

Commognitive conflicts between an instructor and a teacher in a professional development focused on tutoring struggling students in mathematics

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Abstract: This study examines interactions between an instructor and a teacher within a professional development intervention focused on tutoring students with difficulties in mathematics. The study analyzed the discourse of a middle school teacher and her instructor (first author) during a one-on-one professional development designed to make teachers aware of their role in preserving cycles of failure in students' mathematical learning. Using commognitive theory, the analysis surfaces commognitive conflicts within the instructor and the teacher's discourse during their discussions about teaching, and particularly, about tutoring a student whom the teacher described as exceptionally "weak". We found moments of disagreement in which the participants appeared to be discussing the same issue, while actually drawing on different meta-rules, or underlying assumptions about teaching and learning. These moments of disagreement were explored to uncover the implicit pedagogical meta-rules followed by the teacher and the instructor. Our analysis showed that the two drew on distinct sets of meta-rules, which could be organized around two metaphors of learning: a delivery/acquisition metaphor and a growth/construction metaphor.

Keywords: commognition, pedagogical discourse, professional development, struggling students

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1 Rationale

Instruction has a crucial influence on success and failure to learn mathematics. Research highlights the importance of offering opportunities for agentive and meaningful engagement by presenting cognitively demanding tasks to students, requiring explanations and building arguments (e.g., Stein et al., 2008; Walshaw & Anthony, 2008). However, teachers working with struggling students often avoid offering these opportunities. Reasons cited in the literature for this avoidance range from lack of adequate training, concerns about curriculum demands, and teachers' worry about straining students emotionally (Heyd-Metzuyanin, 2015; Kajander et al., 2008; Karsenty, 2018). Nevertheless, rarely has research examined in close detail the pedagogical considerations that teachers have around teaching struggling students and what it takes to shift their perspectives and practices about teaching



these students. Our goal in this study is to better understand what may hinder teachers' progress towards giving struggling students more opportunities to engage agentively and meaningfully with mathematics. We do so through leading a unique one-on-one professional development with a teacher who works with struggling students and closely examining the discussion between her and her instructor (first author).

2 Theoretical framework

Pedagogical discussions around teaching mathematics include two main discourses: one is about mathematics, and the other is about teaching and learning actions. The commognitive framework (Sfard, 2008) conceptualizes both the mathematical teaching-learning interactions as well as the discussions about teaching and learning as discourses: the former is the mathematical discourse while the latter is the pedagogical discourse. This discourse, according to commognition is characterized by certain keywords, routines and narratives (Sfard, 2008). Learning, according to this framework, is the process of becoming a participant in a specific discourse. For example, learning mathematics is the process of becoming a participant in the mathematical discourse, while learning to teach is a process of becoming a participant in a certain pedagogical discourse (Heyd-Metzuyanım & Shabtay, 2019).

Discourses also contain **meta-rules**, which are rules followed by participants, often implicitly (Sfard, 2008. p. 201) such as "To prove a theorem one should start from the givens". Adopting new meta-rules is an important part of learning, yet it often involves **commognitive conflicts** (Ben-Zvi & Sfard, 2007; Sfard, 2007). These commognitive conflicts are situations where the novices and experts seemingly talk about the same thing, yet implicitly follow different meta-rules, leading to failures in communication.

Because of the difficulty in detecting changes in meta-rules, novices faced with them often encounter an **impasse** (Heyd-Metzuyanım & Cooper, 2022). This is a situation where the discourse available to the participant is insufficient for performing a certain task.

2.1 Pedagogical discourse

Stemming their theorizing in the commognitive theory and widening its scope through building on Gee's (2014) discursive framework, Heyd-Metzuyanım and

Shabtay (2019) defined ***Pedagogical Discourses*** as socially originating Discourses¹ that “Shape and orient teachers towards what to teach students, how to teach them, why certain teaching actions are more effective than others, and (...) who can learn (or cannot learn)” (Heyd-Metzuyanin & Shabtay, 2019, p. 543). They identified two distinct Discourses that are common and often conflictual around mathematics teaching. One is the Delivery Pedagogical Discourse² (DPD), which frames teaching as the delivery of knowledge and learning as the acquisition of that knowledge. This often termed the “traditional” Discourse to which teachers have been accustomed since their childhood. The other is the Explorative Pedagogical Discourse (EPD), common in the educational research and pedagogical writings, which values student agency, cognitively demanding tasks and students’ struggle with meaningful mathematics.

Sfard (2019) distinguishes between ***ontic*** and ***deontic*** discourses. Mathematical discourses are ontic: they deal with objects that are treated as existing in the world and "no narrative (...) is to be adopted without undergoing a certain substantiation procedure" (Sfard, 2019, p. 93). In contrast, pedagogical discourses are deontic: they concern values and norms and the adoption of narratives in these discourses "has more to do with people's identity work than with their reasoned choices" (Sfard, 2019, p. 94).

Teacher's Learning as a change in the pedagogical discourse: Nachlieli and Heyd-Metzuyanin (2022) extended the notion of commognitive conflicts to the domain of pedagogical discourse. According to them, teachers and teacher-educators may fail to communicate with each other due to their alignment with different Pedagogical Discourses. For example, these authors identified conflicting meta-rules underlying teachers’ discussions with professional development leaders that had to do with valued teaching actions, valued learning outcomes and who may be able to learn. They stressed the importance of surfacing these conflicts and actively reflecting upon them. Tyskerud and her colleagues (2023) continued this line of work, examining the commognitive conflicts emerging in a professional development program around Lesson Study. These authors emphasized the role of the instructor who is leading the discourse to explicate the conflicts. However, not much has been

¹ Unlike the discourse in little "d" which means language-in-use, the capital "D" points to the Discourse which is socially available through texts and institutions (Gee, 2014).

² Previously termed the Acquisition Pedagogical Discourse (APD).

written about the pedagogical discourses³ of teachers teaching struggling students and the commognitive conflicts that may arise with teacher educators around this subject. Hence, the goal of this research is to examine the pedagogical discourse of such a teacher and the commognitive conflicts between her and her instructor in relation to teaching struggling students.

2.2 Research questions:

1. Were there commognitive conflicts between the instructor and the teacher in relation to teaching mathematics?
2. If commognitive conflicts occurred, what meta-rules underlined these conflicts?

3 Method

The participant at the focus of this case study was Bella⁴, who teaches grades 5 through 8 at an upper elementary school. Bella holds a teaching certificate and has ten years of teaching experience. She also acts as the mathematics coordinator of her school.

The one-on-one professional development program with Bella was facilitated by the first author (Tzila) and focused on collaboratively planning, teaching, and reflecting on tutoring sessions with struggling students. The intervention focused on tutoring sessions held out of the classroom in order to make the teacher-learner interactions as visible as possible for the teacher, as well as to offer maximal opportunities to inquire into students' difficulties.

The tutoring sessions conducted by Bella were with two students from one of her 7th grade classes⁵, whom she identified as needing additional support in mathematics. Overall, the intervention included three cycles. In each cycle, Tzila first video-recorded a tutoring session. After that, there were 2-3 sessions (approximately 45 minutes) in which Tzila and Bella would inquire into Bella's views of teaching the students she chose to tutor and reflect on the recorded tutoring session.

³ the individualized versions of socially existing Pedagogical Discourses.

⁴ All names of participants are pseudonyms.

⁵ Students at the age of 12-13.

3.1 Data collection and analysis:

The data for this study included approximately 7.5 hours of video recordings that were fully transcribed. In addition, the first author maintained a researcher's journal. The selection of episodes for micro-analysis was primarily guided by this journal, with particular attention given to moments in which unexpected or surprising events occurred, as these could mark points of commognitive conflicts.

Based on Sfard (2007) and Nachlieli & Heyd-Metzuyanim (2022), we defined commognitive conflicts in pedagogical discourses as situations in which the participants appear to be discussing the same issue and agreeing (or disagreeing) with each other, while actually operating under different meta-rules. We then used discourse analysis tools (Gee, 2014) to detect the meta-rules underlying these situations. For example, we searched for implicit meanings, declarations that go unquestioned, and hidden metaphors underlying the interlocutors' talk about mathematics and about the students. Following Heyd-Metzuyanim and Shabtay's (2019) definition of Pedagogical Discourses as collections of narratives about how to teach, what to teach and who can learn, we focused particularly on Bella's implicit narratives concerning her teaching, her students, and the mathematical tasks given to these students.

3.2 Context of the episode at the focus of analysis

The episode at the focus of our analysis came from the 6th meeting that Tzila held with Bella. This meeting, as well as the previous one, focused on enhancing Bella's ability to understand students' thinking. To achieve this goal, Tzila chose to observe and discuss with Bella a recording of a previously conducted "mathematical interview" with a student, led by the second author for research purposes. This interview was supposed to surface both the opportunities that such a "think aloud" interview holds for better understanding students' thinking, as well as model for Bella how an authority figure (here, the researcher) could elicit from a student interesting thinking patterns without "telling her" what to do. The part of the interview that Tzila and Bella watched together was around the "Circles Task" (see figure 1), which later formed the target task about which the conflictual discussion arose.

Figure 1. the Circles Task

Here is a sequence of drawings made up of circles



- A. Draw the next two drawings in this sequence.
- B. How many circles are in drawing number 10 in the sequence? Explain.
- C. How many circles are in drawing number 42 in the sequence? Explain.
- D. Formulate a rule to calculate the number of circles in the drawing of any number in the series.

The observation and discussion of the mathematical interview raised Bella's interest in doing such an interview herself and she suggested conducting it with Yoav, a student she described as "very weak yet motivated", and that she was curious about better understanding his difficulties.

The 6th meeting was dedicated to preparing for the interview, including selecting the tasks appropriate for Yoav, anticipating how he would respond and preparing possible interviewing moves by Bella.

We chose to focus our analysis on the last part of the 6th meeting because the discussions were revealing of an impasse that Bella confronted. We added to this an analysis of Bella's narratives in previous meetings to support the detection of meta-rules underlying the conflictual situation between Bella and Tzila.

4 Findings

We start with a description of Bella encountering an impasse about what to do if Yoav says "I don't know". The context of this impasse is a discussion between Tzila and Bella around the Circles Task. Initially, Bella claimed this task was not suitable for Yoav since "weak kids (...) don't understand (it)". Tzila tried to convince her that even if the last sub-tasks, concerning the generalization of operations performed on numbers to an algebraic expression, were difficult for Yoav, he could still engage with the first one or two sub-tasks. As Bella hesitantly agreed, Tzila tried to encourage her to anticipate and plan what she would do if Yoav says "I don't know". Bella reacted with puzzlement and long silences. She could not think of anything she could do in such a situation, other than to direct Yoav how to solve the task.

4.1 Meta-rules underlying Bella's pedagogical narratives

Several meta-rules were detected as underlying the impasse Bella was facing.

The role of giving tasks to students

Bella's puzzled response had to do with the overall goals that she probably saw for the Circles Task. We gain insight into these goals from an instance in the 5th meeting where Bella shared her way of solving such pattern tasks. She reported that she solves the task by first finding the algebraic expression and then finding the specific terms.

"I first do the rule and then I do the-⁶ Um, my rule is like this: 3 plus N minus 1, times 2. I can also do that, like, 1 plus two N... This is my rule, and the circles in number 10 are... 21. Like, I just substitute. And here in drawing 42, um, 42 times 2 is 84, plus 1, is 85".

Bella's attention only to the algebraic expression hints that the only important part of the task for her, pedagogically, was having students find the algebraic expression (question D). As such, the role of the first sub-tasks (drawing the 4th and 5th term and finding the 10th term) seemed to be only a by-product.

According to Bella, tasks are given to examine if students have acquired the rules that have been taught by the teacher. This is seen in her explanation of why she does not want to give the students pattern tasks at the beginning of the 7th grade before they learned about variables and algebraic expressions.

"We do it (pattern tasks) at the beginning of the 7th grade and most of the kids have a really hard time with it, and I really understand them. I don't know, like, how do they want kids to answer that?".

We see here that Bella relates to the task as only reflecting students' success in producing an answer ("have a really hard time with it") and wondering how "they" (implying the curriculum writers) expect students to do so. There is no reference in

⁶ Legend:

'word-' – the speaker stopped in the middle of the sentence.

'word...' – the speaker "stretched" the last word, saying it slowly.

'(...)' – some words were omitted.

'(words)' – some words were added for clarity.

Bella's talk to the task being a means for progressively enabling students to produce the answer to the task.

4.1.1 Expectations from "weak" students

Since from her experience “weak students” rarely succeed with pattern tasks, Bella opposes giving these tasks to such students. She expresses this when rejecting the Circles Task as suitable for Yoav: "(These tasks are not suitable) for average and weak kids- I'm not talking about the strong ones, (...)". Bella differentiates between “strong” students and others, implying that tasks should be given to students only if they can succeed in them.

Bella had previously marked Yoav as a weak student introducing him to Tzila as “a really struggling child”. Although she saw him as having “potential”, mainly due to his motivation and willingness to work hard, she also identified him in the 5th meeting as extremely anxious and suffering from low self-confidence.

Parts of how Bella sees weak students such as Yoav can be understood from the ways she characterized “stronger” students. In the 2nd meeting she talked about the “talented children”, who “aren't intimidated by math. Like, they're mostly successful”. She described such students with some awe. For example, when talking about one of her “talented” students she said: "In the way he got the answer, he got straight to it. As though... it just popped up for him. He understood the meaning of it". In that way, Bella dichotomizes between students who “simply get it” and students who don't. Interestingly, she authors herself as not having been a particularly bright mathematics student, in contrast with her young children and husband, who have “a (natural) tendency” for math.

4.1.2 Teaching and learning

To Bella, the valued learning outcome is that students know the procedures they have acquired and thus succeed in solving the tasks. For example, in the 2nd meeting she described how she knows a student understands saying "He succeeds. He succeeds in doing it (the task)". Bella switches between the verbs "succeed" and "understand" as if they had the same meaning. In other words, for Bella, if the student does not answer the question, it means he simply does not know, and if he answers correctly, then he “understands”.

If students did not acquire the knowledge, Bella says she should re-deliver it. In the same meeting, when asked by Tzila how she plans to help a struggling student, she said:

"I want to take him out (of class for a tutoring lesson) (...) mainly to give him this... motivation boost, like 'Here, you can do it', 'There are some exercises that you do understand'. (...) (I will) sit with him, explain to him slowly".

Thus, the two main tools that Bella sees as promoting struggling students are giving emotional support ("motivation boost") and explaining, perhaps more slowly than in the classroom.

4.2 Commognitive conflicts between Bella and Tzila

Overall, the meta-rules implied by Bella regarding the nature of the task, what it means to learn and to teach, and who can learn (specifically whether Yoav can learn) were drastically different than those of Tzila. Tzila regarded the Circles Task as a means for knowledge construction, not just as providing evidence for the existence of knowledge; regarding who can learn, Tzila saw Yoav as capable of such learning (constructing new knowledge from previous more basic knowledge) and the fact that he would say "I don't know" only provided a springboard for such learning (in fact, in her view, if Yoav already "knew" how to solve the task there would be nothing to learn). Finally, Tzila saw the role of teachers as mediators or cultivators of students' learning, namely helping them construct new knowledge from the stems of existing one.

To illustrate the commognitive conflict that ensued due to the conflicting meta-rules between Bella and Tzila, we provide a short excerpt of their discussion:

Table 1. Transcript excerpt from a discussion between Bella and Tzila

Line	Speaker	What is said (what is done)
159.	Tzila	But now I- Now we're in a situation- Let's say you brought this question to Yoav, and he- We got to question B, (and) he tells you "I don't know". What do you do?
160.	Bella	(3 seconds pause), (smiles and nodes her head slightly, 2 seconds) I don't know. He doesn't know section B? in B?
161.	Tzila	Yes, yes
162.	Bella	I'll tell him, "Keep drawing." (3 seconds pause) or "Think". "Write", like. "Instead of drawing. If you want". "Add two each time"

163.	Tzila	Mhm...
164.	Bella	Like, (I'll tell him) "In drawing 3 you have 7. So, in drawing 4 you have 9 and in drawing- there's 11. Go ahead in this drawing"
165.	Tzila	Mhm... So, what did you do now? Now you've told him what he needs to do to solve the problem (slight smile)
166.	Bella	Yeah. Oh. I'm not going to do that. (but) you're telling me, (what will I do) if he doesn't succeed, like.
167.	Tzila	No, then no. Let's think now of the most extreme (case). I now want as an interviewer, to hear his thinking. So, what do I do when he says "I can't"? (2 seconds pause) Do you understand? We'll play the game...
168.	Bella	Oh... (prepare it) in advance.
169.	Tzila	Yes
170.	Bella	Um... what can I tell him? (smiles)

We first tend to whether the conflict is even visible, namely, whether the different meta-rules can be gleaned from both participants' talk. We see, for example, that in the initiating question of what Bella can do if Yoav "says 'I don't know'" [159] is being repeated, even though Bella claimed she doesn't know what to do in such a situation [160]. By insisting "so, what do I do when he says 'I can't'?" [167] Tzila hints that something can be done by Bella in such a situation, something which Bella has not thought of yet.

We also see the contrast between Tzila's and Bella's use of words around the imagined situation with Yoav. Tzila implicitly differentiates between "telling" or "saying" that one doesn't know and "not knowing" per se [159, 167]. Bella, on the other hand, does not pick this up and instead only talks about Yoav "knowing" or "not succeeding" [160, 166].

Another point of misalignment between the two participants' pedagogical discourse may be detected in the excerpt about the role of the teacher. Tzila attempts to incorporate the responsibilities of interviewers ("I want, as an interviewer, to hear his thinking" [167]) so that Bella will engage with this role and think about the ways by which Yoav can respond. However, Bella does not pick this up and instead uses the verb "tell" ("what can I tell him"? [170]). This implies she finds it difficult to think of any alternative actions to "telling" in the teaching-learning situation.

In response to Bella's confusion in [170] Tzila goes into a relatively long stretch of talk, trying to explain her pedagogical thinking to Bella:

"(...) Like, to ask a question that doesn't direct him to a solution (2 seconds pause). (Do you) get what I'm saying? (...) What questions can we ask him... that

will cause him... to make a little more effort at it... and try to explain... or share with you what he can't or can do. That's actually what we want to happen. Even if we don't get to question D in the end, but at every point where he has difficulty... how we can linger a little longer in this difficulty and not... jump straight to... 'well then, we'll tell you, we'll give you a way'?" [171].

We make several observations about this small speech: First, Tzila moves here from a dialogical attempt to elicit responses from Bella to a more monological approach. This may be a response to the impasse encountered by both participants, where Bella is unable to respond to Tzila's requests. Second, despite the relatively lengthy stretch of talk, the misalignments between Tzila's meta-rules and those of Bella are not easy to detect. Thus, although Tzila's descriptions of the learner are more active than Bella's ("make more effort", "try to explain"), she does not directly challenge Bella's binary view of learning as movement from "not knowing" to "knowing". In fact, some of her comments align with this perspective. For example, when she says: "even if we don't get to question D," she implies that question D is the ultimate goal, without showing how solving questions A, B, C could support reaching it. In this framing, "difficulty" remains the opposite of "knowing," and does not question the binary view Bella holds. Third, in no place does Tzila refer to the process of teaching Yoav as growing or cultivating knowledge. Although her references to Yoav "trying to explain" may be envisioned as instances where Yoav would explain his initial thoughts (which could later be built upon, perhaps with the aid of the teacher), this process is never explicated. Instead, Tzila reverts to more psychological narratives such as "lingering in the difficulty", which relate more to the emotional experience of Yoav than to the mathematical work he may be doing.

Despite its implicitness, we observe that the misaligned discourses seem to raise emotional tension between Tzila and Bella. This is seen in relatively long pauses [160, 162, 167], slightly embarrassed smiles [160, 165, 170], and multiple attempts by Tzila to repair and minimize the tension.

5 Discussion

This study explored the meta-rules underlying commognitive conflicts between an instructor and a teacher, Bella, during a professional development program focused on teaching mathematics to struggling students.

Our findings revealed that Bella's discourse reflected a delivery-oriented approach, in which teaching is seen as transmitting well-defined procedures and

tasks are used to assess whether students have absorbed this knowledge. Bella's meta-rules align with the Delivery Pedagogical Discourse (DPD), a discourse that emphasizes clear teacher's explanations, students' memorizing procedures, and minimizing students' struggle (Heyd-Metzuyanin & Shabtay, 2019).

Bella's dichotomy between "strong" and "weak" students reflects and reinforces the DPD and its meta-rules about struggling learners. These findings echo Nachlieli and Heyd-Metzuyanin (2022) who found teachers aligned with the DPD often regarded low achieving classrooms as unsuitable for cognitively demanding tasks and discourse-rich instruction.

Furthermore, Bella's meta-rules are connected by the metaphor of learning as delivery of knowledge. According to this metaphor, knowledge is a fixed object to be transmitted, and learning is measured by whether the student "absorbed it" or not. If a student fails to perform or answer correctly, the default assumption is that they simply "didn't get it", because the student lacks the (perhaps innate) capacity to absorb the knowledge. Motivation, or effort, can assist in this "absorption" process, but ultimately, one either "gets it" or not.

Tzila's discourse, on the other hand, seems to operate with a different metaphor, which might be called a growth or construction metaphor, emphasizing learning as a process of development that emerges through thinking, exploring, and making sense of mathematical ideas. One of the main differences between these two metaphors is made visible in our findings, where the question of who can learn mathematics relates to the other meta-rules underlying the metaphors. As can be seen in table 1:

Table 2. metaphors of learning and teaching

Learning-teaching metaphor	Delivery / Acquisition	Growth / Construction
Mathematical knowledge	Knowledge is transmitted from the teacher to the student	Knowledge is constructed by the student with help / mediation of the teacher
Who can learn?	Only students that have certain "abilities" can absorb the knowledge delivered to them	Anyone can learn and advance from the point they are, relying on their existing knowledge

The commognitive conflicts between Bella and Tzila remained mostly hidden and implicit. It remains an open question to what extent they could have been directly exposed and addressed during the intervention.

Although this study is limited to a single teacher-instructor case, it highlights the potential of the unique one-on-one intervention that allowed us to closely observe processes of a teacher's learning to teach struggling students. Bella's reflections at the end of the intervention expressed satisfaction and curiosity about her conversations with Tzila, indicating a possible shift in how she thought about teaching struggling students. Still, it remains to be seen in future steps of this larger project, whether her pedagogical discourse has indeed changed.

This case illustrates the tension between maintaining a positive atmosphere between an instructor and a teacher and the need to make conflicts explicit as a means of shifting the teacher's pedagogical discourse. The study also underscores the need to better understand meta-rules of teaching struggling students and the ways to productively shift them.

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