Mathematics teachers' beliefs: on their way into building thinking classrooms

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Abstract: This study investigates lower secondary mathematics teachers' beliefs about mathematics, learning, teaching, and knowledge essential for teaching, prior to their participation in a professional development program that focuses on implementing Peter Liljedahl's "Building Thinking Classrooms" (BTC) approach. We found that teachers a priori beliefs about mathematics, the teaching of mathematics, the learning of mathematics, and mathematical knowledge for teaching served, in some cases, as barrier to the adoption of the ideas and practices of BTC and, in other cases, served as a benefit.

Keywords: mathematics teachers' beliefs, teaching, learning, knowledge for teaching.

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1 Introduction

It is well established that teachers' beliefs influence both their development of knowledge (e.g., Verloop et al., 2001) and their teaching practice (e.g., Hatisaru, 2018; Philipp, 2007). This connection highlights a central challenge in educational reform: fostering shifts towards more progressive pedagogies often meets resistance, as educational change "depends on what teachers do and think – it's as simple and complex as that" (Fullan, 1991, p. 177). Our study investigates this dynamic within the context of **Partners in Practice**, a professional development and research initiative focused on in-service mathematics teachers in Norway. Participants in this project are four practicing teachers who are enrolled in an 18-month professional development project focused on the exploration and implementation of Liljedahl's (2021) "Building Thinking Classrooms" (BTC) framework, which promotes productive student dialogues around mathematical problems. Whereas previous efforts have explored development of the BTC approach and the impact that it has on student thinking, engagement, and experiences, less attention has been paid to the beliefs that teachers bring to such initiatives. It is crucial to understand such pre-existing beliefs, as they are likely to shape teachers' interpretation, adaptation, and enactment of the new pedagogical strategies. The present study aims at filling this gap by approaching the following research question: What beliefs about mathematics, learning, teaching, and knowledge for teaching do lower secondary mathematics teachers hold prior to engaging in building thinking classrooms and how does that impact their engagement with their exploration and implementation of building thinking classrooms? To answer this question, we analyse data from a group interview with teachers just prior to their engagement in the Partners in Practice project. Based on our analysis of data, and on the theoretical foundations for thinking classrooms (Liljedahl, 2021), we discuss how the teachers' beliefs might potentially support or constrain their efforts to build thinking classrooms.

2 Mathematics teacher beliefs

Beliefs are "understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). They are considered more subjective than knowledge – which is often described as beliefs that are justified or certainly true (p. 266) – and, although they are often discussed alongside emotions and attitudes, are more cognitive. Although beliefs have been studied and discussed for a long time in psychology and philosophy, research on teacher beliefs has been criticized. For instance, Skott (2015) argues that the concept is vague and difficult to study. However, despite the challenges, context-dependency, and potential instability of the beliefs construct, we believe that it remains valuable to study teachers' beliefs to better understand teaching practice.

There is considerable research on teachers' beliefs (for beliefs about mathematics and its learning, see e.g., Muhtarom et al., 2024), and beliefs are found to filter information and experiences and to guide action (Fives & Buehl, 2012). Yet, the construct of teacher beliefs is seen as "messy" (Fives & Buehl, 2012), and it has proven difficult to define. Mathematics teachers' beliefs have often been grouped into beliefs about the nature of mathematics, mathematics teaching, and mathematics learning (Table 1, three first columns). Beliefs aligned in the same row in Table 1 are considered theoretically consistent with each other, while those in the same column are viewed as forming a continuum (Beswick, 2012). The three categories of beliefs about mathematics were identified by Ernest (1989) and have been extensively utilized to describe beliefs about the essence of mathematics (left column of Table 1). According to the instrumentalist perspective, mathematics is viewed as “an accumulation of facts, skills, and rules to be used in the pursuance of some external end” (Ernest, 1989, p. 250). The Platonist perspective considers mathematics as a body of pre-existing knowledge. Lastly, the problem-solving perspective regards mathematics as a dynamic human creation.

About thirty years ago, Thompson (1984) argued that the link between teachers’ beliefs about mathematics and their teaching practices had been largely overlooked. She advocated for research focusing on this connection between beliefs and practice, leading to the emergence of several studies with this focus (e.g., Raymond, 1997; Skott, 2001). Following Thompson’s (1984) initiative, there has been growing interest in beliefs about the nature of mathematics, along with an increasing focus on beliefs about mathematics teaching and learning. Research in mathematics education has produced various models and perspectives to explain teachers’ beliefs about learning (Muhtarom et al., 2024; Xie & Cai, 2018), and Van Zoest et al. (1994) identified three key aspects in research on beliefs about mathematics teaching (Table 1, second column). Others, like Ernest (1989), distinguished between beliefs concerning three aspects of mathematics learning (third column).

Table 1. Teachers’ beliefs (adapted from Beswick (2012, p. 30) and Mosvold & Fauskanger (2013, p. 55)).

Beliefs about mathematics	**Beliefs about mathematics teaching**	**Beliefs about mathematics learning**	**Beliefs about mathematical knowledge for teaching**
Instrumentalist	Content-focused, emphasising mastery	Master skills, passive reception of knowledge	Remember content
Platonist	Content-focused, emphasising understanding	Active construction of understanding	Understand content
Problem-solving	Student-focused	Independent exploration	Adjust and differentiate

Mosvold and Fauskanger (2013) proposed adding the fourth column in Table 1 to encompass teachers’ beliefs about the knowledge required to teach mathematics. Previous studies have focused on teachers’ beliefs about knowledge more generally (Buehl & Fives, 2010; Fives & Buehl, 2008), but also on teachers’ beliefs about knowledge necessary for teaching specific subjects, such as physics (Asikainen & Pekka, 2010). In a study of the beliefs of primary and lower secondary teachers regarding the importance of mathematical knowledge for teaching definitions, Mosvold and Fauskanger (2013) found that while the teachers valued knowledge of mathematical definitions for teaching purposes, they did not view it as essential to know the definitions themselves. In a subsequent study, Mosvold and Fauskanger (2014) investigated the same teachers' beliefs about horizon content knowledge and found that the teachers seemed to devalue such knowledge. Koponen et al. (2019) explored pre-service mathematics teachers' beliefs about the knowledge necessary for teaching mathematics. Using network analysis, they revealed that pre-service teachers viewed common content knowledge as essential for teaching and as the basis for understanding specialized content knowledge and horizon content knowledge. Finally, Hatisaru (2024) explored secondary mathematics teachers' views of the knowledge required for teaching mathematics. In that study, the multidimensionality of teacher knowledge was not evident in the responses, with most participants viewing teacher knowledge as a singular dimension.

Although the relationship between teachers' beliefs and their practices is complex and mediated by external factors (e.g., Buehl & Beck, 2014; Fives & Buehl, 2012; Skott, 2015), there is evidence that teachers' beliefs influence their instructional behaviour (Calleja, 2022). This study is grounded in prior findings that highlight the connection between teachers' beliefs and instructional practices. As teachers' beliefs may change during continuing professional development (e.g., Calleja, 2022), it is relevant to explore teachers' beliefs as they are about to enter professional development.

3 Building Thinking Classroom

Liljedahl (2021) has developed Building Thinking Classrooms (BTC) based on the assumptions that students do not engage in much thinking in regular mathematics lessons, and that the organization and structure of classrooms follow a template of public education that is grounded in the industrial revolution, which is not productive. The lack of thinking is particularly troublesome; if students are not cognitively engaged in mathematics classrooms, they are less likely to learn. The second issue is also problematic, and it indicates that contemporary mathematics classrooms have some embedded institutional structures, and these structures are not necessarily productive. Yackel and Cobb (1996) described various kinds of norms that might influence mathematics teaching, which can be difficult to change. Liljedahl (2019) refers to "institutional norms", and his approach to BTC aims at dismantling these institutional norms and facilitating students' thinking. Despite the advancements in education, Liljedahl (2021) argues, classrooms are mainly left unchanged. Students are mostly sitting at their desks, working on paper, whereas teachers are the ones that are standing up – typically in the front of the classroom – and writing on boards. The pattern of teaching as telling and learning as passive reception is still common, and the BTC approach aims at breaking with these current norms. Based on substantial research and development efforts, Liljedahl (2021) presents a set of 14 practices that constitute the BTC approach. Liljedahl went further into the research to study what order these practices should be implemented in. What emerged was a sequence that is developmental in nature – developmental for the teacher and developmental for the students. In what follows is a description of the first three practices that Liljedahl found should be implemented together to begin the process of building a thinking classroom[1].

3.1 The types of tasks we use in a thinking classroom

The first practice highlights the tasks that are used to facilitate students' thinking. Liljedahl (2021) emphasises that the tasks used in BTC have some foundational attributes. Primarily, the tasks must be "thinking tasks", and Liljedahl aims at using tasks that can support the establishment of a "culture of thinking" in mathematics classrooms. Although the aim is not to move away from the curriculum content altogether, Liljedahl often starts by proposing non-curricular tasks that students might find engaging. Later, when students have become accustomed to engaging cognitively in the work of considering and solving mathematical problems, Liljedahl suggests moving back to more regular curriculum tasks. This approach is thus contrary to a more traditional approach where students first spend a lot of time learning and rehearsing the basics before they engage in problem solving.

3.2 How we form collaborative groups in a thinking classroom

Sociocultural theory emphasises that learning is social in nature, and collaboration can have a positive influence on students' learning. On the other hand, Liljedahl (2021) argues, strategic grouping of students does not always work, and letting students select groups on their own might also have negative effects. Students often seem to forget that they have an obligation to actively contribute to the group's learning – or even to do any thinking – and Liljedahl suggests that a shift towards forming visibly random groups might turn around this mindset in relatively short

[1] Our description of the first three practices of the BTC approach in this paper is based on the descriptions on the BTC website: https://www.buildingthinkingclassrooms.com/about-btc

time. Furthermore, random groups seem to have other positive effects, like decreasing students' stress levels and increasing their enthusiasm (Liljedahl, 2014).

3.3 Where students work in a thinking classroom

In addition to random grouping, the third practice is one of the most visible differences between BTC and more conventional ways of organising mathematics classrooms. Traditionally, students spend a lot of time sitting at their desks, solving textbook tasks individually. Liljedahl (2021) suggests a radical break with this norm and organises the classrooms by having students stand in groups around vertical whiteboards. Except they do not need to be whiteboards, they just need to be vertical – this makes it easier for everyone, including the teacher, to see what is written on them – and erasable (non-permanent). So, whiteboards work, but so do blackboards and windows. Liljedahl suggests that erasable boards encourage students to take more risks in their thinking. When the boards are vertical, students must stand up, and this keeps them active.

4 Methodology

Twelve lower secondary mathematics teachers and three teacher educators were involved in **Partners in Practice**. PD sessions focused on exploring key concepts of problem solving, providing participants with hands-on experience in solving mathematical problems, and encouraging them to engage in problem-solving activities with their students, with or without drawing on the practices associated with BTC (Liljedahl, 2021). The discussions within the PD sessions covered topics ranging from how problem solving is defined in the 2020 Norwegian national curriculum to the tasks presented in the textbooks being used. During the problem-solving activities, the emphasis has been on introducing tasks and guiding both group and whole-class discussions as students work as problem-solvers. The teachers participated in twelve PD sessions over a period of 18 months, and they were interviewed in groups before and after these 18 months of collaboration. Data from a two-hour pre-interview are analysed here. While all 14 practices of BTC hold significance, this paper focuses on the first three, as they were the primary emphasis during the teacher meetings analysed. Out of the twelve participating teachers in the PD, four participated in the study: Fred (8th grade, 19 years of experience), Kai (9th grade, first year as a teacher), and Per (9th grade, second year as teacher), Jon (10th

grade, 24 years of experience). Key limitations of this study include its exclusive use of interview data and the relatively small sample size.

There are many ways to denote the way teachers talk about mathematics, learning, teaching, and knowledge important for teaching, but one approach is to study teachers' beliefs through analysis of their discourses (Franke et al., 2007; Hemmi & Ryve, 2015). Data were analysed using the principles of analytic induction (Patton, 2002), which "begins with an analyst's deduced propositions or theory-derived hypotheses and is a procedure for verifying theories and propositions based on qualitative data" (Taylor & Bogdan, 1984, p. 127 cited in Patton, 2002, p. 454). As such, we began a process of extracting teachers' beliefs from the interview data using a constant comparison method like what is used in grounded theory (Glaser & Strauss, 1967). However, whereas grounded theory begins with the data, analytic induction begins with theory. Although the interview data were coded with a theory of beliefs (Table 1) in mind, analytic induction allows for the emergence of themes not anticipated from **a priori** theories. An example is Jon's utterance: "The foundation [for students' mathematical understanding]... lies in extensive practice. And we don't get such extensive practice ... when things are all up in the air and you are supposed to be creative" (line 171). This was coded as a view on students' learning as mastering skills and "content-focused, emphasising mastery" view on mathematics teaching (see Table 1).

5 Findings

To get an overview of participants' beliefs, we first identified the number of utterances for each participant that were coded and used this as the total. Two participants talked more and had more coded utterances (Jon had 116 total utterances with 61 coded, and Fred had 114 total utterances with 40 coded), whereas the other two participants talked less and had fewer coded utterances (Kai had 43 total utterances with 21 coded, and Per had 36 utterances with 16 coded). When considering the beliefs about learning, for instance (Table 2), we notice that 30% of Jon's utterances were coded as **independent exploration**. This constituted 18 of 61 coded utterances for Jon. On the other hand, 38% of Kai's utterances were coded as **passive reception**, and this constituted 8 out of 21 coded utterances for Kai.

Table 2. The participating teachers' beliefs.[2]

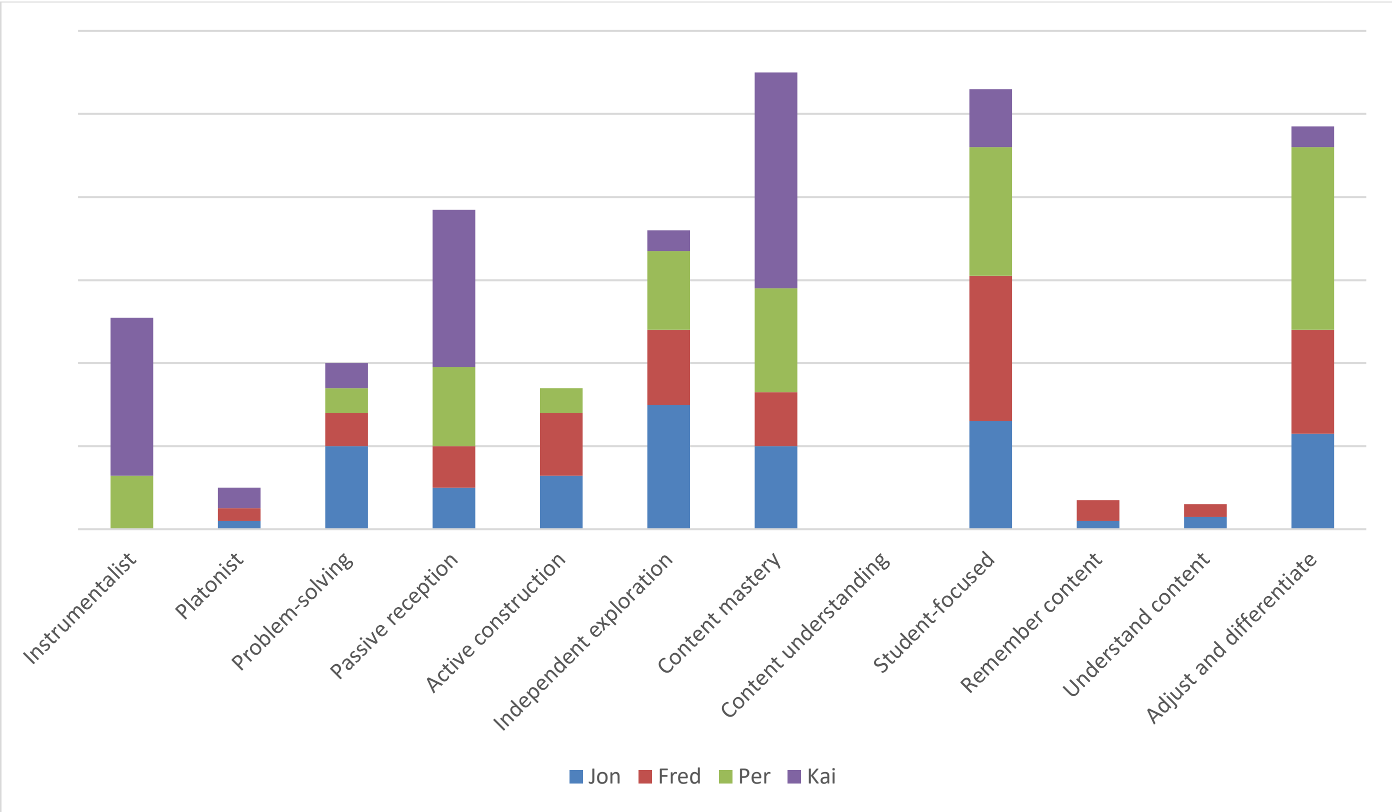
Beliefs	Jon	Fred	Per	Kai	Group
Beliefs about mathematics					
Instrumentalist	0%	0%	13%	38%	7%
Platonist	2%	3%	0%	5%	2%
Problem-solving	20%	8%	6%	6%	12%
Beliefs about learning					
Passive reception	10%	10%	19%	38%	15%
Active construction	13%	15%	6%	0%	11%
Independent exploration	30%	18%	19%	5%	21%
Beliefs about teaching					
Content mastery	20%	13%	25%	52%	23%
Content understanding	0%	0%	0%	0%	0%
Student-focused	26%	35%	31%	14%	28%
Beliefs about knowledge for teaching mathematics					
Remember content	2%	5%	0%	0%	2%
Understand content	3%	3%	0%	0%	2%
Adjust and differentiate	23%	25%	44%	5%	23%

When considering the participants' beliefs, as presented in Table 2, some patterns emerge (see also Figure 1). For instance, Per and Kai displayed more instrumentalist beliefs about mathematics than Jon and Fred, whereas Jon displayed more problem-solving beliefs about mathematics than the others. Kai displayed a larger percentage of beliefs about learning as **passive reception** than the others – but the others still displayed such beliefs in 10–20% of their coded utterances – whereas Jon displayed more beliefs about learning as **independent exploration**. When considering the beliefs about teaching, Kai was again on the one extreme, with 52% of his coded utterances being coded as **content mastery**, and Fred, Per, and Jon had more student-focused beliefs about teaching. Interestingly, however, the teachers had significant distributions of beliefs about teaching on both sides of the spectrum. With respect to their beliefs about knowledge needed to teach, Per, Fred, and Jon showed a significant percentage of

[2] Percentages may not add up to 100% as each utterance could be assigned multiple codes. Each percentage was calculated based on the total number of utterances for that participant or group.

beliefs relating to **adjust and differentiate**, whereas Kai displayed few such beliefs.

Figure 1. Teacher belief profiles[3]



Kai appears to stand out as the more traditional teacher, displaying mostly instrumentalist beliefs about mathematics, considering learning mostly as **passive reception** and teaching as predominantly about **content mastery**. Jon, on the other hand, displays more **problem-solving** beliefs about mathematics, and considers learning as **independent exploration**, and displays a considerable distribution of beliefs about teaching as **student-focused**. Interestingly, however, there is a significant proportion of beliefs about teaching as **content mastery** in Jon's utterances, and he appears to have a mixture of traditional and progressive beliefs. Fred also displays a mixture of beliefs, but his beliefs have a slightly more progressive slant than Jon's. Per displays even more mixed beliefs, with a slight slant

[3] Notice that the segments in the stacked bar chart present the percentage of codes from each participant, relative to his total number of coded utterances.

towards instrumentalist beliefs about mathematics, a mixture of beliefs about learning as **passive reception** and **independent exploration**, and a mixture of beliefs about teaching as **content mastery** and **student-focused**. For the group, there is a mixture of beliefs, but with a slight tendency towards the more progressive beliefs. However, the mix of traditional and progressive beliefs is interesting to notice.

Analysis of the interview data reveals differences between the four teachers' beliefs. Using their beliefs about mathematics teaching and learning as examples, it seems like there is a difference between the two experienced teachers (Jon and Fred) and the two less experienced teachers (Kai and Per). For instance, Kai describes his teaching as similar to the one he experienced as a student, "what I think worked, what worked well" (line 210). He continues by describing his teaching as follows:

> There is a bit of introduction to the topic. Maybe some repetition. Then they work. And then I take up a task again, which they should have done, then we go through it, and then they start working again, in a way. This is often how it goes. I remember that myself, I don't like working for that long myself either. You get a little break you can follow along. Or relax, then, even if you follow along. And then sort of go back to work. But those who want to, they're allowed to continue working while I take on one task.
> (line 210)

Similarly, Per seems to believe in the importance for students "to work with tasks on their own" (line 247) because this is "what they are facing, I mean, there will kind of be an exam. That is what we're preparing them for." But, as a supplement to this "volume training", problem-solving activities are put forward as "very nice." However, our analysis indicates that Per believes in a student-focused explorative approach to teaching: "I try to start [for the first 15 to 20 minutes of his lessons] with something like that ... give a challenge or something that they can discuss with their partner. And then we do it together. Preferably something that is a little on topic. So that they can explore a little on their own" (line 194), but this must be followed by "working." He continues by saying that in his teaching tries to "have some such activities that are perceived as nothing more than working with tasks."

The two experienced teachers, Fred and Jon, also believe that "traditional teaching" is important – as indicated in the following excerpt from the interview:

169. Jon: But I don't think it's a good idea to get completely away from the traditional either.

170. Interviewer: No? Because that's what I was thinking, can you say a little more about that?
171. Jon: Yes, because the basis of … is volume training. And volume training doesn't come into play … when things are up in the air and you have to be creative.
172. Fred: And everything is new all the time.
173. Jon: But discovering something yourself, first. Before you've got all those strings on you. That can be good for a student. To succeed without having to do it in a certain way. And then you'll unfortunately have to go in and hear from the teacher that 'this is the most effective way'.
174. Fred: That's right.

This excerpt illustrates how the two experienced teachers believe that "volume training" (line 171) or mastering skills is an important part of mathematics learning, resulting in teaching emphasising mastery. However, the excerpt also makes visible that the teachers think students must discover something for themselves first (line 173), indicating a student-focused belief about learning and a problem-solving approach to mathematics teaching. Jon also emphasises that his approach to teaching depends on the topic, or if the topic is "suitable for discovering on your own" (line 215). He also believes that the teaching approach depends on students' prior knowledge and that, in eighth grade, the students have "so much of the prior knowledge of the subjects that are there. So, they have the opportunity to do some exploratory work themselves", but that "the new letter arithmetic and algebra and equations in ninth grade" demands teaching approaches that are "very guided."

Although all four teachers say that "working individually on tasks" (Kai, line 87) and "listening to the teacher" (Kai, line 93) is what the students do the most in the mathematics classroom (Table 2), our analysis also reveals a difference between what the teachers say that they do when teaching and what they would like to do. As an illustrative example, seeing teaching as content mastery is visible when Fred says that "working individually on tasks" is what he prefers students do every lesson because of "working condition, because it needs to be trained" (line 77). He says that this is the way his students best develop mathematical understanding – when they have "actually been digesting the content." He continues by highlighting the importance for students to "practice methods from the textbook", to "rehearse them", and that the students must learn to listen to his explanations – even though he tries to limit his own talk to 15 to 20 minutes of a 45-minute lesson. He would, however, like to talk less, because he feels that he talks too much (line 79), but he "gets a thousand questions" if he "does not talk." However, when Fred elaborates on what he would like to do, he emphasized that he would let his students develop their own strategies through problem-solving activities. This is, however, only possible for "the smartest students", "in the time we have" (line 77).

6 Concluding discussion

In this study, we have investigated a group of four lower secondary mathematics teachers' beliefs about mathematics, learning, teaching, and knowledge important for teaching as they are about to engage in a professional development program partly focused on building thinking classrooms (BTC). Below, we discuss how their beliefs might influence efforts to implement BTC – positively or negatively – and we again focus on the first three practices of Liljedahl's (2021) approach. BTC strongly emphasises problem-solving and non-curricular problem-solving tasks (section 3.1). When two of the teachers, Kai and Per, seem to hold instrumentalist beliefs about mathematics, this might provide a particular challenge for implementing BTC as beliefs might filter information and guide actions (Fives & Buehl, 2012) – a challenge important for professional development providers to be aware of when discussing and working on tasks with teachers as they begin to think about implementing BTC. Likewise, the fact that Kai, like two of the other teachers, Fred and Jon, also seem to hold Platonist beliefs about mathematics, indicates that Platonist views about mathematics must be addressed in the professional development program. Although all four teachers' talk also have traces of problem-solving beliefs about mathematics, these beliefs appear most prominent in Jon's talk. Addressing problem solving explicitly in the professional development program will be of utmost importance. Here Jon might be invited to share his beliefs in one way or another.

BTC emphasises collaborative work in (random) groups to stimulate students' mathematical thinking and learning (section 3.2). Beliefs about learning as passive reception, and beliefs about teaching as content mastery – both of which were most visible in Kai's talk – might provide a challenge for efforts to implement new approaches to teaching mathematic (Fives & Buehl, 2012), such as BTC. A certain emphasis on mathematics learning as mastery of skills and passive reception of knowledge is visible in all four teachers' talk. The same is true for content-focused mathematics teaching emphasising mastery of skills. Strong beliefs about learning as passive reception might challenge implementation of BTC because beliefs about learning as independent exploration, and perhaps also beliefs about learning as active construction, correspond better with the idea of using vertical boards as active thinking spaces in BTC (section 3.3). Both Per and Kai seem to emphasise letting students work individually on tasks, and this might conflict with efforts to facilitate collaborative problem-solving in BTC. Moreover, all teachers emphasise mastering skills and "volume training", and such beliefs might also challenge the three core practices of BTC presented in section 3.

Furthermore, the findings indicate that the teachers hold conflicting beliefs that are important to be aware of, and probably to explicitly discuss, when aiming at BTC. For instance, these four teachers seem to have conflicting beliefs between how the teachers teach and how they want to teach, between the need for all students to problem solve, and the belief that only some students can do this. Furthermore, conflicting beliefs seem to exist between believing in volume training and the need for students to discover things and between the desire of the teachers to not talk so much and the increase in questions this creates.

Jon and Fred, who both seem to hold beliefs that appear more in line with BTC, are the most experienced teachers in the group. One can assume, and hope, that the novice teachers might listen to and learn from their more experienced colleagues, and that this might prove important for the success of the group's efforts to implement BTC. One can also hope that the teacher's beliefs about knowledge important for teaching, which tend towards adjusting and differentiating for all four teachers, might prove positive for the professional development efforts. Yet, the mixed beliefs in several of the core categories might constitute a challenge, and some of the beliefs that are more in opposition with the foundational principles and practices of BTC might have to be challenged in order to change teachers' beliefs (Calleja, 2022), and for implementation efforts to be successful.

As this study is limited by its reliance on interview data and by the small sample size, future research should incorporate additional data sources and larger, more diverse samples to build a more robust evidence base.

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References

Asikainen, M. A., & Hirvonen, P. E. (2010). Finnish cooperating physics teachers' conceptions of physics teachers' teacher knowledge. *Journal of Science Teacher Education*, *21*, 431–450. https://doi.org/10.1007/s10972-010-9187-y

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147. https://doi.org/10.1007/sl0649-011-9333-2

Buehl, M. M., & Beck, J. S. (2014). The relationship between teachers' beliefs and teachers' practices. In H. Fives & M. G. Gill (Eds.), *International handbook of research on teachers' beliefs* (pp. 66–84). Routledge. https://doi.org/10.4324/9780203108437-6

Buehl, M. M., & Fives, H. (2010). Exploring teachers' beliefs about teaching knowledge: Where does it come from? Does it change? *The Journal of Experimental Education*, *77*(4), 367–408. https://doi.org/10.3200/JEXE.77.4.367-408

Calleja, J. (2022). Changes in mathematics teachers' self-reported beliefs and practices over the course of a blended continuing professional development programme. *Mathematics Education Research Journal, 34*, 835–861. https://doi.org/10.1007/s13394-021-00366-x

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249–253). Falmer.

Fives, H., & Buehl, M. M. (2012). Spring cleaning for the "messy" construct of teachers' beliefs: What are they? Which have been examined? What can they tell us? In K. R., Harris, S. Graham, & T. Urdan (Eds.), *APA Educational Psychology Handbook* (pp. 471–499). APA. https://doi.org/10.1037/13274-019

Fives, H., & Buehl, M. M. (2008). What do teachers believe? Developing a framework for examining beliefs about teachers' knowledge and ability. *Contemporary Educational Psychology, 33*(2), 134–176. https://doi.org/10.1016/j.cedpsych.2008.01.001

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Information Age Publishing.

Fullan, M. (1991). *The new meaning of educational change*. Teachers' College Press.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory*. Aldine de Gruyter.

Hatisaru, V. (2018). Teachers' beliefs about knowledge of teaching and their impact on teaching practices. In B. Rott, G. Törner, J. Peters-Dasdemir, A. Möller, & Safrudiannur (Eds.), *Views and beliefs in mathematics education: The role of beliefs in the classroom* (pp. 147–159). Springer International Publishing. https://doi.org/10.1007/978-3-030-01273-1_14

Hatisaru, V. (2024). Non-specialist secondary mathematics teachers learning in study groups by engaging with activities of algebra. *Journal on Mathematics Education*, *15*(1), 131–150. https://doi.org/10.22342/jme.v15i1.pp131-150

Hemmi, K., & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education, 18*(6), 501–521. https://doi.org/10.1007/s10857-014-9293-4

Liljedahl, P. (2014). The affordances of using visibly random groups in a mathematics classroom. In Y. Li, E. A. Silver & S. Li (Eds.), *Transforming mathematics instruction: Multiple approaches and practices* (pp. 127–144). Springer.

Liljedahl, P. (2019). Institutional norms: The assumed, the actual, and the possible. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.), *Proceedings of the 43rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–16). PME.

Liljedahl, P. (2021). *Building thinking classrooms in mathematics, Grades K-12: 14 teaching practices for enhancing learning*. Corwin.

Mosvold, R., & Fauskanger, J. (2013). Teachers' beliefs about mathematical knowledge for teaching definitions. *International Electronic Journal of Mathematics Education*, *8*(2-3), 43–61. https://doi.org/10.29333/iejme/273

Mosvold, R., & Fauskanger, J. (2014). Teachers' beliefs about mathematical horizon content knowledge. *International Journal for Mathematics Teaching and Learning,* 1–16. https://www.cimt.org.uk/journal/mosvold2.pdf

Muhtarom, M., Sutrisno, S., Nizaruddin, N., & Murtianto, H. Y. (2024). Research on mathematical beliefs: systematic literature review. *International Journal of Evaluation and Research in* Education, 13(2), 693–704. https://doi.org/10.11591/ijere.v13i2.25968

Patton, M. Q. (2002). *Qualitative research and evaluation methods*. Sage.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Information Age Publishing.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550–576. https://doi.org/10.2307/749691

Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education, 4*(1), 3–28.

Skott, J. (2015). The promises, problems, and prospects of research on teachers' beliefs. In H. Fives & M. G. Gill (Eds.), *International handbook of research on teachers' beliefs* (pp. 13–30). Routledge.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), 105–127. https://doi.org/10.1007/BF00305892

Van Zoest, L., Jones, G., & Thornton, C. (1994). Beliefs about mathematics teaching held by pre-service teachers involved in a first grade mentorship program. *Mathematics Education Research Journal, 6*(1), 37–55. https://doi.org/10.1007/BF03217261

Verloop, N., Van Driel, J., & Meijer, P. (2001). Teacher knowledge and the knowledge base of teaching. *International Journal of Educational Research*, *35*(5), 441–461. https://doi.org/10.1016/S0883-0355(02)00003-4

Xie, S., & Cai, J. (2018). Chinese teachers' beliefs about mathematics teaching. In Y. Cao, & F. K. Leung (Eds.), *The 21st century mathematics education in China* (pp. 413–427). Springer. https://doi.org/10.1007/978-3-662-55781-5_20

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477. https://doi.org/10.5951/jresematheduc.27.4.0458