Pre-service teachers' views on dealing with students' heterogeneous needs in the mathematics classroom

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Abstract: Views of mathematics teachers can influence teachers' noticing, teachers' actions in the classroom, as well as the development of their professional knowledge. Consequently, and against the background of the demands related to dealing with heterogeneous learning prerequisites, teachers' views in this area and their specific self-efficacy are particularly interesting. For its high relevance for practice contexts, the empirical evidence base in this area needs to be broadened. This study responds to the related research need by exploring profiles of pre-service teachers' views on dealing with heterogeneous learning prerequisites and related aspects of self-efficacy. Moreover, interdependencies with global instruction-related views are examined. The results from a questionnaire study with 49 pre-service teachers indicate substantial variation of pre-service teachers' views, which should be taken into account in teacher education. Perspectives for follow-up research are discussed, in which the newly developed reliable scales can provide an access to investigating also the development of teachers' views.

Keywords: teachers' views, heterogeneous learning prerequisites, pre-service mathematics teachers, teacher education.

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1 Introduction

Challenges related to dealing with heterogeneous learning prerequisites in the mathematics classroom and corresponding needs of the students receive increased attention – in particular among pre-service and in-service teachers who often see themselves or expect to be faced with these challenges in classroom practice. For competently dealing with heterogeneous learning prerequisites in the mathematics classroom, teachers need competences of analysing classroom situations and related professional knowledge, as well as awareness, which can be assumed to be related to teachers' views (Kuntze & Friesen, 2018). Moreover, in pre-service teachers' profession-related learning, their views can be expected to play a moderator or filter role, which adds to their significance in the context of teacher education.

Consequently, research into teachers' views related to dealing with heterogeneous learning pre
requisites is needed. The design of profession-related learning opportunities, for instance, may respond to pre-service teachers' views and stimulate the views' further development in goal-oriented ways. Moreover, teachers' views can be expected to directly influence their actions and reactions in the classroom, together with their situation analysis (cf. e.g., Kuntze & Friesen, 2018; Kuntze et al., accepted), which is both meaningful for the quality of how heterogeneous learning prerequisites are dealt with. Somewhat in contrast with these potentially fundamental roles of teachers' views related to dealing with heterogeneous learning prerequisites, the empirical evidence base in this domain is still scarce. Consequently, this paper presents empirical results from a study which explores these views and is intended to derive implications for theory development, as well as the practice of teacher education.

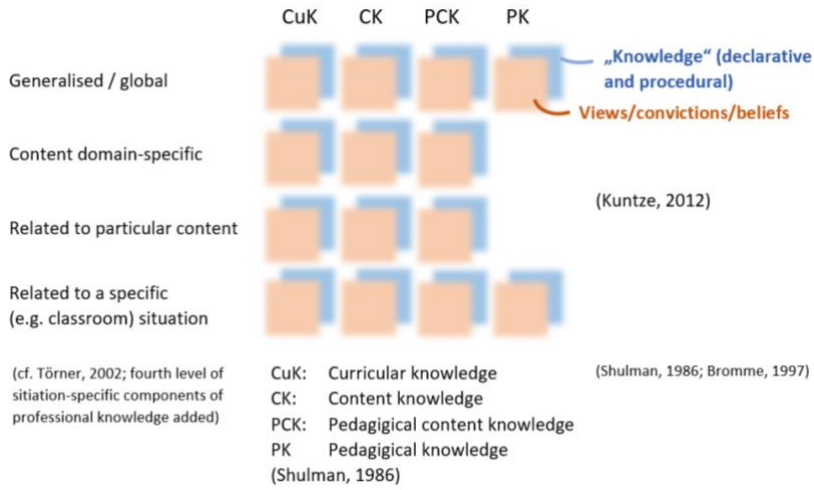
The following sections of the paper will give an overview of the theoretical background of the study (section 2), deduce the research aim (section 3), provide information about methods and sample (section 4), present the results (section 5), and discuss these in a concluding section (6).

2 Theoretical background

Dealing with **heterogeneous learning prerequisites**, i.e., the individually different prerequisites or individual conditions under which individual learning has to take place in different students, is challenging for mathematics teachers (Leuders et al., 2017): The focus on learning prerequisites and the individual thinking of the students, the analysis of learning material and tasks and related decision-making, as well as organisation aspects of orchestrating learning processes, including ways of dealing productively with mistakes (Oser et al., 1999) require a backing in specific declarative and procedural professional knowledge (Shulman, 1986; Kuntze, 2012). This includes corresponding views on teaching and learning of mathematics (Kuntze, 2012; Törner, 2002; Pajares, 1992). According to the model of components of professional knowledge this study builds on (Figure 1, Kuntze, 2012), such views can hardly be separated from other professional knowledge (Pajares, 1992), so that they are considered to be part of mathematics teachers' individual professional knowledge. Furthermore, the model distinguishes rather global views (such as cognitive-constructivist vs. direct-transmission views of teaching and learning

(Staub & Stern, 2002), for instance) and more specific views, which may be related, for example, to dealing with heterogeneous learning prerequisites in specific content areas or in specific classroom situations. This dimension from more global to more content-specific (cf. Törner, 2002) and situation-specific components of professional knowledge appears to be essential against the background of findings from Leinhardt & Greeno (1986), that teachers' professional knowledge and views are often structured in episodic ways, so that contextualised components may play a key role for some elements of professional knowledge. In empirical research concerning teachers' views, these different levels of globality respectively situatedness should hence be taken into account, as well as the degree of potential interrelatedness of situated components of professional knowledge (including views) with more overarching views. In the present study, for instance, well-known global instruction-related views have been included (e.g., Staub & Stern, 2002) together with potentially relevant views on the role of mistakes for developing students' mathematical knowledge (e.g., Kuntze, 2009). Moreover, the situated level can be addressed through analyses of the teachers' views on representations of classroom situations (cf. Buchbinder & Kuntze, 2018) in which dealing with heterogeneous learning prerequisites plays a role (cf. Kuntze et al., accepted, 2021). The expression "dealing with" is hereby understood in a basically neutral way, allowing for rather positive or negative individual views of teachers.

Figure 1. Overview model of components of professional knowledge (Kuntze, 2012)



Note. This model (Kuntze, 2012) is inspired by the levels of globality/situatedness by Törner (2002) and encompasses the spectrum between knowledge and views/convictions/beliefs (Pajares, 1992), for the domains of professional knowledge by Shulman (1986).

Even if research efforts have recently been more and more related to teachers' views on dealing with heterogeneous learning prerequisites (e.g., Bertram & Scherer, 2024), including related self-efficacy aspects (e.g., Winterle & Kampa, 2023), the exploration of different aspects of mathematics teachers' views and of the above-mentioned possible interdependences can make a substantial contribution to the description of predispositions for pre-service teachers' profession-related learning. Consequently, for exploring teachers' views related to heterogeneous learning needs, not only views on whether the requirement of equitable learning chances might imply rather refraining from an individualised and non-equal treatment in the classroom, or views whether heterogeneous learning prerequisites can enrich classroom interaction, for instance, are relevant. Beyond these, views connected to aspects of self-efficacy (Bandura, 1977; Maddux, 1991; Pekrun, 2006) specifically in dealing with heterogeneous learning prerequisites are in the centre of interest. According to research into self-efficacy (cf. e.g., Pekrun, 2006), higher specificity related to the requirements a self-efficacy construct is focusing on, leads to higher correlations with specific achievement variables in the corresponding specific domain. Focusing on specific aspects of self-efficacy in dealing with heterogeneous learning needs and developing corresponding questionnaire scales is hence needed.

3 Research Aim

According to the research needs outlined above, the following research questions are in the focus of this study:

(i) What views related to dealing with heterogeneous learning prerequisites of students do pre-service mathematics teachers hold, including corresponding specific aspects of self-efficacy?

(ii) Can interdependencies with other instruction-related views of the teachers be identified?

4 Methods and Sample

For exploring the research questions, a multiple-choice questionnaire was designed with newly developed scales for teachers' views on dealing with heterogeneous learning prerequisites, including views related to specific self-efficacy aspects in this domain. In line with the theoretical background introduced above and derived from that base, the implemented constructs were the following (cf. Kuntze et al., in press):

- **Equal-treatment orientation:** According to this view, all students should be provided with the same learning chances, which implies equitable treatment by the teacher – a view that may be seen as in conflict with adaptivity to individual learning prerequisites in the sense that learners are offered different learning opportunities, which might restrict their learning results in comparison to other students.
- **Heterogeneity orientation as sure-fire success:** This view expresses the conviction that a positive attitude towards heterogeneous learning needs almost automatically improves the quality of mathematics instruction even in learner groups with significant differences in learning prerequisites.
- **Adaptivity as teachers' obligation:** Teachers with this view see adaptivity as a task that requires the teacher necessarily to provide each student with task material that fits to her/his individual needs.
- **Heterogeneous learning needs as enrichment:** According to this view, heterogeneous learning needs are enriching for the mathematics classroom.
- **Preference for learner groups with few heterogeneous learning needs:** This view reflects teachers' preference for teaching learner groups with less heterogeneous learning prerequisites.
- **Self-efficacy in dealing with heterogeneous learning needs:** This construct addresses specific self-efficacy of the participants in dealing with heterogeneous learning prerequisites.
- **Self-efficacy (tasks and learning material):** This self-efficacy aspect focuses particularly on the aspect of designing tasks and learning material in line with requirements of heterogeneous learning needs.
- **Self-efficacy (analysing learning prerequisites):** This self-efficacy aspect concentrates on the requirements of analysing students' individual learning prerequisites.
- **Self-efficacy (organisation of instruction):** This self-efficacy aspect focuses on requirements related to organising mathematics instruction and to working forms in the classroom that fit to students' different learning pre-requisites.

The scales used a four-point Likert rating scale. Sample items and item numbers can be found in Table 2 in the results section, in which the reliability values of these newly developed scales are reported. These newly developed scales were combined with established instruments (Scales on the generalised/global level of the model in

Fig. 1 are shown in Table 1, e.g., cognitive-constructivist vs. direct-transmission views on the teaching and learning of mathematics according to Staub & Stern, 2002; moreover, views on dealing with mistakes according to Kuntze, 2009, were included), in order to be able to explore interdependencies between different views of the pre-service teachers, in particular across different levels of globality (Fig. 1).

Table 1. Scales related to global instruction-related views included in the study with references

Scale	Sample item	Reference
Cognitive constructivist view on teaching and learning	Students should first understand operations and procedures, before you spend a lot of time on practising them.	Staub & Stern (2002)
Direct-transmission view on teaching and learning	Students learn mathematics best from the presentations and explanations of their teacher.	Staub & Stern (2002)
Entity beliefs about teaching maths (barriers)	In mathematics there are always some students who just won't understand it whatever you do as a teacher.	cf. Stipek et al. (2001)
Entity beliefs about teaching maths (chances)	All students could be good at mathematics provided they worked hard.	cf. Stipek et al. (2001)

Note. The Table presents sample items and references for global instruction-related views scales.

Table 2 shows the newly developed scales and sample items.

A further questionnaire instrument focusing on teachers' views related to dealing with mistakes in the mathematics classroom was also included in the research design (Kuntze, 2009), as such views can be expected to be relevant in the context of dealing with heterogeneous learning prerequisites. This questionnaire had been developed based on theoretical and empirical results from Oser et al. (1999) and Barnet & Sather (1992), in particular. Due to length limits of this contribution, only selected key scales are introduced in the following and analysed in the results section:

- **Learning from mistakes as central for building up mathematical knowledge:** This view sees mistakes as necessary element in the process of building up mathematics-related knowledge.
- **Rationality behind mistakes:** According to this view, possible rational thoughts can or should be assumed behind mistakes.
- **Mistakes as deviating from the chain of thought:** Preference for avoiding a deepened exploration and discussion of mistakes for the fear that

this might deviate from the chain of thought of the lesson and confuse students.

- **Bermuda triangle view:** According to this view, mistakes need not to be discussed in the classroom if other students provide the correct solution.

49 pre-service teachers in their master's degree studies were asked to answer the questionnaire which included the multiple-choice sub-questionnaires introduced above. The pre-service teachers were participants of a seminar and the instrument was administered before the beginning of the seminar. The seminar started after the completion of the questionnaire.

In line with the explorative approach of the first research question of this study, the data on teachers' views on dealing with heterogeneous learning needs of the students was analysed with a cluster analysis (Ward Method), in order to be able to map potential differences in answering patterns. Moreover, the data was analysed for correlations with global instruction-related views of the teachers in order to explore interdependencies, including a focus on evidence whether the newly developed scales represent constructs of their own right in comparison with the overarching, global instruction-related views. The examination of these interdependencies can also contribute to the external validation of the newly developed scales.

5 Results

The first research question concentrates on views the pre-service teachers hold related to dealing with heterogeneous learning prerequisites. The new scales, developed under a continuous validity control by the first and second authors, turned out as reliable (see Table 2) and they could be largely reproduced empirically through a corresponding factor analysis. In particular, the items belonging to the scales could be located together in the factors.

For each scale related to dealing with heterogeneous learning prerequisites, the means and their standard errors are shown in Figure 2. Whereas the means indicate on-average approval of adaptivity as teachers' obligation, of heterogeneity orientation as sure-fire success, and of the view that heterogeneous learning needs are enriching for the mathematics classroom, equal-treatment orientation and preference for little heterogeneity were seen rather negatively on average. The

means of reported self-efficacy aspects turned out as non-positive, the ratings were close to the middle of the respective scales.

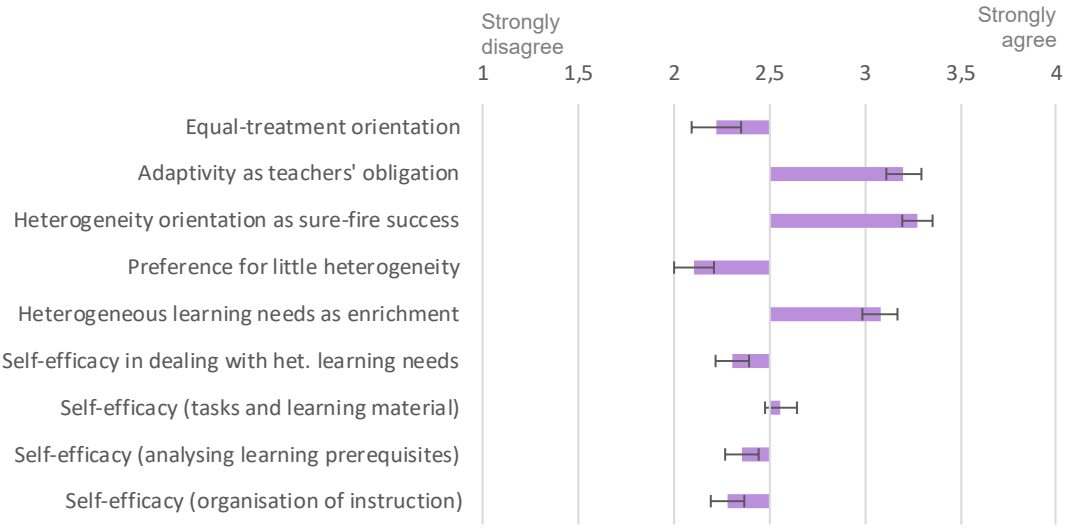
Table 2. Scales related to dealing with heterogeneous learning prerequisites

Scale	Sample item	Item number	α (Cronbach)
Equal-treatment orientation	All students should be treated equally by the teacher, so that everybody is given equal chances.	4	0.89
Heterogeneity orientation as sure-fire success	As soon as a teacher has a really positive attitude towards heterogeneous learning needs, the quality of mathematics instruction will improve also in learner groups with big differences.	4	0.77
Adaptivity as teachers' obligation	Every student should be provided by the teacher with tasks that fit to her/his individual level of ability.	4	0.86
Heterogeneous learning needs as enrichment	If students bring various learning prerequisites in a learner group, then this is an enrichment for the mathematics classroom.	3	0.89
Preference for learner groups with few heterogeneous learning needs	In comparison to classes with heterogeneous learning needs of the students, I would prefer to teach classes in which there are less such heterogeneous learning needs.	3	0.88
Self-efficacy in dealing with heterogeneous learning needs	I think I do not have difficulty in teaching mathematics in learner groups with heterogeneous learning needs.	4	0.80
Self-efficacy (tasks and learning material)	I can create or modify tasks so that every student can contribute to classroom interaction according to her/his learning prerequisites.	4	0.77
Self-efficacy (analysing learning prerequisites)	It is easy for me to notice learning needs of students precisely and to describe them, even if they are very different in a class.	4	0.84
Self-efficacy (organisation of instruction)	It is easy for me to organise mathematics instruction so that the working forms of the students fit to their different learning prerequisites.	4	0.76

Note. The Table shows sample items, item numbers per scale, as well as reliability values.

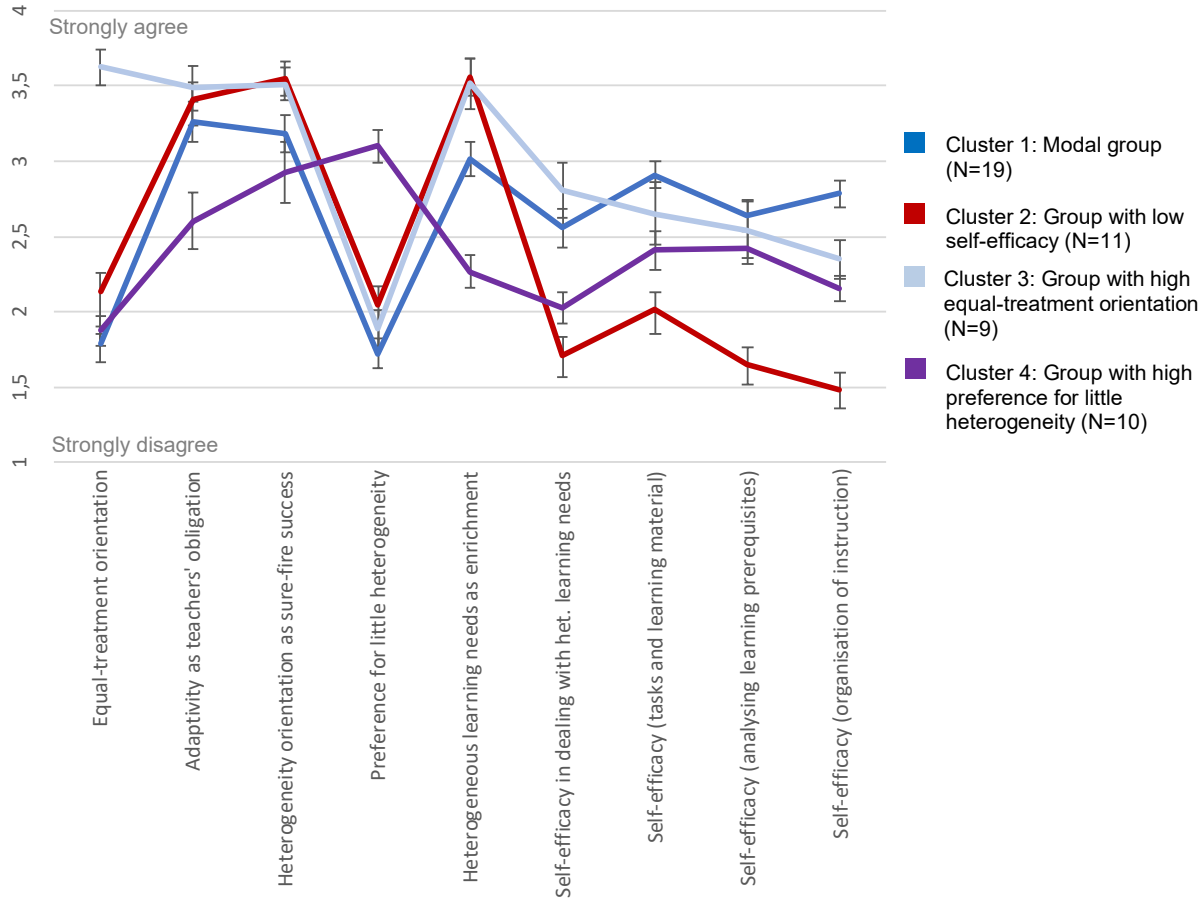
However, these means appear to hide variation within the sample, as the results of the cluster analysis show (Figure 3). A four-cluster solution fitted the data best and indicates groups of pre-service teachers which were labelled according to key features of the profile of their views as reflected in their answers. A **modal group** reflects mainly the qualitative pattern of the data shown in Figure 2, however with the tendency of slightly positive self-efficacy variables. A group labelled as **group with low self-efficacy** saw heterogeneous learning needs as even more enriching than the modal group but shows rather sceptical views in the self-efficacy domains, which are lower than all other pre-service teacher clusters. A relatively small group of pre-ser- service teachers stood out with their *positive* mean value in **equal-**

Figure 2. Pre-service teachers' views, means and their standard deviations



Note. The scale means suggest approval of adaptivity as teachers' obligation, heterogeneity orientation as sure-fire success, and heterogeneous learning needs as enrichment. In contrast, equal-treatment orientation and preference for little heterogeneity are seen rather negatively on average. Means of self-efficacy aspects turned out as non-positive.

Figure 3. Results from Cluster analysis (4 cluster solution), means and their standard errors



Note. Differences between clusters can be seen from non-overlapping standard error of the means intervals.

treatment orientation. The fourth cluster of pre-service teachers labelled as **group with high preference for little heterogeneity** differed in several views' variables from the other clusters, seeing heterogeneous learning needs rather not as enrichment for the mathematics classroom.

The second research question focused on interdependencies between pre-service teachers' views on dealing with heterogeneous learning prerequisites and global instruction-related views of the teachers, as reflected in established and well-known research instruments (e.g., Staub & Stern, 2002). As shown in Table 1, the study also included scales inspired by Stipek et al. (2001), for their potential relevance in the context of dealing with heterogeneous learning prerequisites in the mathematics classroom. Correlations with these global views' scales are shown in Table 3.

Table 3. Correlations between scales related to dealing with heterogeneous learning prerequisites and scales related to global instruction-related views

	Cognitive constructivist view on teaching and learning	Direct-transmission view on teaching and learning	Entity beliefs about teaching maths (barriers)	Entity beliefs about teaching maths (chances)
Equal-treatment orientation	0.13 (0.378)	0.271 (0.063)	0.018 (0.904)	**.368*** (0.010)
Heterogeneity orientation as sure-fire success	**.292*** (0.044)	0.126 (0.393)	-0.167 (0.257)	-0.049 (0.741)
Adaptivity as teachers' obligation	**.347*** (0.016)	0.029 (0.846)	-0.260 (0.075)	0.010 (0.944)
Heterogeneous learning needs as enrichment	0.089 (0.549)	0.076 (0.607)	**.287*** (0.048)	**.304*** (0.036)
Preference for learner groups with few heterogeneous learning needs	0.199 (0.176)	-0.023 (0.876)	**-.376**** (0.008)	-0.075 (0.612)
Self-efficacy in dealing with heterogeneous learning needs	-0.163 (0.267)	**.326*** (0.024)	-0.062 (0.676)	-0.056 (0.706)
Self-efficacy (tasks and learning material)	-0.056 (0.703)	**.321*** (0.026)	0.137 (0.354)	-0.198 (0.176)
Self-efficacy (analysing learning prerequisites)	-0.278 (0.055)	**.366*** (0.011)	-0.095 (0.523)	-0.064 (0.667)
Self-efficacy (organisation of instruction)	**-.354*** (0.014)	**.411**** (0.004)	0.046 (0.755)	-0.148 (0.315)

Note. The significant Pearson correlations (significance 2-sided) are highlighted.

The explorative findings in Table 3 suggest, for instance, interdependencies between the cognitive constructivist view on teaching and learning and adaptivity as teachers' obligation, as well as between the direct-transmission view on teaching and learning and the self-efficacy scales. The "barriers" scale of the entity beliefs about teaching mathematics correlates negatively with the preference of rather non-heterogeneous learner groups, and the "chances" scale of the entity beliefs interdepends with the equal-treatment orientation.

There are also correlations with pre-service teachers' views on dealing with mistakes (Kuntze, 2009; see scale descriptions above): **Adaptivity as teachers' obligation**, for instance, correlates with the views **learning from mistakes as central for building up mathematical knowledge** (r=.389**) and **rationality behind mistakes** (r=.311*), **heterogeneity orientation as sure-fire success** correlates with **"learning from mistakes as central for building up mathematical knowledge"** (r=.309*), and negatively with the **Bermuda triangle view** (r=-.322*) and **mistakes as deviating from the chain of thought**
(r=-.301*; **: p<0.01; *: p<0.05).

6 Discussion, conclusions, and outlook

First of all, we would like to remind of the limitations of this study: the sample being non-representative of German pre-service teachers and of medium sample size, the results should be interpreted with care; the newly developed scales could be subjected to an additional validity check. However, the research questions could be answered. Profiles of pre-service teachers' views on dealing with heterogeneous learning needs and interdependencies with global instruction-related views could be explored and provide new evidence.

The first research question focused on pre-service teachers' views on dealing with heterogeneous learning prerequisites and on specific aspects of self-efficacy. The findings can be interpreted as views which, in majority, acknowledge a need of providing learning opportunities which are adapted individually to the respective needs of the students. Within the potential dilemma between assuring equal learning chances and individualised support through non-identical learning opportunities, the pre-service teachers appear to attribute comparatively lower

importance to the “equal chances” side of the dilemma. The cluster analysis, however, provides insight that this is not the same case for all pre-service teachers: A group of teachers combines a high equal treatment orientation with high approval of the view *heterogeneous learning needs as enrichment for the classroom*, for instance. Also beyond this finding, the results of the cluster analysis point to variance in pre-service teachers’ views related to dealing with heterogeneous learning prerequisites. The clusters of pre-service teachers with low specific self-efficacy and the cluster of pre-service teachers who favour less heterogeneous learning needs suggest needs for improving teacher education in this area. Also, the high agreement with the scales *adaptivity as teacher obligation* and *heterogeneity orientation as sure-fire success* might correspond to a somewhat non-optimal professional knowledge: Pre-service teachers might lack knowledge, e.g., about tasks which are self-adaptive with entries on different levels of complexity. Moreover, pre-service teachers’ potentially lacking insight that also specific professional knowledge is necessary for professionally dealing with heterogeneous learning prerequisites, beyond merely disposing of a positive orientation towards dealing with heterogeneous learning needs could have led to corresponding positive views.

In any case, as the questionnaire has been probed successfully in this study, a novel questionnaire instrument with reliable scales is now ready-for-use for further exploring predispositions of pre-service teachers and for corresponding targeted developments of profession-related learning opportunities taking into account pre-service teachers views. When carrying out evaluation research on such profession-related learning opportunities, the instrument can also open up research pathways towards describing potential changes or developments in pre-service teachers’ views.

Results related to this research need can be expected from a follow-up study to the research reported here, as the pre-service teachers were asked to complete the questionnaire also after having completed the seminar mentioned above.

Further follow-up research is needed concerning the potential interplay with the pre-service teachers’ views and their more situation-related perceptions (cf. Kuntze, 2012; Kuntze et al., accepted). As the coherence between global orientations and situation-related views can be seen as an indicator of expertise (Doerr & Lerman, 2009), such follow-up research could bring a significant contribution, both to theory development and to implications for the practice of teacher education.

The second research question concentrated on potential interdependencies with global instruction-related views, as well as with views on the role of mistakes for

learning and ways of dealing with mistakes in the classroom. First of all, the observations of moderate correlations show that the views of pre-service teachers on dealing with heterogeneous learning prerequisites are constructs of their own right and not just simply aspects of global views of mathematics teachers. Both the moderate size of the observed correlations and the evidence of non-correlations indicate that the interplay between the variables included in the study is complex, however not in contradiction with expectations of potential interdependencies. Even the positive correlations between direct-transmission views and self-efficacy scales can help to adjust profession-related learning opportunities: pre-service teachers with high direct-transmission orientation might profit from interventions which deconstruct strong self-efficacy beliefs, in order to strengthen the perception of profession-related learning needs, as there is often no simple recipe for dealing with heterogeneous learning prerequisites in direct-transmission ways. Vignette-based learning opportunities of reflecting about classroom situations (e.g., Kuntze et al., 2022; Skilling & Stylianides, 2020; Kuntze et al., 2024) could provide solutions in this context.

Entity beliefs (cf. Stipek et al., 2001) have shown to add a facet to Staub & Stern's (2002) well-known instruction-related views in the topic field of this study. The correlation between *equal-treatment orientation* and the *chances* scale of entity beliefs provides a plausible empirical mosaic piece to the picture of teachers' views and might – even beyond the direct scope of this study – explain differences in teaching practice. Earlier findings from a video-based survey (Kuntze, 2007) with in-service mathematics teachers had shown substantial correlations between entity beliefs and teachers' classroom situation-related views. This underscores the potential significance of these global views also for more situated components of professional knowledge. Follow-up research is needed in this regard.

Teachers' views related to the role of mistakes for learning and in classroom interaction appear to add a further plausible facet to the empirical picture: If adaptivity and heterogeneity orientation is foregrounded, the positive perception of the role of mistakes for individual learning and the view, that working on individual mistakes in the classroom is important, correspond to a common teaching goal context, strengthening also the validity of the variables resp. constructs included in this empirical study.

A further follow-up research question concerns the interest to find out about whether and how the views included in this study influence teachers' analysis of

classroom situations (i.e., their noticing in the sense of knowledge-based reasoning, e.g., Sherin et al., 2011; Amador et al., 2021; Kuntze & Friesen, 2016) – which is mostly the base for teachers' actions and reactions in classroom practice. This research focus could bring further evidence for the validity of the newly developed scales. More follow-up questions concentrate on the stability of the views considered here or whether and how they can be developed through learning opportunities in pre-service teacher education. The ongoing research strand this study stems from promises to yield insight related to these follow-up questions in the future.

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