

Integrating flow framework into learning and development systems

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Abstract: This study explores the integration of the flow framework into a Learning and Development Systems (LDS) setting, with the aim of enhancing personalized learning experiences. We propose an adaptation of the flow framework to better reflect the dynamics of LDS environments. The model is tested by analysing a learning path generated by an existing recommendation algorithm, suggesting that traditional approaches based on a priori difficulty might fail to sustain optimal experience. We then implement an adapted recommendation that maintains the flow dynamics throughout the learning materials. The results highlight a trade-off between preserving alignment with flow and ensuring full content coverage. This work suggests that flow-informed design can enhance the personalization of mathematics learning within LDS and highlights the need for further developments in learner profiling and feedback integration.

Keywords: learning and development systems, personalized learning, flow framework, perceived competence, perceived difficulty.

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1 Introduction

Learning and Development Systems (LDS) are advanced digital environments designed to support personalized and adaptive learning by leveraging learner data and algorithmic recommendations to guide users through tailored learning pathways (Gera et al., 2024).

At the core of LDS environments are recommender systems, which generate personalized learning sequences by dynamically aligning learning materials (LM), such as videos, exercises, or assessments, with the learner's evolving profile. While traditional recommender systems have been studied in domains such as e-commerce, recent research has highlighted their growing importance in education, particularly for fostering engagement and personalization (Verbert et al., 2012).

Recent studies highlight the need to move beyond standardized approaches toward adaptive pathways responsive to learners' needs (Albano et al., 2014; Gera et al., 2025). One of the main goals of an LDS is to enhance the learning experience by fostering engagement. In this regard, Csikszentmihalyi's (1990)



theory of optimal experience offers a valuable perspective, as his notion of flow describes a psychological state of full engagement and intrinsic motivation occurring when individuals perceive a balance between the challenge of the activity and their own abilities.

The concept of flow, though initially developed in general psychological research, has increasingly been applied in educational settings as a framework for fostering engagement and effective learning. In mathematics education, for example, Liljedahl notes that “flow is where engagement and, as a consequence, thinking happens” (2020, p. 148). He emphasizes the importance of task design in supporting the conditions for flow, and consequently in fostering mathematical thinking.

However, applications of the flow framework in mathematics education have largely focused on specific tasks, without addressing how to sustain flow across entire learning experiences. At the same time, LDS have not yet incorporated flow principles into their recommendation logics. To address these two gaps, our contribution examines CHUNK Learning (Gera et al., 2019), a representative LDS in the mathematics domain that structures LMs into modular units and generates personalized learning paths (LP) through microlearning content based on a dynamic learner profile. Building on this, we analyze how such LP can be interpreted and strategically adapted using flow-informed principles. This requires reinterpreting the classical flow framework as an “adapted flow” model that aligns with both the specific characteristics of LDS and theoretical perspectives from mathematics education.

The paper is organized as follows. Section 2 outlines the theoretical framework, focusing on the concept of flow and its reinterpretation in the context of LDS. Section 3 describes the methodology adopted to analyze and adapt the recommended LP based on flow. Section 4 presents the empirical analysis of one representative learner profile, comparing the initial and adapted learning trajectories. Finally, Section 5 discusses the implications of the findings and outlines directions for future research.

2 Theoretical framework

Research on affect in mathematics education has increasingly emphasized how emotions, attitudes, and beliefs influence learning processes and outcomes. A foundational contribution is McLeod's (1992) taxonomy of the affective domain, organizing constructs along a continuum of temporal stability and cognitive involvement:

from beliefs (most stable and cognitively oriented) to attitudes (moderately stable and combining affective and cognitive components), to emotions (most transient and predominantly affective).

Building on this, Hannula (2012) emphasizes the interconnected roles of cognition, motivation, and emotion in learning. Cognition supports information processing, motivation governs behavior through goals and intentions, and emotions act as feedback mechanisms for both.

Emotions in learning are not undifferentiated reactions but are typically directed toward specific objects or aspects of the learning environment (Pekrun et al., 2023; Schukajlow et al., 2023). Recent classifications (Pekrun, 2024) distinguish three main categories of emotions relevant to mathematics education: achievement emotions, related to achievement activities or outcomes, and directed toward domains, competencies, strategies, or tasks; epistemic emotions, elicited by cognitive incongruity during knowledge construction, which may lead to enjoyment, frustration, or boredom depending on whether the disequilibrium is resolved (D'Mello & Graesser, 2012); and social emotions, which arise in interpersonal contexts and involve self- or other-related feelings such as pride, shame, or admiration.

For the motivational dimension, Self-Determination Theory (Ryan & Deci, 2002) offers a valuable theoretical framework. It posits that the fulfillment of three basic psychological needs, competence, autonomy, and relatedness, is essential for the development of intrinsic motivation and for fostering positive emotional experiences. Closely related to this is the construct of mathematical self-concept, which refers to students' perceptions of their own ability in the domain of mathematics. Following Pajares and Miller (1994), perceived competence can be seen as an integral part of this self-concept, encompassing both cognitive evaluations and beliefs of self-worth.

Di Martino and Zan (2011) highlight the complex relationship between beliefs, emotions, and attitudes in mathematics learning, showing how perceived competence (Pajares & Miller, 1994), expressed as “I can/I can't,” and emotional disposition (e.g., “I like/dislike mathematics”) influence students' engagement and group interactions. Similarly, Andrà and Liljedahl (2014) show that statements such as “I can” tend to foster engagement, while “I can't” often lead to passivity.

2.1 Optimal experiences and flow

The concept of *optimal experience*, as introduced by Csíkszentmihályi (1990),

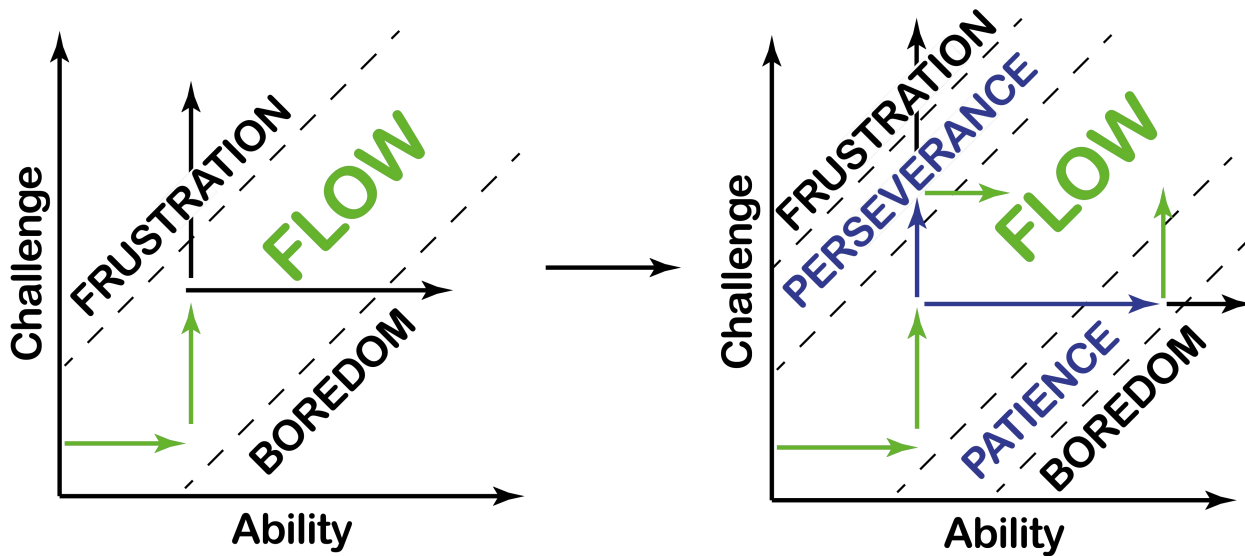
describes a psychological state in which individuals are so engaged in an activity that nothing else seems to matter. The activity is experienced as so intrinsically rewarding that people persist even at personal cost, purely for the desire to engage in it. As Csíkszentmihályi writes:

...the experience is so enjoyable that people will continue to do it even at great cost, for the sheer sake of doing it. (1990, p. 4)

Flow is the term used to describe this optimal psychological state, often considered “the bottom line of existence” (Csíkszentmihályi, 1982, p. 13), characterized by cognitive efficiency, high motivation, and deep satisfaction (Moneta & Csíkszentmihályi, 1996). Flow occurs when individuals perceive a balance between the challenge of the activity and their own abilities. As illustrated in Figure 1 (left), if the challenge is too low respect to the ability, the result is boredom; if it is too high, it leads to frustration. The flow channel lies in the dynamic equilibrium where both dimensions are high and well matched. Recent research on affect in mathematics education confirms that negative deactivating emotions (e.g., boredom) and negative activating emotions (e.g., frustration) can significantly hinder motivation, self-regulation, and engagement (Pekrun et al., 2023; Camacho-Morles et al., 2019).

Building on Csíkszentmihályi’s model, Liljedahl (2018) introduced two additional regions in the flow diagram (Figure 1, right), perseverance and patience, which act as buffers between flow and frustration, and between flow and boredom. These intermediate states allow learners to remain engaged when temporarily outside the optimal balance of challenge and ability, thus preserving autonomy and promoting self-regulated transitions back into the flow channel.

Figure 1. Flow and extended flow diagrams



Note: The left diagram represents the classical flow framework by Csíkszentmihályi (1990). The right diagram illustrates the extended model proposed by Liljedahl (2018), introducing perseverance and patience.

2.2 Flow in the context of LDS

Within the LDS, recommender systems must select and sequence LMs, thereby supporting students in developing both competence and confidence. Inadequate recommendations may lead to disengagement, triggering emotions such as frustration or boredom. In this context, flow can offer a valuable perspective to evaluate and refine the design of personalized learning trajectories.

While the concept of flow has been widely applied in mathematics education to analyze optimal experiences during problem solving and task-based activities (Williams, 2001; Liljedahl, 2018, 2020), LDS environments typically include a broader spectrum of LMs such as explanations, worked examples, guided exercises, interactive experiments and assessment tasks. These resources do not always involve direct task execution, but often require students to engage in learning new mathematical knowledge.

Since flow is characterized by cognitive efficiency, high motivation, and deep satisfaction (Moneta & Csíkszentmihályi, 1996), it has often been associated with intrinsically motivated engagement. We interpret this motivational component in light of Self-Determination Theory (Ryan & Deci, 2002), which identifies the fulfilment of the need for competence as a fundamental condition for intrinsic motivation and positive emotional experience. In this perspective, we define ability in the flow as the learner's perceived competence, that is, the belief in their own capacity to understand and manage the proposed material (Pajares & Miller, 1994).

Turning to the “challenge” dimension of the flow, research has traditionally conceptualized difficulty as an objective property of the task or activity, often included as a variable in experimental designs (Efklides et al., 1998). This assumption also underlies most recommender systems, where difficulty levels are typically assigned a priori by content authors and applied uniformly across all learners. However, this approach does not account for the learner’s perceived difficulty, which may vary significantly depending on individual characteristics and prior experiences.

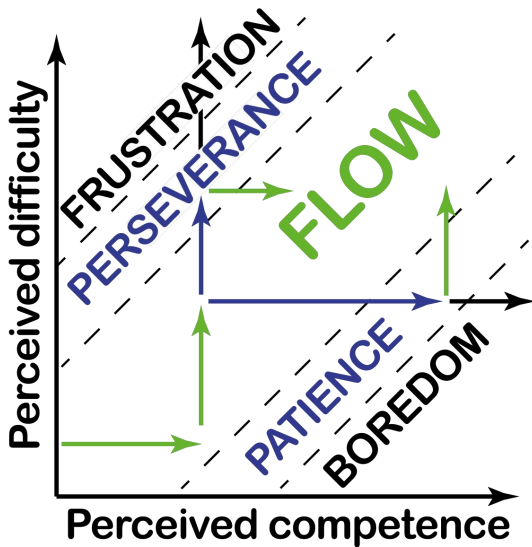
According to its definition in the literature, perceived difficulty is typically assessed after the activity has been completed (e.g., Spagnolo & Saccoletto, 2024). However, recommendation systems, where learning paths must be generated before the learner interacts with the content, also require an a-priori estimation of perceived difficulty, but as a proxy for how difficult a learning material might be for a specific learner, based on learners’ goals, preferences, and background knowledge.

To support this operational need, we propose a method to estimate perceived difficulty a priori, grounded in the notion of cognitive disequilibrium. Originally introduced in Piaget’s theory of cognitive development (Piaget, 1964), cognitive disequilibrium refers to the mental imbalance that occurs when new information fails to align with existing cognitive schemas. D’Mello and Graesser (2012) reintroduce cognitive disequilibrium and connect it to the activation of epistemic emotions (Pekrun, 2024), particularly in processes of knowledge acquisition and exploration within digital learning environments, referring to frustration, boredom, and flow. Lodge et al. (2018) further emphasize its role in conceptual learning more broadly, highlighting how states of disequilibrium can trigger affective responses that influence engagement and cognitive processing.

In the context of LDSs, we therefore propose to interpret the “challenge” dimension of the flow in terms of an a priori estimation of perceived difficulty, defined as the level of cognitive disequilibrium subjectively experienced by the learner during the active processing of a LM, including but not limited to tasks, and expressed through epistemic emotions.

We thus introduce the “adapted flow model” (see Figure 2), in which we replace ability with perceived competence, and replace challenge with perceived difficulty. This shift aims to align the flow framework with the dynamics of LDS, where engagement depends on the active processing of diverse LM.

Figure 2. Adapted flow for LDS environments model



Note: The figure represents the adapted flow model, which replaces the original dimensions of ability and challenge with perceived competence and perceived difficulty.

Having redefined the core dimensions of flow—perceived competence and perceived difficulty—we now turn to the practical implications of this framework for the design and analysis of personalized learning paths.

This leads us to the following research questions: How can recommended LP be interpreted through the lens of the adapted flow? And how might recommended paths be adapted, based on this redefinition of flow, to potentially support more optimal learning experiences?

3. Methodology

The nature of the research question requires the use of mixed methods. That is, we use quantitative data to fit into the qualitative framework of the adapted flow model. To describe such a mixed method, we present the set of data, explaining the construction of perceived competence and perceived difficulty. Finally, we describe the procedure used to adapt the learning trajectories based on our model.

3.1 Data source and learning path generation

The data used in this study came from a mathematics course taught by one of the authors at Naval Postgraduate School. During the course, the instructor used the chunk learning for the teaching of the class and collected the data through survey. The profiles of the students are drawn according to the methodology reported in

Gera et al. (2025). These profiles take into account self-assessments of cognitive levels with respect to the main topics covered by the available LMs, as well as learning goals, preferences regarding content types (e.g., educational, research, professional skills), preferred media formats (e.g., video, slides, PDFs), and time constraints.

The initial recommendation system, developed by Mochocki and Reith (2025), generates optimized LP based on detailed learner profiles. Each LM is annotated with metadata such as topic, content type, author-assigned difficulty, format, a list of prerequisite contents and additional pedagogical or contextual descriptors. Learning path generation follows a two-stage process: first, a subset of LMs is selected based on the similarity between learner profiles and content metadata; then, a sequencing algorithm organizes the selected materials according to an increasing difficulty progression, intended to align with the learner's initial cognitive level. Prerequisite constraints are incorporated to ensure that foundational content precedes any dependent LM.

3.2 Constructing the flow dimensions

To analyse how each recommended path aligns with the learner's subjective experience, we operationalized the two core components of the adapted flow model: perceived difficulty and perceived competence. These variables were constructed for each learner and LM pair, based on the information provided in the learner profiles and the metadata associated with the LMs. The construction of these quantities follows the theoretical definitions introduced in Section 2.

The a priori perceived difficulty, understood as the level of cognitive difference experienced by a learner during the active processing of a given LM, was estimated by computing the degree of misalignment between the learner's preferences and the characteristics of the LM. This misalignment was computed as the mean dissimilarity across three aspects: the semantic alignment between the learner's goals and the LM topics, the match between preferred and assigned content types, and the correspondence between preferred and actual media formats.

Lower alignment across these dimensions suggests higher potential for cognitive disequilibrium, and thus a higher perceived difficulty. Conversely, a stronger match across the three indicators indicates a lower perceived difficulty, as the learner is more likely to engage with materials that are aligned with their expectations and cognitive schemas.

The initial perceived competence of each learner with respect to a given LM was computed as the average of the cognitive levels self-assessed by the learner for each of the content addressed by that LM. For instance, if an LM covers three content activities and the learner has provided a cognitive self-rating for each, the initial perceived competence for that LM corresponds to the mean of these ratings.

However, perceived competence is not considered static throughout the learning path. Following the logic of the flow framework, increasing levels of challenge should be paired with a gradual development of the learner's skills over time (Csíkszentmihályi, 1990; Liljedahl, 2018). In our model, this principle is operationalized by treating perceived competence as a dynamic variable that is updated after each LM, depending on the perceived difficulty of the preceding material.

When learners engage with materials that match or slightly exceed their competence, their perceived competence increases. Conversely, when the perceived difficulty is too high or too low, perceived competence remains unchanged.

3.3 Adapting recommended paths

To investigate how recommended LP could be restructured in accordance with the adapted flow model, we designed a reordering algorithm that aims to maintain each learner within or near his or her specific flow channel throughout the learning path as new content is being suggested. The algorithm considers the perceived competence and perceived difficulty associated with each LM, while enforcing prerequisite constraints to preserve content coherence.

The ordering procedure for our methodology follows these main steps:

1. Entry point selection
 - Identify all LMs that do not depend on any prerequisite elements within the subset of LMs under consideration—that is, those whose required prior knowledge is not covered by other LMs in the current set.
 - Among these, the LM with the lowest combined value of perceived difficulty and perceived competence is selected as the starting point.
2. Path construction step by step
 - Evaluating the learner's state, based on estimated perceived competence and perceived difficulty of the previous LM.

- Selecting the next LM, ensuring prerequisite constraint are satisfied, as follows:
 - If the learner is closer to the lower boundary of the flow region (risk of patience/boredom), the next LM is selected with a slightly higher perceived difficulty to sustain engagement.
 - If the learner is closer to the upper boundary (risk of perseverance/frustration), the next LM is chosen to match the previous difficulty level, allowing perceived competence to increase and potentially bringing the learner back toward the optimal zone.
 - If no LM satisfies these primary criteria, fallback strategies are employed: The acceptable range of perceived difficulty is gradually widened to find a suitable LM candidate.

The fallback strategies introduce flexibility, which helps accommodate the limitations of the LM pool and the prerequisite structure, even if it leads to departures from the ideal flow trajectory.

A consequence of this strict application of flow-based ordering is that some LMs may be excluded from the final sequence if they cannot be integrated without violating either the flow or prerequisite constraints.

4. Data analysis and results

This section presents a two-step analysis. First, we interpret the initial recommended paths through the lens of the adapted flow model. Then, we show how the same set of LMs can be adapted using the methodology described in the previous section.

The analysis is based on 12 learner profiles constructed as described in Section 3.1, each including cognitive self-assessments, learning goals, content preferences, and declared time availability. A total of 356 unique LMs, covering 67 content areas.

The number of LMs recommended to each learner ranged from 2 to 153, depending primarily on the declared time availability, but also influenced by the alignment between author-assigned difficulty and self-assessed competence, prerequisite coherence, and individual preferences.

To illustrate the flow-based analysis and adaptation procedure, we focus on one representative case for in-depth examination, as the patterns observed in the initial recommendations and their flow-based adaptations were consistent across all

analysed profiles. To support the representativeness of this case, we also report two additional examples (Figures 4 and 5), where similar patterns emerge across different learner profiles.

This learner was assigned a relatively high number of LMs (80) in the initial recommended path, having declared one of the highest available time ranges on the platform. The profile was characterized by the following features:

- Expressed no preference regarding media formats;
- Indicated a preference for content types categorized as readings or articles;
- Reported medium to low self-assessed cognitive levels across most content areas, with a majority rated at level 1 or 2, and only a few at levels 3 and 4.

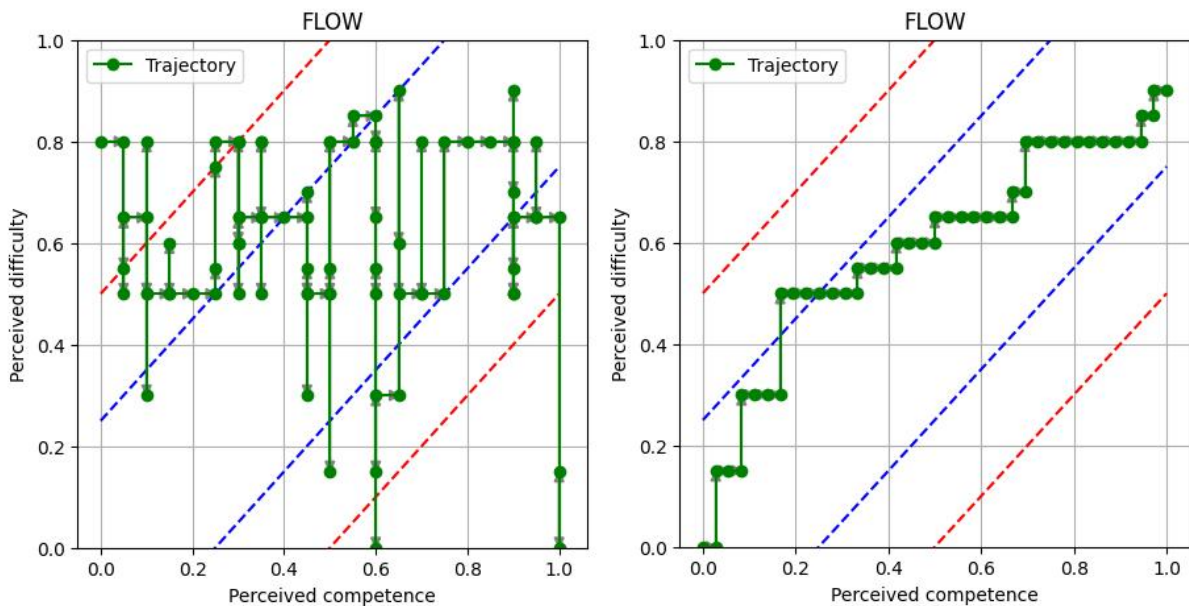
To visualize the learner's evolving experience within the adapted flow model, we plotted the trajectory of estimated perceived competence and perceived difficulty across the sequence of LMs recommended by the system. Each point represents the learner's state at the moment of engagement with a specific LM, and the path is traced according to the recommendation order. The x-axis corresponds to perceived competence, while the y-axis indicates perceived difficulty, dynamically updated as described in Section 3.2.

Figure 3 shows the learning trajectory of the selected learner profile before and after the adaptation. In the left panel, which represents the initial recommended path, the trajectory clearly does not align with the logic of the adapted flow model. The learner's trajectory frequently exits the optimal flow region, suggesting potential moments of frustration or boredom according to the model. The perceived difficulty rises and falls continuously along the path, preventing a steady growth in perceived competence and leading instead to frequent stagnation points in the learner's experience.

Notably, the very first LM has a perceived competence of 0 but a perceived difficulty of 0.8. This discrepancy stems from the fact that LM4, although assigned a low a priori difficulty by the authors, does not align with the learner's profile and preferences—resulting in a high perceived difficulty and an initial state of frustration due to cognitive disequilibrium. This mismatch reflects a fundamental difference in design perspective, as the system prioritizes alignment between author-assigned difficulty and the learner's cognitive level, whereas our approach focuses on the learner's perceived difficulty as a subjective experience.

In contrast, the right panel shows the adapted LP, generated using our flow-based methodology. The trajectory progresses more gradually, maintaining a closer alignment with the central flow region. Perceived competence increases incrementally, and perceived difficulty follows a smoother and more controlled evolution. Although a brief excursion into the perseverance zone is observed, the learner promptly returns to the flow region.

Figure 3. Initial and adapted LP within the adapted flow



Note: Comparison between the initial recommended path (left) and the adapted learning path (right) for a single learner profile. Each point represents a LM positioned according to its perceived competence (x-axis) and perceived difficulty (y-axis). The trajectory reflects the order of presentation. The background shows the adapted flow model: the area between the blue dashed lines defines the optimal flow region; areas between red and blue lines represent intermediate states (perseverance above, patience below); and outer regions correspond to frustration (top) and boredom (bottom).

To satisfy both the logic of the adapted flow model and the prerequisite constraints, 11 LM from the initial recommendation could not be integrated into the final sequence. This outcome highlights the inherent trade-off between preserving a coherent and engaging experience and ensuring full content coverage, raising important considerations about the balance between adaptation and completeness in personalized learning design.

Figures 4 and 5 show the initial and adapted trajectories of two other students, confirming similar patterns. In Figure 4 (right), a brief excursion toward frustration occurs due to the absence of LM with intermediate perceived difficulty between two steps, but it is quickly compensated, bringing the learner back into flow.

Figure 4. Initial and adapted LP for Student 2

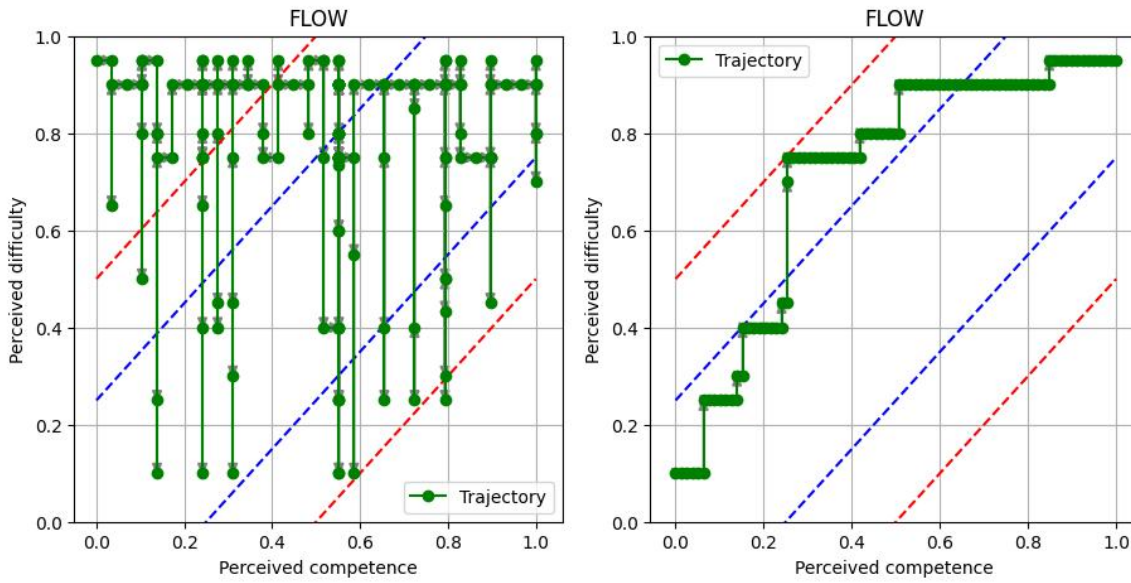
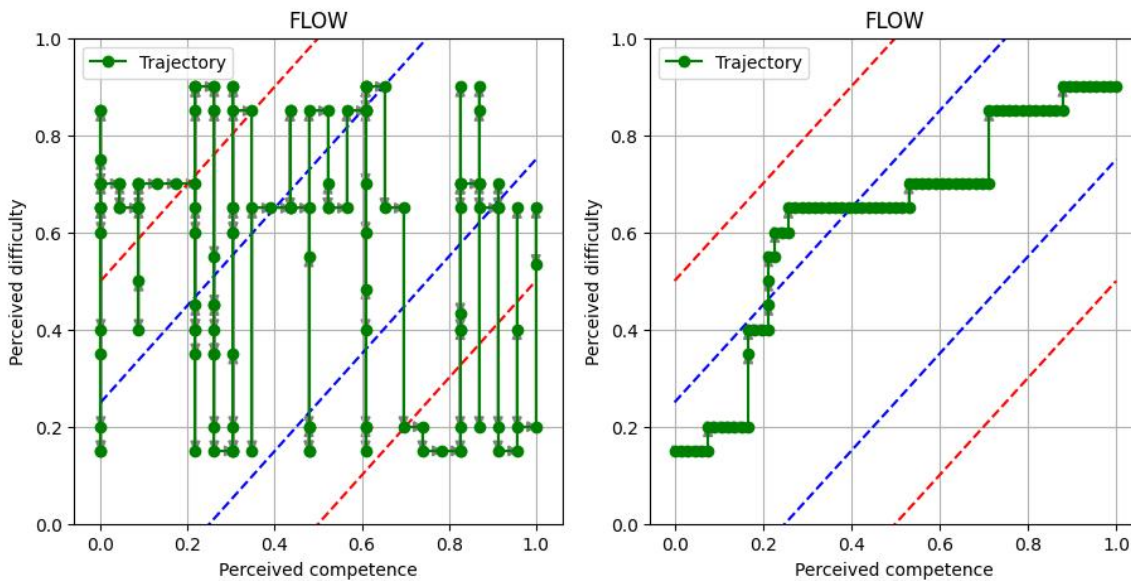


Figure 5. Initial and adapted LP for Student 3



5. Discussions and conclusion

The goal of this research was to introduce a different perspective of how an LDS may support personalized learning experiences by integrating the flow framework as a guiding lens. To this end, we proposed an adaptation of the flow framework to LDS delivering mathematical content, aiming to better model the evolving learner experience across diverse types of LMs. This reinterpretation was motivated by the need to align the flow dimensions with the characteristics of LDS and with

perspectives from mathematics education, which involve not only assessment but also the active processing of content through explanations, exercises, simulations and other resources.

We reinterpreted the flow dimension of “ability” with “perceived competence”, according with the Self-Determination Theory (Ryan and Deci, 2002), which identifies the need for competence as a fundamental condition for intrinsic motivation. This perspective is also supported by the research in mathematics education which emphasizes how perceived competence influences students’ engagement and persistence (e.g., Di Martino and Zan, 2011).

Similarly, we reconceptualized “challenge” as “perceived difficulty”, redefined in this context as the level of cognitive disequilibrium (Piaget, 1964) experienced by the learner during the active processing of a given LM. This redefinition is consistent with the work of D’Mello and Graesser (2012), who describe cognitive disequilibrium as a trigger for epistemic emotions: if resolved, it can lead to epistemic enjoyment, while if unresolved, it may result in frustration or boredom.

Once the flow dimensions were reconceptualized, we resorted to the adapted flow model to analyze in a descriptive way 3 LPs generated by the recommendation algorithm developed by Mochocki and Reith (2025). While the system demonstrates a robust structure and an effective use of learner profiles, our analysis identified a key limitation when assessed through the lens of the adapted flow model. Specifically, the such an algorithm prioritizes alignment between the learner’s cognitive level and the a-priori difficulty assigned by content authors. As a result, perceived difficulty fluctuates considerably along the path, leading to repeated stagnation in the development of perceived competence.

We then exploit the adapted flow model in a normative way, as described in Section 3.3, to propose a LP that better adheres to the theoretical conditions for flow. In this case, LMs were sequenced with a gradually increasing perceived difficulty, enabling the learner to stay within or near the flow region, according to the model. However, enforcing both the flow and prerequisite constraints led to the exclusion of several LMs from the original learning path.

While our study focused on a specific case, the approach is generalizable to any LDS that collects learner profile data, offering a perspective for evaluating and adapting learning paths that take into account the evolving nature of the learner’s experience.

Several aspects, however, require further refinement. In our methodology, perceived difficulty and initial perceived competence are indirectly estimated from the learner's initial profile and LM metadata. In particular, the current estimation of perceived difficulty relies on learner profiles defined in Gera et al. (2025). Future work should therefore explore how to redesign these profiles and related data collection processes, so as to align more closely with our a priori estimation of perceived difficulty, grounded in the notion of cognitive disequilibrium.

Another important direction for future research concerns the validity of our a priori estimation of perceived difficulty. The model incorporates this dimension using learner profiles and LM metadata, but it remains a theoretical approximation based on cognitive disequilibrium. Additionally, perceived competence is currently modeled as a monotonically increasing variable. Yet, as shown by Andrà et al. (2015), the learner's sense of "I can" may fluctuate throughout the activity, influenced by emotional reactions, social interactions, and the interpretation of failure or success. To address both limitations, future developments should integrate explicit feedback strategies, allowing the system to validate the estimated values and to more accurately model the evolving nature of learners' perceptions.

In summary, our contribution shows that adapting the flow model to LDS might meaningfully inform the construction and adjustment of personalized learning paths. While the approach holds promise for improving learner experience and engagement, its full potential will only be realized through continued refinement of both modeling assumptions and system interactivity.

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