioned. In three cases, the drawings reflected a temporary emotional condition (state), whereas in six cases a stable condition (trait). No drawing contained more than one negative expression. Furthermore, two drawings (7% of drawings) represented the emotional climate in the geometry classroom as negative since only negative features (e.g., "shut up", "zzz") with some neutral ones (e.g., arms closed downwards) were illustrated, which, however, reflected a temporary condition (state). In total, two Grade 5 students (7% of drawings) represented the emotional climate in the geometry classroom as neutral since only neutral features were illustrated or mentioned (e.g., hands on the table, eyes with no expression, mouth drawn as a straight line, mathematical statements such as "I'm drawing a prism."). Only one drawing was rated as unidentifiable (4% of drawings). In this drawing, there were no facial or body expressions, and speech and thought bubbles could not be identified. Likewise, the interview did not provide any further information.

To summarize, in the sample of Grade 5 students' participant-produced drawings, a positive classroom climate was portrayed most frequently, but did not predominate. In most of these drawings, a positive emotional climate was a stable condition (trait). The percentage of drawings illustrating an ambivalent classroom climate was slightly lower than those illustrating a positive classroom climate. Also, these drawings predominantly reflected a stable condition (trait). Only a few drawings illustrated a negative or a neutral emotional classroom climate. The former, however, reflected a temporary condition (state).

4.1.4: Emotional classroom climate in Grade 6. Almost half of the drawings (46%, n = 13) of Grade 6 students represented the emotional climate in the geometry classroom as positive since often only positive features (e.g., smile, arms open upwards, engaging arms, wide open eyes, eyebrow upward slant, smiley) with occasionally also neutral ones (round eyes with no expression, “A triangle.”) were depicted. The interviews revealed that in all cases the drawings represented a stable condition (state). Furthermore, 12 drawings (43%) by Grade 6 students represented the emotional climate in the geometry classroom as ambivalent since both positive (e.g., protagonists smiling, arms are open upwards, “That’s easy!”, raising arm), as well as negative features (e.g., mouth turned downward, closed eyes, “Oh no!”, “zzz”), were illustrated. The interviews revealed that in all cases but one, the drawings represented a stable condition (trait). The drawing shown in Figure 4 is an example of a drawing that was rated as ambivalent.

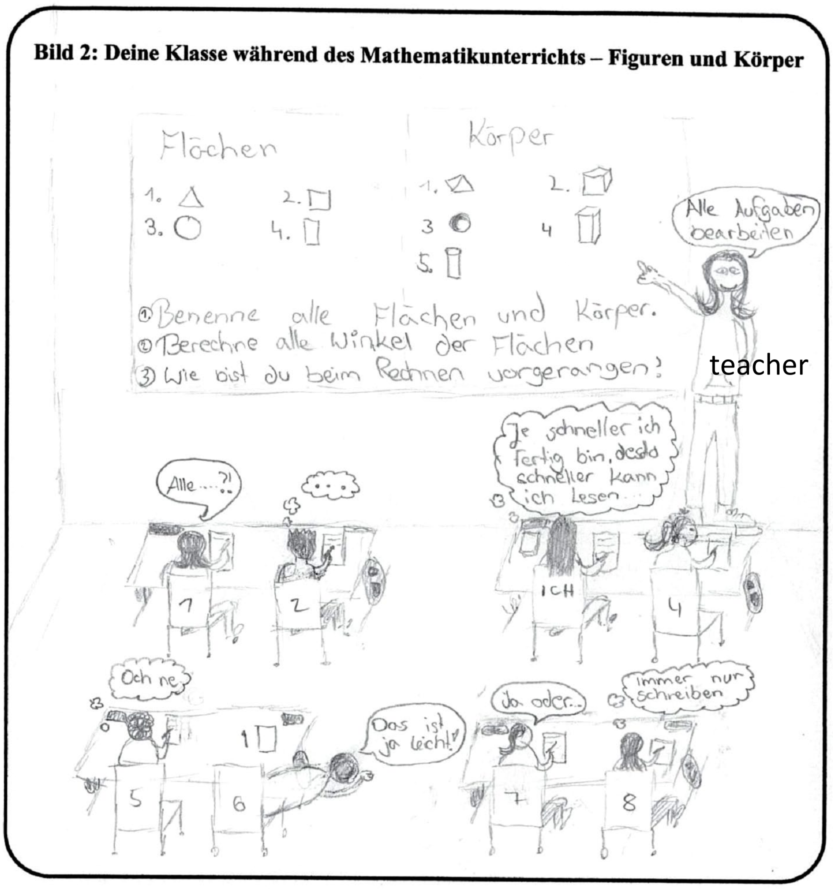

Figure 4. A Grade 6 student’s drawing of an ambivalent emotional classroom climate.

Specifically, the teacher is depicted as smiling (i.e., positive emotional state). Her arms are in motion as she is pointing at the board and assigning the students the following task “Do all the tasks.” (i.e., neutral feature). In total, eight children are illustrated in the drawing. Two children, namely child 4 and child 6 embody positive emotions since child 4 is smiling, and the arms of child 6 are open upwards with a speech bubble “That is easy”. Two children, namely child 2 and child 7 reflect a neutral emotional state since only two neutral statements are illustrated in the thought bubble (i.e., “…”, “yes or…”). Lastly, four children elicit negative features regarding the emotional classroom climate. Child 1 is illustrated with a speech bubble “All … [of them]?!”, child 3 (“Ich = me”) with a though bubble “The faster I finish, the faster I can read.”, child 5 with a thought bubble “Oh no.”, and child 8 with a thought bubble “Always just writing.”. All of these reflect dissatisfaction with the classroom activities, and in that manner express negative emotions towards the geometry lesson. The interview revealed that even though the drawing illustrated a particular geometry lesson, that this reflected geometry lessons in general, and hence a permanent condition (trait).

No drawing from Grade 6 students depicted a negative classroom climate. Two drawings only (7%) represented a stable neutral emotional classroom climate in geometry lessons, which contained protagonists with neutral body language (e.g., hands on the table), as well as neutral thought and speech bubbles (“A circle.”, “A cube has 6 faces.”). There was no evidence of positive or negative emotions. Lastly, one drawing was rated as non-identifiable since it did not show any form of emotional representation.

To summarize, in the sample of Grade 6 students’ participant-produced drawings, a positive classroom climate was portrayed with the highest frequency, but it did not predominate. In all cases, a positive classroom climate was a stable condition (trait). The percentage of drawings illustrating an ambivalent classroom climate was very similar to those illustrating a positive classroom climate. These drawings also reflected a rather stable condition (trait). None of the drawings reflected a negative emotional classroom climate.

4.2 Emotional classroom climate in primary geometry education through students’ lenses: Similarities and differences

Here, the focus was to evaluate the distribution of emotional classroom climate categories. Given the cross-sectional design, it is not possible to show a progression of the emotional classroom climate over the school period. For that reason, the results are discussed with respect to the similarities and differences of the distribution regarding the different grade levels.

As shown in Table 4, a positive classroom climate was most frequently coded (48% of drawings). This was independent of the grade level, where almost all the students' drawings included at least one positive emotional feature. This was followed by ambivalent, neutral, and negative emotional classroom climates, with 33%, 10%, and 5% of drawings, respectively. In total, only 4% of drawings were rated as unidentifiable. With respect to the positive classroom climate a decrease from the lower (60% in Grade 3) to the higher grades is observable (46% in Grades 5 and 6), reaching its minimum in Grade 4 (42%). In other words, drawings portraying a positive classroom climate in geometry lessons were only dominant in Grade 3 (i.e., more than 50% of drawings). However, independent of the grade level almost all the drawings reflected a positive emotional classroom climate as a stable condition (trait). With respect to the ambivalent classroom climate, an increase from the lower (24% in Grade 3) to the higher grades is observable (43% in Grade 6), which – independent of the grade level – reflected in almost all drawings a stable condition. Thus, Grade 6 students more often drew negative facial expressions, body postures, thought or speech bubbles in their drawings of geometry lessons. A difference can be noted in the depictions of negative features; in the lower grades, it is mostly teachers shown in a negative mood, whereas in the higher grades it is students who are shown in a negative mood. A neutral classroom climate was the third least coded emotional state with 10% of drawings. Apart from Grade 4, where five students (15% of drawings) depicted their geometry lessons as neutral, only two students (7%-8% of drawings) in Grades 3, 5, and 6 depicted their classroom climate as neutral. In all grades, almost all interviews confirmed this being a stable emotional condition in geometry lessons (trait). A standout difference can be seen regarding the percentage of negative emotional classroom climate. Among the third-graders, with one exception, no drawing was drawn with purely negative features. This single drawing, however, represented a state situation of emotion. None of the children in Grade 6 drew a picture with negative features. In Grades 4 and 5, three and two drawings, respectively, showed a negative classroom climate. With respect to the former, a state situation of emotion was revealed in the interviews, whereas the Grade 5 drawings represented a trait situation of emotion. Overall, the emotional classroom climate could not be identified in five drawings (1-2 drawings per grade level) as no identifying characteristics were either drawn or described.

Thus, when comparing all four grade levels, it can be observed that the greatest difference concerns the distribution of positive and ambivalent classroom climates. There was no significant difference in the proportion of drawings rated as negative. This may be due, among other things, to the fact that the proportion of ambivalent drawings is higher in Grade 6 (i.e., a larger proportion of drawn individuals with negative features) than in Grades 3 to 5.

5 Discussion and conclusions

In the last section, the key aspects of geometry education with respect to emotional classroom climate are discussed. Lastly, the limitations of the study are considered, and some possible future research directions are provided.

5.1 Emotional classroom climate in primary grade geometry

The study reported here focused on children's perceptions of the geometry classroom learning milieu from an emotional perspective (Evans et al., 2009). For this purpose, a coding manual was created by adding to an existing one (Zambo, 2006), and rated the drawings five emotional classroom climate categories as suggested by Laine et al. (2013, 2015, 2020).

The results indicate that a positive emotional classroom climate is prevalent in geometry lessons in all primary grade levels as was also reported by Reindl and Hascher (2013). That there are no negative trait representations in the drawings is consistent with the findings of the study by Glasnović Gracin and Kuzle (2018). What is striking is the high percentage of drawings illustrating an ambivalent classroom climate in Grade 6, which is accompanied by a lower percentage of positive ones. Dahlgren Johansson and Sumpter (2010) reached a similar conclusion, and also noticed a decrease in positive emotions in Grade 5 compared to Grade 2. A possible explanation for this finding could be a child's optimism, which is much more pronounced in younger students than in older ones (Hasselhorn, 2005). In all grades, there were one to two drawings in which the emotional classroom climate was unidentifiable. This does not necessarily mean that these students did not know or were unable to draw their own or their classmates' emotions (Lucquet, 1913, 1923, in Anning & Ring, 2004); it may be that these few students took the task less seriously, and saved time by generalizing the children drawn in the drawings.

The lower percentage of drawings evaluated as positive as well as an increased percentage of ambivalent drawings in Grade 6 indicates a negative trend in the positive emotional classroom climate. However, it needs to be clarified whether this may be limited to geometry teaching only. With respect to mathematics teaching in general, clearer results exist for this trend. In the studies of Laine et al. (2013, 2015, 2020) the emotional climate was mainly positive in Grade 3 classrooms, and the emotional climate in Grade 5 more negative, although there were large differences between classrooms. Thus, the findings with respect to emotional classroom climate in primary grade geometry are largely consistent with the previously conducted studies in mathematics (Dahlgren Johansson & Sumpter, 2010; Laine et al., 2013, 2015, 2020; Reindl & Hascher, 2013). It may be that a negative trend in the positive emotional classroom climate is driving this phase of schooling in general (i.e., independent of the subject or subject-specific area).

The emotional classroom climate depicted in the drawings was strongly characterized by "affective" traits. According to the analysis of the interviews, only 16 out of 114 drawings (14%) were characterized by "situational context-bound states" which can independent of their nature (positive and negative) directly influence the classroom climate (Hannula, 2012). Furthermore, another factor influencing the emotional classroom climate in mathematics classes is the teacher (e.g., Evans et al., 2009; Harrison et al., 2007). In almost all cases (ca. 90%), the teacher was portrayed with positive or neutral features, which was confirmed in the interviews. From the interviews, it was clear that the teacher's admonitions or bad mood (e.g., yelling), had a negative influence on the students' emotional experience in geometry lessons which was also reported by Hannula (2012).

5.2 Limitations of the study and future research directions

This study was an exploratory qualitative study using purposive sampling with 114 students from two German federal states, and for that reason cannot be generalized to a broader population but is rather illustrative of other similar samples. These limitations suggest a possible next step in research, namely to conduct a study with a larger data sample in a wider variety of settings (e.g., federal states or countries) and using alternative sampling strategies (e.g., maximum variation sampling, probability sampling), so that a researcher could create a more thorough description of the perceptions students have of the emotional classroom climate in geometry lessons, which can then be generalizable to a population. In addition, drawings from entire classrooms across different grades and schools may provide a more holistic insight into the collective emotional climate in primary grade geometry lessons as was done in the work by Laine et al. (2013, 2015, 2020) and Tuohilampi et al. (2016). This would, in addition, allow for comparisons between different grades and schools. Since this was a cross-sectional study and for that reason, the development of emotional classroom climate could not be researched, in order to map the course of the emotional climate in the classroom, a longitudinal study from the beginning of school to the transition to secondary school of each individual reference group could be aimed at. Also, in further studies, other subareas of mathematics (e.g., arithmetic) could be examined more closely as well as differences in the emotional perceptions concerning these. Lastly, the study design did not allow the making of direct inferences between the students' perceptions of the emotional classroom climate in geometry and those of the teacher. The particular role of the teacher can be further explored in future research by using additional data sources, such as the teacher's drawing of a geometry lesson.

Drawings and the processes by which they are made have opened up a new way of gaining insight into students' perceptions of emotional classroom climate in primary grade geometry. Nevertheless, there were some drawbacks. The drawings showed considerable differences in quality. Due to the heterogeneous development in childhood, it is important to ensure that the children in the lowest grades are able to do what is required of them in terms of drawing (Billmann-Mahecha & Drexler, 2010). Furthermore, the representations are strongly dependent on the motivation of the students. The drawings of Grade 5 and 6 students in particular were often very simple and could only be interpreted correctly by taking the interviews into consideration. It may be useful to see if this type of research matches the interests of this age group. By talking to participants before they begin the actual task, it could be established whether the respective child likes to draw, or more specific instructions or incentives could be given.

Despite the inventory, the analysis of the drawings has proven to be a challenging task. As Blumer (1969) noted, the analysis of drawings is understood as interpreting the meanings that the students had given to the situations and objects they had presented. Thus, in order to avoid the coder's own interpretation, not only analyst triangulation is needed, but also methodological triangulation such as participant-produced drawings (Kuzle & Glasnović Gracin, 2020; Kearney & Hyle, 2004). This allowed each student to interpret his or her drawing, which consequently allowed an in-depth understanding of what the child had drawn, and a more accurate representation of their experiences and emotions. Methodologically, the semi-structured interview guide can be modified and extended, especially with regard to the "affective" state and trait. Furthermore, the coding manual developed by Zambo (2006) can be further developed. Here, each drawing offers new data and sometimes contains different characteristics, which should be recorded in the manual.

By relating the study results to teaching practice, some implications for geometry lessons can be drawn. After evaluating the results, it became apparent that the students experience quite different emotions in their geometry classrooms. These can be for instance positive, negative, short-term, or relatively stable. For practice, this means that everything that happens in the classroom, every statement, gesture, facial expression, and behavior can have a direct impact on the emotional classroom climate. Similarly, the teachers' actions in the classroom are instrumental in shaping students' attitudes toward mathematics (Harrison et al., 2007). If they evoke negative emotions in the students through their attitude, facial expressions, gestures, or behavior, it can have a detrimental effect on the students' attitudes toward mathematics instruction. Often, short-term emotions are related to the demands of mathematics instruction. It is particularly important that short-term negative emotions do not become entrenched. The lessons and the teacher's interaction with the students in the classroom must be reflected on regularly so that any problems can be quickly identified and remedied. Here, drawings may also be used as a method of evaluation and feedback (Borthwick, 2011). They help students to express themselves better and provide the teacher with an insight into how the students perceive the emotional classroom climate taking all protagonists into consideration. As such, students' drawings and their interpretations are productive ways of promoting dialogue about the working atmosphere (i.e., teaching and learning) between young people and their teachers (Anning & Ring, 2004).

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Effect of resource-based instructions on pre-service biology teachers' motivation toward learning biology

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Linking motivation and learning is central to understanding students' motivation toward learning and learning itself as complex cognitive phenomena. Some studies focused on students' motivation toward learning biology in general; however, the shortage of studies on the effect of animation-based instruction and small-group laboratory activities as Resource-based Instructions (RBIs) on pre-service biology teachers was realized. The present study aimed to determine the effect of resource-based Instructions on pre-service biology teachers' academic motivation toward learning biology at private and public Universities in Rwanda. Pre-service biology teachers were grouped into three groups at a public teacher traning University and received a pre-and post-assessment. Quasi-experimental, pre and post-test control group design was used at a public university, while a repeated measures design was used at a private university. The standard academic motivation scale for learning biology (AMSLB) yielded a Cronbach alpha coefficient of 0.71 before use. The t-Test was computed to measure the statistically significant difference between the pre- and post-assessment scores and group of RBI interventions. Multivariate analysis (MANOVA) was computed to measure the effect of RBIs vis à vis the AMSLB factors. Findings revealed no statistically significant difference (df=18, *p*=.458) in the motivation of learning biology of pre-service teachers before and after learning via traditional instruction at a public university. However, a statistically signficant difference was found with animation instruction (df=18, *p*=.002) and lab instruction (df=18, *p*=.014). The motivation of learning biology increased at a public university than at a private university. However, animations and small-group lab activities increased pre-service biology teachers' intrinsic and extrinsic—career motivation of learning biology at both universities. Therefore, the study recommends using RBIs to improve pre-service biology teachers' motivation toward learning biology.

Keywords: Academic motivation, resource-based instructions, learning biology, pre-service biology teachers, university, Rwanda

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1 Introduction

Biology is a science subject that informs the world about all aspects of life. Its teaching and learning increase knowledge of life sciences (Özbaş, 2019). Some studies reported that students showed a good interest in learning biology (Koul et al., 2011; Prokop et al., 2007); however, some difficulties in learning biology like teachers teaching strategies and lack of learning stimulus resources, among others, were pointed out

(Çimer, 2012). Teaching and learning biology require a motivating teaching and learning environment where student's involvement is taken into account. This is imperative based on the biological concepts that many are experimental in nature and challenge interested students to learn and work on the concepts (Cuthbert, 2005; Dohn et al., 2016; Şen et al., 2014).

Linking motivation and learning is central to understanding students' stimulus toward learning and learning themselves as complex cognitive phenomena (Jurisevic et al., 2008). Cuthbert (2005) defined learning as ontogenetic adaptations that mean the changes in an organism's behavior due to the regularities from its environments. Jurisevic et al. (2008) added that motivation to learn is a behavioral factor defined by different elements of motivation like interests, goals, attributes, self-image, and external enticements. The literature emphasized that motivation to learn is a crucial factor in learning science and interactions between different learning domains like cognitive and affective with intrinsic or extrinsic motivation (Shin et al., 2017).

In the present study, academic motivation was discussed in three different factors pointed by self-determination theory (Ryan & Deci, 2000) as intrinsic motivation, extrinsic motivation, and *amotivation*. Different studies Ayub (2010); Covington & Müeller, (2001); Jurisevic et al., (2008); Reiss (2012) and Ryan & Deci (2000) discussed intrinsic and extrinsic motivation to learn. All authors defined the terms and came up with a similar description stating intrinsic motivation as an act of doing something because it is inherently interesting, enjoyable, or satisfying to someone. Simply intrinsic motivation is characterized by doing things without any reason or expected benefits. At the same time, extrinsic motivation was seen as pursuing something without its own sake. In other words, to engage in an activity with an end of achieving the goal. For instance, students extrinsically may be motivated to perform better in a competition or test to achieve a good grade, please their parents, get a reward, or skip a punishment (Gilakjani et al., 2012).

According to Ryan and Deci (2000), *amotivation* is the third factor or type of motivation that announces the absence of an individual intention to act. Amotivated students do not accord any disparity to a learning activity, feel incompetent toward activity, lose interest, and find no enjoyment or reason to do an activity. In their queries, there is "why to join the school," a behavioral question that may result in low academic achievement or school dropout as advocated that students with low performance showed low motivational belief (Ekici, 2010).

Apart from *amotivation*, motivation has a crucial role in education. Şen et al. (2014) stated that motivational belief in students exerts a direct impact on their academic achievement. Ayub (2010) added that intrinsic motivation significantly impacts students' academic learning and competency, while career motivation, a form of extrinsic motivation, plays an essential role in supporting students in choosing science subjects for learning, especially STEM Choice (Shin et al., 2017). Chua and Karpudewan (2017) added that the extent of motivation in students toward active learning environments like laboratories predicts their attitude toward science learning. Hence, investigating the effect of factors influencing motivation in students like university students is an imperative need.

Different factors have been shown to influence students' motivation to learn biology, and among others, instructional methods influence learning motivation differently. Keraro et al. (2007) advocated that students showed a high motivation toward learning biology after being treated by cooperative concept mapping teaching approach, while in the study by Özarslan and Çetin (2018), biology projects proved negative effect on students" motivation toward learning biology. Online teaching did not display a significant difference in improving student motivation toward learning biology in comparison to traditional instructional methods (Bulic & Blazevic, 2020). Hence, it is imperative to test the motivational level in pre-service biology teachers' after being treated by animation-based instructions and small-group laboratory activities. For instance, a study by Mukagihana et al. (2021) found that students were motivated during learning microbiology through small groups and were excited to manipulate computer animations. Such instructions also demonstrated a rise in students' positive attitudes toward learning biology (Mukagihana et al., 2021). In the latter study, the students were motivated during learning through the teaching and learning bucket model. This is a lecturer backing and learners owning learning model (Ndihokubwayo et al., 2021) interested students in collaboratively constructing improvised materials (Ndihokubwayo et al., 2019) and presenting their outcome to the whole class. Kibga et al.'s (2021) study showed a significant increase in students' curiosity among secondary schools in Tanzania due to the implementation of hands-on activities as an instructional strategy. In other studies, innovative collaborative instructional strategies moderated students' verbal ability and their achievement in biology (Adejimi et al., 2021) and motivation toward learning biology (Dohn et al., 2016; Hewitt et al., 2019).

In general, studies by Dohn et al. (2016), Mnguni (2018), Özbaş (2019), and Kişoğlu (2018) were interested in finding out the students motivation toward learning biology; however, the literature lacks sufficient studies on pre-service biology teachers motivation toward learning biology. Besides, the effect of different instructional methods on student's motivation toward learning biology was found out (Bulic & Blazevic, 2020; Corkin et al., 2017; Dyrberg et al., 2017; Hewitt et al., 2019; Keraro et al., 2007; Özarslan & Çetin, 2018), but minor studies focused on the effect of resource-based instructions on pre-service biology teacher's motivation toward learning biology. Likewise, a recent study done in higher learning institutions in Rwanda (Mukagihana et al., 2020) showed low use of resource-based instructional tools prevailed to teach and learn biology. Therefore, to bridge the gap, this study aimed to determine the effect of animation-based instruction and small group laboratory activities as resource-based instructions on pre-service biology teachers' motivation to learn biology.

Self- determination theory (SDT), as described by Cook & Artino (2016) and Deci & Ryan (1985), guides the study. The theory postulates that innately human is motivated and need to be self-directed in activities they find inherently enjoyable. This reflects on intrinsic motivation, which primarily is not influenced but innate. Sometimes, humans may be influenced to do an inherently enjoyable activity to earn an instrumental value that generates extrinsic motivation in various forms such as career, goals, societal values, rewards, and others. The theory relates to this study which sought to determine the effect of resource-based instructions on pre-service biology teachers' motivation toward learning biology. In the study, motivation is conceptualized in its three different factors as intrinsic motivation, *amotivation*, and extrinsic motivation in two forms, career goals, and social values.

Motivation toward learning proved its essential role in learning science concepts (Shin et al., 2017). The present study significantly contributes by adding in literature the motivational level of pre-service biology teachers to learn biology at private and public teacher training Universities. Besides, it informs about the effect of animation-based instruction and small group laboratory activities on pre-service biology motivation to learn biology. Therefore, we aimed to measure the effect of animation-based instruction and small-group laboratory activities as resource-based instructions on pre-service biology teacher's motivation toward learning biology.

The study answered two research questions:

1. What is the motivation of pre-service biology teachers toward learning biology at private and public teacher training Universities in Rwanda?
2. What effect do animation-based instruction and small group laboratory activities as resource-based instructions have on pre-service biology teacher's motivation toward learning biology?

We do hypothesize that:

H01: There is no statistically significant difference between pre-service biology teachers' motivational level taught by traditional methods and those taught by animation-based instruction or small group laboratory activities as resource-based instructions.

H02: There is no statistically significant difference in motivation toward learning biology between pre-service biology teachers at private and public universities.

2 Methodology

2.1 Participants and sample

The participant in the study consisted of fifty (50) pre-service biology teachers in year two at the University of Technology and Arts of Byumba (UTAB), a private university with biology education programs and one hundred and eighty (180) year two pre-service biology teachers assigned to the study from a population of 528 at University of Rwanda College of Education (URCE), a public teacher training University. Thus a purposive sampling was used. The research unit and innovation at URCE granted ethical clearance, and the universities granted data collection approval before the conduct of the study, which was held from November 2020 to March 2021.

2.2 Research design

A survey design was embedded in a quasi-experimental non-equivalent control group design and repeated measures design to check the effect of resource-based instructions on pre-service biology teachers' motivation to learn biology. Repeated measures design is a longitudinal research design involving multiple measures of the same variable in which change over time is assessed (Creswell, 2014). Survey design is one of the procedures in quantitative research that support researchers to measure individuals' different aspects like emotions, attitudes, and opinions (Creswell, 2015). The survey was used to collect the data before and after treatment and permitted the researcher to measure the motivational level of pre-service biology teachers to learn biology as the effect of treatment.

2.3 Research instruments

The instrument used to collect the data is an adapted Academic Motivation Scale for Learning Biology (AMSLB) developed by Aydin et al. (2014). The scale comprises 19 statements distributed into four motivation factors or subscales named "Intrinsic motivation with six statements, *amotivation* with five statements, Extrinsic motivation – Career with four statements, and Extrinsic motivation – Social with four statements. Statements are scored from 1 to 5 with 1= Strongly disagree (SD), 2= Disagree (D), 3= No opinion, 4= Agree (A), and 5= Strongly agree (SA). All statements are positive except five statements in the *amotivation* factor. However, these items were similarly scored as others (SD, D, NO, A, and SA). We did not reverse the scales to avoid participants' confusion. Before using the scale, the items were rearranged in subscales or factors in an orderly manner. Thus "Intrinsic Motivation has statements 1 to 6, *amotivation* statements 7 to 11, and Extrinsic Motivation – Career statements 12 to 15, and extrinsic motivation –Social statements 16-19 (see Appendix A).

Statements in extrinsic motivations career factors were rephrased to relate them to pre-service biology teachers as university students. Furthermore, the instrument was subjected to one expert judge at Masinde Muliro University of Science and Technology and one at URCE for validity checking.

The AMSLB adapted to the Rwandan context was found to have significant reliability. Before using it, we tested it with 35 pre-service biology teachers at a university that did not participate in the study. We computed a correlation statistic using SPSS 23 and found a coefficient of 0.71 of Cronbach alpha. This informs that AMSLB is reliable, and its statements are internally consistent.

2.4 Data collection procedures

At the University of Technology and Arts of Byumba (UTAB), the participants consisted of a single group of fifty (50) pre-service biology teachers. The participants received a pre-assessment by answering the Academic Motivation Scale for Learning Biology (AMSLB) before receiving any treatments and a post-assessment after each treatment. They were treated by starting with traditional methods of teaching (Lecturer method), followed by treatment by animation-based instruction, and lastly by small group laboratory activities. It means that a post-assessment by AMSLB alternated with treatment in repeated measures as described in Creswell (2015). All 50 participants did not participate in the study; some attended pre-assessment but did not answer all post-assessment. Due to this, by data filtering, only 33 participants answered pre-assessment and all post-assessment by AMSLB.

Contrary, at the University of Rwanda College of Education (URCE), 179 pre-service biology teachers were available on the starting day of data collection. They were randomly assigned to three groups as the control group (N=60) and two experimental groups. The first (N= 59) was treated by animations-based instruction, and the second (N= 60) was treated by laboratory method using small-group laboratory activities. One instructor carried out the instruction. This helped us minimize the instructor's threat of validity. Pre-service biology teachers in each group received a pre-assessment by administering AMSLB before receiving treatment and answered the same AMSLB as post-assessment after interventions. The intervention lasted for a semester, starting from November 2020 to March 2021 at both Universities. Concept of introduction to microbiology (history of microbiology, its scientists and their discoveries, types of microorganism), method and techniques for microorganism (gram staining), method of pure culture isolation (streak, spread, and pour method) were discussed.

In the control group, the course took place in the classroom. The instructor used a projector and drawing on a whiteboard, especially diagrams such as structures of bacteria, cell walls, etc. The animation group used animated lessons from YouTube. Introduction to microbiology was projected, and where necessary instructor intervened for more explanation. For other concepts, every concept had its animation. The animated video contained graphical images of lab practical and moving text. Thus, participants heard, watched what was being done, and then read related text. When having a question, the instructor stopped, and where challenges occur, she intervened. Small group lab group mainly studied in the lab. Group of 2 to 3 students (pre-service biology teachers) spent an amount of time in the lab. We first introduced lab rules, introduction to microbiology using a projector and showing the materials, provided lab procedure for each technique showing materials, reagents, and procedure. The instructor played the role of guidance while pre-service biology teachers (students) were conducting experiments.

2.5 Data Analysis

Upon coming from the research field, we entered data in MS Excel 2016. The first column was filled with pre-service biology teachers' codes, while the first row was filled with Academic Motivation Scale for Learning Biology (AMSLB) statements (STAT). We entered the number scored by each participant—from 1 as strongly disagree to 5 as strongly agree—under each statement/item. One file contained data from URCE, while another file contained data from UTAB pre-service biology teachers. We first computed the average scores for each statement across all the participants before exporting the data into SPSS version 23. We analyzed the data using this software. We first computed mean scores from each intervention at URCE—where three groups [Control group that was taught using lecture method, first experimental group that was taught using animation-based instruction, and second experimental group that was taught using small group laboratory activities] were tested twice via pre-and post-test design. At UTAB, a single group of the participant was tested four times via repeated measures [(a) before assessment, (b) after being taught by lecture method, (c) after being taught using animation-based instruction, and (d) after being taught using small group laboratory activities]. After computing the mean score of each group, the t-Test was computed to measure the statistically significant difference between the pre-and post-test (motivation assessment) and a group of RBI interventions. Lastly, multivariate analysis (MANOVA) was computed to measure the effect of resource-based instructions (RBIs) vis à vis the AMSLB factors.

3 Findings and Discussion

Table 1 shows the mean scores of pre-service biology teachers' answers on the Academic Motivation Scale for Learning Biology (AMSLB) before and after each intervention at both University of Rwanda College of Education (URCE) and the University of Technology and Arts of Byumba (UTAB). The means scores are computed on the Likert AMSLB scale. Thus, the lowest score is 1, while the highest score is 5. At both universities, pre-service biology teachers showed a good intrinsic motivation before interventions in each group, with an average of pre-assessment above 4.0, in general interventions did not change their intrinsic motivation toward learning biology except in the control group after treatment with the traditional method, mean scores AV=4.50 before treatment and mean scores AV=4.56 after treatment at URCE. The mean score decrease for intrinsic motivation factor was realized at UTAB after intervention by small group lab activities (see Table 1). This is attributed to the effect of repeated testing. Although pre-service biology teachers generally hold a high intrinsic motivation, UR-CE pre-service biology teachers are intrinsically motivated than UTAB pre-service biology teachers (See Figure 1).

Even though pre-service biology teachers hold intrinsic motivation, they also expressed an amount of *amotivation* toward learning biology. Both traditional methods of teaching, animation-based instruction, and laboratory methods through small group lab activities decreased *amotivation* toward learning biology in all groups of pre-service biology teachers at URCE. This was not the case at UTAB due to repeated testing, where pre-service biology teachers taught in series of interventions continuously showed *amotivation* after treatment by traditional method and by small group lab activities see (Table 1). *Amotivation* was high in pre-service biology teachers at UTAB than URCE pre-service biology teachers (see Figure 1).

Traditional animation-based instructions and small group laboratory activities improved extrinsic motivation-career in pre-service biology teachers at UTAB after each intervention than it did at URCE; however, pre-service biology teachers at URCE hold high extrinsic motivation motivation-career than those at UTAB. Table 2 shows the average mean scores of extrinsic motivation-career before and after each intervention at both Universities. Generally, extrinsic motivation-social did not increase after interventions in all groups at both universities; however, this motivation was very high in pre-service biology teachers at UTAB than URCE before and after interventions.

Table 1. Mean scores from AMSLB across Universities and Resource-based Instructions (RBIs)

		URCE						**UTAB**			
		Control Pre	**Control Post**	**Animation Pre**	**Animation Post**	**Lab Pre**	**Lab Post**	**Pre-assessment**	**Traditional**	**Animation**	**Small group Lab**
Intrinsic motivation	STAT1	4.60	4.65	4.36	4.25	4.32	4.28	4.42	4.48	4.33	4.09
	STAT2	4.55	4.58	4.46	4.29	4.37	4.37	4.42	4.36	4.21	4.12
	STAT3	4.65	4.65	4.29	4.15	4.60	4.31	4.42	4.52	4.39	4.15
	STAT4	4.62	4.65	4.47	4.34	4.45	4.28	4.30	4.39	4.48	4.21
	STAT5	4.63	4.68	4.47	4.29	4.43	4.32	4.18	4.27	4.18	4.03
	STAT6	3.95	4.13	4.03	4.02	3.83	3.88	2.88	2.09	2.42	2.85
	Average	**4.50**	**4.56**	**4.35**	**4.22**	**4.33**	**4.24**	**4.11**	**4.02**	**4.01**	**3.91**
Amotivation	STAT7	1.28	1.30	1.86	1.78	1.83	1.80	2.21	2.33	2.18	2.30
	STAT8	1.60	1.52	1.83	1.80	1.83	1.85	2.30	2.22	2.12	2.24
	STAT9	1.33	1.32	1.64	1.71	1.77	1.55	2.03	2.09	2.09	2.12
	STAT10	1.67	1.62	1.81	1.81	1.85	1.75	2.12	2.24	2.30	2.33
	STAT11	1.58	1.52	1.93	1.80	1.68	1.63	3.30	3.85	3.39	3.15
	Average	**1.49**	**1.45**	**1.82**	**1.78**	**1.79**	**1.72**	**2.39**	**2.55**	**2.42**	**2.43**
Extrinsic motivation -	STAT12	4.23	4.27	4.19	3.98	3.93	3.95	3.94	4.18	3.94	3.91
	STAT13	4.55	4.43	4.17	3.98	4.27	4.15	3.70	3.55	3.67	3.88
	STAT14	4.12	4.07	3.85	3.80	3.60	3.67	3.79	3.82	3.70	3.82
	STAT15	4.22	4.18	3.90	3.88	3.82	3.90	3.27	3.39	3.58	3.97
	Average	**4.28**	**4.24**	**4.03**	**3.91**	**3.90**	**3.92**	**3.67**	**3.73**	**3.72**	**3.89**
Extrinsic motivation -	STAT16	3.20	3.22	3.34	3.07	3.12	2.97	3.88	3.61	3.48	3.58
	STAT17	3.77	3.90	3.59	3.46	3.83	3.67	3.85	3.36	3.73	3.67
	STAT18	2.75	2.50	2.44	2.53	2.72	2.52	3.64	3.25	3.48	3.58
	STAT19	2.85	2.62	2.42	2.51	2.43	2.52	3.61	3.69	3.72	3.61
	Average	**3.14**	**3.06**	**2.95**	**2.89**	**3.03**	**2.92**	**3.74**	**3.48**	**3.60**	**3.61**

The t-Test of paired samples showed no statistically significant difference (df=18, p=.458) before and after learning via traditional instruction at URCE. However, it was shown by animation instruction (df=18, p=.002) and lab instruction (df=18, p=.014). Therefore, we reject the null hypothesis that there would not be a statistically significant difference in the motivational level of pre-service biology teachers taught by the traditional method and those taught by animation-based instruction or small group laboratory activities. Although the motivation of learning biology increased at URCE, this was not the case at UTAB. The t-Test of paired samples showed no statistically significant difference (df=18, p=.660) between pre-assessment and traditional instruction (df=18, p=.750) between traditional and animation instruction, and (df=18, p=.832) between animation and lab instruction.

The fact that animation-based instruction and small-group laboratory activities did not increase intrinsic motivation in pre-service biology teachers at UR-CE as did the traditional teaching method explains that pre-serve biology teachers in the two experimental groups learn biology for their own sake. Their motivation to learn biology is innate in them rather than stimulated by environmental factors. Inherently, they find biology enjoyable and interesting and learn with no purpose of avoiding like a failure or earning instrumental value. Being treated with resource-based instructions or not, they are always intrinsically motivated to learn biology. Their colleagues who were taught by traditional methods tend to increase their intrinsic motivation toward learning biology; this may mean that for their learning they like the traditional method of teaching or that they are used to learn with it or that traditional methods (lecture) are the easiest engaging instructional method for them to learn biology. This type of instructional method does not bring new or attractive instructional resources in a classroom environment that may challenge or stimulate students to learn with a mind to earn extrinsic incentives; thus, it increases pre-service biology teachers' innate enjoyment from learning biology. These findings are not consistent with the findings of Bye et al. (2007), who reported a high intrinsic motivation in students taught by non-traditional instructional methods.

Teaching pre-service biology teachers by series of interventions did not show a statistically significant difference in instructional methods. This does not mean that used instructional methods have no effect on motivation but rather may improve motivation toward learning biology at the same level. This similar statistically significant effect of animation-based instruction, small group laboratory activities, and traditional methods on motivation toward learning biology tells that at UTAB, pre-service biology teachers gained little motivation after learning by traditional, but did not increase by followed interventions. This lack of improvement may be caused by the repetition of learned content that characterizes interventions and creates a boring learning environment.

We hypothesized no statistically significant difference in motivation toward learning biology between pre-service biology teachers at a private university (UTAB) and a public university (URCE). Hence the hypothesis is consistent with the statistical results that proved that the difference between pre-service biology teachers' motivation toward learning biology at URCE and UTAB was not statistically significant (df=1, *p*=123) prior to RBI intervention. The finding informs that pre-service biology teachers at private (UTAB) and at public university (URCE) are committed to learning biology and that both may be equally interested and skilled in learning biology. This implies that both pre-service biology teachers may be similarly competent in teaching biology at secondary schools after their studies.

Through the general linear model, MANOVA results are displayed in Figure 1. Four factors of AMSLB are displayed in the same figure at both universities. Although the difference between pre-service teachers at URCE and UTAB was not statistically significant (df=1, *p*=123), the four factors made this significant (df=3, *p*=.003). Pre-service teachers at URCE and UTAB possess high intrinsic and extrinsic—career motivation of learning biology; however, URCE possesses such motivation higher than those at UTAB.

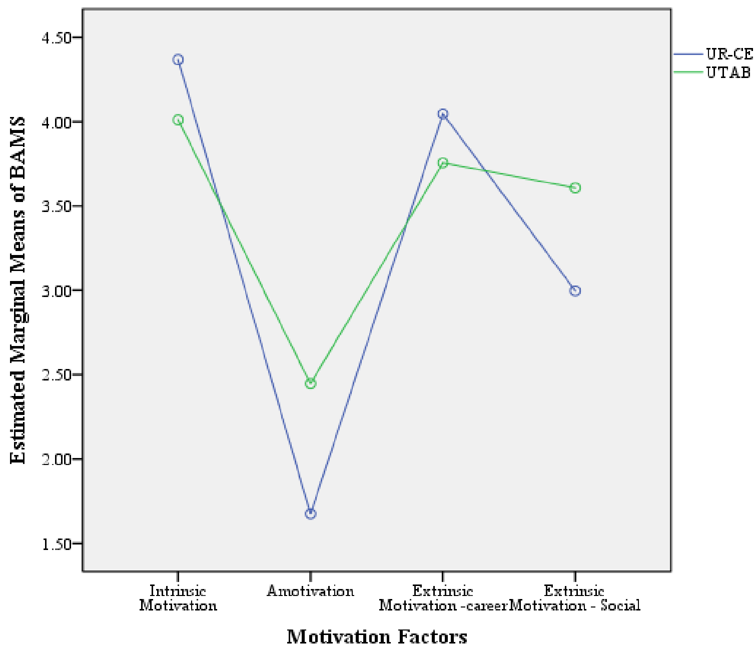

Figure 1. Academic Motivation Scale for Learning Biology (AMSLB) at different Universities

Contrariwise, both pre-service biology teachers at URCE and UTAB possess low *amotivation* and extrinsic—social motivation of learning biology. UTAB possesses such motivation higher than those at URCE. The overall Levene's Test of Equality of Error Variances confirmed that the AMSLB factors displayed a high statistical significance between AMSLB factors (F=6.475, df1=3, df2=15, *p*=.005) at URCE. At the same time, this difference was not statistically significant (F=1.794, df1=3, df2=15, *p*=.191) at UTAB.

For further analysis, we analyzed each of four factors in AMSLB across RBIs intervention among pre-service-teachers both at URCE and UTAB (Table 2)

Table 2. Effect of RBIs on pre-service biology teachers' motivation toward learning biology, specifically across AMSLB Factors

	RBIs Intervention	Intrinsic motivation	Amotivation	Extrinsic motivation - career	Extrinsic motivation – social
URCE	Traditional [Pre vs Post]	0.038	0.045	0.135	0.221
	Animation [Pre vs Post]	0.002	0.173	0.047	0.279
	Lab [Pre vs Post]	0.066	0.063	0.400	0.096
UTAB	Pre-assessment vs Traditional	0.428	0.357	0.398	0.039
	Traditional vs Animation	0.488	0.381	0.469	0.172
	Animation vs Lab	0.404	0.485	0.041	0.489

***Statistically significant difference at 0.05 level**

The fact that *amotivation* decreased in pre-service biology teachers at URCE after all interventions and increased in UTAB after intervention by the traditional method, animation-based instruction, and small group laboratory activities may result from the difference in the research designs applied during interventions. At URCE, a quasi-experimental of nonequivalent group control group design was used. This permitted them to be assigned to different groups where each group received treatment by only one instructional method. This helped them to focus on the usefulness of a single instructional method than in Pre-service biology teachers at UTAB, where they received series of interventions one after another by different instructional methods. Series of interventions at UTAB might create a boring and challenging learning environment as they repeated the same microbiology content by changing instructional methods. The same reason was also reported by students in a study by Planchard et al. (2015) found that boring or redundancy is one of the demotivating factors toward learning.

On the side of students, the cause of a decrease in *amotivation* at URCE and an increase in UTAB may be attributed to their perceptions and appreciation that may be different on the used instructional methods. The findings do not tell that animation-based instruction and small group lab activities demotivate private university students toward learning biology as they are active instructional methods, but instructions should be carefully applied by avoiding research designs that involve assessment repetitions and learning content repetitions.

Though extrinsic motivation-career was high in pre-service biology teachers at URCE than in those at UTAB, instructional methods improved extrinsic motivation-career in pre-service biology teachers after interventions at UTAB than in pre-service biology teachers at URCE. This improving effect of instructional methods at UTAB may explain low extrinsic motivation-career in Pre-service biology teachers at UTAB before joining university. It may also explain the strong effect of resource-based instructions that stimulated students to learn biology education as their future career, generating income when becoming professional biology teachers. The presence of high extrinsic motivation-career in pre-service biology teachers at URCE tells that they joined university with a commitment to learning biology with a defined learning goal or purpose of their future life.

A no statistically significant effect of the traditional method on the overall motivation of pre-service biology teachers toward learning biology at URCE is explained by teacher-centered characteristics of this instructional method that do not promote student's self-learning, self-determination, or stimulate them extrinsically to learn with goal orientation. This tells that by the traditional method, pre-serve biology teachers' motivation to learn biology may not continuously improve; instead may remain constant or tend to decrease. Contrary, the statistically significant effect of animation-based instructions and small group laboratory activities on motivation toward learning biology may result from the fact that those instructional methods are active, engaging, and attractive, thus may improve all aspects of motivation toward learning. The findings line with Bye et al. (2007), who also noticed an improvement in students' motivation to learn biology when active instructional methods are applied in the teaching and learning process. The implication is that resource-based instructions may improve students' motivation toward learning.

The fact that pre-service biology teachers at URCE were statistically significantly different based on their intrinsic motivation, *amotivation*, extrinsic motivation – career, and social may results from their orientations to Universities after their secondary schools. Those pre-service biology teachers are oriented following different factors but mainly based on their performance in national exams. One may be oriented to a teacher training university, which is not their first choice or oriented in biology education which may not also be their subject of choice. This may create variability in their motivation toward learning biology, where some may be intrinsically motivated to learn biology while others are not. Some students may be extrinsically motivated with career goals, others may be extrinsically motivated with social values, and others may be demotivated to learn biology. This might be why the same difference was not statistically observed in pre-service service biology teachers at UTAB, where they join university and biology education subjects based on their real choice. The implication is the production of secondary school biology teachers with different motivations that may lead to the remarkable difference in competency among pre-service biology teachers who graduated from URCE.

4 Conclusion and Recommendations

The present study tested the effect of animation based-instruction and laboratory methods through small activities as resource-based instructions on pre-service biology teachers' motivation toward learning biology. The study was conducted at the University of Technology and Arts of Byumba (UTAB), a private university with Biology education programs, and at the University of Rwanda College of Education (URCE), a public teacher-training university. A survey design was used to collect the data during interventions by resource-based instructions. The findings revealed no statistically significant difference in motivation toward learning biology between pre-service biology teachers at private university (UTAB) and public university (URCE). However, a statistically significant difference in motivation factors between universities was revealed. There was no significant difference in motivation toward learning biology in pre-service biology teachers taught by traditional methods at URCE. However, statistically, animation-based instruction and small group laboratory activities improved the overall motivation of pre-service biology teachers toward learning biology; therefore, they are recommended for teaching pre-service biology teachers motivation toward learning biology. At UTAB, no statistically significant difference (df=18, p=.660) between pre-assessment and traditional instruction, between traditional and animation-based instruction (df=18, p=.750), and between animation and lab instruction (df=18, p=.832). The similarity may result from the repetition of learned content that characterized interventions and may create a boring learning environment that did not increase pre-service biology teachers' motivation after each intervention series. The study is biased against participants at the public university. It might have been better to have used three universities and grouped each university to each of the interventions or stick to the public university alone because of the population reported. Therefore, we recommend using other research designs rather than repeated measures that involve multiple tests and interventions on a single group of participants. Instructional methods improved motivation factors in pre-service biology teachers, statistically traditional methods improved intrinsic motivation and reduced *amotivation* in pre-service biology teachers at URCE. At the same time, animation-based instructions increased both intrinsic and extrinsic motivation career but did not reduce *amotivation* toward learning biology.

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Appendix A. Adapted Academic Motivation Scale for Learning Biology (AMSLB)

	Level of agreement				
Items	SD	D	NO	A	SA
Intrinsic Motivation					
1. I enjoy making discussions on biology subjects					
2. Learning new things in the biology subjects that I am interested in is enjoyable.					
3. I enjoy sharing the new things that I learn in biology.					
4. Biology subjects interest me.					
5. I enjoy learning biology subjects.					
6. I enjoy reading magazines and texts related to biology					
Amotivation					
7. To be honest, I don’t see any reason for learning biology.					
8. Actually, I don’t think the subjects that I learn will be useful for me in the future					
9. Honestly, I don’t know why I should learn biology.					
10. I have no idea. I don’t understand how useful the things I learn will be.					
11. In fact, I don’t like participating the activities in biology.					
Extrinsic Motivation – Career					
12. I learn biology because it is related to the profession that I chose for my future.					
13. I learn biology because it is important in my choice of profession.					
14. I learn biology to get a good job in the field of biology.					
15. I learn biology to be able to make better choices for my further studies					
Extrinsic Motivation – Social					
16. I learn biology to show my family that I’m successful in biology.					
17. I learn biology to prove myself that I can be successful in biology					
18. I learn biology to show that I’m better than the other students.					
19. I want to be praised by the people around me.					

A calculus student's understanding of graphical approach to the derivative through quantitative reasoning

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The concept of derivative is used in many areas including applied problems and requiring mathematical modelling in different disciplines. One of the most important approaches for teaching the derivative is to support students in visualizing the concept. Also, it is necessary to shift researchers and teachers' focuses to students' dynamic mental actions while learning derivative in order to conduct effective teaching process. With this necessity, I focused on the perspective of quantitative reasoning related to the graphical approach to the derivative. This study aims to reveal a calculus student's mental actions related to the graphical approach to the derivative. The data were collected from a first-year calculus student engaged in the task requiring graphical interpretation of the derivative. Results showed that the student's understanding of the slope shaped her inferences about the tangent line because the quantity of ratio is prior knowledge for learning the instantaneous rate of change. Besides, as the student had the idea of correspondence related to the concept of function, she had difficulties in interpreting the global view of the derivate. This result suggests that having global view of the derivative requires a strong understanding of function and rate.

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1 Introduction

The derivative of a function is a fundamental concept for the basis of calculus (García et al., 2011) and is used in many areas including requiring mathematical modeling of several situations in different disciplines such as engineering, physics, economics, etc. This concept was historically constructed as a way to represent rate of change which explains how one quantity changes in relation to another quantity (Weber et al., 2012). Thus, understanding the derivative requires a wide intuitive base of examples and related perceptions, especially concerning the concept of the rate of change in real-life problems (Weigand, 2014). Thompson (1990) has emphasized the idea that a rate is conceived of as constituting a functional relationship may be a foundation for the derivative in calculus because it is consistent with conceptions of a single-variable derivative evaluated at a point (i.e., an instantaneous rate of change) as being the slope of a tangent line. Especially, visualizing a graph supports students to construct a tangent as an instantaneous rate of change by zooming in on a sufficiently small portion of the curve (Tall, 2010). Tall (1997) has related to the graphical representation of the derivative with the qualitative actions of visualizing and conceptualizing. Students might thus transfer their meanings to the instantaneous rate of change which is an important cognitive action for learning derivative and engage in sense making of the derivative beyond applying the derivative formulas (Samuels, 2017). The slopes of secant line and tangent line are quantities which have to be measured necessarily in quantification process of derivative graphically. Students first think the slope of secant line and then use the values of the slopes for different secant lines to get the slope of the tangent line at a point. The relationship between the derivative of a function at a point and the slope of the tangent line at that point forms a foundation for understanding the derivative as function (Asiala et al., 1997). As they progress, students can visualize the quantities on the graph and imagine the slope of all tangent lines at any points on the curve. Considering that many calculus students have difficulties in visualizing of the rate of change of two quantities (Hausknecht & Kowalczyk, 2008), the graphical representation of derivative becomes important for visualization and learning. Additionally, students need to progress developmentally based on graphical tasks about the derivative in order to interpret it graphically. Because many students interpret a graphical problem about the derivative algebraically, when a curve and its tangent line are given (Asiala et al., 1997). However, it would require that students be able to meaningfully connect the ideas of slope (instantaneous rate of change) in both algebraic ratio and geometric ratio for perceiving as an internal concept (Nagle et al., 2019). Based on these ideas, I focused on the graphical approach to the concept of derivative and the quantities related to the graph. On hand, this study proposes a task implementation for supporting students' mental actions regarding instantaneous rate of change, on the other hand, presents reveal a calculus student's mental actions related to the graphical approach to the concept of derivative.

Most studies on graphical representation of the derivative have been based on using a dynamic mathematic software (i.e. Borji et al., 2018a; Delos Santos & Thomas, 2005). Since high school teachers or college instructors may not have any opportunity to use technological tools in terms of facilities or technological knowledge, a learning task challenging student to imagine the dynamic process of the derivative without using any software is of importance. The task in this study can provide teachers to teach the derivative dynamically without using any mathematics or geometry software. Additionally, presenting a typical calculus student's mental actions while engaging in a such task may give readers opportunities to revise and use this kind of task while teaching the derivative. Since the derivative is constructed by human pattern-finding tendencies, rather than being self-existent (Jones & Watson, 2018), the results from a calculus student's mental actions may help mathematics educators to understand students' developmental progression related to the derivative and to design different teaching processes to support in making connections among different representations. Thus, I seek to respond this research question: "What are a calculus student's mental actions related to the graphical interpretation of the derivative by quantitative reasoning?"

1.1 Literature Review: The Concept of Derivative

There are several studies on students' understanding of the derivative (Asiala et al., 1997; Borji et al., 2018a; Borji et al., 2018b; Delos Santos & Thomas, 2005; Firouzian, 2013; García et al., 2011; Habre & Abboud, 2006; Jones & Watson, 2018; Kendal & Stacey; 2000; Kertil, 2014; Park et al., 2013; Serhan, 2000; Verhoef et al., 2015; Zandieh, 2000). Some researchers (Asiala et al., 1997; Zandieh, 2000) explained the developmental progressions during learning the concept of derivative while some (i.e. Delos Santos & Thomas, 2005; Habre & Abboud, 2006; Verhoef et al., 2015) studied on the teaching process of the derivative. Asiala et al. (1997) who examined calculus students' graphical understanding of the derivative suggested the graphical paths and coordinated these paths with the analytic paths (p.10):

1. The action of connecting two points on a curve to form a chord which is a portion of the secant line through two points together with the action of computing the slope of the secant line through the two points.
2. Interiorization of the actions in point to a single process as the two points on the graph get closer and closer together.
3. Produce the tangent line as the limiting position of the secant lines and also produce the slope of the tangent line at a point on the graph of a function.

Similarly, Zandieh (2000) has developed a framework for exploring students' understanding of derivative. For four contexts (graphical, verbal, paradigmatic physical, symbolic), Zandieh has elaborated three layers (ratio-limit-function) during learning process and stated that these layers could be seen as dynamic process and as static objects which are linked in a chain:

> The ratio process takes two objects (two differences, two lengths, a distance and a time, etc.) and acts by division. The reified object (the ratio, slope, velocity, etc.) is used by the next process that of taking a limit. The limiting process 'passes through' infinitely many of the ratios approaching a particular value (the limiting value, the slope at a point on a curve, instantaneous velocity). The reified object, the limit, is used to define each value of the derivative function. The derivative function acts as a process of passing through (possibly) infinitely many input values and for each determining an output value given by the limit of the difference quotient at that point. The derivative function may also be viewed as a reified object just as any function may. (p.107)

While students are studying on the graphical context at the ratio layer, they find the slope of a secant line through two points on the graph. Secondly, they use the concept of limit to find the slope of the tangent line at a point by thinking of approximation points on the curve to a specific point. Finally, comprehending the derivative as a function requires understanding that the slope is different for different values of the independent variable. Borji et al. (2018b), one of the studies analyzing students' mental actions about the derivative, examined fourteen university students' understanding on the graph of the function and its derivative by using the perspectives APOS (Action, Process, Object, Schema) and OSA (Onto-Semiotic Approach) which they saw them complementary. Their results showed that ten students were at the intra level which is the lowest level of development of the Schema. These students at that level could perform some mental constructions of the genetic compositions considered as actions, but they did not consider some of the propositions of the epistemic configuration and could not sketch the graph of the derivative function. Nagle et al. (2019) proposing a framework for slope using APOS expressed that, for the construction of a derivative as a function f', where at each point on the graph of f, a student has the dynamic imagery of secant lines approaching the tangent line at the point (requiring a Process of slope) and of tangent lines moving along the graph of f with the slopes corresponding to the values of f' (resulting from Actions on a Process of slope). These frameworks provide important insights us how students have mental actions in the learning process of the derivative.

Additionally, there are considerable evidence that the calculus and high school advanced algebra students have many difficulties about the derivative (e.g. Firouzian, 2013; Kendal & Stacey, 2000; Sánchez-Matamoros et al., 2015). Firouzian (2013) found that most calculus students did not explain the derivative with the idea of slope of the tangent line and instantaneous rate of change. Sánchez-Matamoros et al. (2015) assert that students cannot understand the relation between the limit and the derivative and also relation between derivative of a function at a point and global view of derivative. Kendal and Stacey (2000) stated that only the most capable students were successful at numerical, graphical and symbolic representations of the derivative. These challenges bring to light the necessity of effective teaching process prompting students to have deeper meanings of the derivative.

Some of these researchers who focused on the teaching process of the derivative have offered using the tasks supported by technological tools (Borji et al., 2018a; Delos Santos & Thomas, 2005; Habre & Abboud, 2006; Kendal & Stacey, 2000; Verhoef et al., 2015) while some have asserted that mathematical modelling activities have helped students to have conceptual learning of the derivative (Kertil, 2014; Park et al., 2013). Delos Santos and Thomas (2005) conducted a case study with two students and they examined the effectiveness of a module using graphical calculators on students' understanding of the derivative concept. They have observed that the students move from a procedural perspective to a more concept-oriented view of derivative through this module. Habre and Abboud (2006) focused on ten undergraduate students' understanding of the function and its derivative during a reformed Calculus-I including graphing calculators and a dynamical calculus software program. After all courses, some students explained the derivative as being the slope of the tangent line at a point and some of them explained it in terms of the instantaneous rate of change. Similarly, Borji et al. (2018a) used Maple software to improve university students' graphical understanding of derivative. They conducted an experimental study in which they designed three activities based on APOS-ACE (Action, Process, Object, Schema-Activities in class, Classroom discussion, Exercises) framework and emphasized interiorization of the process of calculating the slope of a secant line drawn by connecting two points on a curve. The results showed that the students in the experimental group had a deeper understanding of derivative. These researches acknowledge that the dynamically learning environments involving process meaning of the derivative might support students' mental actions. The results from the previous researches present students' understanding of derivative after they completed the learning process. In this study, I alternatively focus on the student's quantitative reasoning while engaging in a new task for her. Since the student's mental actions of the derivative shaped while studying on the task, this study gives significant approaches related to learning processes of the derivative to the mathematics teachers to help their students to have conceptual understanding or to the researchers to conduct larger teaching experiments. As a different aspect from the above studies, this study draws on the quantitative reasoning which can serve as the conceptual root stalk for many different approaches to algebra and calculus and make sense the relations among the quantities (Smith III & Thompson, 2007).

1.2 Theoretical Perspective

In this section, I outline the perspective of quantitative reasoning that guided my graphical approach to the derivative and then explain how I use this perspective.

Quantitative reasoning is the analysis of a situation into a quantitative structure—a network of quantities and quantitative relationships (Thompson, 1990, p.12). It involves an individual's mental actions when conceiving of a situation, constructing measurable attributes of the situation and constructing about relationships between conceived quantities (Moore, 2013). In this perspective, since relations among quantities are important, quantitative reasoning is closely related to the kind of reasoning customarily emphasized in algebra instruction (Thompson, 1993). The Common Core State Standards for Mathematics (CCSSI, 2010) emphasizes the importance of students' quantitative reasoning, which is defined as making sense of the quantities in the problem situations and the relations between these quantities, in other words, as the mental actions necessary for students to learn mathematics. Accordingly, quantitative reasoning can be considered as an aim of teaching, and mental operations and conceptual structures that enable quantitative reasoning should be elaborated (Thompson, 1990).

Thompson (1990) elaborated the quantitative reasoning by defining several concepts such as quantity, quantification, quantitative operations, value. A quantity is a quality of something that one has conceived as admitting some measurement process (Thompson, 1990). In this study, the horizontal distance (purple line segment in the Figure 1) and the vertical distance (orange line segment) between two points which the secant line (red line) intersects on the curve are quantities. By comparing these quantities multiplicatively, the slope of the secant line (average rate of change $[(f(x+\Delta x)-f(x))/\Delta x]$ or steepness) are produced. Comparing quantities multiplicatively is quantitative operation which is the conception of two quantities being taken to produce a new quantity (Thompson, 1990).

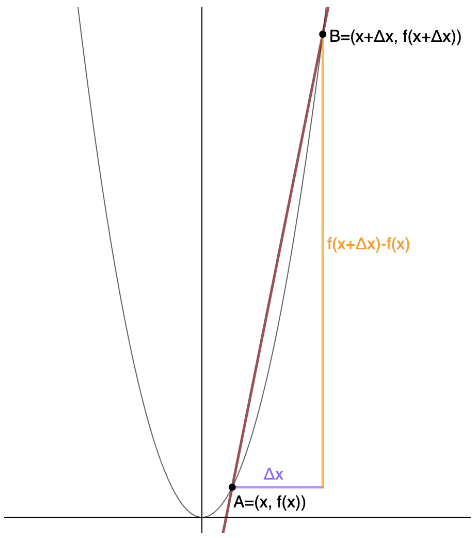


Figure 1. Secant line on the curve f(x)=x^2

The process of measuring the value of the slope of secant line on the curve is quantification. This measurement process is precursor step in constructing the slope of tangent line (instantaneous rate of change) which is another quantity. In this process, by moving a point through curve towards the fixed point, a new secant line which its steepness (slope or rate of change) is changing is formed. As the point A approximates to the point B in the Figure 2, since changing of the change in x-axis is less than changing of the change in y-axis based on the quadratic function, steepness is increasing, and a new secant line is formed. If students are given the opportunity of forming the secant lines based on this idea, their emergent shape thinking which "involves understanding a graph simultaneously as what is made (a trace) and how it is made (covariation)"(Moore & Thompson, 2015, p. 785) which is may be improved. Secant lines and tangent line are constructed based on emergent This movement also leads to think a rotating secant line. The activity of rotating this secant line around the fixed point on which tangent line would be drawn or moving the point A towards the point B through curve prompts to reason about the idea of approximating (Figure 2).

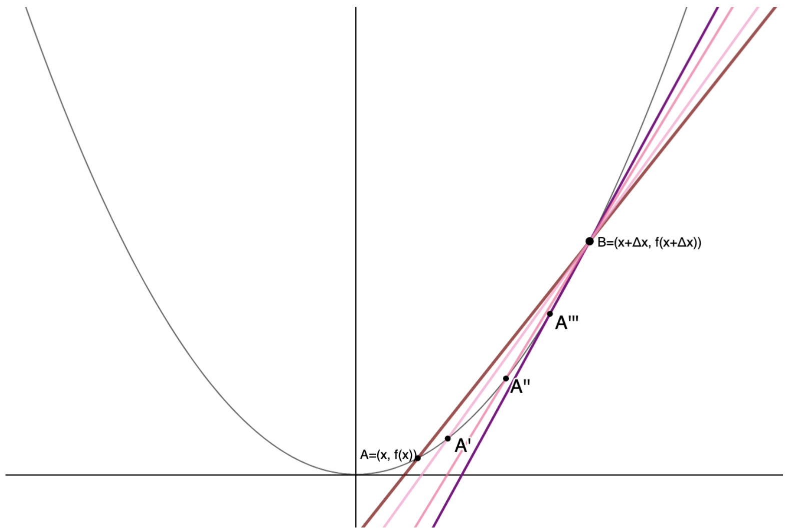


Figure 2. The activity of moving the point A to the point B

These activities are important in constructing the slope of the tangent line at the point B (new quantity). Based on these activities, the tangent line and its slope cannot visible but imaginable. The instantaneous rate of change is the result of an approximation producing average rate of change over smaller and smaller intervals. Since this approximation process is related to the concept of limit, quantifying the instantaneous rate of change can be by calculating the limit of average rate of change:

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

After students construct the slope of tangent line at a point, they think that slopes of different tangent lines on the curve change and notice derivative as a function. These actions based on the perspective of quantitative reasoning are related to the ideas from Zandieh's (2000) and Asiala et al's (1997) frameworks. However, since quantitative reasoning as a way of defining the mental actions of a student who understands a situation, constitutes quantitative situations, associates, organizes and uses these quantities to make the problem situation meaningful (Weber et al., 2014), it helps us to understand students' dynamic mental actions which support their conceptual learning by allowing them to make connections between different concepts and evaluate situations within quantitative structures.

2 Methodology

The methodology for this study was a qualitative case study having the aim of revealing a calculus student's graphical understanding of the derivative. The case was the calculus student's mental actions based on the quantitative reasoning. The participant, pseudonym named Amelia (female), was a typical first-year calculus student and was willingness to participate to the study. I carried out a task-based interview with Amelie to understand her mathematics. Goldin (2000) emphasized the characteristics of task-based interviews different from conventional interviews:

> ...task-based interviews make it possible to focus research attention more directly on the subjects' processes of addressing mathematical tasks, rather than just on the patterns of correct and incorrect answers in the results they produce. Thus, there is the possibility of delving into a variety of important topics more deeply than is possible by other experimental means topics such as complex cognitions associated with learning mathematics, mechanisms of mathematical exploration and problem solving, relationships between problem solving and learning, relationships between affect and cognition, and so forth. (p. 520)

In the study, the task-based interview helped me to understand Amelie's complex mental actions related to the derivative. The task was presented to Amelia out of the class by a paper before she did not engage in a learning process of derivative as a part of the calculus course. I conducted the interview in one session lasting about two and a half hours. While she was engaging in the task, I continually asked the underlying reasons of her thoughts and I videotaped all process.

2.1 The Task and Data Collection

The focus of the task is on the graphical interpretation of the derivative. The steps of the task are as follows (Figure 3):

Step 1: How do you find slope of a line? Explain.

Step 2: Interpret this function, $f: \mathbb{R} \to \mathbb{R}, f(x) = x^2$? What can you say about it? Explain.

Step 3: Sketch the graph of this function.

Step 4: Determine a point A on the graph and draw a tangent line at a point A.

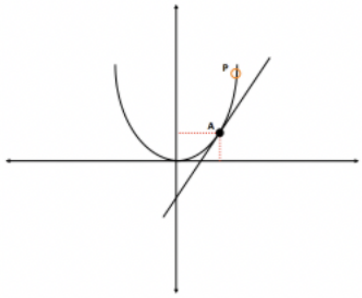
Step 5: Find the slope of this tangent line. Explain your steps.

Prompt questions:

- What information is given about the line?
- What do you know about this line?

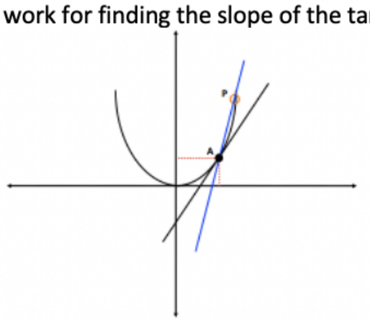
(If the student has not a logical idea for finding the slope, continue with Step 6)

Step 6: Assume that there is a ring (P) on the graph. Is it possible to remove the ring through the curve? What do you think?



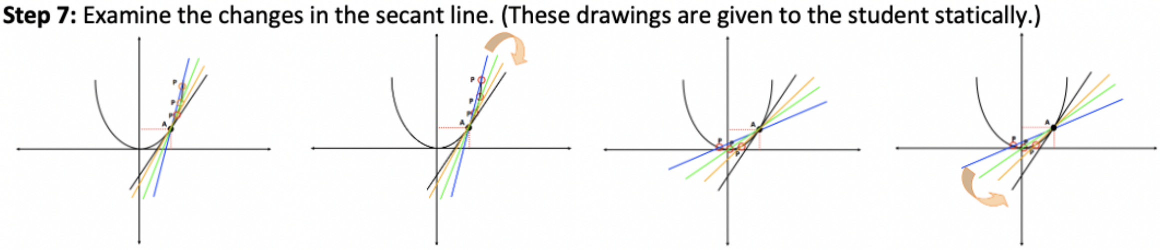
Prompt questions:

- What do you think about removing the ring for finding slope of the tangent line?
- What can you say about the function considering this movable action?
- What does moving of the ring work for finding the slope of the tangent line?



(If the student has not a logical idea for finding the slope, continue with Step 7.)

Step 7: Examine the changes in the secant line. (These drawings are given to the student statically.)



Step 8: Which steps do you follow for finding the slope of tangent line at point of (1,1)?

Step 9: Find the slope of tangent line at the different points (for x=0, x=2, x=-1 et al.) on the graph and draw a table representing matching values of slope to values of x-axis of points.

Prompt questions:

- What does the matching point out?
- Can you find the slope of the tangent line at all points on the graph depending on their values of x? Explain.
- Is there a value of slope corresponding to every x-value?
- What does this relation mean?

Step 10: Construct the function representing the value of slope of tangent line at any point depending on the value of x-axes of that point.

Step 1–4. In the first step of the task, students are expected to think the slope of a line. By this step, students need to activate their knowledge about the slope as a constant rate of change. In the further steps they might build new ideas on their knowledge regarding the quantity of slope. In the task, the function is a quadratic function because most calculus students know the curve representing its graph. The aim of second step is to reveal students' meanings related to the quadratic function. For instance, they may have a misunderstanding that any segment drawn through two points on the parabola is linear and such an understanding may be a block for their reasoning about the slope of secant line and tangent line. The third step requires students to draw the graph of the function. The factors such as drawing graph, determining a point on the graph should not lead to some obstacles so that they can reason about the slope of a tangent line and derivative. Students are asked to draw a tangent line on it after they draw the graph.

Step 5–7. In the 5th step, it is assumed that students might have difficulties about reasoning about the slope of a tangent line because they can calculate the slope of a line by using two points on the line. However, they only have limited information about the tangent line. They may easily determine the point on the curve on which the tangent line is drawn. But this point is not identified at this stage. For this reason, teachers can ask prompt questions to them. These questions may trigger students to think the tangent line as a line and to notice that they need two points at least to interpret the slope of the tangent line. Then, they continue to work on the task by assuming that there is a ring on the curve and that they can move the ring on the curve. In this study, the images given to students are static. In different teaching processes, they may be asked to work in a dynamic technological environment such as GeoGebra. The ring on the curve is representing another point and students thus might form a secant line through the ring and the point on which the tangent line is drawn. Also, since the ring can move through all points of the curve, students may notice continuity for the function. In other words, the idea of moving a ring through the curve helps students to make inferences that the function is continuous. Since being continuous is necessity for differentiable, using this imaginable activity is of importance. As the ring moves, the changes in the x-axis and in the y-axis change differently because of the quadratic function and thus the slope of the secant line changes. When students interpret rate of change of the secant lines before drawing the lines, they construct secant lines based on emerging shape thinking. This thinking supports them in using the slope of secant lines to get the slope of the tangent line. Students need to make inferences about the secant lines in this step but if not, in the 7th step they examined the graphs given. It is assumed that this action might support students' learning for the relation between average rate of change and instantaneous rate of change. In this step, several prompt questions may be asked to students. These steps are important for the students to realize quantitative operations significant ideas for the slope of tangent line.

Step 8. In this step, students work on quantification process for a slope of the tangent line at a specific point (1,1). They determine another point on the curve correspondence to the ring and gradually they approximate it to the (1,1) on the curve. They need to think on the points which their x-values are smaller than 1 (as the x-value of the specific point of (1,1)).

Step 9–10. After interpreting the slope of a tangent line at a specific point on the curve by the idea of instantaneous rate of change, students are asked to think the slope of tangent line at different points. They are expected to match x-values of the points to the slope of tangent lines drawn on that points. Thus, it is assumed that they might notice a new function and have a global view of the derivative.

2.2 Data Analysis

The data analysis process consisted of ongoing analysis simultaneous to the data collection process and retrospective analysis after the interview. By the ongoing analyses, I focused on the student's activities and quantitative operations. This process supported me an insight about the student's thoughts and was precursor for the retrospective analysis.

After interview, I did retrospective analysis. The retrospective analysis method provided me to examine how the student has thought and to understand what actions have shaped her thoughts (Battista & Clement, 2000). For retrospective analysis, I first transcribed the video camera recordings verbatim and examined the student's gestures and talks based on the perspective of quantitative reasoning. I divided the transcription into the parts accordance with the steps of the task. I first coded the student's talks about the slope in terms of the rate of change. In this stage, I focused on her talks by considering the idea of the multiplicative comparison which is an important operation for the slope. Then, I continued the analysis process of her ideas about the slope of a tangent line in a descriptive way. For next steps of the task, I focused on the student's quantitative operations, quantitative relations and the quantities and I coded her talks by continuous comparative analysis. I initially examined her quantitative operations and then, the quantities and concepts she constructed by these quantitative operations. By this coding, I aimed to reveal the quantities she produced during the interview. Based on the coding process, I modelled the student's mental actions in the process of quantification.

3 Results

In the section of results, I first present the student's knowledge about the slope of a line. Then, I illustrate her initial understanding of the slope of a tangent line at a point on the curve. Lastly, I provide evidence on the student's quantification and quantitative reasoning about graphical approach to derivative.

3.1 Amelie's Understanding of Slope of a Line

In the task, I initially asked the student's knowledge about the slope of a line. By this question, I also aimed to reveal whether the student had an understanding of slope related to the idea of rate of change. She explained that she could find the slope with the ratio of y-intercept value over x-intercept value. Based on this explanation, I thought that she did not think the amounts of change for slope and only considered intercept points of line with the coordinate system. In order to elaborate Amelie's thinking, I asked her to explain ideas further. She drew a coordinate system and indicated what she meant on it [Excerpt 1].

Excerpt 1.

> Amelie: This is a line. I determine two points on this line and identify one of them as (x1, y1), the other one as (x2, y2). In order to find the slope of this line, I calculate the value of x2- x1, then the value of y2- y1. The ratio of y2- y1 over x2- x1 would be equal to the slope of the line.

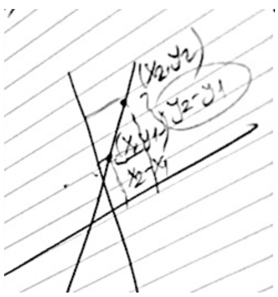


Figure 4. Amelie's diagram representing the slope

[She writes the equation $\frac{y_2-y_1}{x_2-x_1}=m$]

Amelie mentioned the ratio of amounts of change in x and y values and she wrote the equation for this ratio [$\frac{y_2-y_1}{x_2-x_1}=m$]. However, she explained the slope of the line statically and only considered two points she determined on the line. Also, she did not make any explanation about anything referring that slope could be defined as the amount of change in y-axis for every one-unit change in x-axis.

3.2 Amelie's Initial Understanding about Slope of Tangent Line

Amelie easily drew the graph of the function [$f(x)=x^2$] in the third step and a tangent line at a point on the parabola in the fourth step.

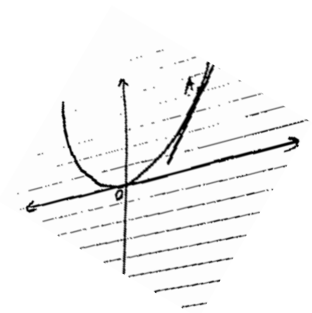


Figure 5. Amelie's diagram for the tangent line

Initially, while she was finding the slope of the tangent line on the parabola, she needed to write algebraically the equation of this tangent line or to determine two points on the tangent line. Since one of these points [A] was also on the curve, she had to identify another point. "...If I know two points on the line, I can draw it and find its slope. I know one point on the line [indicating A] but I cannot find the other one." Just as I hypothetically thought that Amelie might have difficulties for this process, Amelie did not proceed to next step for reasoning the slope. When I pushed her to think of given information about the line, she was still attempted to find the second point on the x-axis. Because of her confusion, Amelie thought the angle formed by the intersection of the line and the x-axis. Her aim was to use the value of the angle for the tangent. I can say that her approach was not paving way towards finding the slope also, because there was not enough information about the angle and the line. She thus could not reason about calculating the value of the slope since there was inadequate information about all ways in finding the slope of tangent line. Her fruitless attempts were helping her to feel the existence of alternative ways in finding the slope of the tangent line. This task in the step was thus so valuable.

3.3 Amelie's Mental Actions During Teaching Process

I can say that the step in which reasoning the slope of tangent line is engaged is 6th step. In this step, the first aim was to encourage the student to have an understanding about the idea of continuity of the function with the idea of the ring. Continuity of function is a necessary characteristic for the derivative but not enough. For this function, since the ring can be moved through the curve, the function has a derivative at every point in its domain. When I asked Amelie to assume that there was a ring on the graph, she mentioned continuity of the function in its domain and existence of moveableness for the ring through the all curve. Moving the ring through the curve would support the student in imagining the secant lines dynamically.

She then continued the task with Step 7. In this step, while she was examining the first graph, she determined a point correspondence to the ring on the first secant line. Thus, this step challenged her to engage in the quantification process for only one secant line. The slope of the first secant line she calculated was a quantity. By examining the graphs given on the task step by step, she noticed that the rate of change for the secant line gradually would become like to the rate of change for the tangent line at the point A [Excerpt 2].

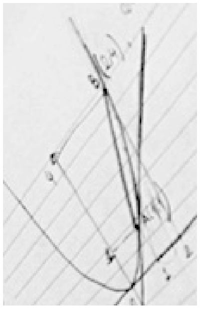
Excerpt 2.

> Amelie: This is point B [the first B point on the blue line], this is point B [indicating the point on the green line], then this is B [indicating the point on the orange line], and then this is point B [indicating the point on the black secant line]. When the ring is at this point [indicating the point A], the slope of the secant line is equal to the slope of the tangent line!

Amelie who had examined the changing graphs reasoned on the situation by considering the points on the secant lines. She qualitatively approximated the point B on the curve to the point A. The task prompted her to make inference about the relation between the secant line and tangent line by the means of moving the ring and rotating the secant line as to being look like the tangent line. Based on these activities, she interpreted that the point B gradually would become to be the point A.

She focused rotating the first secant line around the point A after idea of approximating and imagined as if a new secant line was formed at every stage. Since the point A was not identified in this step, she did not assign a value to slope of the any secant line. As it can be understood in the Excerpt 2, she interpreted the changing situation and its result. Amelie's interpretation indicated that she noticed that the slope of the secant line would gradually equal to the slope of the tangent line. The action of rotating prompted her to think of the slope as a steepness.

In the next step, she was asked to draw the tangent line at the point A(1,1) on the parabola and to find its slope. By depending on the previous step, she first determined the point as (2,4) and draw a secant line through (1,1) and (2,4). Then she calculated the value of slope of this secant line as an average rate of change.



$$m = \frac{4-1}{2-1} = 3$$

Figure 6. Amelie’s diagram related to the slope of the secant line

She determined different points gradually approaching to 1 in way that their x-values were bigger than 1. These points were (3/2, 9/4), (5/4, 25/16), (1.1, 1.21) and (1.01, 1.0201). She drew the secant lines passing through (1,1) and one point which she determined. She then found the values of the slopes of these lines via a calculator and created a table including the values of the slopes and the points (Table 1).

Table 1. Amelie’s table for the changing slope values of the secant lines

Point	Slope
B (2,4)	$m_{BA} = 3$
C (3/2, 9/4)	$m_{CA} = 2.5$
D (5/4, 25/16)	$m_{DA} = 2.25$
E (1.1, 1.21)	$m_{EA} = 2.1$
F (1.01, 1.0201)	$m_{FA} = 2.01$

She then determined the points gradually approaching to 1 which their x-values were smaller than 1. Similarly, she drew the secant lines by using these points and created a table by computing the slopes of the secant lines by using a calculator (Table 2). This process involved numerical examinations based on her ideas from the previous step as a quantifying process of the slopes (as rate of change).

Table 2. Amelie's table for the changing values of slopes

Point	Slope
G (0.5, 0.25)	$m_{GA} = 1.5$
H (0.9, 0.81)	$m_{HA} = 1.9$
I (0.99, 0.9801)	$m_{IA} = 1.99$

While the points were approximating to the point A through the curve, she first focused on x-values of the points and then calculated the y-values depending on x-values. Based on this quantifying process, Amelie concluded that slope of the tangent line at (1,1) would be 2. Determining the points by approximating (1,1) and calculating the values of the slopes of the secant lines supported the student in making inference about the tangent line and its slope.

Excerpt 3.

> **Researcher:** Why did you examine the slopes of the secant lines through the points which their x-values were smaller than 1?
> **Amelie:** If the values of the slopes [indicating the lines through latter points] approximated to 3 and these values of slopes [indicating the lines through former points] approximated to 2, the tangent line would not have been the same. This function would be a piecewise function. Because this function is continuous, the values of the slopes have to approximate the same value. I calculated for checking it. It is 2.

Amelie related this approach with the continuity of the function. Her this idea was derived from moving the ring through the curve. As Amelie imagined moving the ring at the beginning of step 6, she could justify her explanation related to the approximating to the point A from the points with smaller x-values and the points with bigger x-values.

In this process, Amelie's developmental progression was revealed based on her mental actions. There were two different layers which the later one depended on the former one.

In the former layer, the student constructed the relation between the secant lines and the tangent line. While she was forming this relation, her mental actions were as follow:

1. Approximating the points on the curve to the specific point on the curve.
2. Rotating the secant line around the specific point.

These two mental actions supported her in understanding how to be formed the tangent line on the curve. She then needed the quantification of the slope of the tangent line. This quantification process also included the mental actions in detail as follow:

MA1: Approximating the x-values of the points to the x-value of a specific point
MA2: Thinking y-values changing depending on x-values
MA3: Matching the x-values of each points to the y-values
MA4: Comparing x-values of points with each other additively
MA5: Comparing y-values of points with each other additively
MA6: Comparing multiplicatively the amount of change in the x-values in reference to the specific point with the amount of change in the y-values. (There are two quantities, the amount of change in the x-values (Δx) and the amount of change in the y-values (Δy))
MA7: Compare the ratio corresponding to slopes of tangent lines at different points.

Amelie initially considered the points which their x-values were bigger than the x-value of the point A(1,1). She then had similar approaches for the points which their x-values were smaller and completed the quantifying process by comparing the two approximation values.

3.4 Amelie's Algebraic Representation of the Derivative

The task also encouraged Amelie to write algebraically the slope of the tangent line. The action of approximating to a specific point on the coordinate system triggered her to call on the concept of limit. While using the limit, she also generalized the slope of the secant lines as the average rate of change. While she was calculating the slopes of the secant lines, she converted her inferences from the graphical approach to the algebraic expression [Excerpt 4].

Excerpt 4.

Amelie: I am searching the slope of the tangent line, I thus examine the slopes of the secant lines. Because I gradually approximate to the given point, I can use the limit.
Researcher: How do you use the limit?
Amelie: I need find the limit of the slope of secant lines, $\Delta y/\Delta x$.
Researcher: Could you write this statement algebraically?
Amelie: There are the value $f(x+a)$ and the value $f(x)$ for the independent variables. I subtract x from $x+a$ and I would write as

$$\frac{f(x+a)-f(x)}{(x+a)-(x)}$$

Amelie wrote a new mathematical expression $\frac{f(x+a)-f(x)}{(x+a)-(x)}$ as to be equal to $\frac{\Delta y}{\Delta x}$. This expression was more descriptor to see how she thought the average rate of change. She approximated the points to the given point over smaller and smaller intervals. She algebraically wrote this procedure with the limit of $\frac{f(x+a)-f(x)}{(x+a)-(x)}$ as $x+a$ approaches x. She did not mention that the interval would approach to 0. She stated “When I assign values for x, I would find the y-values and the change in the y-values. I then examine the ratio of this change [indicating the change in the y-values] over this change [indicating the change in the x-values]. I would find the slope in this way. Then, I approximate this point [indicating any point on the curve] to this point [indicating point A]. Finally, I use the limit for finding the result of this approximation.”

3.5 Global View of Derivative

Amelia initially worked on the slope of the tangent line at point (1,1) and she found that the value of the slope was 2. This was not indicating her global view for the derivative yet. In the step 9 and step 10, she thought the derivative at different points on the curve in order to construct a global view. She similarly calculated the slope of the tangent lines at different points and matched the value of slope to the x-value of a point [Excerpt 5].

Excerpt 5

> **Amelie:** These values [indicating slopes] are double of these values [indicating the x-values of the points].
> **Researcher:**What can you say else?
> **Amelie:** x transforms to 2x.
> **Researcher:** Okay, think this relation. There is only one slope of the lines at these points. What do you recall mathematically? Are there tangent lines at the all points on the curve and also their slopes?
> **Amelie:** Yes.
> **Researcher:** Do the slopes have only one value?
> **Amelie:** They have one value.
> **Researcher:** When you consider these characteristics, what can you say?
> **Amelia:** This relation is a one-to-one function.

While she was interpreting the values, she stated that the values of the slopes were double of x-values of the points by matching them. However, she could not relate this relation with the function and could not construct the global view of the derivative. In this step, I asked leading questions and mentioned about the characteristics providing the definition of the function. After all, she could say “This relation is a one-to-one function”.

By this task including graphical approach to the derivative, although the student constructed an understanding of the derivative at a point, she had difficulties about thinking the derivative as a function.

4 Discussion and Conclusion

In this study, I illustrate a calculus student’s mental actions based on the perspective of quantitative reasoning while engaging in the task including the graphical approach to the concept of derivative. Results indicated that using the task involving imagining the situations without technological helps might prompts students in reason about the derivative graphically. this task-based interview also provided me to observe the student’s actions during learning process.

Although Amelie algebraically explained how to find the slope of a line, she did not reason about the slope of a tangent line when one point on the tangent line was given. One reason of this obstacle could be derived from Amelie’s understanding the slope as a ratio but not a rate. As it can be seen in the Figure 4, she had simultaneous view of algebraic ratio to the geometric ratio (Nagle et al., 2019) but she did not use the idea of slope in calculus meaning yet in this stage. Considering that she could explain the quantity of slope in terms of two points on the line, she did not have the concept of the rate which is a reflectively abstracted constant ratio (Thompson, 1994) and did not have an understanding of it as an object (Nagle et al., 2019). Dubinsky (1991) stated that when properly understood, reflective abstraction appears as a description of the mechanism of the development of intellectual thought (p. 98). In that reason, reflective abstraction of the quantity of ratio is crucial in learning the instantaneous rate of change. In interpreting the derivative as a slope of tangent line, the student’s schema about the slope was significant. Although, in this study, I did not examine her understanding of the rate in detail, I observed that Amelie interpreted average rate of change while searching the slope of the tangent line. Students have to associate slope with the steepness of the tangent line of the curve at a point in order to interpret derivative graphically (Christensen & Thompson, 2012). Also, it is important for students to relate and justify the slope's corresponding numerical and physical properties (Nagle et al., 2019). The image in the task representing changing the secant lines as to be the tangent line helped the student to think of the slope of the secant line as a steepness physically and numerically. The idea of steepness pushed the student to form the relationship between the slope of the secant line and the tangent line. However, while Amelie was examining moving of the ring through the parabola, she did not create the secant lines by considering amount of the change between two points on the curve. Therefore, in the step 7, I showed the secant lines formed depending on the movement of the ring. She then interpreted the slopes of the secant lines. This result indicates that Amelie's thinking about the secant line was a static shape thinking which she made inferences based on the secant lines' appearance or shape (Moore & Thompson, 2015). The movement of ring catalyzed her thinking of the change in the slopes of the lines and approximation to the tangent line. The calculus student's understanding of graphically derivative was consistent with the findings from Asiala et al. (1997) and Zandieh (2000). While these researchers presented more broad actions, I articulated her actions by elaborating them based on the perspective of quantitative reasoning. For example, Asiala et al. (1997) explained, as a first layer, that the students connected two points on a curve to form a secant line and computed the slopes of the secant lines. Differently, this study presented that the student had several actions (MA1–MA6) to reason about the secant lines in the quantification process. In order to interiorize of the actions in point to a single process as the two points on the graph get closer and closer together (Asiala et al., 1997, p.10), the student in this study had the actions of approximating the points on the curve to the specific point on the curve and of rotating the secant lines to form construct the secant lines.

Amelie reasoned about the graphs by the idea of approximation. If she worked on the concept of derivative physically, she might have had more difficulties in the learning process. Physical approach necessarily to have deeply rate of change because it is embedded in this idea (Zandieh, 2000). Since the graphical approach supported the student to have visual understanding of the concept, she could progress on the task. Ellis (2011) has emphasized that visual representations of mathematical relationships help students to make inferences and generalizations. In this regard, presenting the task which the student could imagine the situation by the means of the visual representation prompted the student's mental actions. Also, this process may support her in improving the idea of rate of change and slope after the teaching process. This reverse process may detail be researched in the future studies.

When she was first asked to think the slope of tangent line, Amelia related this quantity to the derivative. However, she did not explain its underlying reason because she did not have knowledge about the relation between the secant lines and tangent line. Students generally have procedural understanding of mathematical concepts in the high school teaching process (Ferrini-Mundy & Graham, 1991). The results related to the student's initial understanding showed that she procedurally learned derivative in the high school. Based on this result, high school mathematics teachers give importance to the quantities of average rate of change and instantaneous rate of change to support their students to have conceptual learning of the derivative.

Student thought that the secant line would look like the tangent line as the ring approximates to the fixed point. Considering that the concept of derivative is explained by the concept of limit, the student's thinking on approximating to a point prompted her to relate this case with the concept of limit. Oerthman (2004) has emphasized that students' spontaneous reasoning about approximation can serve as a productive foundation for limit concept and subsequent development of other major concepts in calculus including the derivative. The results also showed that the student could imagine the instantaneously changing on the distance between the x-values of points on the curve and this mental action supported her to be able to write the derivative algebraically. The approach in the study would help the student to interpret the derivative in a corner point and it can be drawn more tangent lines at a corner point, and he function had no derivative at this point. The task might be meaningful for the students ignoring that the function had no derivative at the corner point and sketching one graph for the function f' that was continuous and differentiable at that point (e.g. Borji et al., 2018b).

Besides of these results, I saw that there were some obstacles for having global view of the concept of derivative. The global view of derivative requires strong understanding about the concept of function (Sánchez-Matamoros et al., 2015). In the interview, the student examined the slopes of tangent lines at different points on the curve and matched the dependent variable (slopes) to the independent variable (x-values of the points) in this process. However, she could not interpret the relationship as a function without support.

This study presents the results from one typical calculus student's mental actions. A research involving students at different level can be done and results can be compared with each other. Also, the task can be made more detail and students' learning paths while engaging in the task can be examined through a teaching experiment.

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Verkko-oppimisympäristöjen kehittäminen tekoälyn avulla: Tulevaisuusvisio matematiikan opetuksen täydennyskoulutuksesta

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Verkko-oppimisympäristöjen suosio kasvaa kiihtyvällä tahdilla maailman laajuisesti ja samaan aikaan tarve uusille lähestymistavoille verkko-oppimisen ja -opetuksen kehittämisessä ja tutkimuksessa on kasvanut. Vaikka lokitiedot mahdollistavat verkkokäyttäytymisen tutkimisen, on tätä mahdollisuutta hyödynnetty verkko-oppimisen ja -opetuksen tutkimuskontekstissa verrattain vähän. Tutkimuksessa hyödynnettiin lokitietoja verkkokurssin arvioinnissa ja kehittämisessä MOOC-ympäristössä. Tässä artikkelissa kuvataan oppimisprosessia lokitietojen avulla sekä pohditaan sitä, minkälaista ymmärrystä oppimisesta lokitiedot voivat tuottaa ja miten tällaisen tiedon voisi tulevaisuudessa valjastaa oppimisen tueksi. Tutkimusaineisto koostui matematiikan opettajien (N=58) täydennyskoulutusverkkokurssin lokitiedoista. Tutkimustulokset osoittavat, että syvällisemmät tai enemmän aikaa vaativat aktiviteetit keskeyttävät yhtenäisen opiskelun herkemmin kuin esimerkiksi videoluennot. Lyhytkestoiset videot ja nopeasti vastattavat kyselyt sen sijaan sitouttavat osallistujia yhtenäiseen opiskeluun. Vaikka suoristustavoissa on yksilöllisiä eroja, verkkokurssin kehittämistarvetta on mahdollista arvioida lokitietojen avulla. Esitämme artikkelissa vision siitä, kuinka tulevaisuudessa lokitiedot voisivat automaattisesti analysoitua, järjestelmä tunnistaisi oppimisprofiileja ja verkko-oppimisympäristö muokkautuisi automaattisesti tunnistettujen opiskelutaipumusten mukaan. Kun prosessiin yhdistetään tekoäly myös profilointialgoritmi kehittyisi automaattisesti käyttäjädatan kasvun myötä.

Avainsanat: täydennyskoulutus, verkko-oppiminen, lokitiedot, käyttäjädata, tekoäly

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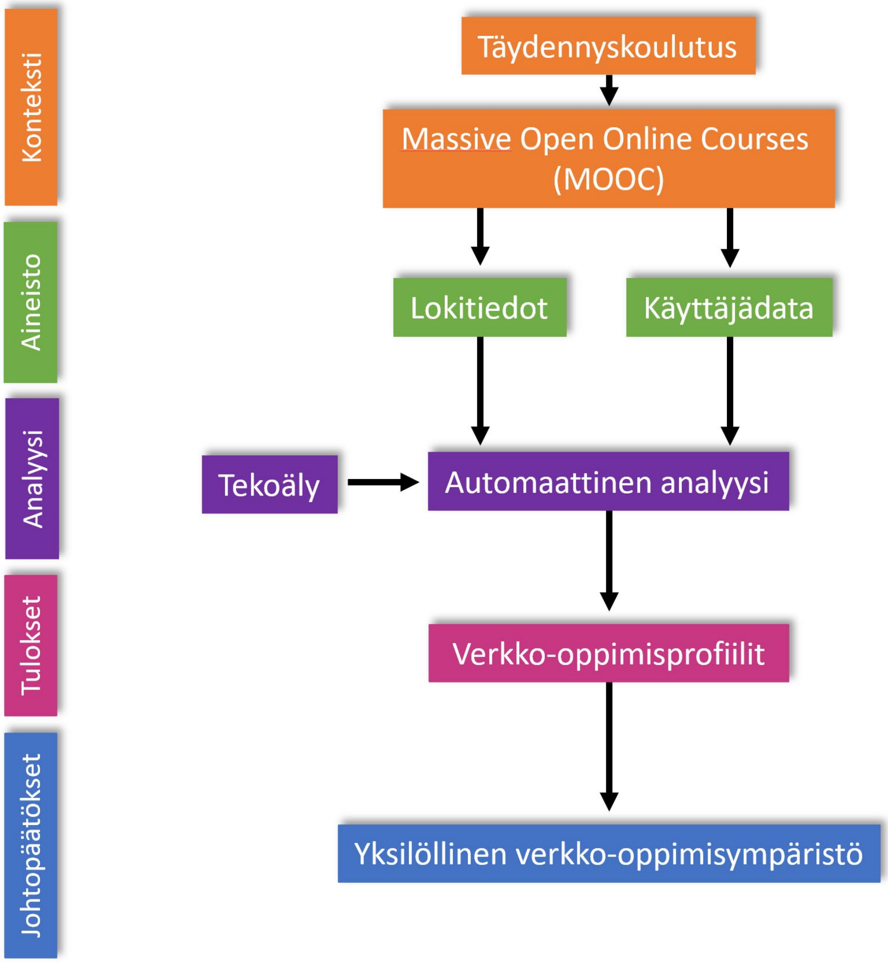


1 Opettajien täydennyskoulutus verkossa

Laajat kansainväliset tutkimukset osoittavat, että opettajien saamalla koulutuksella on yhteys heidän tiedolliseen osaamiseensa (Schmidt, Houang & Cogan, 2011). Kouluttautumisen myötä opettajien matemaattinen ja pedagoginen osaaminen kasvaa, opetus kehittyy ja matematiikan oppimistulokset paranevat myös koulutasolla. Esimerkiksi matematiikan osaamisen TIMSS[1]-tulokset ovat korkeampia niissä maissa, joissa opettajien matemaattinen ja pedagoginen osaaminen (arvioituna matematiikan tehtävien ja matematiikan opetukseen liittyvien tilannearvioiden kautta) on korkeampaa (Schmidt, Houang & Cogan, 2011). Matematiikan perus- ja täydennyskoulutuksen vaikutukset ulottuvat siis koulutasolle saakka (Blömeke, Busse, Kaiser ym., 2016; Kaiser & König, 2019). Vaikka laajat kansainväliset opettajankoulutusohjelmien vertailututkimukset tuottavat tietoa

[1] Laajassa kansainvälisessä TIMSS-tutkimuksessa (Trends in International Mathematics and Science Study) arvioitiin kahdeksannen luokan oppilaiden matematiikan osaamista.

opettajankoulutusohjelmien eroista (Schmidt, Houang & Cogan, 2011), opettajien perus- ja täydennyskoulutuksen kehittämiseen tarvitaan kuitenkin tutkimusperustaista tietoa koulutusohjelman vaikuttavuudesta (Hsieh, Law, Shy ym. 2011).

Verkko-oppiminen on mullistanut täydennyskoulutuksen kenttää merkittävästi (Cheng ym., 2014). Koulutusmuotojen monipuolistamiseksi verkkoympäristöjä on kehitetty jo vuosikymmeniä myös opettajankoulutuksen tarpeisiin. Vastaavaa työtä tehdään nyt myös Suomessa täydennyskoulutustarjonnan kehittämiseksi. Opetushallitus on rahoittanut vuosina 2018–2022 toteuttavia kehittämishankkeita, joiden tavoitteena on viedä matematiikan opetuksen täydennyskoulutusta verkkoympäristöön. Verkkokurssit ovat helposti saatavilla ympäri vuorokauden sijainnista riippumatta ja ne tarjoavat uudenlaisia oppimismahdollisuuksia, joten verkko-opetusmuodon toivotaan aktivoivan myös niitä opettajia, jotka eivät aktiivisesti osallistu täydennyskoulutuksiin. Heitä on opettajakunnassa noin viidennes opettajista (Lavonen & Mahlamäki-Kultanen, 2016).

Verkko-opetukseen liittyy kuitenkin haasteita. Esimerkiksi Massive Open Online Course (MOOC) -verkkokursseille osallistujia voi olla määrällisesti lähes rajattomasti. Kansainvälisissä tutkimuksissa verkkokurssien osallistujamäärät ovatkin varsin suuria, mutta samaan aikaan keskeyttäneiden osuus on tyypillisesti noin 90 % (Nawrot & Doucet, 2014). Myös verkkokurssien opettajat kohtaavat uudenlaisia haasteita, sillä verkkokursseilla yksilöllisten erojen huomiointi tai sosiaalisen vuorovaikutuksen vahvistaminen on haastavaa (Terras & Ramsay, 2015; Yu, 2015). Verkko-oppimiskokemusta heikentää osallistujan kielteinen asenne teknologiaa kohtaan (Rogers, 2000), heikot tietotekniset taidot (Berge, 2002, Mailizar, Abdulsalam & Suci, 2020), teknologisten välineiden heikko saatavuus (Brzycki & Dubt, 2005), tuen puute (Muilenburg & Berge, 2005) ja tietotekniset ongelmat (Ali & Magalhaes, 2008). Toisaalta verkkokurssien etuina voidaan pitää esimerkiksi sitä, että verkkokurssit ovat saatavilla missä ja milloin vain, verkkokurssit ovat kustannustehokkaita ja oppijan on helppo palata opetussisältöihin uudelleen (Yu, 2015). Erityisesti opettajien täydennyskoulutuksessa MOOCeista on saatu lupaavaa näyttöä odotusten toteutumisen ja relevanssin suhteen (Herranen, Aksela, Kaul & Lehto, 2021).

Vaikka verkko-opetukseen liittyy monia haasteita, voidaan verkossa nähdä myös uudenlaisia mahdollisuuksia oppimiseen. Yi-Shun Wang (2003) on esittänyt verkkokurssilla vaikuttavien tekijöiden teoreettisen mallin, jonka mukaan tekninen rajapinta ja sen ominaisuudet, oppijayhteisö, oppisisältö ja mahdollisuudet oppimisen yksilöllistämiseen muodostavat oppimiskokemusta kuvaavan kokonaisuuden. Näillä tekijöillä on yhteys oppijan kurssityytyväisyyteen ja niillä oletetaan myös olevan yhteys verkkokurssin jälkeiselle toiminnalle (Wang, 2003). Täydennyskoulutuksen vaikuttavuuden näkökulmasta kiinnostavaa on se, kuinka hyvin verkkokoulutus vastaa yksittäisen opettajan ammatillisiin odotuksiin, kehittymistarpeisiin ja kuinka koulutuksesta saadut uudet opetusideat siirtyvät käytäntöön (Chen, 2010; Herranen ym., 2021). Juuri kokemus siitä, että verkkokurssilla opitusta sisällöstä on hyötyä esimerkiksi työtehtävien kannalta, on keskeistä verkkokurssin loppuun suorittamisessa (Rodríguez-Ardura & Meseguer-Artola, 2016). Esimerkiksi Lun ja Chioun (2010) tutkimuksessa työssäkäyvät opiskelijat suhtautuivat kurssisisältöihin kriittisemmin kuin täysipäiväiset opiskelijat, mikä viittaa siihen, että opiskeltavan sisällön relevanssia punnitaan työssähyödynnettävyyden näkökulmasta (ks. myös Herranen ym., 2021).

Verkkoympäristössä oppimateriaali kuuluu keskeisesti oppimiskokemusta muovaaviin tekijöihin (Wu & Lin, 2012). Verkkokursseilla opetussisältöihin tulisi kiinnittää enemmän huomiota, ehkä jopa enemmän kuin ympäristön suunnitteluun sinänsä (Papachristos ym., 2014). Kokemus verkkokurssin hyödyllisyydestä näyttäisi syntyvän nimenomaan merkitykselliseksi koetun sisällön eikä kurssiympäristön kautta, joskin oppimisympäristön käyttäjäystävällisyys on oppimiskokemukseen vaikuttava tekijä (Lu & Choiu, 2010). Oppijan tunne siitä, että hän pystyy hallitsemaan oppimisympäristöään ja kykenee hyödyntämään vuorovaikutuksen välineitä kommunikoidakseen oppimisympäristössä edistävät niin ikään myönteisiä oppimiskokemuksia (Rodríguez-Ardura & Meseguer-Artola, 2017).

Vaikka verkkokurssit tarjoavat väylän myös suomalaisen täydennyskoulutustarjonnan monipuolistamiseen, uuden äärellä tarvitaan tutkimustietoa verkossa tapahtuvan täydennyskoulutuksen vaikuttavuudesta. Usein valtaosa tutkimusresursseista suuntautuu opettajien peruskoulutuksen vahvistamiseen, jolloin täydennyskoulutuksien tutkimusperustaista kehittämistyötä tehdään huomattavasti vähemmän (Wang, Wang & Shee, 2007; Chen, 2010). Koska verkkokurssit saattavat aktivoida myös niitä opettajia, jotka eivät perinteiseen täydennyskoulutukseen osallistu, voivat täydennyskoulutuksen haasteet olla moninaisempia. Esimerkiksi tutkimuksen kohteena olevassa täydennyskoulutusohjelmassa joka kolmas opettaja osallistui verkossa tapahtuvaan täydennyskoulutukseen ensimmäistä kertaa (Löfström ym. 2021). Opettajat kokivat myös, että arjen kiire ja ajanpuute ovat keskeisimpiä syitä siihen miksi täydennyskoulutuksen opit eivät päädy kouluopetukseen.

Tässä tutkimuksessa pureudutaan siihen, kuinka opettajat suorittavat täydennyskoulutusverkkokurssia ajallisesti. Tutkimuksessa analysoimme verkkotehtävien suoristusmääriä, -aikoja ja -ajankohtia. Verkkokurssin oppimisympäristöön tallentuvista lokitiedoista selvitämme yksilöllisesti kurssipudokkuutta (verkkokurssin aikana tehtyjen tehtävien lukumäärä), kurssikuormitusta (verkkokurssiin käytetty kokonaisaika), tehtäväkuormitusta (yksittäiseen tehtävään käytetty keskimääräinen aika), kiinnittymistä (yhdellä kirjautumiskerralla tehtyjen tehtävien lukumäärä) ja yhtenäistä opiskeluaikaa (yhdellä kirjautumiskerralla verkkoympäristössä käytetty kokonaisaika). Näihin tietoihin perustuen vastaamme tutkimuskysymyksiin:

1. Millainen on opettajien oppimisprosessi matematiikan opetuksen täydennyskoulutusverkkokurssilla ajallisesta, kurssi- ja tehtäväkuormituksen sekä tehtäviin kiinnittymisen näkökulmasta?
2. Millaisia yksilöllisiä eroja opettajien ajankäytössä, kuormituksessa ja tehtäviin kiinnittymisessä on matematiikan opetuksen täydennyskoulutusverkkokurssin suorittamisessa?

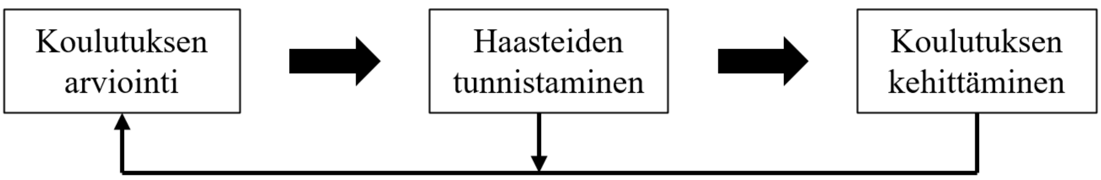
Koska opetussisällöillä ja oppimateriaaleilla on vaikutus verkko-oppimiseen (Wu & Lin, 2012; Papachristos ym. 2014), analysoimme myös sitä, kuinka opetussisältö ja erilaiset verkkotehtävätyypit (ohje, video, kysely ja keskustelu, pohdinta tai soveltava tehtävä) vaikuttavat verkko-opiskelun yhtenäisyyteen. Tarkastelemme erityisesti verkko-opiskelun keskeytysten suhdetta annettuihin verkkotehtäviin. Kolmas tutkimusta ohjaava kysymys on

3. Kuinka koulutuksen opetussisältö ja verkkotehtävätyypit vaikuttavat verkko-opiskelun yhtenäisyyteen?

Hypoteesimme mukaan kuhunkin tehtävään käytetystä opiskeluajasta on mahdollista tunnistaa esimerkiksi sellaiset videot, joita ei tyypillisesti katsota kokonaan tai tehtävät, joita ei varsinaisesti pohdita vaan vastaus annetaan heti. Koska analyysistä saatavia tuloksia voidaan verrata eri verkkotehtävien tavoitteisiin, on koulutuksen järjestäjän mahdollista arvioida ovatko verkkotehtäviin käytetyt opiskeluajat tarkoituksenmukaisia. Näin analyysistä saatava tieto auttaa koulutuksen järjestäjää arvioimaan verkkokurssin kehittämistarvetta. Toteutetun tutkimuksen innoittamana esitämme lopuksi vision tulevaisuuden verkko-oppimisympäristöstä, jossa järjestelmä analysoisi automaattisesti oppijoiden opiskelutaipumuksia, muodostaisi niistä oppimisprofiileja ja muokkaisi verkkoympäristöä yksilöllisesti tunnistettujen opiskelutaipumusten mukaan oppimista paremmin tukevasi. Kun prosessia kehitettäisi tekoälyn avulla, käyttäjädatan kasvu parantaisi oppijoiden profilointia ja johtaisi yhä älykkäämpiin verkkoympäristön yksilöllistämisratkaisuihin. Tekoäly on jo muuttanut verkko-oppimisen yksilöllistämistä tukevia ratkaisuja (Roll & Wylie, 2016), mutta suomalaisessa opettajien täydennyskoulutuskontekstissa tätä on pohdittu toistaiseksi vähemmän.

2 Verkkotäydennyskoulutus kehittämistutkimuksen kohteena

Tutkittavaa matematiikan opetuksen täydennyskoulutusta on kehitetty kehittämistutkimuksen menetelmällä. Kehittämistutkimuksen strategia etenee siten, että ensin koulutusta arvioidaan, mikä avulla tunnistetaan koulutuksen haasteet ja lopuksi tunnistetuille haasteille etsitään ratkaisua kehittämisen kautta (Kuva 1). Kehittämistutkimusta on hyödynnetty erityisesti teknologiavälitteisissä koulutuskonteksteissa, ja se soveltuu strategiaksi myös koulutusohjelmien kehittämiseen matematiikan opetuksen kontekstissa (Wood & Berry 2003). Kehittämistutkimus on luonteeltaan iteratiivista ja tuottaa jatkuvasti uutta tietoa koulutusohjelman kehittämiseen (Amiel & Reeves, 2008; Kervinen ym., 2016). Kehittämistutkimus soveltuu siten myös käynnissä olevien koulutusohjelmien kehittämiseen. Wood ja Berry (2003, p. 197) kuvaa strategiaa osuvasti: kehittämistutkimus on kuin ”lentäisi lentokoneella ja korjaisi sitä samaan aikaa”.



Kuva 1. Kehittämistutkimus arviointiperustaisena ja iteratiivisena prosessina

Tutkimuksen aiemmassa vaiheessa täydennyskoulutuksen vaikuttavuutta arvioitiin kyselytutkimuksen avulla, jonka tuloksena osallistujien aikahaasteet ja niiden moninaisuus nousivat esille (Koponen, Löfström & Portaankorva-Koivisto, 2020). Kun koulutukseen osallistuneilta opettajilta kysyttiin, millaiset tekijät estävät heitä viemästä koulutuksesta opittuja asioita käytäntöön, nosti 60 % opettajista esiin työhön liittyvän kiireen ja ajanpuutteen (Koponen, Löfström & Portaankorva-Koivisto, 2020). Koska koettu kiire ja ajanpuute saattavat vaikuttaa myös verkkokurssien suorittamiseen, tässä tutkimuksessa on pureuduttu tarkemmin siihen, miten osallistujat käyttävät aikaansa ja miten he toimivat verkko-oppimisympäristössä. Kehittämistutkimukselle ominaista on juuri se, että tunnistetut haasteet toimivat lähtökohtana koulutuksen jatkoarviointiin tai -kehittämiseen. Myös vaiheiden mukainen raportointi on ominaista kehittämistutkimukselle (Juuti & Lavonen, 2006).

Kehittämistutkimukselle tyypillistä on, että se toteutetaan aidossa koulutuksellisessa kontekstissa (Anderson & Shattuck, 2012). Tutkittavassa täydennyskoulutusohjelmassa verkkokurssit itsessään muodostavat opetuksellisen intervention, jota kehitetään osapuolilta saadun palautteen perusteella (vrt. Anderson & Shattuck, 2012). Kehittäminen on usein iteratiivinen prosessi, jossa kehittämisen kohteena oleva interventio hioutuu sitä mukaan, kun sen toimivuudesta saadaan lisää tietoa. Tutkimuksen kohteena oleva täydennyskoulutusverkkokurssi on toteutettu jo kymmeniä kertoja, mutta lokitietojen tarkastelu tuli ajankohtaiseksi vasta kyselytutkimuksen nostettua esille osallistujien kokemat aikahaasteet.

Usein kehittämistutkimuksen kimmokkeena on jokin jännite, haaste (Wang & Hannafin, 2005) tai muutokset toimintaympäristössä (Juuti & Lavonen, 2006). Yhtenä motiivina on esitetty oppimisnäkemyksen ja sen käytännön toteutuksen väliset ristiriidat juuri verkko-oppimiskontekstissa (Wang & Hannafin, 2005). Tutkittavassa täydennyskoulutusohjelmassa jännitteen aiheutti osallistujien kokemat arjen aikahaasteet (Koponen, Löfström & Portaankorva-Koivisto, 2020) ja toisaalta koulutuksen järjestäjien epätietoisuus siitä, kuinka kuorimittavia tai aikaa vieviä verkkokurssin tehtävät ovat. Tunnistettu haaste voikin siis toimia koulutuksen kehittämisen lähtökohtana tai käynnistää koulutuksen jatkoarvioinnin (Kuva 1).

Kehittämistutkimusprosessin tuloksena syntyvä pedagoginen tieto valjastetaan sekä teoreettisen tiedon lisäämiseen että käytäntöjen kehittämisen (Juuti & Lavonen, 2006). Tutkimuksen tavoitteena on tuottaa tietoa täydennyskoulutukseen osallistuvien toiminnasta verkkoympäristössä ja sitä kautta antaa välineitä kehittää koulutusta osallistujia paremmin palvelevaan muotoon.

3 Menetelmä

3.1 Konteksti

Tutkimuksen aineisto kerättiin matematiikan opetuksen LUMATIKKA-täydennyskoulutusohjelmasta. LUMATIKKA on Helsingin yliopiston järjestämä ja Opetushallituksen rahoittama täydennyskoulutusohjelma varhaiskasvatuksesta toiselle asteelle työskenteleville opettajille. Täydennyskoulutusohjelma (15 op[2]) koostui kaikkien luokka-asteiden opettajille yhteisestä osasta (3 op), luokka-astekohtaisesta osasta (6 op), sekä valinnaisesta osasta (6 op). Koulutuksen ensimmäisen osan tavoitteena oli kehittää laaja-alaisesti osallistujien matemaattista ja pedagogisia osaamista, toisessa osassa syvennyttiin opetusastekohtaiseen didaktiikkaan ja kolmannessa osassa osallistujan oli mahdollista valita verkkokurssi tarkempaan teemaan liittyen kuten esimerkiksi, kuinka yhdistää ohjelmointi, taide tai liikunta matematiikan opetukseen. Taustalla on sosiokonstruktivismiin pohjautuva näkemys oppimisesta.

Tämän tutkimuksen aineisto kerättiin koulutusohjelman ensimmäisestä osasta. Tutkimuksen kohteena oleva verkkokurssi sisälsi teoreettisen ja soveltavan osan, joiden opetussisällölliset teemat on kuvattu Taulukossa 1. Teoriaosuuden ensimmäinen teema esitteli aiempaa tutkimustietoa suomalaisten oppilaiden matematiikan osaamiseen liittyen. Seuraavat viisi teemaa keskittyivät matemaattisiin taitoihin, niiden tukemiseen sekä niihin vaikuttaviin tekijöihin. Kolme seuraavaa teemaa käsitteli ongelmalähtöisen matematiikan opetuksen toteuttamista.

[2] Laskennallisesti yksi opintopiste (op) vastaa noin 27 tunnin työpanosta.

Taulukko 1. Verkkokurssin opetussisällöt teemoittain

	Teema	Opetussisältö
Teoriaosa	Tutkimustietoa matematiikan osaamisesta	Matematiikan oppimistuloksia kansainvälisissä ja kansallisista tutkimuksissa
	Matemaattiset taidot ja niiden tukeminen	Matematiikan perustaidot
		Matematiikan oppimisvaikeudet
		Kielitietoinen matematiikan opetus
		Matematiikan oppiminen aivotutkimuksen näkökulmasta
		Motivaation merkitys oppimiseen
	Ongelmalähtöinen matematiikan opetus (teoriassa)	Ongelmanratkaisutaidot matematiikan opetuksessa
		Tutkivan oppimisen tukeminen opetuksessa
		Oppimateriaalien erilaiset käyttömahdollisuudet
Soveltavaosa	Ongelmalähtöinen matematiikan opetus (käytännössä)	Ongelmalähtöisen opetuskokeilun suunnittelu, toteutus ja raportointi

Verkkokurssin suoritus rakentui erilaisten tehtävien suorituksista (Taulukko 2). Jokainen opetussisältöteema sisälsi vähintään yhden videoluennon ja tehtäväaktiviteetin. Esimerkiksi teoriaosuudessa osallistuja katsoi videoluentoja opetussisältöteemaan liittyen, jonka päätteeksi osallistujaa pyydettiin vastaamaan Testaa tietosi -kyselyyn. Kysely sisälsi videoihin liittyviä väittämiä, joihin hyväksytty suorittaminen vaati 80 % tarkkuuden. Videoluentojen ja kyselyiden lisäksi verkkokurssi sisälsi keskustelu- ja pohdintatehtäviä, sekä soveltavia tehtäviä. Keskustelutehtävissä osallistujat lähettivät oman kommenttinsa annettuun tehtävään, sekä kommentoivat muiden osallistujien ajatuksia. Pohdintatehtävissä opettajille annettiin kysymys mietittäväksi, mutta omaa pohdintaa ei tarvinnut palauttaa verkkoympäristöön. Soveltavissa tehtävissä edellytettiin kyseisen teeman videoluennoista saatujen tietojen hyödyntämistä tai soveltamista omien kokemusten pohjalta. Soveltavassa osuudessa opettajat suunnittelivat ja toteuttivat ongelmalähtöisen opetuskokeilun oman opetusryhmän kanssa. Toteutuksen jälkeen opetuskokeilusta tehtiin raportti ja kokemusta reflektoitiin yhdessä muiden osallistujien kanssa kurssialueella. Teoriaosuus voitiin suorittaa esimerkiksi ilman omaa opetusryhmää, kun taas soveltavassa osuudessa oman opetusryhmän kanssa toteuttava opetuskokeilu, raportointi ja reflektointi oli suorituksen edellytyksenä. Koko verkkokurssin suorittaminen sisälsi yhteensä 50 tehtävää, joista teoriaosuuteen kuului 40 tehtävää ja 10 tehtävää soveltavaan osaan.

Taulukko 2. Verkkokurssin sisältämien aktiviteettien lukumäärät

	Ohjeet	Luento	Keskustelu	Pohdinta	Soveltava	Testaa tietosi	Opetuskokeilu
Yhteensä	3	26	3	3	6	7	2

Koko verkkokurssin suorittaminen sisälsi yhteensä 50 tehtävää, joista teoriaosuuteen kuului 40 tehtävää ja 10 tehtävää soveltavaan osaan.

3.2 Aineisto ja analyysi

Tutkimukseen osallistuneet opettajat (N=58) suorittivat täydennyskoulutusohjelman ensimmäistä osaa syksyllä 2019 MOOC-verkkoympäristössä (Massive Open Online Course). MOOC-ympäristön alustana on Blackboard Open LMS, joka pohjautuu Moodleen. Koska MOOC-ympäristöltä edellytetään, että verkkokursseja voidaan toteuttaa automatisoidusti ilman opettajaa, on MOOC-ympäristöissä yleensä Moodlea kattavammat valikoimat työkaluja ja lokitietoja. Esimerkiksi tutkittava MOOC-ympäristö[3] tallensi yksilöllisesti päivämäärän ja kellonajan jokaiselle napin painallukselle. Näin ollen avatut linkit voidaan näyttää esimerkiksi sinisen värin sijasta harmaalla ja tehdyt tai tekemättömät tehtävät voidaan näyttää osallistujille erilaisina. Toisaalta lokitiedot mahdollistavat, että käyttäjien navigointia ja tehtävien suorittamista voidaan jälkikäteen tutkia.

Tässä tutkimuksessa rajauduttiin tarkastelemaan *Aktiviteettien suoritus* –tietoja eli kurssiosallistumista ja lokitietoja. Näissä tiedoissa on kirjattuna tehtävien suorittamisen päivämäärä ja kellonaika sekunnin tarkkuudella. Verkkokurssin osallistujia informoitiin verkkokurssilla tehtävästä tutkimuksesta ja siitä, mitä tietoja kerätään ja miten tietoja käytetään (koulutuksen kehittäminen ja tutkimus). Suostumus tutkimukseen osallistumisesta kerättiin kyselytutkimuksen yhteydessä. Tutkimukseen osallistuminen oli vapaaehtoista. Osallistujilla oli oikeus peruuttaa tutkimukseen osallistumisensa koska tahansa syytä ilmoittamatta ja ilman seurauksia. Lokitietojen tarkastelusta ei synny tunnistamiseen liittyvää haittaa. Lokitietojen tarkastelusta ei ole välitöntä hyötyä yksittäisille osallistujille, mutta tutkimuksen avulla täydennyskoulutusta voidaan kehittää ja laajemmalla tasolla tulokset voivat antaa tutkimukseen perustuvia kehittämisideoita muihin täydennyskoulutusohjelmiin.

[3] Aineistonkeruun aikana käytettiin Blackboard Open LMS 3.5 Maintenance Pack 2 versiota.

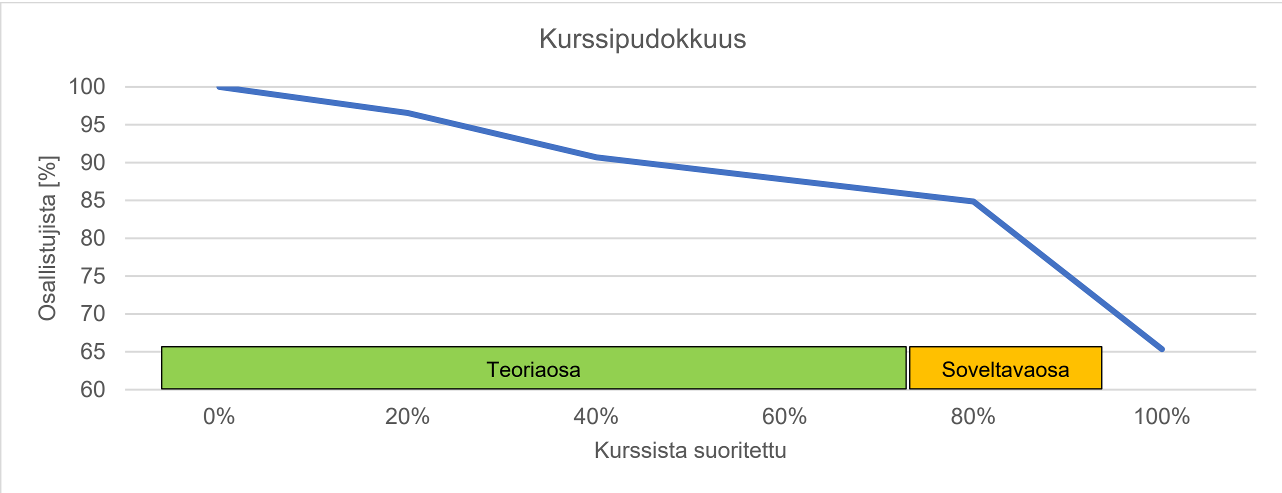
Koko verkkokurssin suorittamiseen kulunut aika määritettiin vertaamalla ensimmäisen ja viimeisen aktiviteetin suoritusaikoja. Yksittäisen aktiviteetin suorittamiseen kulunut aika määritettiin laskemalla erotus kahden peräkkäisen aktiviteetin suoritusaikojen välillä. Mikäli seuraava aktiviteetti oli suoritettu yli 3 tunnin kuluttua laskettiin ne suoritetuiksi eri kirjautumiskerralla. Kolmen tunnin raja-arvoon päädyttiin alustavan analyysin perusteella, sillä vaikka yhden tehtävän tekemiseen kului osallistujalta tyypillisesti alle tunti, niin tietyillä soveltavilla tehtävillä ja pohdintatehtävillä tehtävään käytetty aika oli järjestelmällisesti huomattavasti suurempi. Kun tarkastelimme näitä ns. työläämpiä tehtäviä ja otimme tarkasteluun vain ne osallistujat, jotka olivat edenneet muita hitaammin myös muissa tehtävissä, päädyimme arvioon, että vaativampiin tehtäviin on käytetty aikaa noin 2 tuntia. Ottaen huomioon, että yhden tehtävän tekemiseen kuluvan keskimääräinen aika oli alle tunnin, päädyimme valitsemaan raja-arvoksi kolme tuntia. Raja-arvon määrittäminen oli tarpeen, jotta voitiin tarkastella, kuinka monta kertaa verkkokurssialueelle on kirjauduttu tehtäviä tekemään, kuinka monta tehtävää on tehty kerrallaan ja kuinka kauan tehtävien tekemiseen on käytetty aikaa. Raja-arvon avulla voidaan myös nähdä, minkä tehtävän kohdalla tehtävien tekeminen on jätetty kesken ja jatkettu toisena ajankohta. Tuloksissa keskiarvoa on käytetty tyypillisen suorituksen kuvaamiseen ja vaihtelua keskihajonnan sekä pienimmän että suurimman arvon avulla.

Verkkokurssin sisällön vaikutusta suorituskäyttäytymiseen tarkasteltiin tutkimalla tehtäviin kulunutta aikaa tehtävätyypeittäin (vrt. Taulukot 1 ja 2). Suoritusaikojen avulla tarkasteltiin tiettyyn tehtävätyyppiin kulunutta aikaa ja tehtävätyypin yhteyttä yhtenäisen opiskelun keskeyttämiseen. Erityisenä kiinnostuksen kohteena olivat videopohjaiset luennot, sillä videoiden kestoa voitiin verrata tehtävän suorittamiseen käytettyyn aikaan. Yhteydet videon keston, tehtävään käytetyn ajan ja opiskelun keskeyttämisen välillä laskettiin Pearsonin korrelaation avulla.

4 Tulokset

4.1 Kuinka matematiikan opetuksen täydennyskoulutusverkkokurssi tyypillisesti suoritettiin?

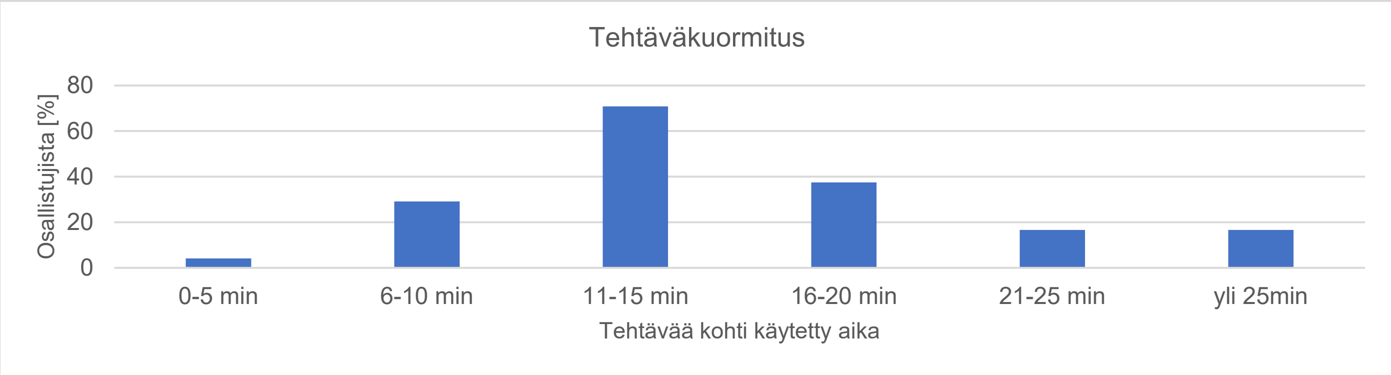
Kurssipudokkuus. Ensimmäisenä analysoitiin verkkokurssin tehtävien suoritusmäärien kehitystä. Suorittajamäärät laskivat verkkokurssin edetessä niin, että voimakkainta verkkokurssin keskenjättäminen oli soveltavassa osassa, jossa keskenjättäjien määrät kasvoivat suhteessa teoriaosaan (Kuva 2).



Kuva 2. Tutkimukseen osallistuneista opettajista (N=58) 81% suoritti teoriaosan ja 59% koko verkkokurssin

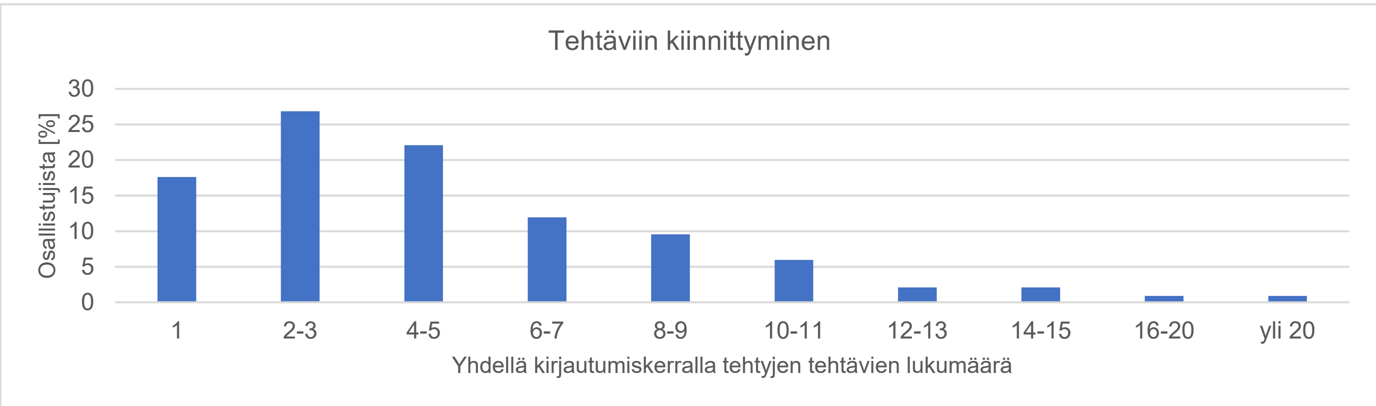
Kurssikuormitus. Seuraavaksi tarkasteltiin verkkokurssin suorittamiseen kuluvaa kokonaisaikaa. Teoriaosa suoritettiin keskimäärin 63 vuorokaudessa (n. 2 kk) ja koko verkkokurssi 103 vuorokaudessa (n. 3 kk). Teoriaosa oli monessa mielessä kevyempi suorittaa kuin soveltava osuus. Soveltavan osuuden suorittamiseen sisältyi muun muassa oman opetusryhmän kanssa toteutettava opetuskokeilu. Soveltavaan osuuteen sisältyvän opetuskokeilun suunnittelun, toteutuksen ja raportoinnin havaittiin hidastavan verkkokurssin nopeaa suorittamista ja lisäävän suoritusaikaa noin kuudella viikolla (1,5 kk).

Tehtäväkuormitus. Suoritusaikojen perusteella voitiin laskea tehtävien tekemiseen käytetty keskimääräinen aika. Jotta tehtävien suoritusajat ovat vertailukelpoisia, analyysissä keskitytään vain niiden opettajien (N=34) suorituksiin, jotka suorittivat verkkokurssin kokonaisuudessaan. Näin vertailtavat suoritusajat ovat lähtöisin samoilta henkilöiltä läpi analyysin. Osallistujat tekivät keskimäärin 4,6 tehtävää yhdellä kirjautumiskerralla ja yhteen tehtävään kului keskimäärin aikaa 10-15 minuuttia (Kuva 3).



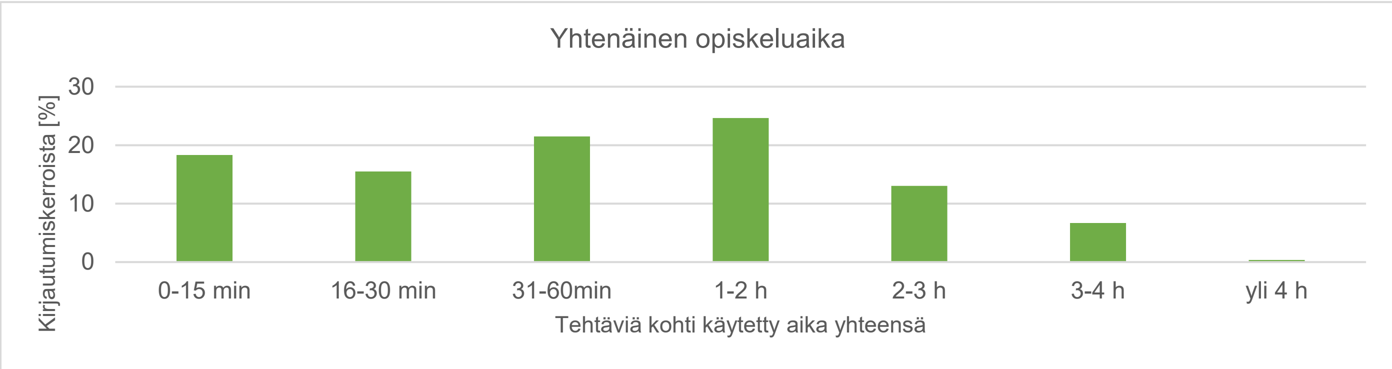
Kuva 3. Koko verkkokurssin suorittaneet opettajat (N=34) käyttivät yhteen tehtävään tyypillisesti opiskeluaikaa 10–15 minuuttia

Tehtäviin kiinnittyminen. Yhdellä kirjautumiskerralla tehtiinn useimmiten useampi tehtävä, sillä kirjautumiskerroista 18 % sisälsi yhden ja 82 % useamman tehtävän tekemisen kerralla. Mikäli tehtäviä suoritettiin enemmän kuin yksi kerrallaan niin, yhdellä kirjautumiskerralla tehtiin keskimäärin 5,8 tehtävää. Kuvaan 4 on havainnollistettu yhdellä kirjautumiskerralla tehtyjen tehtävien lukumääriä. Myös yksittäisellä kirjautumiskerralla suoritettujen tehtävien tekemisessä oli havaittavissa yksilöllisiä eroja, mutta useimmiten tehtäviä tehtiin alle kymmenen kerrallaan.



Kuva 4. Koko verkkokurssin suorittaneet opettajat (N=34) tekivät yhdellä kirjautumiskerralla tyypillisesti 2-10 tehtävää kerrallaan

Yhtenäinen opiskeluaika. Yhtenäinen opiskeluaika oli joka kolmannella puoli tuntia tai alle ja yli 50 % osallistujista yhtenäinen opiskeluaika vaihteli 1–3 tunnin välillä. Yli neljän tunnin käyttäminen tehtävien tekemiseen kerralla oli marginaalista. Mikäli kirjautumiskerralla tehtiin useampi kuin yksi tehtävä, tehtävien tekemiseen käytettiin keskimäärin 71 minuuttia kerrallaan. Kuva 5 havainnollistaa vaihtelua yhtenäisen opiskeluajan suhteen.



Kuva 5. Koko verkkokurssin suorittaneet opettajat (N=34) käyttivät opiskeluun aikaa tyypillisesti 1-2 tuntia, yhdellä kirjautumiskerralla

Yhtenäinen opiskeluaika oli joka kolmannella puoli tuntia tai alle ja yli 50 % osallistujista yhtenäinen opiskeluaika vaihteli 1–3 tunnin välillä. Yli neljän tunnin käyttäminen tehtävien tekemiseen kerralla oli marginaalista.

4.2 Millaisia yksilöllisiä eroja oli havaittavissa verkkokurssin suorittamisessa?

Vaihtelu verkkokurssin suoritusajassa. Siinä missä keskiarvot kuvaavat tyypillistä suoritustapaa, muiden tunnuslukujen avulla voidaan tehdä päätelmiä suoritusten yksilöllisistä eroista (mm. keskihajonta ja minimi- sekä maksimiarvo). Samalla kun teoriaosuus suoritettiin keskimäärin 63 vuorokaudessa oli osallistujien välinen keskihajonta 45 vuorokautta (Taulukko 3). Tämä tarkoittaa, että suoritusajat vaihtelivat[4] tyypillisesti 18 ja 108 vuorokauden välillä (vaihtelua siis 2,5 viikosta 3,5 kuukauteen). Vaihtelu pysyi sen sijaan melko samana, vaikka osallistujat suorittivat koko verkkokurssin. Tämä tarkoittaa, että omassa opetuksessa toteutettava opetuskokeilu ei lisännyt vaihtelua.

[4] Tulkinnassa hyödynnetään empiiristä 68-95-99.7 sääntöä, jonka perusteella 68% tapauksista on yleensä ensimmäisen ”keskihajonta-askeleen” päässä keskiarvosta. Näin ollen säännön mukaisesti suurin osa aineistossa esiintyvästä vaihtelusta sijoittuu keskiarvon molemmin puolin keskihajonnan mittaiselle etäisyydelle. Säännön kehitti ranskalainen matemaatikko Abraham de Moivre 1700-luvulla.

Taulukko 3. Vaihtelu teoriaosan ja koko verkkokurssin suoritusajoissa (N=34 opettajaa)

	Minimi	Keskiarvo/-hajonta	Maksimi
Teoriaosuus suoritettu aikavälillä [vuorokautta]	4	63/45	179
Koko verkkokurssi suoritettu aikavälillä [vuorokautta]	20	103/46	195

Vaihtelu kirjautumiskerroissa. Yksilöllisiä eroja oli havaittavissa verkkokurssiin käytetyn ajan lisäksi myös osallistujien kirjautumiskäyttäytymisessä (Taulukko 4). Ääripäitä tarkastelemalla havaittiin, että vähimmillään verkkokurssi suoritettiin viiden ja enimmillään 24 kirjautumiskerran aikana. Tyypillinen vaihtelu kirjautumiskerroissa oli 7-15. Niiden kirjautumiskertojen kokonaismäärä, jolloin osallistujat tekivät vain yhden tehtävän, vaihteli tyypillisesti 1-5 välillä. Kun taas niiden kirjautumiskertojen kokonaismäärä, jolloin tehtiin useampi tehtävä, vaihteli tyypillisesti 2-10 tehtävän välillä. Kirjautumiskertojen lukumäärällä ja kirjautumisen aikana tehtyjen tehtävien välillä on luonnollisestikin yhteys. Mikäli osallistuja suoritti verkkokurssin vähillä kirjautumiskerroilla, tuli hänen tehdä useampia tehtäviä kerrallaan ja päinvastoin.

Taulukko 4. Vaihtelu kirjautumiskertojen kokonaismäärässä (N=34 opettajaa)

	Minimi	Keskiarvo/-hajonta	Maksimi
Kirjautumiskertoja kokonaismäärä	5	11/4	24
Kirjautumiskerrat, jolloin teen yhden tehtävä	1	3/2	13
Kirjautumiskerrat, jolloin teen useamman tehtävän	2	6/4	25

Vaihtelu tehtäviin ja verkkokurssiin käytetyssä ajassa. Aiemmissa tarkasteluissa havaittiin yksilöllisiä eroja tehtävien tekoon käytetyn ajan ja yhdellä kirjautumiskerralla tehtyjen tehtävien määrän suhteen. Taulukkoon 5 on kuvattu verkkokurssin osallistujien ääripäät suhteessa koko verkkokurssiin sekä yksittäiseen tehtävään käytettyyn aikaan. Tyypillisesti yhtä tehtävää kohti käytetty aika vaihteli 9-21 minuutin välillä ja koko verkkokurssiin käytetty aika 6-14 tunnin välillä.

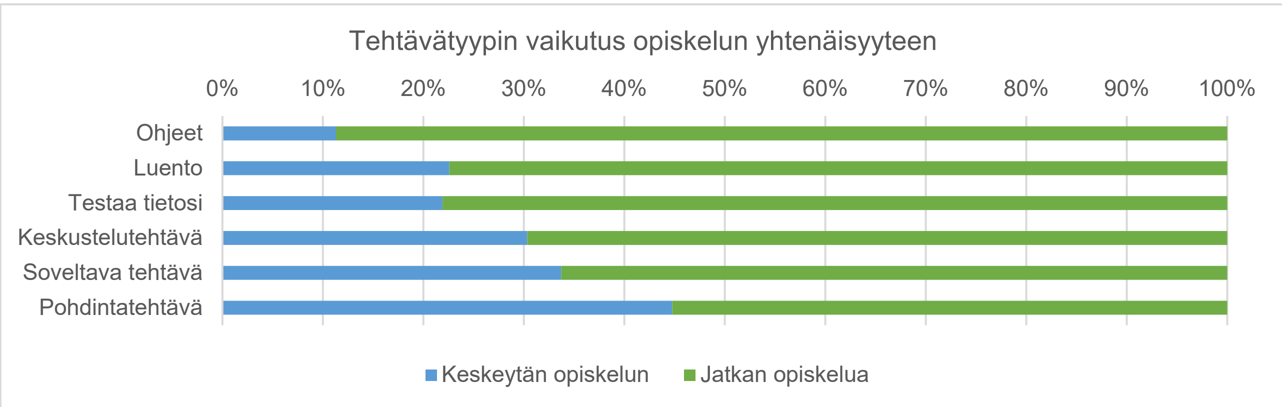
Taulukko 5. Vaihtelu tehtäviin ja verkkokurssiin käytetyssä kokonaisajassa (N=34 opettajaa)

	Minimi	Keskiarvo/-hajonta	Maksimi
Käytän aikaa yhden tehtävän tekemiseen [minuuttia]	5	15/6	26
Käytän aikaa koko verkkokurssiin [tuntia]	4	10/4	21

Yksittäiseen tehtävään käytetty aika oli likimain suoraan verrannollinen koko verkkokurssin suorittamiseen käytetyn ajan suhteen. Osallistujien kesken oli havaittavissa suuria eroja yksittäiseen tehtävään käytetyn ajan suhteen, jolloin myös koko verkkokurssin opiskeluun investoitu aika eroaa merkittävästi ääripäiden välillä. Esimerkiksi koko verkkokurssiin käytetty opiskeluaika oli yli viisinkertainen ääripäiden välillä.

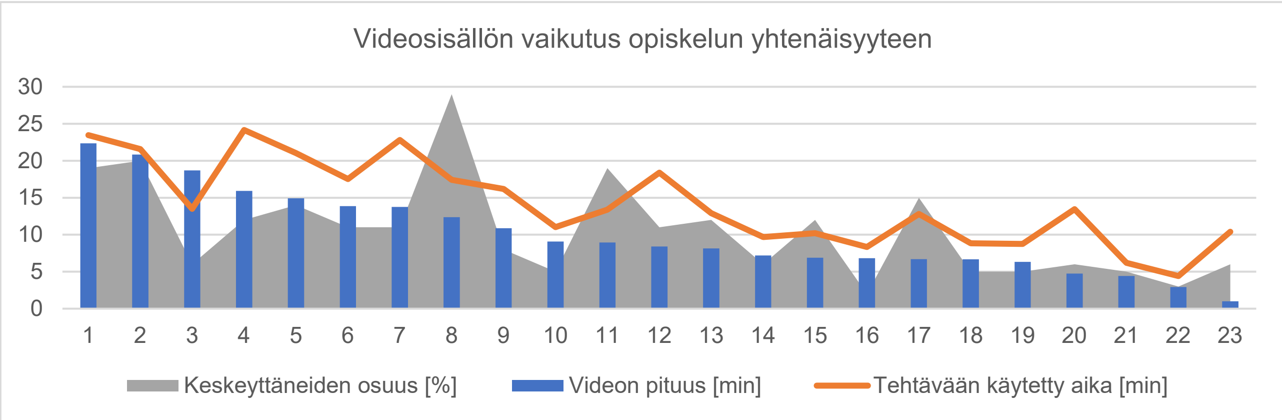
4.3 Kuinka tehtävätyypit vaikuttivat verkkokurssin suorittamiseen?

Tehtävätyypin vaikutus yhtenäiseen opiskeluun. Tehtävien suoritusajankohtien perusteella voidaan myös selvittää, minkä tehtävän kohdalla opiskelu on keskeytynyt ja siirtynyt toiseen ajankohtaan. Kuvasta 6 voidaan havaita, että ohjeita jätetään harvimmin toiseen opiskelukertaan. Testaa tietosi -kyselyt keskeyttivät yhtenäisen opiskelun yhtä harvoin kuin videoluennot. Itse tuotettua tekstiä vaativissa tehtävissä, todennäköisuus sille, että tehtävä tehdään myöhemmin, kasvoi. Esimerkiksi pohdintatehtävissä keskimäärin noin 45 % jätti tehtävän tekemisen myöhempään ajankohtaan. Toisaalta tulos saattaa indikoida oppimisen näkökulmasta hedelmällistä prosessia, sillä pohdintatehtävissä osallistujien tuli syventää pohdintaansa videoissa esitetyistä näkökulmista.



Kuva 6. Yhtenäisen opiskelun jatkaminen ja keskeyttäminen tehtävätyypeittäin

Videoiden ja opiskelun välinen yhteys. Analyysissä verrattiin videoiden kestoa suhteessa tehtävään käytettyyn aikaan sekä näiden aikamääreiden suhdetta tehtävien teon keskeytymiseen (Kuva 7). Siinä missä kuva 6 osoittaa, että videoluentojen kohdalla yhtenäinen opiskelu keskeytyi tyypillisesti 23 % opiskelukerroista, kuva 7 antaa tarkempaa tietoa videoiden katselusta. Kuvasta voidaan havaita, että esimerkiksi videoluennon 3 kohdalla tehtävään käytetty aika on vähemmän kuin videon pituus (oranssi viiva jää sinisen palkin alapuolelle). Tämä tarkoittaa, että videon katsomiseen käytettiin keskimäärin vähemmän aikaa kuin itse video kestää eli toisin sanoen kyseistä videota ei ole tyypillisesti katsottu loppuun saakka.



Kuva 7. Videon pituuden vuorovaikutus tehtävään käytettyyn aikaan ja tekemättä jättämiseen.

Harmaat piikit videoiden 8, 11, 15 ja 17 kohdalla tarkoittavat, että trendi poikkeaa tavanomaisesta. Näiden videoiden kohdalla opiskelu useimmiten keskeytyi ja tehtävien tekeminen jätettiin usein toiseen opiskelukertaan. Trendi poikkeaa myös videon 3 kohdalla, jossa harmaa piikki on toisinpäin. Se tarkoittaa, että yhtenäistä opiskelua on jatkettu keskimääräistä paremmin. Mielenkiintoista tuloksessa on se, että kyseessä on juuri video 3, jota ei tyypillisesti katsottu loppuun saakka. Tulos tarkoittaa sitä, että vaikka osallistujat ovat jättäneet tämän videon kesken, he ovat keskimääräistä paremmin jatkaneet opiskelua seuraavaan tehtävään. Videoiden 4, 7, 12, 17, 20, 23 kohdalla on taas piikki (oranssi viiva). Se tarkoittaa, että kyseisiin videoihin on käytetty videon keston lisäksi keskimääräistä enemmän aikaa.

Havaintojen tarkempi tarkastelu osoittaa, että videon pituus korreloi tehtävän tekemiseen käytetyn ajan kanssa ($r=.806$, $p<.01$), tehtävän tekemiseen kuluva aika korreloi keskeyttämisen kanssa ($r=.624$, $p<.01$) samoin kuin videon pituus ($r=.538$, $p<.01$). Tuloksen perusteella voidaan sanoa, että mitä pidempi video on, sitä todennäköisemmin verkko-opiskelu keskeytyy. Kun tulosta verrataan kuvan 6 tuloksiin on todennäköistä, että keskustelutehtävät, soveltavat tehtävät ja pohdintatehtävät koetaan myös aikaa vieviksi. Näin ollen tulokset voisi kiteyttää seuraavasti: mitä työläämpänä yksittäinen verkkotehtävä koetaan, sitä todennäköisemin verkko-opiskelu keskeytyy.

5 Keskustelu: lokitiedoista hyötyä verkkokurssien kehittämiseen ja opiskelutapojen tunnistamiseen

Tutkimuksessa tarkasteltiin lokitietojen avulla, kuinka opettajat suorittavat täydennyskoulutuksen verkkokursseja MOOC-verkkoympäristössä. Tällä hetkellä lokitietoja käytetään pääasiassa järjestelmän toiminnan ylläpitämiseen, jotta järjestelmä erottaa käyttäjäkohtaisesti esimerkiksi tekemättömät tehtävät tehdyistä. Vaikka lokitiedot mahdollistavat verkkokäyttäytymisen tutkimuksen, on niitä hyödynnetty verkko-oppimisen ja opetuksen tutkimuksessa vähän.

Aineiston perusteella näyttäisi siltä, että mitä työläämpänä tai aikaa kuluttavampana verkkotehtävän tekeminen koetaan, sitä todennäköisemmin yhtenäinen verkko-opiskelu keskeytyy. Ohjeet, videot ja kyselyt keskeyttävät opiskelun harvemmin kuin soveltavat tehtävät, keskustelu- tai pohdintatehtävät. Tutkittavassa verkkokurssissa tehtävät oli järjestetty siten, että opiskeltavasta teemasta oli useita videoita, joiden päätteeksi tuli joko pohdintatehtävä, keskustelutehtävä tai testaa tietosi –kysely. Tutkimustuloksemme osoittavat, että osallistujat jatkavat yleensä opiskelua uuteen aiheeseen kyselyn jälkeen, kun taas pohdinta- ja keskustelutehtävät usein katkaisivat opiskelun ja osallistujat jatkoivat verkko-opiskelua toisena ajankohtana. Tulos on oppimisprosessin näkökulmasta mielenkiintoinen ja se selittyy mahdollisesti tehtävien kuormittavuudella tai niihin kuluvalla ajalla. Mikäli osallistuja arvioi, että kyselyyn vastaamiseen ei mene pitkään hän suorittaa sen heti, kun taas pohdintatehtävän kohdalla saatetaan kokea, että tehtävä vaatii syventymistä ja siten enemmän aikaa, jolloin tehtävien teko keskeytyy helpommin. Ajan ja kuormittavuuden välillä on havaittu yhteys MOOC-ympäristössä (Champaign ym., 2014). Keskeytys pohdintatehtävissä saattaa johtua myös siitä, että asioita halutaan pohtia ja ”jättää hautumaan” ennen vastaamista, mikä voikin oppimisen kannalta olla osoitus niin kutsutusta hedelmällisestä ponnistelusta (engl. *productive struggle*, ks. Champaign ym., 2014 ja Gardner & Brooks, 2018, s. 160). Videoluentojen keston ja keskeyttämisen välinen yhteys puoltaa kuitenkin sitä, että keskeyttämisen taustalla saattaa olla tehtävän kuormitus tai siihen kuluva aika. Tulosten perusteella mitä pidempi videon kesto on, sitä todennäköisemmin yhtenäinen opiskelu keskeytyy. Lyhyen videon katsomisen jälkeen osallistuja siirtyy katsomaan seuraavaa videota, kun taas pidemmän videon kohdalla todennäköisyys tehtävien tekemisen keskeyttämiselle kasvaa. DeBoerin ja Breslowin tutkimus (2014) osoitti, että MOOC-kurssissa menestymistä ennakoi paremmin kurssivideoiden välissä tapahtuvat aktiviteetit kuin itse kurssivideoiden katseluun käytetty aika.

Mikäli tämä pitää paikkansa, ja katselun keskeytyksen jälkeinen aika hyödynnetään hedelmällisiin oppimisponnisteluihin, voi tällaisessa ympäristössä tapahtunut katselun keskeytys olla signaali ”hyvästä” oppimisesta. Aihe edellyttää lisätutkimusta. Opiskeluajan löytäminen suhteessa tehtävien työmäärään on tunnistettu verkko-opiskelun keskeiseksi haasteeksi lukuisissa muissa tutkimuksissa (Rogers, 2000; Berge 2002; Brycki & Dudt, 2005; Muilenburg & Berge, 2005; Ali & Magalhaes, 2008). Myös tutkittavassa täydennyskoulutusohjelmassa osallistujat mainitsivat kiireen tai ajanpuutteen keskeisimmäksi syyksi siihen miksi täydennyskoulutuksesta opitut tiedot ja taidot eivät aina päädy hyötykäyttöön (Koponen, Löfström & Portaankorva-Koivisto, 2020).

Vaikka osallistujien kokemaa kurssikuormitusta tai tehtävien työmäärää ei lokitiedoista voida suoranaisesti nähdä, voimme tarkastella osallistujien eroja tehtäviin sekä verkkokurssiin käytetyssä ajassa. Tulokset osoittavat, että tehtäviin käytetty aika kumuloituu verkkokurssiin käytetyn kokonaisajan kanssa. Kun nopeassa tahdissa etenevä suorittaa nopeasti kaikki tehtävät on verkkokurssiin käytetty kokonaisaika merkittävästi pienempi kuin toisessa ääripäässä. Hitaammin etenevillä verkkokurssin käytetty kokonaisaika oli enimmillään viisinkertainen nopeisiin suorittajiin nähden. Käytännössä tämä tarkoittaa, että siinä missä nopeasti etenevä suorittaa verkkokurssin esimerkiksi kuukaudessa toisen kurssisuoritus kestää viisi kuukautta. Tätä kautta voidaan melko luotettavasti päätellä, että tehtäviin ja verkkokurssiin käytetyillä ajoilla on todennäköisesti yhteys koettuun työmäärään ja kuormittavuuteen.

Tehtäviin ja verkkokurssiin käytetty aika saattaa heijastella myös erilaisia tapoja opiskella. Opiskelulle saattaa löytyä aikaa harvemmin ja se voi olla luonteeltaan pohdiskelevampaa, kun taas toisella on mahdollisuus edetä kurssitehtävissä nopeaan tahtiin. Kun otetaan huomioon aiemmin kuvattu tulos, jonka mukaan todennäköisyys yhtenäisen opiskelun keskeytymiselle kasvaa osallistujan käyttäessä enemmän aikaa tehtävien tekemiseen, on mahdollista, että pohdiskelevampi oppija kokee tehtävien olevan yksi toisensa jälkeen kuormittavia, jolloin verkkokurssin suorittaminen näyttäytyy työläämpänä kuin muille osallistujille. Näin ollen kirjautumismääristä, -ajankohdista ja tehtäviin käytetyistä ajoista suhteessa muihin osallistujiin voi olla mahdollista tunnistaa potentiaaliset kurssipudokkaat. Esimerkiksi jos osallistuja käyttää suhteettoman paljon aikaa yksittäisiin tehtäviin ja hänen on lisäksi vaikea löytää aikaa opiskelulle, on opiskelu todennäköisesti satunnaista pikemmin kuin säännöllistä, tehtävät saatetaan kokea työläinä ja lisäksi verkkokurssisisällöt saattavat unohtua ja niihin palaaminen vaikeutuu. Vastaava ilmiö on tunnistettu kansainvälisissä tutkimuksissa: mitä pidemmäksi verkkokurssin suorittamisen kokonaisaika kasvaa sitä suurempi on keskeyttäneiden osuus (Nilsen, 2019).

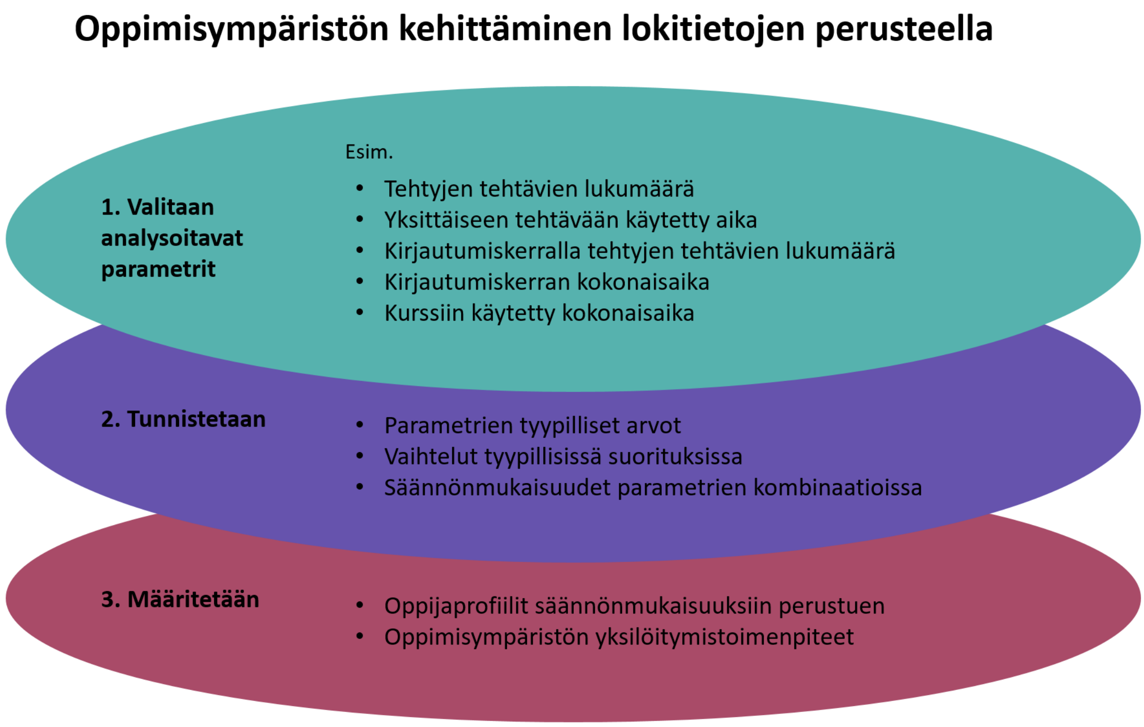
Tutkimustulosten perusteella täydennyskoulutus suoritettiin nopeammassa ajassa ja koulutuksen keskeyttämisprosentti oli pienempi teoriaosuudessa kuin soveltavassa osuudessa. Teoriaosuus sisälsi pääasiassa videopohjaisia luentoja ja automaattisesti tarkentuvia tehtäviä. Soveltavassa osuudessa osallistujan tuli tehdä omassa opetuksessa opetuskokeilu, raportoida siitä verkkoon ja keskustella kokeilusta muiden osallistujien kanssa. Teoriaosuus voitiin suorittaa esimerkiksi ilman omaa opetusryhmää, kun taas soveltavassa osuudessa oma opetusryhmä oli suorituksen edellytyksenä ja lisäksi aikaa tuli varata opetuskokeilun suunnittelulle, toteutukselle ja reflektoinnille. Teoriaosuus oli siis monessa mielessä kevyempi ja nopeampi suorittaa kuin soveltava osuus. Motivaatiota teoriaosuuden suoritukseen saattoi lisätä myös se, että opettajilla oli mahdollisuus saada todistus täydennyskoulutusverkkokurssin suorituksesta vain teoriaosasta. Vaikka edellytykset ja vaativuus olivat korkeammat soveltavalle osalle kuin teoriaosalle voidaan keskeyttämisprosenttia pitää molemmissa osuuksissa suhteellisen matalana. Tutkittavassa verkkokurssissa keskeyttämisprosentti oli teoriaosuudessa 19 % ja soveltavassa osuudessa 41 %.

Tutkimuksemme osoittaa, että lokitiedoista saatavan tutkimustiedon avulla voidaan puuttua useisiin verkko-oppimisen haasteisiin. Lokitiedoista saadaan tietoa oppijoiden yksilöllisistä eroista, joiden tunnistaminen on aiemmin tutkimuksissa todettu haasteelliseksi (Terras & Ramsay, 2015; Yu, 2015). Osallistujien yksilölliset erot opiskelurytmissä ja tehtävien tekonopeudessa voidaan tunnistaa ja samaa rytmiä suosivat voitaisi ryhmäyttää. Lokitiedoista saadaan tietoa tehtävien tekemiseen käytetyistä ajoista, joilla on todennäköisesti yhteys myös kuormittavuuden kokemukseen. Ajan löytämisen haaste ja kuormituksen kokeminen on todettu merkittävänä haasteena useissa tutkimuksissa (Ali & Magalhaes, 2008; Berge, 2002, Bryzycki & Dudt, 2005; Muilenburg & Berge, 2005; Rogers, 2000). Mikäli kuormittavuuteen tekijät ja ajankäyttöön liittyvät haasteet tunnistetaan, ne saattavat vähentää myös verkkokurssien ongelmakohdaksi tunnistettua keskeyttämistä (Nawrot & Doucet, 2014; Nilsen, 2019). Siinä missä Yu (2015) kuvaa verkkokurssien eduiksi niiden helppoa saatavuutta ja osallistujien mahdollisuutta palata opetussisältöihin uudelleen, lokitiedot tarjoavat täsmällistä tietoa väitteen totuusarvosta. Lokitiedoista saadaan selville palaavatko osallistujat sisältöihin uudelleen, mihin sisältöihin uudelleenpalaaminen kohdistuu sekä milloin ja miten opiskelu tapahtuu. Papachristos ym. (2014) esittävät huolen, että verkkokurssien opetussisältöihin tulisi kiinnittää enemmän huomiota. Lokitiedoista saadaan selville esimerkiksi mitkä opetussisällöt keskeyttävät herkimmin yhtenäisen opiskelun tai millaisia videoita ei tyypillisesti katsota kokonaan.

Tutkimuksemme luotettavuutta lisää se, että analysoitu lokitietoraportti sisälsi sekunnin tarkkuudella tiedon tehtävien suorituksesta. Tätä tietoa voidaan pitää itseraportointia huomattavasti luotettavampana, sillä tutkimukseen osallistuneiden olisi ollut käytännössä mahdotonta itse arvioida tehtäviin käytettyä aikaa tällä tarkkuudella. Sen sijaan on hyvä tiedostaa, että oppija on saattanut tehdä tai ajatella jotain muuta verkossa opiskellessaan, jolloin esimerkiksi tehtäviin kuluva aika kasvaa. Kvantitatiivisen tutkimuksen periaatteiden mukaan mitä suurempi tutkittava lokitietoaineisto on, sitä merkityksettömämpiä yksittäiset poikkeavuudet ovat. Lokitiedoista saatava data kasvaa nopeasti, sillä esimerkiksi tässä tutkimuksessa 58 osallistujan 50 tehtäväsuoritusta sisältää 2 900 datapistettä. Toisaalta kvantitatiivisen tutkimuksen periaatteet asettavat myös rajoitteita lokitietotutkimukselle. Esimerkiksi lokitietojen avulla voidaan tutkia osallistujien vuorovaikutuksen määrää ja kestoa verkkoympäristössä, mutta lokitiedoilla ei päästä tutkimaan kohtaamisten laatua. Tässä mielessä kvantitatiivisten ja kvalitatiivisten menetelmien yhdistäminen on hyödyllistä jatkossa. Esimerkiksi juuri vuorovaikutuksen määrä ja laatu on koettu verkkokursseilla haastavaksi (Berge, 2002; Muilenburg & Berge, 2005; Terras & Ramsay, 2015; Yu, 2015), joten lokitietojen osoittama vuorovaikutuksen määrä voisi olla lähtökohtana laadulliseen arviointiin.

6 Tulevaisuusvisio: oppimisprofiilit ja verkkoympäristön kehittäminen tekoälyn avulla

Kehittämistutkimuksen strategian mukaisesti kuvailemme seuraavaksi, kuinka tuloksia voisi hyödyntää verkkokurssien kehittämiseen. Esitämme vision, jossa lokitiedoista saatavaa tietoa hyödynnetään oppimisprofiilien tunnistamisessa ja oppimisympäristön yksilölliseen muokkaamiseen, ja johon myös tekoälyä sovellettaisi prosessin kehittämiseksi (Kuva 8).



Kuva 8. Lokitietojen avulla verkko-oppijat voidaan profiloida, jolloin verkkoympäristö voisi automaattisesti muokkautua oppijan yksilöllisten opiskelutaipumusten mukaan.

Ensimmäisenä käytössä olevista lokitiedoista valitaan analysoitavat parametrit. Parametrit voivat olla mitä tahansa verkkokäyttäytymiseen liittyvää, joista tietoa on saatavilla ja jotka jollakin merkityksellisellä tavalla liittyvät oppimistoimintaan. Parametrien valinnan jälkeen lasketaan niiden tyyppiarvot ja vaihtelut niiden välillä. Tässä tutkimuksessa analysoituja parametrejä on esitetty kuvassa 8. Näille parametreille laskettiin keskiarvo kuvaamaan tyypillistä suoritusta keskihajonnan sekä pienimmän ja suurimman arvon avulla.

Jo pelkästään tämän analyysin avulla voidaan tehdä päätelmiä tehtävien ja opetussisältöjen kehittämistarpeesta (Kuva 7). Jos järjestelmä ohjelmoidaan analysoimaan automaattisesti videoiden katselukertojen kestoa suhteessa videoiden kestoon, voidaan videoista tunnistaa ne, joita ei tyypillisesti katsota kokonaan. Tällöin oppimisympäristö voisi tiedottaa koulutuksen järjestäjää videoista, joita ei tyypillisesti katsota kokonaan. Mikäli koulutuksen järjestäjällä olisi tällainen tieto, videon ongelmakohtia olisi mahdollista arvioida. Video voidaan editoida, jakaa pienempiin osiin tai poistaa osuuksia. Niin ikään järjestelmä voisi ilmoittaa tehtävistä, joiden tekemiseen osallistujilta kuluu tyypillisesti suhteettoman paljon aikaa, jolloin tehtäväkuormittavuutta on mahdollista arvioida uudestaan. Myös erityisen kiinnostavista ja sitouttavista tehtävistä koulutuksen järjestäjän olisi hyvä olla tietoinen.

Seuraava askel onkin kääntää katseet opetussisältöjen sijasta osallistujien erilaisuuteen. Tämä tarkoittaa, että analyysi laajennetaan koskemaan oppijoiden välisiä eroja. Analyysi muuttuu hieman vaativammaksi, sillä tavoitteena on tunnistaa säännönmukaisuudet parametrien yhdistelmässä. Jokainen valittu parametri luo luokittelevan näkökulman oppijoiden erilaisuuteen. Tavoitteena on siis luokitella oppimistapoja ei vain yhden vaan kaikkien käytettyjen parametrien mukaan. Esimerkiksi kirjautumiskertojen lukumäärä ja ajankohta luovat näkökulmia opiskelun rytmittämiseen, kun taas tehtäviin käytetty aika luo näkökulman opiskelun pohdiskelevuuteen. Näin on mahdollista tunnistaa erilaisia oppimisprofiileja. Koska oppimisprofiilit määritetään aineistolähtöisesti, mitä enemmän käyttäjädataa on käytössä, sitä tarkemmin osallistuja luokittuu yhteen, sopivaan oppimisprofiiliin.

Tämän jälkeen verkkoympäristö voidaan ohjelmoida muokkautumaan yksilöllisesti verkko-oppimisprofiilia vastaavaksi. Wangin (2003) mukaan verkkoympäristön yksilöllistäminen on keskeinen verkko-oppimiskokemukseen vaikuttava tekijä (Wang, 2003). Esimerkiksi säännöllisesti kaksi tuntia kerrallaan opiskelevalle järjestelmä voisi muokata tehtävät sellaiseen järjestykseen, että yhden tehtäväpaketin tekemiseen kuluu keskimäärin kaksi tuntia. Osallistujalle, joka vuorostaan opiskelee tyypillisesti koko päivän kerrallaan, järjestelmä voisi kysyä toiveita taukojen määristä ja rytmittää tehtävät aikataulutoiveen mukaisesti. Teknisiä vaikeuksia kohtaavan tai eksyksissä olevan käyttäjän järjestelmä tunnistaisi epätyypillisistä klikkauksista, jolloin järjestelmä voisi tarjota teknistä tai navigointia-apua sitä kaipaavalle. Kun osallistuja käyttää tehtäviin enemmän aikaa suhteessa muihin osallistujiin, järjestelmä voisi automaattisesti keventää tehtäviä, jakaa niitä pienempiin osiin tai kutsua koulutuksen järjestäjän avuksi pohtimaan ratkaisua tilanteeseen. Rodríguez-Ardura ja Meseguer-Artola (2017) ovat esittäneet, että opiskelijan kognitiivisen kuormittumisen huomioiva oppimisprofilointi voisi auttaa tehtävien sopivassa palastelussa ja rytmittämisessä. Oppimisprofilointi voisi auttaa pienryhmien muodostamisessa, jolloin samaa rytmiä suosivat löytäisivät toisensa helpommin.

Rollin ja Wileyn (2016) katsausartikkeli tekoälytutkimuksesta oppimisen kentällä nostaa esille yksilöllisten ratkaisujen tarpeen sekä oppimisanalytiikan mahdollisuudet oppijan oppimiskokemuksen muovaajana. Myös Karsenti (2019) nostaa esille tekoälyn merkityksen esimerkiksi keskeyttämisvaarassa olevien oppijoiden tunnistamisessa. Ongelmakohta on kuitenkin se, että prosessi ei itsestään tuota uutta tietoa, esimerkiksi uusia verkko-oppimisprofiileja. Profilointi tarkentuu, mutta profiilit perustuvat alkuperäiseen aineistoon. Tietokone ei tee päätöksiä siitä mitä parametrejä valitaan, miten tunnistetut säännönmukaisuudet tulkitaan, miten oppimisprofiilit määritellään ja millaiset toimenpiteet oppimisympäristön muokkaamiseen määritellään. Jos prosessia vahvistetaan *koneoppimisella* niin tietokone voi oppia tekemään tämänkaltaisia päätöksiä (Dey, 2016; Jordan & Mitchell, 2015). Koneoppiminen on eräs tekoälyn muoto, jossa järjestelmä käyttää olemassa olevaa dataa myös omien algoritmien kehittämiseen (Chang, Cohen, & Ostdiek, 2018). Koneoppiminen soveltuu esitettyyn prosessiin (Kuva 8), sillä oppimisprofiilit määritetään aineistolähtöisesti. Mitä enemmän käyttäjädataa on, sitä paremmin tietokone oppii tunnistamaan säännönmukaisuudet, jolloin profilointi ja sen myötä mahdollisuudet oppimisprosessin räätälöintiin, tarkentuvat. Tämä voi osaltaan helpottaa opettajien täydennyskoulutukseen osallistumisen esteitä (ks. Lavonen & Mahlamäki-Kultanen, 2016) ja koettuja aikahaasteita (Koponen, Löfström & Portaankorva-Koivisto, 2020).

Koneoppiminen ei tapahdu ilman ihmisen apua. Koneoppiminen voidaan jakaa kolmeen muotoon; ohjattu oppiminen, vahvistusoppiminen ja ohjaamaton oppiminen, joissa kaikissa ihmisen rooli on erilainen (Dey, 2016). Ensimmäisessä vaiheessa järjestelmää kehitetään *ohjattuna oppisena* (engl. supervised learning). Tämä tarkoittaa, että koulutuksen järjestäjä tekee päätöksen ”oikeasta ratkaisusta” ja kone oppii sen myötä. Jos esimerkiksi järjestelmä tunnistaa, että osallistuja ei ole kirjautunut verkkoympäristöön kahteen viikkoon, järjestelmä kysyy mitä tilanteessa tulisi tehdä. Kun koulutuksen järjestäjä tekee päätöksen, että osallistujalle voisi lähettää sähköpostimuistutuksen koulutuksesta, järjestelmä oppii tunnistamaan missä tilanteissa sähköpostimuistutus on tarpeen. Seuraavassa vaiheessa sovelletaan *vahvistusoppimisen* periaatetta (engl. reinforcement learning). Tämä tarkoittaa, että järjestelmä toimii eri tilanteissa parhaaksi katsomallaan tavalla. Koulutuksen järjestäjän tehtävänä on valvoa tietokoneen tekemiä päätöksiä. Tämän vaiheen jälkeen järjestelmä toimii jo hyvin itsenäisesti. Viimeisessä *ohjaamattoman oppimisen* (engl. unsupervised learning) vaiheessa järjestelmän annetaan toiminnan lisäksi myös kehittää itse itseään. Koska esimerkiksi oppimisprofiilien määrittämisen taustalla olevat algoritmit ovat syntyneet aineistolähtöisesti, käyttäjädatan kasvun myötä järjestelmä oppisi parantamaan käytössä olevia profilointialgoritmeja. Tämä edellyttää algoritmien säännöllistä arviointia eettisestä näkökulmasta (Morley ym., 2021).

Lopulta keskeistä on, että lokitietoja ja käyttäjädataa hyödynnetään osallistujia kunnioittaen ja heidän oppimistaan edistäen. Mikäli tietoja hyödynnetään oppimis- tai muussa tutkimuksessa, on sen pohjauduttava tietoon perustuvaan suostumukseen (ellei siitä poikkeamiselle ole erityisiä perusteluja, ks. TENK, 2019). Eettisenä kysymyksenä nousee esille myös se, mitä luokitellaan tai profiloidaan ja miten se tapahtuu; puhutaanko esimerkiksi oppijoiden toimintaan perustuvista oppimisprofiileista (vrt. engl. learning profile esim. Kashive, Powal & Kashive, 2021) vai oppijaprofiileista (vrt. engl. learner profile, esim. Zou ym., 2017), joka voidaan ymmärtää oppijan henkilökohtaisiin ominaisuuksiin perustuvana luokitteluna. Lopuksi, vaikka esimerkiksi koneoppimisen seurauksena luodut profiilit ja niihin sovelletut oppimispolut perustuvat oppijan jälkiin verkossa, on mahdollista, että yksilö ei tunnista itseään ehdotetuissa valinnoissa, poluissa tai ohjeissa. Oppimistoiminta muodostuu hyvin monesta tekijästä, jotka voivat eri yksilöillä olla erilaisia, vaikka itse toiminta verkossa näyttää olevan samansuuntaista. Tästä syystä on tärkeää miettiä, minkälaisia muita oppimiskokemusta tukevia elementtejä verkko-oppimisympäristössä tarvitaan. Oppijoiden osallisuus oman oppimisprosessin muovaamisessa on (edelleen) keskeinen kysymys.

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Communicating mathematics through images: A multimodal study of Year One students' meaning-making when working with mathematics textbooks

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This article focuses on how Swedish Year One students (age 7–8) make meaning when working with images in mathematics textbooks. Images include textbook images, but also students' self-drawn images used as support for calculation. The focus was (1) what the images in the exercises were designed to offer (the designed affordance), and (2) what the students discovered when working with them. The data material consisted of video transcripts of 18 students working with subtraction exercises from mathematics textbooks. The results showed that the students sometimes discovered the designed affordance and sometimes did not. The students who discovered the designed affordance sometimes used the image when performing the calculations, while others did not. Some students expressed that images in mathematics textbooks are for those who find mathematics difficult, and that completing exercises without using the images was desired. The students' approaches to images were discussed in two specific cases: First, the students' desire to use mathematical symbols rather than images may lead to students not discovering the mathematics content that the exercise is designed to offer. Second, the use of mathematical symbols rather than images may lead to students not discovering themselves as mathematical individuals.

Keywords: Affordance; design; images; mathematics textbooks; meaning-making

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1 Introduction

We make meaning using many different images every day, including images found on our smartphones, in newspapers and books, on television, flyers, and traffic signs, to mention a few sources. So, images play an essential role in communication (Kress & van Leeuwen, 2006). Making meaning when encountering images is thereby of great importance. This article focuses on images of a specific kind: images in mathematics textbooks.

The visual has always played a role in school (Jewitt, 2014). Images symbolise the concrete world, while mathematical symbols (numbers and other symbols expressing mathematics operations, e.g., equals sign) represent the abstract (O'Halloran, 2005). Mathematics highly relies on visual information (e.g., Arcavi, 2003; Presmeg, 2006), and visualisation is central in mathematics education (Arcavi, 2003). However, research on visual learning in mathematics education is comparatively new (Presmeg, 2020). Cooper and Alibali (2012) suggested more research about how students interpret different kinds of visual representations.

An image is always coded and needs to be interpreted; it is perceived as transparent if the individual already knows its code (Kress & van Leeuwen, 2006). This is often implicit knowledge; people can rarely put images into words directly and explain how we interpret an image (Kress & van Leeuwen, 2006). This underpins the importance and complexity of studying images in educational settings. Kress (2010) formulated this by stating, “Images are (as yet, relatively) difficult to describe and analyse since, unlike writing, they are rarely composed of clearly discrete constituent entities, as words are” (47). Images are also used in different ways within a given textbook (Norberg, 2021). Thus, mathematics textbook images can be considered challenging to perceive, and since working with primary school mathematics textbooks generally involves working with images, this is crucial.

Furthermore, mathematics teaching largely involves working with mathematics textbooks. It is well known that the teacher plays an important role in teaching (e.g., Pansell & Andrews, 2017; Segerby & Chronaki, 2018). But, regardless of how the teacher stages their teaching, mathematics teaching involves students’ individual work with the textbook (Boesen et al., 2014; Österholm, Bergqvist, Liljekvist & van Bommel, 2016). Therefore, it is important to study students’ individual work with the textbook, which is the focus of this article.

Images in mathematics textbooks are often designed to offer specific content to guide the learner to a particular meaning-making (Norberg, 2019, 2021). This is because the mathematics textbook is a teaching resource aiming for specific learning, which means that the particular meaning being made while working with the textbook is important. This specific offer is theoretically understood as the designed affordance. The designed affordance points out the affordance the learner needs to discover in order to solve the exercise correctly. The concept of affordance derives from Gibson’s (1986) work and emphasises the potential meaning and the relation between textbook image and learner. Hence, it is important to highlight the designed affordance and the meaning made when studying students’ work with mathematics textbooks’ images to determine whether students discover the designed affordance. Suppose the student does not discover the designed affordance. In that case, this is of great importance for textbook authors, illustrators, publishers, and teachers as it would imply that the students do not complete the exercise as intended. Here, it is important to state that textbooks include images not related to calculation, images with a more decorative aim; those images are not in focus in this article.

The study presented in this article stems from an overarching interest in students' meaning-making when working with mathematics textbooks. A previous study (Norberg, accepted for publication) stated the importance of the images in the mathematics exercises in students' meaning-making. This directed my interest to analyse further the specific meaning students made when working with the images they encountered in the exercises. The study derives from an approach that focuses on meaning-making and multimodality (Kress, 2010; Selander & Kress, 2010). Multimodality recognises meaning in all resources for communication (modes), such as images, writing, and speech. It might be seen as a bit narrow in focus to select one of the modes included in the textbook, but this was made because the image mode appeared to be of particular interest. This can be understood as an in-depth study on one of the modes included in the multimodal text. This article will also discuss how working with mathematical textbooks' images might influence students' perceptions of themselves as mathematical or not.

This article's aim is to get a deeper understanding of students' meaning-making when working with mathematics textbooks, focusing on images connected to calculation in Swedish Year One textbooks. As the textbook is designed to offer specific content, the study will be related to that content, described as the designed affordance. Attention is directed to both the textbook's images and images drawn by students while working with the textbook. This is done with a delimitation to the mathematical content of subtraction. The following research questions are addressed:

- How do the students relate to images in their meaning-making?
- How do the students' meaning-making relate to the textbooks' designed affordances, focusing on images?

2 Literature review

Mathematics textbook images have been studied for a long time. In the research field, different concepts have been applied to images used for calculation in mathematics textbooks: pictures, illustrations, visual representations, and images. Sometimes, different categories of images are mentioned, such as iconic images and symbolic images. In this study, the concept of image is used because it aligns with the theoretical approach that understands image as one of the modes in a multimodal text. Images form a large part of the content of mathematics textbooks for younger students (Norberg, 2021). The exercises' mathematical content is often found in the image, while the image rarely answers a task (Norberg, 2021).

More than 40 years ago, researchers stated that students must learn how to read images (Levie & Dickie, 1973) and develop strategies to guess what answers teachers wanted (Byrne & Mason, 1976). Also, students need to perceive the eventual actions depicted in the images to use them (Campbell, 1978, 1979, 1981). Campbell found that students did not always find the mathematical content that was supposed to be seen in the images, and concluded that images could be supportive, but only when students understood them. Altogether, mathematics textbook images have been an object of study in mathematics education for a long time. These studies are still relevant in many respects, and the questions they ask remain valid.

Studies on students' work with images in mathematics textbooks have different focuses, including various image types (Presmeg, 1986), gender (Moser & Hannover, 2014), visually impaired students (Sedaghatjou, 2018; Emerson & Anderson, 2018), or eye-tracking connected to specific mathematical content (Jr-Hung Lin & Lin, 2014; Roy, Inglis, & Alcock, 2017). Several studies have used a multimodal approach (Freeman et al., 2016; Sedaghatjou, 2018; Teledahl, 2017; Wilson & Landon-Hays, 2016), and two of these also a social semiotic approach (Teledahl, 2017; Wilson & Landon-Hays, 2016), which is consistent with the study reported in this article. Wilson and Landon-Hays (2016) studied how six middle-school teachers used images to teach students aged 11–13 four subjects, of which mathematics was one. Their study used a case-study design and the following concepts from Kress and van Leeuwen's (2006) visual grammar to analyse the data: framing, vantage point, subject of image, orientation, background, and colour. The results showed that, in mathematics, humans were more seldom pictured than in the other subjects. Typical images were figures or shapes that the researchers described as generalised images, which were often black and white and most often on white backgrounds.

Students use mathematics textbooks in different ways (Teledahl, 2017) and the same affordances help some students and hinder others (Moyer-Packenham et al., 2016). Moyer-Packenham et al.'s (2016) conclusion derives from analysing 100 students age 3–8 using mathematical apps on touch-screen devices. Teledahl (2017) analysed students' (9–12 years old) writing when problem-solving. The results showed that the students used images, writing, mathematical symbols, and layout in different ways. Teledahl (2017) also found that images in mathematics textbooks sometimes contained contextual information with the problem to be solved (e.g., the farmer was illustrated even though she was not part of the calculation).

The use of images when working with mathematics and if they support the students is debated. Two recent Turkish studies showed that students are supported by images when solving problems (Ulu & Akar, 2016; Usta, Yilmaz, Kartopu & Kadan, 2020). Ulu and Akar (2016) studied 370 9–10-year-old students and found that the use of images meant that more students answered the problems, and that more students answered correctly. Usta, Yilmaz, Kartopu and Kadan (2020) studied 108 9-10-year-old students. An experimental group received image support and a control group were offered the problems without images. The results showed that the experimental group did better than the control group. This could be compared to Dewolf, van Dooren, Cimen and Verschaffel (2013), who found that images had no impact on students solving word problems. 635 total students, age 10–11, from Belgium and Turkey participated. The result showed that images did not have an effect on how the students solved word problems, and the authors discussed three possible reasons for this. First, the students may not have looked at the images. Second, the students may not have found support in the images. Or, third, the students may have found support in the images, but made no use of that support due to former experiences of solving word problems. In another study of visual representations in problem-solving, Cooper and Alibali (2012) compared 93 American higher education students' performance using diagrams and images. They saw that images helped some of the students, depending on the students' backgrounds.

Students use the images as support for solving tasks regardless of the design and the actual purpose of the illustrations. This was shown by Jellis (2008), where 128 students aged 7–8 years from three schools participated. The students had difficulty deciding whether the information in the illustration was relevant or irrelevant to solving the task. This meant that the information in the illustrations sometimes misled the students.

Students have greater use of images they draw themselves than those offered in the problems, Ulu and Akar (2016) stated. Teledahl (2017) found that the majority of students drew images when problem-solving. Freeman, Higgins, and Horney (2016) saw that younger students drew images more often than older students, in a multimodal approach studying 42 students age 8–13 writing mathematical notes using digital technologies. Moreover, Teledahl (2017) categorised the self-drawn images as either iconic (e.g., hens or pigs) or symbolic (e.g., dots or lines). The images were often complemented with numbers and words, and sometimes a sequence was drawn. The result also showed that the problem-solving often started when the student began drawing the image; sometimes, the students manipulated the image by erasing or crossing out objects. Almost a third of all students used images as part of their answers, rarely without complementation from other resources such as writing and mathematical symbols. Students drawing images were also studied by Jones (2018). This study focused on higher education students' prototype images using 205 surveys and 23 interviews. The prototype images were useful but needed to be complemented with other images or nonvisual representations. Jones also problematised that a prototype image may give a generalised view of the mathematical content while a more complex image rather makes perception harder, as it contains a lot of information.

Similar to the present study, a study of more than 400 Japanese students, age 6–12, interpreted images showing different subtraction situations (Kinda, 2010). Subtraction as a mathematical operation (understood as "something is taken away") was easier for students to recognise than a mathematical operation recognised as two amounts being compared. The comparison situation tended to be interpreted as a form of the taking-away situation.

In sum, textbook images have long been an object of study. There are various types of research on the subject. Studies have shown that images are essential when working with textbooks but that they are complex to perceive and that the same image can be helpful for some learners but not others. There are also studies showing that images do not impact students' work. This article will contribute new knowledge on how students make meaning when working with images in mathematics textbooks. With a deeper understanding of students' meaning-making, young students' perspective on the use of images in mathematics textbooks could be broadened. Thus, mathematics education with a focus on images could be developed.

3 Conceptual framework

The study reported in this article is based on a multimodal design theoretical approach (Selander & Kress, 2010), which refers to the social semiotic field (see, for example, Kress, 2010) where meaning-making is essential. Meaning-making is an activity in a social and cultural context in which an individual seeks to understand the world (e.g., Kress, 2010; Selander & Kress, 2010). Meaning-making is always multimodal, implying that communication is made through various modes (e.g., speech, gesture, text, and image). Modes are understood as resources for communication and meaning-making (Kress, 2014). All modes offer potential for meaning-making (Jewitt, 2016; Kress & van Leeuwen, 2001), and "different modes offer different potentials for meaning-making" (Kress, 2010, p. 79), with advantages as well as limitations (Jewitt, Bezemer, & O'Halloran, 2016).

Mathematics has been claimed to be a multimodal subject (O'Halloran, 2005), a fundamental point of departure for this article. The subject has a long history of using various modes for communicating mathematical content. For instance, specific mathematical content can be communicated in different ways, such as using a bar chart, writing, or mathematical symbols in a diagram. As mentioned in the introduction, this article reports on a multimodal study in which particular focus is given to one mode, rather than a study in which only one of the modes is studied. This means that the analysis focused on how students made meaning when working with mathematics textbooks concerning the image mode. This study also, as mentioned earlier, analyses the part of teaching that involves students' individual work with the mathematics textbook. This does not mean that the teacher's important role is neglected, but that the student's individual work is given attention. The intention is not to provide a micro-level analysis but rather to study how students work with selected exercises in a mathematics textbook focusing on the images. To understand students' working with textbook images, the concepts of meaning-making, design, and affordance will be described in the following paragraphs.

Meaning-making is a creative activity in which the individual redesigns already existing representations (Selander, 2017; Selander & Kress, 2010). In the present study, meaning-making occurs when the students work with the textbook images. The textbook images, when redesigned by the students' meaning-making, produce new representations. The students' meaning-making is always new and can never be a replica of the textbook author's meaning-making of the same content (Bezemer & Kress, 2010). The present study focuses on students' meaning-making when working with textbook images and relates them to the images designed affordances. Designed affordance is defined as the affordance needed to solve the exercise so that the mathematical content focused in the exercise is discovered.

The design concept concerns how teaching resources are designed and how individuals are involved in designing their learning situations (Selander & Kress, 2010). Design refers to both objects and conditions for communication and is understood as a communicative process in which the individual, through her involvement, depicts representations (Kempe & Selander, 2008; Selander & Kress, 2010). In this article, the concept of design refers to how the textbook is framed and the specific affordance, the designed affordance, that the student is intended to discover.

Affordance is considered a communication resource and the potential meaning that can be created (Gibson, 1986). In the present study, interest is directed to both the textbook's designed affordance and the affordance the students discover. Danielsson and Selander (2016) described affordance as offered meaning potential, or opportunities and limitations. There may be more given affordances for an individual's meaning-making when working with a resource, but different individuals can also discover different affordances (van Leeuwen, 2005). For example, a chair is likely to be found as something to sit on, but also as something to use to lock a door. According to Selander and Kress (2010), there will always be new potentials waiting to be discovered. Jewitt (2016) stated that affordance is a controversial and debated concept in multimodal research because it may have partly different meanings. For this article, I focus on the dualistic significance of the concept of affordance. On the one hand, interest is directed to the images designed affordances and, on the other hand, the students' meaning-making.

To achieve an in-depth understanding of students' meaning-making when working with mathematics textbook images, the results will be discussed in relation to how the textbook can help shape students' perceptions of themselves as mathematical individuals.

4 Method

This article seeks to understand how students make meaning when working with images in mathematics textbooks. A previous study (Norberg, accepted for publication) suggested that images were vital in the students' meaning-making. Images play a crucial part in students' meaning-making when they work with their textbooks. To study this, the images' designed affordances (the potential meaning-making the student needed to discover to solve the exercise correctly) were analysed and the students' meaning-making. Various types of data were used: textbook exercises, video transcripts in which students work with the same exercises, and the students' own representations (i.e., their answers to the mathematics exercises). In this section, data collection and the framework for analysis will be described.

4.1 Collection of data

18 individual video transcripts of Year One students (age 7–8) were collected to study their work with mathematics textbook images. The school was chosen from a convenience sample and is located in a medium-large city in Sweden. All of the students whose caregivers gave consent for inclusion in the study participated. Before data collection, I spent one week in the class, getting to know the students, so that the presence of a new adult would not feel uncomfortable for them. I have 12 years of experience as a compulsory-level teacher, which made interactions with the students quite natural. The aim was to study students in a situation that was as realistic as possible. Most natural would have been to study students in the classroom, during mathematics lessons. Since mathematics textbook work is often individual and silent, recordings of entire classes would provide little insight into the meaning-making of individual students. Therefore, I sat with one student at a time, in a room next to the classroom, in order to focus on the students' meaning-making in detail. This approach allowed me to watch the student working and ask the student follow-up questions.

The video material consists of 450 minutes of film, approximately 25 minutes per student, ranging from 19 to 44 minutes. The videos were recorded using a tablet. I chose a tablet because this was familiar to the students, who used tablets in their everyday learning. Therefore, the presence of a tablet did not draw any particular attention from the students; rather, it provided an undramatic way of documenting the students in their meaning-making. The tablet was placed obliquely above the student who sat at a table beside me. This allowed both the student, the textbook, and me to be seen in the image. The student started working on the exercise on her own. After some time, to understand how the student made meaning from the exercise, I asked questions of an investigative nature, such as "Can you tell me how you went along on this side?", "How did you know how to work with this exercise?", or "I saw that you did something with this image here, can you show me?" If the student had difficulty getting started with the exercise, I provided support in the form of questions such as "Can you use the images to solve the exercise?" or "Why do you think these dots are here?" (while pointing to the image in the task).

The textbook series used in the studied class is well known and used across Sweden. It consists of two levels: *Favoritmatematik* (Favourite Mathematics) *1A* and *1B* (Ristola, Tapaninaho, & Tirronen, 2012a, 2012b) and *Mera favoritmatematik* (More Favourite Mathematics) *1A* (Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima, 2013). *Mera favoritmatematik* is considered a more challenging textbook series than

Favoritmatematik. The exercises were chosen based on the results of a quantitative study (Norberg, 2021) using the following criteria: They should address subtraction as an arithmetic operation, and the exercises should be commonly used and show breadth according to how the different modes were used. Also, the mathematical content should not be new for the students; therefore, no exercise from *Mera favoritmatematik 1B* was chosen. The exercises (Figures 2–8 in the results below) were colour-copied and handed out to the student, one at a time. The exercises are shown with the publisher's permission. For a definition of exercise and task, see Figure 1 below.

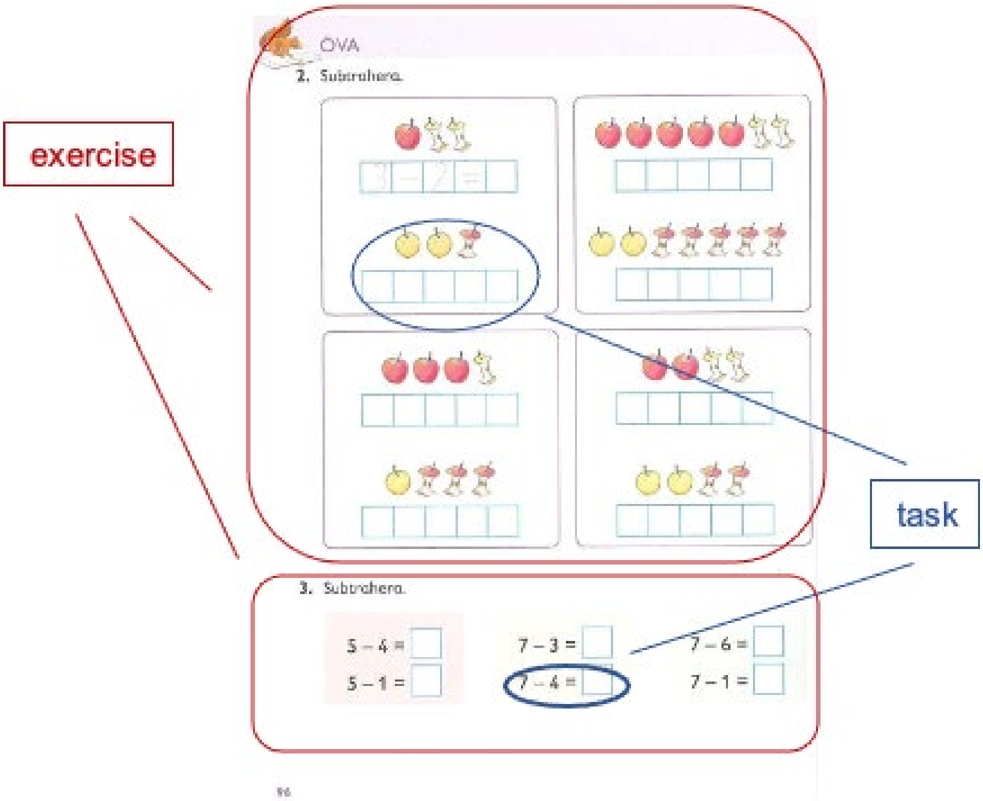


Figure 1. Exercise and task. Ristola, Tapaninaho, and Tirronen (2012). Favoritmatematik 1A. p. 96. Illustrator: Rajamäki, M.[1]

[1] All textbook pages are shown with the publisher's permission.

4.2 Framework for analysis

An analysis in three steps was conducted to understand the students' meaning-making. First, a textbook analysis of the seven exercises was made to capture the use of images and the affordances designed by textbook authors—that is, the intended affordances to discover a specific mathematical content. Then, the video material and the students' representations (on the copied textbook exercises) were analysed. Lastly, the designed affordance and the students' meaning-making were compared. The analysis evaluated each student's meaning-making when working with textbook images and subtraction as mathematical content.

4.2.1 Textbook analysis

In the first step, the textbook exercises were analysed to answer the following question: "Which images exist?", "How are the images used in the exercises?", and "Which affordances are designed into the exercises?". The last question refers to the affordances the students should discover to solve the exercises, the designed affordances. To answer how subtraction is addressed, the mathematical content of the exercises and the mode or modes that carry information for solving the exercise were studied. The teacher's guide was studied to obtain these answers. Here, how subtraction could be addressed in the textbooks will be clarified. Subtraction as an arithmetic operation can be addressed in a specific subtraction situation or not. Fuson's (1992) categorisation of subtraction was used to distinguish subtraction situations: categories included *subtraction as change/take from* (take away) and *subtraction as comparison*. Examples of this are shown in Exercise 1 (Figure 2); for instance, exercise 1A shows a *change/take from situation*, where the apples have been eaten. Exercise 1B involves subtraction without a specific subtraction situation; the information does not contain a specific situation but rather refers to subtraction in general. It is important to note that there are no exercises showing *subtraction as comparison* in these textbooks, and therefore no such examples are reported in this article.

4.2.2 Analysis of video transcripts and representations

In the second step, the video transcripts were first transcribed based on different modes and using three headings: speech, image, and body language. In the image column, the students' use of images, and cases in which they drew an image to support their calculations, were documented. The students' own representations were placed in front of me and supported the analysis. In the third step, I focused on whether the student solved the exercises according to the designed affordance, in terms of subtraction content. Also, how students expressed their approaches to the mathematics textbook images was documented. Finally, the transcripts were coded by reading through the material several times and highlighting using a colour code. The research questions guided the coding of the data. Then condensed meanings were summarised in a matrix, from which four main categories emerged.

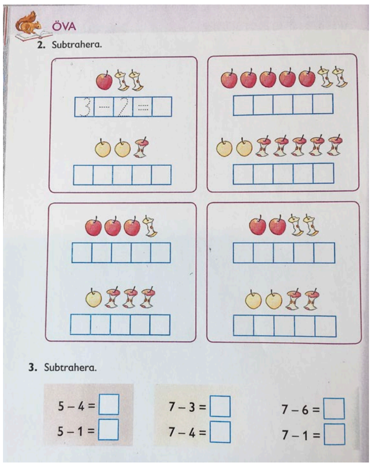
5 Results

The aim was to understand Swedish Year One students' meaning-making while working with mathematics textbooks focused on subtraction and images. First, a short description of the images in the chosen textbook exercises will be given, then the results will be reported of the students' meaning-making when working with those exercises.

5. 1 The textbooks' use of images and the images' designed affordances

The studied images are used in various ways, as (a) an event, (b) a resource for calculation, or (c) a guide box. There are also images showing context, images that are not connected to the calculation, and image-like elements in the exercises. These are described in this section (see also Table 1 below).

Some images show (a) *an event* (Exercises 1 and 2). In these images, something has already happened: The apples have been eaten, or the dots have been crossed out. The student needs to visualise the situation prior to the image to perceive the images according to their designed affordances; for instance, in the first task, in exercise 1, "First there were three apples, and then someone ate two of them. Now there is one apple left." Or, in exercise 2, "The crossed-out dots means that they are no longer there".



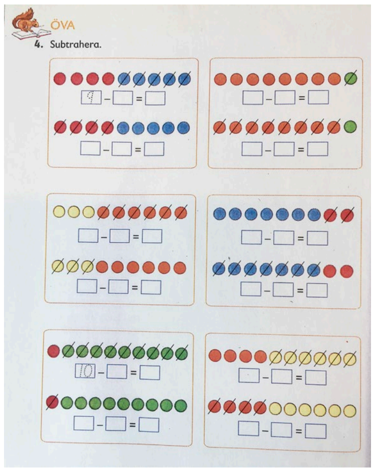
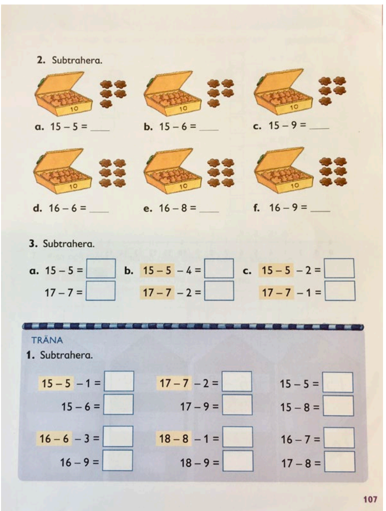
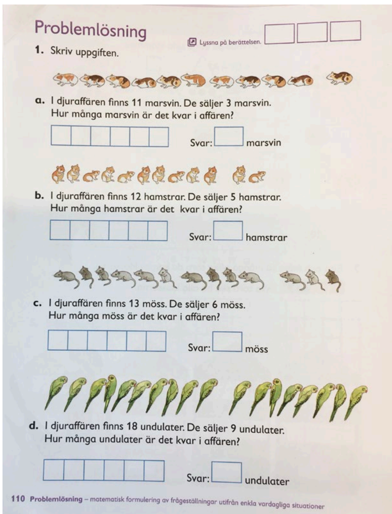


Figure 2. Exercise 1A and 1B. Ristola, Tapaninaho & Tirronen (2012). Favoritmatematik 1A. p. 96. Illustrator: Rajamäki, M.[2]

Figure 3. Exercise 2. Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima (2013). Favoritmatematik 1A. p. 140. Illustrator: Rajamäki, M.

Other images could be used as (b) *resources for calculation* (Exercises 3–7). These images can be used as support for solving tasks, but in contrast to the previous image type, these images are static, and nothing has happened. The images can serve as counters, for instance, the gingerbread cookies in exercise 3A, the various animals in exercise 4, or the pencils in exercise 5, below.





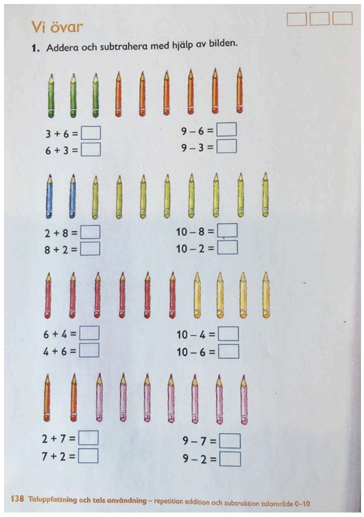


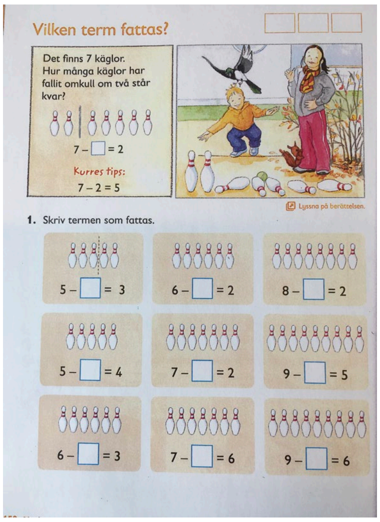
Figure 4. Exercise 3A, 3B and 3C. Ristola, Tapaninaho & Tirronen (2012). Favoritmatematik 1B. p. 107. Illustrator: Rajamäki, M.

Figure 5. Exercise 4. Ristola, Tapaninaho & Tirronen (2012). Favoritmatematik 1B. p. 110. Illustrator: Rajamäki, M.

Figure 6. Exercise 5. Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima (2013). Favoritmatematik 1A. p. 138. Illustrator: Rajamäki, M.

[2] All textbook pages are shown with the publisher's permission.

Two of the exercises also contain images in (c) *guide boxes* (Exercises 6 and 7) with information about how to solve the exercise. Here, the images provide task-solving support. They explain how to understand the content that follows. Exercises 6 and 7 also include *contextual images* that show the counters in a real-world situation.



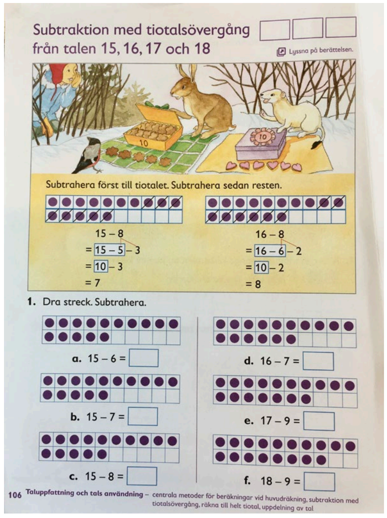


Figure 7. Exercise 6. Ristola, Tapaninaho & Tirronen (2012). Favoritmatematik 1A. p. 150. Illustrator: Rajamäki, M.
Figure 8. Exercise 7. Ristola, Tapaninah & Tirronen (2012). Favoritmatematik 1B. p. 106. Illustrator: Rajamäki, M.

Sometimes, the exercises contain *images not connected to the calculation*. This is shown in exercises 1 and 2 by the squirrel in the upper left corner, and in exercise 4 by the striped stick. There are also *image-like elements,* such as coloured or striped squares around tasks or lines. These sometimes point out one task (Exercise 6), two tasks that are connected (Exercises 1 and 2), an exercise (Exercise 3C), a guide box (Exercise 6 and 7), or indicate different columns (Exercise 7). Furthermore, exercise 1's heading includes a coloured square, and all exercises include squares for writing numbers and other mathematical symbols. Colour is sometimes used in the images to demonstrate the numbers in the tasks (Exercises 2 and 5). The table below summarises the image data.

Table 1. The textbooks' images and the images' designed affordances

Exercise	Textbook, page	The images' designed affordances	Image-like elements connected to the solving of the tasks	Other images not connected to the calculation
1A	*Favoritmatematik* 1A, 96	Subtraction event as change/take from	Striped squares around tasks	Square in the heading Squirrel
1B		No image or subtraction situation	Coloured squares around tasks	No
2	*Mera favorit-matematik* 1A, 140	Subtraction event as change take from	Striped squares around tasks	Squirrel
3A	*Favoritmatematik* 1B, 107	Resource for subtraction calculation as change/take from	No	No
3B		No image or subtraction situation	Coloured squares around part of tasks	No
3C		No image or subtraction situation	Coloured squares around part of tasks and tasks	Striped stick
4	*Favoritmatematik* 1B, 110	Resource for subtraction calculation as change/take from	No	No
5	*Mera favorit-matematik* 1A, 138	Resource for subtraction calculation as change/take from	No	No
6	*Favoritmatematik* 1A, 150	Resource for subtraction calculation and guide box as change/take from	Coloured squares around task and guide box	No
7	*Favoritmatematik* 1B, 106	Resource for subtraction calculation and guide box as change/take from	Square around guide box	No

5.2 The students' meaning-making

Here, the students' meaning-making when working with the images will be considered. Analysis produced four categories: (1) *The student discovers the designed affordance but does not use the image,* (2) *the student discovers the designed affordance and uses the image,* (3) *the student discovers the designed affordance and uses the image after support from the researcher,* and (4) *the student does not discover the exercise's designed affordance*. These four categories, and any associated subcategories, are described in the following.

5.2.1 The student discovers the designed affordance but does not use the image

In this category, the student does not use the images but instead uses mathematical symbols. This is shown below in relation to exercise 6, where the designed affordance is to practice subtraction as change/take from, which is shown in the guide box, and where the images can be used as resources for calculation.

> The student begins by reading the guide box and then writes a mathematical symbol in each empty box but makes no use of the images. I ask him if it is possible to use the image when doing the calculations. He answers: “Yes, you can use the image. Then you draw a line here, between these two” (draws a line in the right place between two bowling pins).

This is understood as the student already knowing the content to be taught, and therefore not needing to use the image as support for his calculation. From the information in the guide box, the student understood that the exercise focused on subtraction as change/take from and described to me how the image could support the calculations.

5.2.2 The student discovers the designed affordance and uses the image

Another way to make meaning when working with the exercises is to use the image for the calculation. This is done in two different ways, shown in (a) and (b) below. First, for example, in exercise 7, where the designed affordance is to practice subtraction as change/take from guided by the guide box and using the images as resources for calculation.

> The student reads the guide box. Then she looks at the first task and the number to be subtracted “15 – 6 =__”. She crosses out the correct number of dots, 6, counts the remainder, and writes the number “9” in the empty box.

So, (a) *the images are used as a resource for calculation;* the images support the calculation shown in mathematical symbols. The other way is shown in exercise 2, where the designed affordance is to understand the image as depicting an event of subtraction as change/take from.

> The student looks at the first task. She counts the crossed-out dots and writes “5” in the first empty box. Then she counts the remaining dots and writes “4” in the empty box after the equal sign.

In this exercise (b), *the images show an event,* they are designed to show a calculation that has already been made, shown in images. This differs from the example above, where the images can be used to do the calculation, but the calculation itself is demonstrated in mathematical symbols. So, the students need to be aware that images are used for different purposes.

5.2.3 The student discovers the designed affordance and uses the image after support from the researcher

In this category, the student does not discover the designed affordance by herself, but does so with support from the researcher. The support consisted of posing questions to the student to help the student realise that the images held information that could be used when solving the exercise. An example of this was when a student worked on exercise 4, where the designed affordance was to use the images as resources for calculating subtraction as change/take from.

> First, the student reads the text of task 1a aloud: "In the pet shop, there are 11 guinea pigs. They sell 3 guinea pigs. How many guinea pigs are left in the shop?." Then she solves the exercise using mental arithmetic, which is a bit challenging for her. She struggles but continues her work. When the first task is solved, she continues with the same approach while working with the second task. After solving that task, I ask her, "Why are there images here, do you think?" She answered, "Oh, why didn't I think of that! Ah, you can use them as a number line, here [points to the row of hamsters in task 1b] 1 to 12, because there are 12 hamsters."

So, with the help of the researcher's question, the student discovered the designed affordance, which she did not by herself. Without my question, the student did not find the images as supportive for the calculation. When she continued working on the next task in this exercise, she used the image to support her calculation.

5.2.4 The student does not discover the image's designed affordance

In the categories above, the students, in different ways and to different extents, discover the designed affordances. In this last category, the student does not discover the exercise's designed affordance on her own even after receiving support in the form of questions from me. This means that the student instead discovers other ways to solve the exercises. This is done in three different ways, shown below.

The first way of not discovering the designed affordance is when (a) *the student discovers an affordance in the image other than the intended one.* This is exemplified by a student working with exercise 1, where the designed affordance is to use the image as depicting an event of subtraction as change/take from. The student does not discover that the apples have been eaten and that she should count the remaining apples. Rather:

> The student looks at the task including an image showing five red apples and two yellow apple cores. She counts the red apples and writes the number "5" in the first box below the row of apples. Then, she writes "–" in the next empty box. After this, she counts the cores and writes "2" in the third box and an equal sign after that. Then she writes the answer "3" in the last empty box.

This means that the student interprets the apples as the minuend and the cores as the subtrahend. When she works with tasks that with this method get a negative number, she inverts the two numbers in the calculation (for instance, 1 – 3 is changed to 3 – 1); and says that "It sometimes could be like that".

The second way of not discovering the designed affordance is when (b) *the student discovers no support in the image but searches for it*. In those cases, the student understands that the image is supposed to support the calculation but does not know how. This is exemplified by the student below, working with exercise 5. The designed affordance is to use the images as resources for calculation of subtraction as change/take from:

> The student starts working on the tasks by counting the pencils in the image a few times. She frowns and looks up. After that, she puts her fingers up and uses them to solve the tasks. For instance, she puts nine fingers up when solving the calculation "9 – 6 =__". Then she puts six down, counts the remaining, and writes "3" on the empty line. Afterwards, I asked her, "Is it possible to use the pencils instead of the fingers for counting?" The student answered: "Yes, although I found no way."

This exemplifies that the student knows that the image should be supportive for the calculation, but the image's designed affordance does not prompt this for the student. She solves the tasks with support other than from the images.

The third way of not discovering the designed affordance is when (c) *The student discovers no support in the image but creates her own image and uses that image to solve the exercise*. For example, in exercise 3A, where the designed affordance is to use the image as a resource for calculation of subtraction as change/take from, the student solves the tasks in another way:

> The student first looks at the images in exercise 3A. After a while, he says: "This is a bit hard." I asked him: "Are you trying to use the image for counting now?" "Yes," he answered. Then he draws rows of lines in the margin of the paper and uses them for calculation. He begins with the first task, "15 – 5 =", counts lines up to fifteen and then faintly crosses out five lines, counts the remainder, and writes "10" as his answer.

So, this subcategory has similarities with subcategory (b) in that the student creates his own support. Of note is that the students who drew their own images did so in the margin of the paper and/or erased the image after using it. This is interpreted as drawing your own images is not a desirable way of solving the tasks.

5.3 Students' expressions about using images

Several students spoke about using the images to solve the exercises. Some students also expressed that those who use the images find mathematics hard, and that solving the exercises without using the images is the desirable way of solving them. For example:

> In response to my question, "Can you use the image to do the calculation?" one of the students answered, "Yes, but you don't need that if you are good at math."

This approach was also shown in action, when some students used the image to solve the exercise. When they worked with the exercise, I saw that they used the images, but when I asked how they proceeded with the exercise, they did not mention the image. Only when they were specifically asked if they used the image, did students say so. For instance:

> *Researcher:* How did you know how to do this exercise?
> *Student:* I read here (she points to the written instruction) and then I counted.
> *Researcher:* How did you do when you counted? Do you remember?
> *Student:* I counted in my head.
> *Researcher:* Mm. It looked as if you also used the images. Did you?
> *Student:* Yes, I did.

The same approach was seen with finger-counting by some of the students. They did not mention that they used finger-counting when I asked them how they worked. When I asked them about how they used their fingers, they explained that they used them for calculation and described how. Based on the above, using images when working with mathematics textbooks can be understood as non-desirable. An interpretation based on this is that the desirable way of solving the tasks is by using mathematical symbols.

5.4 Summary

In sum, the results show that the students sometimes discovered the designed affordance and sometimes did not. Some students discovered the designed affordance by themselves or with help from me. Some students used the images for calculation, whereas others used mathematical symbols instead. Some students did not discover the designed affordance, despite searching, and some drew their own images instead of using the image in the task. Lastly, some students expressed that using images is for those who find mathematics hard, and that solving the exercises without using the images is the desirable way.

6 Discussion and conclusions

The aim of the study was to understand students' meaning-making within the context of Swedish Year One mathematics textbook images, when practicing subtraction. The research questions were designed to determine how the students' meaning-making related to images and the textbooks' designed affordances, focusing on the image mode. The discussion will focus on the students' work with the mathematics textbooks' images, including the work of students who did not discover the designed affordance, and those who discovered the designed affordance but did not use the images. Also, the students' expressed verbally and practically that using images to solve the exercises is for those who find mathematics hard. This will be especially highlighted. This section will encompass a discussion about how the textbook can be part of shaping students' perceptions of themselves as mathematical individuals, or fail to do so.

Students who did not discover the image's designed affordance were conscious that the image was supposed to give support, but could not discover how. This could be compared to the finding of Jellis (2008), that students used the image to solve the task, whether it should be used or not. This allows the conclusion that the designed affordance is not communicated well enough, and the student is unable to make meaning towards the designed affordance. When the images do not communicate the designed affordance well enough, the students need support from elsewhere, typically from a teacher (or a researcher, as in here reported). This relates to Levie and Dickie's (1973) findings from almost 50 years ago, namely that students must learn how to read images. Thus, teachers need to ensure that students understand the design of the exercise when students work individually with the textbook.

The result also showed that some students chose to use mathematical symbols instead of images, when possible. This is interpreted as the students not needing the images' support; the student knows the mathematical content well enough. But this is argued to have consequences regarding how mathematics is viewed by students and, by extension, how a mathematical individual is viewed. This view of mathematics is also connected to the students' opinions about mathematical images, namely that they are for those who find mathematics difficult, which will be discussed below. Some students expressed that the desirable way of solving the exercises is to do so without using the images, because this signals good knowledge of mathematics. Teledahl (2017) found that most of the students drew images when problem-solving, and that this often supported their problem-solving. Thus, representing mathematics with images may be essential to many students, and should therefore be encouraged when working with the textbook. However, in doing so, the idea of mathematical symbols as the "desirable" mode needs to be challenged. Using mathematical symbols rather than images may obscure the exercise's mathematical content from the students, as this content is often described in the images. For instance, it is possible to describe a comparison using images, but not with mathematical symbols. Using mathematical symbols alone risks teaching students only subtraction, without including different kinds of subtraction situations, which could result in a merely basic understanding of the content. The students must discover the different kinds of subtraction situations, which are sometimes hard for students to recognise, especially the comparison situation (Kinda, 2010). Thus, teachers have significant responsibility to offer different ways of representing mathematics, as do the authors and illustrators responsible for textbook design.

Mathematics textbooks mostly require answers in the form mathematical symbols, which implies that this mode has a particular status. Mathematical symbols' special status can be related to Teledahl's (2017) study, where almost a third of the students used images in their answers. As an interpretation of the findings of Teledahl (2017) as well as the present study, some students would likely choose to answer textbook exercises using images whenever possible. Suppose that students feel the expectation to use mathematical symbols, but would nevertheless benefit more from using other modes (e.g., images). In that case, students may perceive other modes as a sign of failure. This may, in the long run, affect their perceptions of themselves as mathematical. Another example of using images is when students drew their own images. They did so in the margins and sometimes erased them. I interpreted this as showing that students' own images are not considered as showing their mathematical skills, and that mathematical symbols are the most desirable way to demonstrate the solutions. The students express, already by age 7–8, that mathematical symbols matter most. This can pose a risk. All students can discover themselves as mathematical and represent mathematics in different ways. But what happens to those who do this through modes other than mathematical symbols? Unfortunately, these students may decide to give up on mathematics; they may feel that the subject of mathematics is not for them, which would be devastating both for the individual and society at large. In conclusion, striving to use mathematical symbols rather than images may prevent students from discovering themselves as mathematical individuals.

In summary, images are always coded and can only be perceived by those who already know the code (Kress & van Leeuwen, 2006). Therefore, and in light of the results of the present study, images in mathematics textbooks should receive greater attention from teachers, textbook authors, textbook illustrators, and publishers. Textbook images are complex to perceive, and this complexity needs to be highlighted. Furthermore, the present study showed that, whenever possible, students chose to use mathematical symbols when solving the exercises, instead of images. As articulated by students, the hierarchy between images and mathematical symbols could imply that some students may miss out on learning situations if mathematics is taught in modes that they are not confident in. Therefore, this article makes an important contribution by highlighting the role of the image mode in mathematics textbooks and broadening current perspectives on the use of images in mathematics textbooks for young students.

7 Pedagogical implications

Concerning the claim that textbook images should receive greater attention in mathematics teaching, teachers are advised to teach students how to use the images (and other communication resources) in mathematics textbooks. Because images can be complex, students will need guidance. In other words, students should only work individually with mathematics textbook images once they know how the images should be used to discover the designed affordance.

The teacher plays a crucial role in guiding students' meaning-making with mathematics textbook images, as do textbook authors, illustrators, and publishers. When writing and developing textbooks, authors and illustrators need to be more aware of the difficulties that students experience in order to increase the supporting role of images and thus facilitate students' learning.

Also, textbooks do not invite students to draw their own images (Norberg, 2021). Rather, they prompt the student to respond with mathematical symbols. A mathematics textbook could encourage students to use multimodal representations by leaving space for them to draw an image as support, or by presenting images and mathematical symbols as equally valid ways to answer a given task. This will encourage students to perceive themselves as competent in mathematics.

Mathematics is considered a multimodal subject (O'Halloran, 2005); furthermore, students age 7–8 are at the beginning of their mathematics education. Therefore, they have the right to express mathematics in modes they feel confident using. Mathematics education strives to teach students to use mathematical symbols. Nevertheless, the youngest students should be allowed to use all modes and consider all modes of equal value in meeting students' needs.

The reported study also has a further implication, in that it challenges the distinct status given to mathematical symbols. One solution is to advocate a mathematics education that more often allows multimodal representations. This could, for example, be done by including exercises in which different modes can be used to answer tasks, or exercises that request the student to answer using various modes. These will help students discover themselves as mathematical individuals.

Competing Interests

The author has no competing interests to declare.

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