What do teachers think about students' approach to collaborative problem solving? Comparison about teachers' beliefs at different school grades

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Abstract: This research shows a study involving students and teachers engaged in a collaborative problem-solving activity. We asked students to solve an OECD-PISA task and answer a metacognitive questionnaire that allowed them to review the entire problem-solving process. Teachers from different school grades replayed the same experience as the students. Then teachers answered another questionnaire designed with the aim of understanding how they imagined students behaved during the problem solving, both in the individual and collaborative phases. Preliminary results seem to show a meta-didactical conflict highlighted by discrepancy between teachers' beliefs about students' approach to collaborative problem solving and students' statements.

Keywords: problem solving, teachers' professional development, beliefs, OECD-PISA

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1 Introduction and theoretical background

Problem solving and, in general, problem handling, is one of the required competences for students in the 21st century (Niss & Højgaard, 2019). Metacognition is essential in problem solving because it is instrumental in building an appropriate representation of a given problem and monitoring the solution processes for solving it (Garofalo & Lester, 1985; Schoenfeld, 2016). It is also related to the decisions that a problem solver makes between different cognitive strategies when finding the solution (Casalvieri et al., 2024), decisions which relate to his/her personal beliefs and values (Radmehr & Drake, 2017). By beliefs, we mean "psychological understandings, premises or propositions about the world that are believed to be true" (Philipp, 2007, p. 259). Beliefs and values about learning, and problem solving are important in the encoding and retrieval of content knowledge (Radmehr & Drake, 2017). In addition, metacognitive experiences have an effect on decisions which students make in learning situations regarding effort allocation, time investment or strategy use (Efklides, 2008).

Collaboration can further the problem-solving process. The distinct ideas of group members can foster greater creativity and quality of solutions. PISA defines collaborative problem-solving competency as the "capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills and efforts to reach that solution" (OECD-PISA, 2017, p. 134). Students, interacting with each other, explain, argue and debate, and this can foster the development of critical thinking and problem-solving skills (Pruner & Liljedahl, 2021) in accordance with a socio-cultural approach (Vygotsky, 1978). Online environments can foster collaboration (Stahl, 2005), encourage the production of arguments by students (Albano et al., 2021a), and promote inclusion (Giberti et al., 2022; Capone & Spagnolo, 2019).

Some studies highlight that teachers at different school levels have different visions with respect to the construction of reasoning and problem solving (see, for example, Philipp, 2007). In addition, mathematics teachers operate with visions of teaching and reasoning for students that do not always correspond to student performance (Nathan & Koedinger, 2000). Since these views can have a significant impact on teachers' practices and students' learning experiences, we compared teachers' and students' beliefs and behaviors with respect to the same problem-solving activity, also interpreting the results considering the concept of meta-didactical conflict (Arzarello & Ferretti, 2021). Our aim is to highlight whether there is a common view (or not) between teachers and students. More in detail, the aim of this paper is to understand what primary and middle school teachers' beliefs are about high school students' approach to collaborative problem solving. In the problem-solving process, students actively construct personal beliefs mostly related to teachers' practices (Beswick, 2007). Based on this evidence, it would be possible to design effective teaching interventions.

2 Methodology

We asked grade 10 students (in Italy it is the second year of high school) to solve an OECD-PISA task. First, students had a few minutes to read the task individually. Then they discussed in an online environment, i.e., WhatsApp groups, to reach a shared solution. Finally, students individually answered a metacognitive questionnaire that allowed them to review the entire problem-solving process. We chose to use a chat for communication to encourage the production of written texts. Indeed, in the chat context, the exchange of text messages is favored, and this is a relevant aspect in learning since "writing promotes awareness and also supports control and reviewing processes" (Albano et al., 2021b, p. 33). In particular, we used WhatsApp because it is a popular tool among Italian teenagers and is an inclusive tool as it allows for sending texts, emoticons, audio, and pictures. Primary and middle school mathematics teachers replayed the same experience as the students. Then they answered another questionnaire designed with the aim of understanding how they imagined students behaved during the problem solving, both in the individual and collaborative phases. Thus, they, reflecting among themselves on the possible behavior of grade 10 students, could become aware of the competencies needed to be able to deal with collaborative problem-solving situations in a vertical curriculum perspective (e.g., argumentative and communication competencies).

The trial involved 44 mathematics teachers (from different Italian schools and regions) participating in a mathematics teachers professional development program: 36 primary school teachers and 8 middle school teachers. The participants worked in presence. They were divided into 8 random groups consisting of 5 or 6 members each. Each member of each group logged into their corresponding "WhatsApp Group" using a QR code. The activity included three phases: a resolution phase of the task, a discussion phase in small groups, and a collective discussion phase. Below, we describe the first two phases in detail, which are the focus of this paper.

2.1 Resolution phase of the task

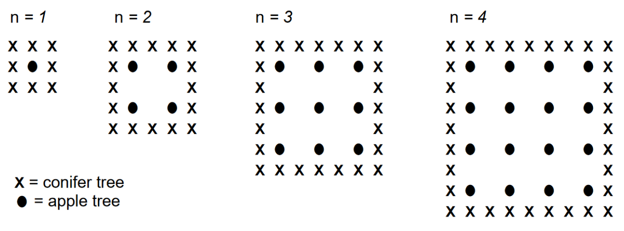
The purpose of this phase was to let teachers relive the experience of 57 grade 10 students from three different high schools. Students worked from home, divided into small groups and communicated with each other using WhatsApp groups. Students and teachers had to solve OECD-PISA tasks (Table 1). This is a version of the OECD-PISA tasks "Apple trees" (OECD-PISA, 2006).

Table 1. Version of task OECD-PISA "Apple trees", source https://www.oecd.org/pisa/38709418.pdf

APPLE TREES

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees:



Task 1: Complete the table.

n=	Number of apple trees	Number of conifer trees
1	1	8
2	4	
3		
4		
7		
20		

Justify your answers.

Task 2: Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees? Explain how you found your answer.

To simulate the situation experienced by the students, teachers were placed in the classroom "away from each other". They had the possibility of communicating with members of their group only via WhatsApp. Just as for the students, once tasks (available on a Padlet accessible via QR code) were displayed, each teacher was given about 3 minutes to read it on his/her own. Thereafter, members of each group had about 20 minutes to discuss within the WhatsApp group and answer the tasks. The delivery for each group was as follows: "*Now discuss within the WhatsApp group and answer tasks you have visualized on the Padlet. You have about 20 minutes. You can chat, share photos or audio. The purpose is to reach a final answer, shared and agreed by the whole group. That answer, written on WhatsApp, must also be posted on the Padlet*". Similarly, to what happened with the students, each participant was able to view the other groups' posts on the Padlet only after our approval of the answers, which took place after each group submitted its answers to tasks.

2.2 Discussion phase in small groups

The purpose of this phase was twofold: each group of teachers should ponder on what they have experienced together, and should imagine the process that led students to answer tasks. This phase began with a video projection of the Padlet with the answers to tasks from each teaching group. Members of each group took place near each other within the classroom in order to interact verbally and then they answered a questionnaire administered as a Google Form. The delivery for each group was as follows: "*You now have about 20 minutes to complete a questionnaire together. Imagine going through the process that led students to solve the task*".

The questionnaire was designed so that each group of teachers, dressing in the shoes of students struggling with solving the same task, could:

- imagine going through the process that led students to the resolution of the task, from the moment of individual reading to the collaborative moment of discussion aimed at finding a shared answer;
- reflect on any difficulties faced by students relative to task resolution (also related to text comprehension) and how students rated collaboration in the problem-solving process as a successful strategy.

The questionnaire was divided into 3 sections: "individual moment before collaboration", "collaborative moment", and "general reflections". Table 2 and Table 3 show (for comparison) the questionnaire questions posed to both teachers and students regarding the individual and collaborative moments, on which we focus in this paper. Questions 1, 3, and 5 were multiple-choice (max. 3 answers); questions 3 and 6 were Yes/No type; question 9 was Likert scale type (1 to 4); the others were completely open-ended. Teachers were shown neither student responses to the questionnaire nor student conversations in WhatsApp groups.

3 Preliminary findings

From the collected data, three possible mismatches can be investigated: between students' statements and students' actions; between students' actions and teachers' beliefs; between students' statements and teachers' beliefs. In this paper, we investigated the possible third mismatch. More in detail, we analyzed the data with the aim of understanding primary and middle school teachers' beliefs about grade 10 students' approach to collaborative problem solving. For this purpose, we analyzed teachers' answers to the questionnaire and compared them with students' answers to the similar questionnaire (see Table 2 and Table 3). For the open-ended questions, we identified categories-content throw Informed Grounded Theory (Thornberg, 2012). We analyzed the data by reporting the percentages of answers provided by both teachers and students. We first analyzed the data referring to the "individual moment before collaboration" (Table 2) and then to the "collaborative moment" (Table 3). In the analysis, we focused more on any mismatches between teachers and students' answers.

3.1 Individual moment before collaboration

Following Table 2 shows answers (compared) of teachers and students to the questions related to the individual moment before collaboration. We have highlighted those points where a mismatch between teachers and students' answers is evident.

Table 2. Teacher and students' answers to questions about the individual moment before collaboration.

Teachers		**Students**	
In your opinion, in the initial individual moment, when students read tasks by themselves,		**In the initial moment when you read tasks by yourself,**	
1. what elements did they focus on?		1.what elements did you focus on?	
100%	image	73,7%	image
87%	table	33,3%	table
25%	initial text before the image	35,1%	initial text before the image
37,5%	task 1	21,1%	task 1
12,5%	number	14%	number
0%	task 2	14%	task 2
2. were there any words or parts of the text that were not clear? Which ones?		2. were there any words or parts of the text that were not clear? Which ones?	
62,5%	Give meaning to n; for a number n of rows; the letter n as an indicator of a quantity; any n number		
12,5%	Legend reading	5,3%	Images that represented apples and conifers.
37,5%	The word "rows"	1,8%	Text before image
12,5%	The word "coniferous"	1,8%	The word "coniferous"
12,5%	The expression "faster"	3,5%	Second task
		87,6%	Text was clear
3. did they think about the solution on their own before discussing it with the group on WhatsApp?		3. did you think about the solution yourself before discussing it with the group on WhatsApp?	
75%	Yes	77,2%	Yes

4. from what elements did they start to look for the solution?		4. from what elements did you start to look for the solution?	
87,5%	Image	77,2%	Image
37,5%	Task 1; Table	43,9%	Task 1; Table

Table 2 seems to show that in the individual moment before collaboration, most students seem to have focused on the image of apple trees and conifers (item 1 in Table 2). However, they seem to have observed all parts of the question, including "question 2". This was not predicted by teachers, for whom the students' focus would have been exclusively on the graphic parts (images and table), and thus on "task 1". No teachers mentioned "task 2" as one of their answers. In addition, there seems to be a strong mismatch between answers given by teachers and students regarding "words or parts of the text that were unclear" (item 2 in Table 2). For many teachers, the presence of words in the text, such as "rows", could create difficulties for students who may not know the correct meaning. The biggest difficulty might be about the meaning given to the letter n as an "indicator of any quantity". Instead, almost all students stated that the "text was clear" and that they understood all the meanings of the terms in the text. Regarding items 3 and 4 in Table 2, however, there seems to be an alignment between teachers and students' answers.

3.2 Collaborative moment

Following Table 3 shows answers (compared) of teachers and students to the questions related to the collaborative moment.

Table 3. Teachers and students' answers to questions related to the collaborative moment

Teachers		**Students**	
In your opinion, in the moment students started to work in groups,		**In the moment you started to work in groups,**	
5. what elements of the question did they focus on?		5. what elements of the question did you focus on?	
87,5%	Image showing the arrangement of apple trees and conifers	84,2%	Image showing the arrangement of apple trees and conifers
75%	Table	73,7%	Table
37,5%	Initial text before the image showing the arrangement of apple trees and conifers	35,1%	Initial text before the image showing the arrangement of apple trees and conifers
37,5%	Task 1	52,6%	Task 1
37,5%	Number	33,3%	Number
25%	Task 2	22,8%	Task 2
6. did they share with the group the solution they thought about on their own in the individual moment?		6. did you share with the group the solution you thought about on your own in the individual moment?	
87,5%	Yes	77,2%	Yes (task 1)

		70,2%	Yes (task 2)
12,5%	Other: Not everyone has shared	5,3%	I did not share; Other: I thought it was wrong and did not say it (task 1)
		3,5%	I did not share (task 2)
0%	They did not think about a solution on their own	17,5%	I haven't thought of a solution (task 1)
		26,3%	I haven't thought of a solution (task 2)
7. What happened during working group? How did the students get to the solutions?		7. Describe what happened during working group and how you and your group got to the answers.	
62,5%	Shared activity; sharing of ideas; shared reasoning	79%	Shared activity; sharing of ideas; shared reasoning
25%	By testing; by trial and error	7%	By testing
12,5%	They trusted the most "good" student	14%	We chose the most correct solution
8. were there any students who changed their minds about the solutions during the working group on WhatsApp? If yes, what led them to change their minds?		8. were there elements that made you change your mind about the solutions during the working group on WhatsApp? Which ones?	
50%	Yes, after sharing the reasoning	28,1%	Yes, after sharing the reasoning
25%	Yes, getting led by the "better" classmates	0%	Yes, getting led by the "better" classmates
25%	No	71,9%	No
9. In your opinion, how effective did students feel the collaboration in arriving at the solutions? (Likert scale)		9. How effective do you think the collaboration was in arriving at the solutions? (Likert scale)	
0%	1 (Not at all)	1,8%	1 (Not at all)
0%	2	8,8%	2
25%	3	31,6%	3
75%	4 (Very much)	57,9%	4 (Very much)

Regarding the collaborative moment, the mismatch between teachers and students' answers is evident. For all teachers, students thought of an answer in the individual moment, and for almost all teachers, (i.e., 87.5%), students shared those answers with their groups (item 6 in Table 3). On the other hand, the data show that 15 students (26.3%) stated that they did not think of an answer to task 2 on their own and 10 students (17.5%) did not think of an answer to task 1 either. Remarkably, 3 students (5.3%), although they thought of an answer individually, stated that they did not share it. Even one student stated, "I thought it was wrong and didn't say it". For many teachers (25%), students reached a solution by "testing, trial and error" (item 7 in Table 3). For most students (79%), on the other hand, the solution was co-constructed, that is, it was the result of a "shared activity", "sharing ideas", and/or "shared reasoning". No stoudent reported that they trusted the best student in the group, rather that they chose together with the group "the most correct solution". A strong mismatch between students and teachers' answers also relates to item 8 of Table 3. For half of the teachers (50%), students were likely to change their minds from their original thought “after sharing reasoning” with their groups. Also, for many teachers (25%), students were “led by their better classmates”. This is not confirmed by students’ answers. Only 28.1% of them said they changed their minds during the discussion “after sharing the reasoning” with their groups, and none said they trusted their “better” classmates. This seems to explain the difference in teachers and students’ answers regarding the perceived effectiveness of collaboration (item 9 in Table 3). The majority of teachers (75%) perceived collaboration among students to be “very much” effective in finding a solution. The total of teachers’ answers was concentrated on Likert scale values 3 and 4. For students, collaboration was effective, but “very much” effective for only 57.9% of them. More than 10% of students’ answers focused on Likert scale values 1 and 2. This critical point would need verification with what actually happened in the WhatsApp chat.

4 Discussion and conclusion

Research on teachers’ beliefs is mainly due to two aspects (Skott, 2015): first, some researchers have tried to understand classroom processes from teachers’ perspectives and recommend enhancements (e.g., Nespor, 1987); second, some researchers have tried to understand teachers’ perspectives (e.g., Elbaz, 1983). Analysis of the questionnaire and comparison of students’ and teachers’ answers seem to show how teachers’ beliefs and students’ statements (that is, what students state, not what actually happen) in mathematics often do not overlap. There seems to be a conflict (a meta-didactical conflict in the sense of Arzarello and Ferretti, 2021) that, if not made explicit and overcome, can have serious consequences for the success of teaching/learning processes in the classroom.

Preliminary findings seem to provide insights that are helpful in designing future effective educational interventions that promote collaborative problem solving and support students’ metacognitive development. From this point of view, this study can lead to pedagogical implications. Indeed, teacher professional development based on student thinking can help teachers to create learning environments that foster improved student achievement (Philipp, 2007). However, in this paper, we considered teachers’ and students’ answers to questionnaires but did not consider what actually happened in WhatsApp groups. In the future, through the analysis of the chats, we will try to compare what teachers and students state in the questionnaire with what they actually did. In addition, we will try to extend this research to other students and teachers from different school grades. The aim will be to understand whether there are (and what are) differences between teachers’ and students’ beliefs and behaviors in different school grades regarding collaborative problem solving. This could contribute to research on the different views of teachers and students from different school grades with respect to the construction of reasoning (Nathan & Koedinger, 2000) in collaborative problem-solving processes.

Research ethics

Author contributions

UDI: Data curation, Formal Analysis, Investigation, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization.
CS: Conceptualization, Formal Analysis, Investigation, Methodology, Project administration, Resources, Supervision, Writing – original draft, Writing – review & editing, Validation.

All authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy.

Conflicts of Interest

The authors declare no conflicts of interest.

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