Emotions expressed by prospective teachers solving a mathematical problem

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Abstract: In mathematics education, increased attention is being paid to affective processes and how they relate to cognitive processes. In addition, emotional education is increasingly present in the education curricula of many countries to achieve an integral learning. Seeking to integrate these two areas, we ask prospective teachers to identify the emotions they feel when solving a mathematics problem. From the valence, the name and the concrete situation of the mathematical task that has led them to feel this emotion, reflection and emotional identification take place. Based on the valence, we grouped the results into four different profiles and the main emotions identified. The study has allowed us to analyse the type of emotions that prospective teachers feel when solving a mathematical problem, as well as the vocabulary and emotional knowledge to manage these emotional processes in the classroom. Prospective teachers' inability to adequately name or describe their emotions will have an impact on their emotional well-being now and in the future as teachers.

Keywords: affect, emotions, mathematics education, prospective teachers, problem solving.

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1 Introduction

In the field of mathematics education, increasing attention is being paid to affect and how it influences and interacts with cognitive processes when dealing with classroom and learning situations. Affective constructs such as emotions and motivation are important prerequisites before, during and in the pursuit of learning and achievement (Schukajlow et al., 2017) and can mediate effects of teaching on cognitive outcomes, such as knowledge (Schukajlow et al., 2023). Therefore, there is a need to study and analyse students' and teachers' emotions at specific moments in the teaching and learning of mathematics. Although we are going to focus on emotions, it is important to note that we understand affect as a system of variables (Andrá et al., 2023), or constructs that interrelate or overlap in a complex way (Grootenboer & Marshman, 2016). Everything that happens in the mathematics classroom is imbued with emotions and feelings (Roth & Walshaw, 2019), and only the subject themselves can explain the complex relationships between their thinking

and mathematical activity while they are performing it. We do not discuss emotions in isolation but in the context of the relationship between them and with other affective and cognitive systems (Pepin & Roesken-Winter, 2015).

Emotional education is a topic of growing importance in many countries. In Spain, Organic Law 3/2020, of 29 December, considers this emotional education essential for the development of pupils, including emotional management, self-esteem and social integration. In addition, at the Primary Education stage, the socio-affective sense is directly specified in the area of mathematics. The last two specific competences in mathematics talk about emotions: identifying and managing them when facing mathematical challenges and building a positive identity as a student of mathematics (Royal Decree 157/2022, 1 March). For this reason, we believe it is essential to work with prospective teachers who are studying Didactics of Mathematics. Teachers will not be able to guide their students in emotional management if they themselves are not able to identify their own emotions and how they inform or influence their daily work and specifically in the resolution of mathematical tasks. We cannot forget that emotions continuously mediate between social events and the individual's responses and experiences (Frenzel et al., 2021). In fact, an alarming phenomenon of negative feelings towards mathematics and its teaching has been identified among students in the grades of education (Coppola et al., 2012). Hence the importance of making this emotional reflection and even creating new pleasant experiences in relation to mathematics. Moreover, educating emotionally competent citizens will help us to create a society with greater emotional well-being and mental health (Gomis et al., 2021).

All this leads us to the research question of our investigation: What kind of emotions do prospective teachers explicitly feel during the individual solving of a mathematics problem? In addition, we will encourage prospective teachers to review their emotional vocabulary and to relate each of the emotions to the mathematical task. This reflection will enable them to connect their thinking and personality with mathematical activity, providing insight into this relationship influenced by internal and external factors.

2 Theoretical framework

Each person has his or her own affective system (Andrà et al., 2023) and, based on this system, will respond in one way or another to the mathematical situation he or

she is facing. In our case, we are going to focus on the emotions experienced and expressed by the prospective teachers while solving a mathematical task.

In order to describe emotional processes, it is important to specify the valence, temporal stability and object on which they are focused (Schukajlow, 2017). For this reason, we chose a theory, the OCC model, which falls within the cognitive approach and, as such, assumes that emotions are activated by a special type of cognitive activity (Marbán, 2022). In relation to valence, we included Russell's (1980) two-dimensional circumplex model in the study. The model allows any affective word to be defined as a combination of pleasure and activation components (Russell, 1980). We do not describe the model in more depth because it will not be used in the analysis, but it does allow us to confirm the information that the prospective teachers indicate in relation to the emotion they make explicit and the valence they assign to it. This information will help us to describe the emotional process experienced together with the mathematical cognitive activity being carried out, which is the purpose of our research. In fact, these two models help us cover the dimensions identified by Pekrun (2024) for grouping emotions by focusing on the object of the emotion, its valence, and its activation. In addition, our models allow subjects to reflect on their own emotional process and help them to name it at the exact moment it is happening.

2.1 Cognitive theory of emotions: OCC model

This model is included in the “appraisal theories” that emerge to explain that emotions respond to the person's appraisals of his or her environment and well-being (Moors et al., 2013). Specifically, the model focuses on characterising and classifying emotions on the basis of the type of cognitions that trigger them or are responsible for their occurrence (Ortony et al., 1996). This is very important because it allows us to focus on the mathematical task and allows students to relate the emotions they feel to the problem they are solving.

We align ourselves with the definition given by the authors that emotions are valenced reactions to events, agents, or objects, the nature of which is determined by the way in which the triggering situation is interpreted (Ortony et al., 1996). Although all students face the same problem and under the same conditions, their interpretations of the situation may vary, giving rise to different emotions. This is the contribution that cognition makes to the emotional system (Ortony et al., 1996).

On the other hand, the OCC model does not pay much attention to the different linguistic expressions associated with each type of emotion (Ortony et al., 1996). This is also important for us because it allows us to unify the answers given by the students, even if they do not use the word that best defines the emotion they are feeling. Table 1 shows the classification proposed by the OCC model according to where the subject of analysis focuses (reactions to events, agents, and objects, or a simultaneous combination of event and agent). Written in italics is a representative of each type of emotion, which tries to be as neutral as possible and to group together the different linguistic expressions of the group. Then, based on the subject's interpretation, the triggering condition of the emotion is specified, which we will also use to classify the emotions expressed in the study.

Table 1. Classification based on information from the OCC theory (Ortony et al., 1996)

Reactions to events	
Well-being	*Joy*: pleased about a desirable event
	Distress: displeased about an undesirable event
Fortunes-of-others	*Happy-for*: pleased about an event desirable for someone else
	Sorry-for: displeased about an event undesirable for someone else
	Resentment: displeased about an event desirable for someone else
	Gloating: pleased about an event undesirable for someone else
Prospect-based	*Hope*: pleased about the prospect of a desirable event
	Fear: displeased about the prospect of an undesirable event
	Satisfaction: pleased about the confirmation of the prospect of a desirable event
	Fears-confirmed: displeased about the confirmation of the prospect of an undesirable event
	Relief: pleased about the disconfirmation of the prospect of an undesirable event
	Disappointment: displeased about the disconfirmation of the prospect of a desirable event
Reactions to agents	
Pride: approving of one's own praiseworthy action	
Shame: disapproving of one's own blameworthy action	
Admiration: approving of someone else's praiseworthy action	
Reproach: disapproving of someone else's blameworthy action	
Reactions to objects	
Love: liking an appealing object	
Hate: disliking an unappealing object	

Combined reactions to event and agent
Gratitude: pleased about a desirable event and someone else's praiseworthy action
Anger: displeased about an undesirable event and someone else's blameworthy action
Gratification: pleased about a desirable event and one's own praiseworthy action
Remorse: displeased about an undesirable event and one's own blameworthy action

Note. The table presents the main groups and subgroups into which emotions are organised. The type of cognition that triggers them is also indicated.

Emotional vocabulary is not usually very high among pupils and is one of the factors to be improved to form emotionally competent citizens, with greater emotional well-being and mental health (Gomis et al., 2022). Using this model allows us, on the one hand, to classify the emotion, even if the pupils do not use the most appropriate word, and on the other hand, to provide them with different words to increase their vocabulary and emotional reflection. However, while we take away the importance of the word used to define the emotion, we emphasise the relationship of the emotion with the triggering situation and the mathematical task. This allows us to understand the relationships between a person's thoughts and needs and the specific activity they are performing (Roth & Walshaw, 2019), which is necessary in studies of affect in the field of mathematics education.

3 Methodology

Educational research is a systematic enquiry designed to support educational judgements and decisions in order to improve educational action (Bassey, 1999). We will follow a qualitative methodology based on an interpretative paradigm focused on the individual (Cohen et al., 2000) with an exploratory intention. Based on the emotions expressed by our students, we will describe the emotional processes identified and interpret the different emotional profiles found in the students when they solve a mathematical task.

3.1 Sample and Context

The data was collected in a classroom session with Primary Education students at a Spanish public university. We are interested in prospective teachers solving a task in their usual environment and then analysing the emotional reactions they report feeling. The data will be collected in the classroom by giving them a sheet of paper so

that they can individually record the emotions they are feeling as they solve the corresponding mathematical task.

The sample consisted of prospective teachers enrolled in the Mathematics and its Didactics I course in the Primary Education Teaching Degree programme, which focuses on teaching arithmetic. Although there are 50 prospective teachers enrolled, not all of them attend class, and specifically, 39 participated in this study, coinciding precisely with those who attend class regularly. The prospective teachers are in their second year of university. The sample size was selected using non-probability convenience sampling: the group of prospective teachers to which the research team has access.

3.2 Instrument

The instrument for this study consisted of the following task: prospective teachers were asked individually to solve the following problem (Figure 1), showing the process of solving it in as many ways as possible. The mathematical-didactic content that was intended to be developed with prospective teachers consisted of different auxiliary counting techniques, which would enable them to count all the guests at a family meal without counting anyone twice.

At the same time as each resolution, the prospective teachers were asked to indicate the emotions they were feeling, recording the order in which they appeared, together with the specific situation related to the mathematical task that had triggered them. They are given a separate sheet of paper to indicate their emotions and place them on the axes of Russell's model according to their valence and intensity. The time spent on the task was longer than it would have been for the resolution, allowing sufficient time to be devoted to identifying emotions.

Figure 1. Statement of the mathematical task to be solved by the students.

During a family meal there is a grandfather, a grandmother, two fathers, two mothers, an aunt, five sons and daughters (two sons and three daughters), three grandsons and granddaughters (two granddaughters and a grandson), two brothers, three sisters, a father-in-law, a mother-in-law and a daughter-in-law. How many people attended the family meal?

Note. The figure shows the picture of the problem provided to the students.

3.3 Analysis

According to Ortony et al. (1996), there are four types of evidence that can help us understand emotions: language, self-reports, the type of behaviour elicited, and physiological signals. In our case, we can say that we will focus on the first two, analysing the emotion, its valence and the situation that the subject identifies as triggering it; in other words, the cognitive assessment that leads to the elicitation of the emotion. To do this, a content analysis is carried out (Krippendorff, 2019), taking as categories of analysis the emotions proposed by the OCC model (Table 1). These categories facilitate coding for subsequent emotional classification and grouping into profiles. This was addressed independently by both researchers, who were able to discuss those codifications on which there was initially no agreement. This model allows us to work with the cognitive part of the emotion, showing where prospective teachers focus and the different profiles that we find when faced with a type of mathematical task.

For the analysis and classification of the emotions, first of all, they are copied into a table keeping the order and the word used by each prospective teacher in the resolution of the task. From this table and following the OCC categories, a new table is created in which a representative is designated for each group of words referring to the same type of emotion. As we have explained in the theoretical framework, each of the emotional groups that appear in Table 1 are structured according to the triggering condition that appears after the chosen word or representative. If any doubt is generated in this process, the information given by the prospective teacher in relation to the aspect of the task that generates the emotion is used. In addition, students place each of the emotions on coordinate axes following the model of Russell (1980), which represents the cognitive structure of affect. This reinforces, in case of doubt, the valence that the subjects give to each specific emotion expressed by them, which is represented on the horizontal axis. We show here below the chosen representative and some of the other words chosen by the prospective teachers to designate the emotion felt:

- JOYFUL: cheerful, happy, motivated, astonished, enthusiasm.
- DISTRESS: sad, unhappy, disoriented, confused.
- FEAR: anxious, overwhelmed, nervous, petrified/blocked, stressed out.
- SATISFACTION
- RELIEF: calm, tranquility, peace.

- DISAPPOINTMENT: frustration, despair, disillusionment.
- PRIDE
- SELF-REPROACH/ SHAME: self-doubt, insecurity about knowledge (confused).
- LIKE/ LOVE: enthusiasm.
- HATE: tiredness, laziness.
- ANGER: rage, furious.

We find it interesting to mention that the word 'surprise' appears five times, which is not considered an emotion in the OCC model. According to the situation that the prospective teachers mention as a trigger for this surprise, we have classified it sometimes as 'joy' and sometimes as 'distress'. An example of 'joy' is PT4 because when reading the exercise it seems easy; and an example of 'distress' is PT8 who explains that he/she cannot understand the task and there are many options.

Another emotion that appears on many occasions is 'confusion' and in all cases prospective teachers assign it a negative valence. This is a very interesting emotion from the point of view of learning; since, as Schukajlow et al. (2023) say, not all negative emotional constructs are negatively related to academic achievement in mathematics. On most occasions when this word is used, the focus is on being an undesirable event because the goal is apparently not achieved: a different result than expected, a complicated statement, which in our case we encompass in 'distress'. On the other hand, when prospective teachers focus on the person, on not having the necessary tools to solve it, we label it as 'self-reproach'.

We specify the type of emotions found according to the categories of the OCC model used. In terms of positively valenced emotions, we find several from the group of events because something desirable occurs that is related to the prospective teacher's objectives. We find 'joy' because they understand the problem, they know how to solve it, they think they have done it well; 'satisfied' because of the mathematical work done or 'relief' because they reach the expected result or they know how to go about working on the mathematical exercise. In this same group, those with negative valence express that the event is undesirable because it prevents them from achieving some goal related to the mathematical task. We find 'distress' because they do not understand the problem or do not know how to solve or explain it; 'fear' because they do not know if it is right or they have made a mistake; 'disappointed' because no matter how hard they try they do not manage to reach the result or they are blocked. From the group of reactions to agents, base on the OCC

model, the subject focuses on the action of the responsible person and assesses it according to his or her norms and beliefs. These emotions rarely appear and we only find with positive valence “pride” and with negative valence “shame”. Also very rarely do the emotions of ‘love’ and ‘hate’ appear in terms of attraction to the mathematical object or of doing the task requested. Finally, from the group combining attribution and well-being emotions, or one can also say in combination of the first two groups, only ‘anger’ appears when prospective teachers get angry because they are not able to do the exercise.

4 Results

Once all emotions have been classified by analysing their content and categorising them with the designated representatives, we grouped the prospective teachers in our sample into four different profiles according to the valence of the emotions expressed during the resolution of the mathematical task. To explain each of the profiles, the results of one of the prospective teachers belonging to each of the groups or profiles found are presented. In this way, each profile will be exemplified by the emotions shown by a specific prospective teacher (PT#).

4.1 Results of profile P1

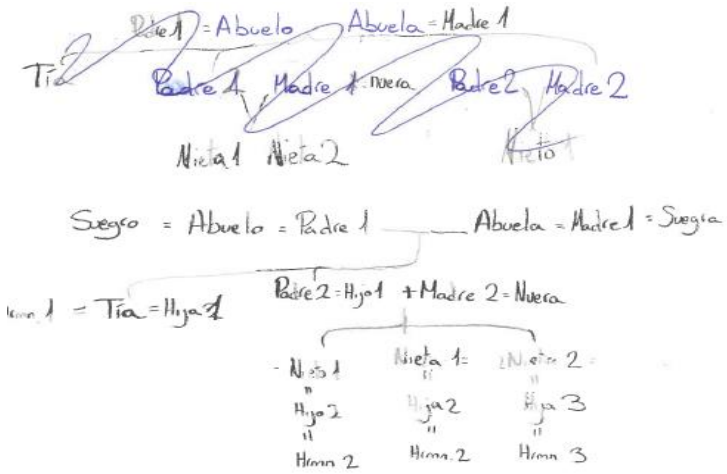
This profile of PTs (n=7) is characterised by showing at the beginning of the problem-solving process a positive emotion, such as ‘tranquillity’ or ‘happy’, and ending the task with a negative emotion such as ‘distress’, ‘anger’, ‘self-reproach’ or ‘fear’. We exemplify this profile with PT32. This PT shows the emotion ‘being at ease’ when writing down the data of the problem, as she thought that she had been able to extract it well from the statement at first. PST32 considers each piece of data independently and does not link the fact that a mother can also be a daughter. Subsequently, she feels ‘confused’, as she had first thought that the problem was very simple, but then she saw that the way she had thought of solving it was not the right way. This happens when you try to graphically represent the problem data to show the solution. Finally, she was ‘frustrated’ because she could not find a way to solve it after trying several times.

4.2 Results of profile P2

In the case of PTs of this profile type (n=15), they start with a negative emotion and

end the task resolution with a positive emotion. Among the initial emotions we find ‘disappointment’, ‘distress’, ‘fear’, ‘anger’ or ‘dislike’. Most of them, despite these initial emotions, end the resolution with the emotion ‘satisfaction’. We evidenced these changes through PT20. This prospective teacher shows ‘displeasure’ for the long day he has been exposed to. Subsequently, once he starts solving the problem, he shows ‘confusion’ because of the large amount of data provided to solve the problem. Next, he continues with a new negative emotion ‘fear’, for not knowing what to do with the data and having failed to solve it. Finally, having been able to solve the problem he only expresses ‘pride’. Figure 2 shows the exact moment when the emotion changes. In the first part, crossed out by the prospective teacher, he realises that he cannot find the correct answer, while the second part is part of the process that leads him to the correct solution.

Figure 2. The moment of resolution when the change in emotion occurs.



4.3 Results of profile P3

Profile 3, made up of 7 PTs, is characterised by not showing changes in the type of emotion during the whole resolution process. In addition, it is important to highlight that 6 of them feel negative emotions during the course of the task. An example of this can be found in PT34. This prospective teacher, with the first reading of the problem, shows ‘confusion’. Subsequently, he shows ‘disappointment’ at not being able to solve the problem. Finally, he states that he is ‘unhappy’ because he opts for a possible solution but does not know if it is right. PST34 finds a way to solve the proposed problem through a family tree, which, according to him, allows him to see the components of the family. However, he maintains negative emotions when

encountering problems with his aunt, siblings, grandchildren, and children. This profile represents prospective teachers who may need additional emotional support and different teaching approaches to build mathematical confidence.

4.4 Results of profile P4

The fourth profile is formed by those prospective teachers (n=10) who, in spite of changing the type of emotions, from positive to negative or vice versa, during the problem-solving procedure, end up obtaining the same type of emotion at the beginning and at the end of this process. An example of this is PT41. This prospective teacher is 'happy' because the exercise, in the reading, seems easy to him. In fact, it solves the problem in two different, and incorrect, ways, adding up all the numerical data that appears. Subsequently, he starts to be 'unhappy' because of how easy the problem seems to him and because the teacher said that it gives a lot of play. He then became 'disappointed', as he had misunderstood the exercise and it did not work. At that moment, he felt 'fear', because he was thinking and it didn't work. At this point, a third resolution begins, using a list while drawing a picture. Although he initially realises the difficulties of this new resolution, it is what leads him to the correct solution. It is precisely the latter that changes the type of emotion, he perceives himself to be 'proud' because he has achieved it after a lot of effort, and to be 'happy' for having succeeded.

5 Conclusions

In accordance with the analysis model used, the OCC model or cognitive theory of emotions, we can specify what type of emotions prospective teachers report feeling when solving a mathematical task. It is important to mention that using the OCC model has helped us to unify and classify prospective teachers' emotions. In many cases the word chosen by them did not define the emotion they were feeling or was not well expressed. To understand emotions, the first step is to be emotionally aware and for this it is essential to know how to name emotions and have a good emotional vocabulary (Gomis et al., 2021). Furthermore, doing so at the exact moment it occurs makes them realise that emotions arise in relation to the mathematical task and influence how they approach it. According to Ortony et al. (1996), feeling an emotion is an important way of classifying memory. Managing emotions and solving tasks will help them maximize the effectiveness of their responses to similar tasks

and emotions.

On the other hand, the emotional work carried out has helped prospective teachers to see how emotions influence learning and in this particular case, mathematical work. Thought is entirely affective (Roth & Walshaw, 2019). We know that unpleasant emotions have a negative effect on intellectual performance and, furthermore, students who understand and regulate these emotions better are more successful academically (Fernández-Berrocal & Cabello, 2021). The prospective teachers themselves realised that some of the emotions they felt had caused them to block and not dare to try; and later they saw that they could have done things to solve it. Prospective teachers' awareness of the importance of using and managing emotions appropriately will enable them to use emotions in the teaching process and with their students in the near future. As Coppola et al. (2012) say, the affective sphere has to be taken into account for a mathematics teacher to teach effectively. This leads us to consider monitoring their emotional processes throughout the course and possibly designing activities that generate pleasant emotions and greater mathematical confidence in prospective teachers.

A significant limitation of the study is the inability to conduct a more in-depth analysis with follow-up interviews to analyze the emotional processes experienced and adequate emotional management. This would be particularly interesting with participants in profile P3, but they are anonymous and voluntarily declined to participate in this work.

Informed consent statement

Informed consent was obtained from all research participants. The research project has been approved by the Ethics Committee of Rey Juan Carlos University, with internal registration number: 041220246292024.

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