

# Promoting flexible mathematical thinking with growth mindset, deliberate practice, and serious games

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*Learned Custos Professor Minna M. Hannula-Sormunen, my esteemed opponent Associate Professor, Erin Ottmar, honorable members of the audience, I will now give a short introductory lecture on the topic of my dissertation.*

What is your earliest mathematical memory? Here's mine: When I was a little child, going to the market with my mom was very special. We would shop for the groceries daily, and Mom would tell me what the budget for the day was. We would always buy groceries for lunch and dinner first, then I would get to decide what I wanted for my breakfast. As we went from stall to stall, she would tell me how much each product cost and asked me to keep track of the total.

Sometimes, when I wanted a special side dish or a dessert for dinner, she would ask me "*Do you think we have enough cash for it today?*", and that was when I needed to really think if there would be enough money also for my breakfast. As a child, deciding what's for breakfast probably would be the biggest decision of the day. Little had I known how much math was involved in this decision.

As a kid, for me math was fun. However, as many others, there were also moments in my school years where I thought to myself "*When would I ever need this math?*". I am certain that I am not the only one. For many, mathematics often conjures images of complex equations, daunting examinations, and perhaps, memories of classroom struggles. Let's be honest: for many of us, our relationship with math is... complicated. But what if we could change this narrative? What if math became a realm of exploration, challenge, and even... fun? Can we make math popular again?

Today, I'll be sharing insights from my doctoral dissertation, which delves into the transformative potential of game-based learning in mathematics education. Specifically, we'll discuss the design principles for learning environments that promote flexible mathematical thinking. In this presentation, I'll highlight the Number



Navigation Game, showcasing its unique role in promoting adaptive expertise through deliberate practice among students. Additionally, we'll discuss the importance of cultivating a growth mindset.

## **Theory**

Traditional math classrooms often emphasize routine tasks. While these tasks lay a strong foundation, they limit the exploration of alternative solutions. This is an example of the typical routine tasks that are often presented in textbooks or students' homework. Such exercises are also supposed to be completed in a short amount of time. This approach can hinder students from building a comprehensive knowledge base, leading to mere routine expertise rather than adaptive expertise.

Adaptive expertise is the ability to apply knowledge in unconventional situations. In other words, it means the ability to solve non-routine tasks, dynamically navigate, and flexibly apply acquired strategies in new, different contexts. This is an example of the non-routine tasks in the Number Navigation Game.

However, the traditional classroom settings, dominated by textbooks and routinized exercises, often don't nurture this adaptive mindset. Instead, they often portray mathematics as a fixed domain with absolute rules.

Since 1992, Schoenfeld outlined and demonstrated a broad conceptualization of mathematical thinking. He suggested that the nature of the math classroom largely determined students' beliefs about that subject, and consequently, their beliefs would shape their behaviors.

For instance, he argued, students who believe that if they struggle with a task for a few minutes, or cannot solve it immediately, they would not have persisted even though they could have worked out that problem. Without preserving and reattempting to solve the task, students also limit themselves from discovering new connections or reflecting on different strategies, those that are less obvious or less straightforward. When the classroom emphasis is on finding the "one" or the "right" solution, it leaves little room for discussion about alternative strategies, learning from their mistakes, or reflections on other possible connections to the real world. Schoenfeld advocated for more non-routine problem-solving tasks, which could be described as "adaptive expertise" as part of mathematic instructional goals.

This brings me to my dissertation's focus: How can we bridge this gap and foster adaptive expertise?

Promoting adaptive expertise in elementary mathematics education has been a challenge. Questions loom about its feasibility for all students, given their varied mathematical competencies. How can we systematically present students with non-routine problem-solving tasks, those that allow them to engage in opportunities to think and reflect on possible alternative strategies on their own, and those that give them the chance to contemplate and struggle?

The answer might lie in the concept of deliberate practice. According to Ericsson et al., (1993), it's a form of practice that's not just repetitive but is also purposeful and systematic. It requires one to step out of their comfort zone, to engage deeply, and most importantly, to reflect and adapt. Deliberate practice has been proven to be the key component in explaining extraordinary development in various domains.

In math education, Lehtinen and his collaborators (2015) showcased the potential of embedding deliberate practice principles in the design of a game-based learning environment named Number Navigation Game. Through the game, they provided an open-ended learning environment, allowing students to explore various arithmetic strategies in a game-based setting.

The game captures this essence of deliberate practice, presenting students with novel contexts and immediate feedback. It's not just a game; it's a tool designed to stretch mathematical boundaries and promote the adaptive expertise we've discussed.

The Number Navigation Game stands as a collaborative effort of our mathematical research group. While prior research has focused on its development and explored its motivational aspects, my dissertation focuses on a distinct path: to explore the fascinating intersection of game-based learning environments, specifically through the lens of the Number Navigation Game, and uncover the design principles for learning environments that promote flexible mathematical thinking.

## **Aims**

Therefore, I have two main aims in my dissertation.

The first objective is how adaptive expertise can be promoted with deliberate practice, and whether this can be done by using the Number Navigation Game. I wanted to know how students interact with this game, not just as playful activities, but as a rigorous training ground for mathematical adaptability. How does gameplay translate to genuine mathematical growth? How does a student's interaction with

the game foster not just routine expertise, but also adaptive expertise? Can we find evidence of students engaging in deliberate practice through gameplay?

Since the nature of deliberate practice is demanding and takes place just beyond learners' abilities; therefore, it requires students' deep engagement, persistent attempts to improve performance, and a positive attitude towards unpleasantness when faced with difficult tasks. These are the distinguished traits of having a Growth Mindset. Given the association between a growth mindset and persistent learning behavior, the second objective of my dissertation is to explore ways to cultivate growth mindset in mathematics classrooms. This is vital for integrating game-based learning into conventional math classrooms and realizing the goal of adaptive expertise in mathematics.

### **Methods & sub-studies**

To accomplish these aims, my work comprises three sub-studies, which are divided into two parts. Part 1 involves the studies of the Number Navigation Game.

In Study I, the focus is on the game's development process, emphasizing students' experiences and interactions through four different versions of the game from 2014 to 2016 through a mixed-methods design. I was particularly intrigued by how different design choices impacted these experiences, striving to strike a balance between gaming elements and mathematical learning objectives.

The results indicated that improvements in the game's usability and clarity were successful in sustaining more beneficial and immersive gaming experiences. Additionally, it appears that although there is a distinct benefit in providing better aesthetics in game-based learning platforms, motivating elements don't always produce the expected effects on players' gaming experiences. Future work is required to understand the precise merit of extrinsic elements in maintaining players' enthusiasm, motivation, and situational interest in game-based learning environments. This study provided a foundational understanding of the Number Navigation Game's design intricacies and its alignment with educational goals.

Building on the foundation of Study I, Study II continued to focus on the game's experiential aspects, particularly in relation to deliberate practice. A core tenet of deliberate practice is acute concentration, especially when confronting tasks slightly outside one's current ability. This concentration, in the gaming realm, is often referred to as flow.

However, there's a nuanced difference. Flow represents a state where time might seem distorted when one engages in a pleasant experience on an optimal level of challenge, and one's sense of self might fade. Deliberate practice, on the other hand, demands conscious effort, and full concentration, and is often a challenging experience.

The hypothesis was that in a mathematical game-based environment like the Number Navigation Game, evidence of deliberate practice might manifest as steady but slow progress, combined with a modest flow experience and a heightened sense of challenge. Using gaming analytics of students' game performance in the Number Navigation Game, self-report gaming experience, and pre-and-post mathematical measures, we speculated that students engaging in deliberate practice would show a gradual improvement in game performance while experiencing a low-flow experience and an amplified feeling of challenge.

A sequence of growth mixture models was conducted to record patterns of game performance. The results indicate that the Number Navigation Game offers some participants opportunities to be involved in deliberate practice. Based on the four profiles, it is noticeable that the improving performers group consistently improved their gameplay at every measured point in spite of their subpar initial performance. This group also showed strong academic outcomes in other mathematical measures and recorded a lower-level flow experience. Based on these statistical results and theoretical considerations, the data provided direct evidence that the improving performers participated in deliberate practice in the Number Navigation Game.

Now we found evidence that for some students, interaction with the game fosters adaptive expertise, and they also deliberately engage in challenging practice provided by the game.

We also know this kind of practice is difficult to sustain for a long period of time because consistently pushing our boundaries is tough, but it's driven by a key belief: the growth mindset, which shapes how we tackle challenges.

In part 2 of my dissertation, I investigated the current state of growth mindset interventions in mathematics education through a systematic review.

A growth mindset is the belief that our abilities are malleable and can be improved with dedication and effort. This stands in contrast to the "fixed mindset", where abilities are perceived as innate and unchangeable. Students with a fixed mindset see mistakes as proof of their limited ability. Those with a growth mindset

embrace challenges, persist during setbacks, and see effort as vital for mastery. In contrast, fixed-mindset individuals avoid challenges and view mistakes as failures.

In math education, a belief in the “math brain”—as something you do or do not possess—is widely popular. “*I am not a math person*” is not only popularized but also capitalized widely. Unfortunately, research has shown that teachers and students are likely to feel that math achievement is due to inborn ability more so than other academic domains.

Revisiting the broad conceptualization of mathematical thinking put forward by Schoenfeld (1992), it is worth noting that teachers’ and students’ “fixed” beliefs about mathematics are not new.

For this reason, in Study III, a systematic review of mindset interventions in primary and secondary school math classrooms was conducted. This review examined the types of interventions that have been implemented in math classrooms that foster the mindsets of both teachers and students. It answers the question of when a growth mindset intervention works in a math classroom.

The findings indicate that math-focused interventions that incorporate a growth mindset directly into the content significantly improve student performance compared to general interventions. This highlights the need to address domain-specific beliefs, especially in math, where notions of an inherent “math brain” can impede learning, for both teachers and students.

This study also highlighted the importance of opportunities for students to have deep engagement with math content beyond a superficial level. Just telling students to “have a growth mindset” is not enough. Mindset interventions that happen in isolation from changes in classroom practices or other interactions often do not result in sustainable effects. This means more emphasis on problem-solving tasks that are multidimensional and allow for multiple solution paths. It is important that the learning environment encourages effortful practices, which means learning from mistakes and showcasing progress to students.

## **Conclusions**

My dissertation underscores the need for a paradigm shift in our approach to mathematics education. Traditional math tasks often prioritize rote learning and procedural mastery, this approach neglects the significance of open-ended, non-routine problem-solving which enables students to reflect, analyze, and explore alternative solutions. By grounding math education in the principles of deliberate practice, as

seen in the design of the Number Navigation Game, we can offer students more complex challenges that are suitably demanding, individually tailored, and intrinsically motivating.

This research serves as a bridge between the theoretical discussions around mathematical thinking and the tangible, practical applications offered by serious game-based learning. Through the design and analysis of the Number Navigation Game, we unearthed the potential of deliberate practice in fostering adaptive expertise. Moreover, the findings shed light on the importance of game design considerations, and how technological advancements can be harmonized with pedagogical strategies to enhance mathematical understanding.

Teachers' roles and mindsets are also critical for promoting flexible mathematical thinking and creating an open learning environment in which mistakes and errors are normalized and considered part of the learning process.

As educators, we must ensure that tools like the Number Navigation Game are not just standalone solutions but are seamlessly integrated into the broader educational landscape. This means creating supportive, mistake-friendly environments, fostering a growth mindset, and ensuring that both teachers and students perceive math as an explorative and dynamic discipline.

Looking ahead, there's much to explore. However, one thing is clear: if we can effectively combine the right tools, mindset, and challenges, it is possible to make mathematics popular and fun again.

As we wrap up today's discussion, I'd like to leave you with a quote that resonates with the heart of my topic and offers a perspective to reflect upon by mathematician George Polya: "*It is better to solve one problem five different ways, than to solve five problems one way.*" – Thank you.

*I respectfully ask you, esteemed Associate Professor Erin Ottmar as the Opponent appointed by the Faculty of Education, University of Turku for the public defense of my doctoral dissertation, to present your criticisms of my doctoral dissertation.*

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