Improvement Without Change in Meta-rules: The Challenge of De-Ritualization

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Abstract: In this study, we explore the changes in mathematical participation of a student who underwent a one-on-one tutoring intervention, led by the first author, aimed at disrupting her ritual participation in the algebraic discourse. The student, Fani (pseudonym), was a 7th grader going on to 8th grade, who was on the verge of being transferred to a lower track in mathematics. Our intervention aimed to de-ritualize her discourse, that is, to assist her in becoming more agentive and to produce narratives about mathematical objects rather than follow procedures. To assess the effectiveness of the intervention, we compared the pre- and post-intervention interviews with Fani using the dimensions of ritual and explorative participation, including agentivity, syntactic/objectified mediation, bondedness and focus on procedure or on result. Our analysis shows that Fani's enactment of algebraic tasks remained predominantly ritual both before and after the intervention. Nevertheless, the number of correct responses increased. Examining the interaction in the interview revealed that the instructor's prompts, although designed to encourage justification and substantiation, mostly failed to produce de-ritualization due to different meta-rules followed by Fani than the canonical ones.

Keywords: Commognition, de-ritualization, meta-rules, algebra.

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Upon moving from elementary, arithmetics-focused mathematical learning to middle-school algebra, students often encounter difficulties, leading them to "ritual cycles of failure" (Heyd-Metzuyanim, 2025). These cycles involve the student focusing solely on authority-pleasing while imitating procedures, without reflection or self-authored justifications (Heyd-Metzuyanim, 2015). Despite ritual participation being quite common, not much is known about how it may be disrupted. In the reported study, we designed a one-on-one intervention aimed at helping students who were at the early stages of ritualization – what we term the cusp of failure - to de-ritualize their participation. We aimed to help their participation become more explorative, directed at producing mathematical narratives about objects rather than meaningless manipulation of symbols.

1 Theoretical framework

Much research on student failure in algebra tends to isolate cognitive, social, or affective factors, without accounting for their interdependence. For instance, some studies focus on cognitive barriers such as conceptual misunderstandings (e.g., Kieran, 2006), while others emphasize social dimensions like teaching methods or marginalization (e.g., Gresalfi, 2009), or affective factors such as anxiety and self-efficacy (e.g., Ashcraft & Kirk, 2001). While each of these perspectives offers valuable insights, they often overlook how these domains interact. In contrast, the commognitive framework (Sfard, 2008) conceptualizes mathematical learning as concomitantly involving cognitive, affective and social dimensions, which all work together to form success (or failure) to learn.

Commognition defines learning as becoming a central participant in a discourse, a process marked by shifts in how routines are performed (Sfard, 2008). Routines consist of a task (the perceived obligation) and a procedure (the steps taken to fulfill that obligation). When students are new to a discourse, such as algebra, their participation is mostly *ritual* (Baccaglini-Frank, 2021; Lavie et al., 2019; Sfard, 2008; Sfard & Lavie, 2005). Sfard (2008) and Lavie et al. (2019) provide the theoretical grounding for this view, defining ritual participation and its role in early learning. Sfard & Lavie (2005) illustrate it through empirical examples from early numerical discourse. Baccaglini-Frank (2021) extends the concept to high school algebra, highlighting its implications for low-achieving students. Being unfamiliar with the objects of the discourse, students are bound to repeat procedures modeled by the teacher. Over time, with successful learning, this ritual participation should give way to explorative participation, where students generate and justify their own mathematical narratives.

The shift from ritual to explorative participation, referred to as de-ritualization, can be recognized through changes in how students perform mathematical routines. These changes include: (a) Substantiation—students justify their narratives based on formerly established ones; (b) Bondedness—there is logical continuity between steps in the procedure; (c) Performer's agentivity—students make independent choices; and (d) Goal-focus—students remain oriented toward the problem's objective and verify results (e) Objectified mediation– the narratives are authored about discursive objects rather than about empty signifiers. Objectified mediation reflects the culmination of the process of objectification. For objectified mediation to occur, students need to treat different realizations of mathematical objects as signifying "the same" discursive object and treat it as though it "exists" in the world. For example, x becomes in their discourse a signifier of an object existing independent such as "a number", "an unknown" or "a variable" (Heyd-Metzuyanim et al., 2022; Lavie et al., 2019).

While ritual participation is a natural and necessary entry point into a new discourse, extensive ritual participation is bound to lead, at a certain point, to failure in mathematics, particularly in tests and evaluations. Thereafter, the institutionalized mechanisms of tracking, may afford less and less opportunity for students to de-ritualize their discourse (Solomon, 2007; Zevenbergen, 2003).

Most research aligned with the importance of de-ritualizing students' discourse has assumed that instructional practices that afford students agency, and that provide tasks with multiple solution paths, is the best way to help students advance to more explorative forms of participation (Lipper, & Karavi, 2023; Viirman, & Jacobsson, 2023). At the same time, however, research within the commognitive framework has warned against full reliance on students' reasoning, emphasizing the role of the teacher in critical points of mathematical learning (Nachlieli, & Elbaum-Cohen, 2021). Such critical points have been named **meta-level learning**.

Meta-level learning concerns the adoption of new, often implicit, meta-rules for endorsing mathematical narratives or enacting routines. These are the rules of the mathematical discourse that govern what counts as legitimate activity in this particular discourse (Sfard, 2008). Sub-discourses of mathematics, such as arithmetic or algebra, greatly differ in their meta-rules (Sfard, 2007). Thus, when students are required to shift from one discourse to the next, **commognitive conflicts** may often occur (Sfard, 2007). These are situations where the students and the teacher (or expert) use the same keywords, yet their use is governed by different meta-rules. For example, a key meta-rule in algebra (unlike in arithmetic) is that the equal sign can function as a sign of equivalence of two expressions (e.g. $2x + 1 = x + 3$), while in the arithmetic discourse, the equation sign is often interpreted by students as signifying "the result is" (as "the result of 2 +1 is 3") (Knuth, et al., 2006).

Whenever experts in the discourse relate to a sign (or keyword, e.g. "equal") in accordance to the new meta-rules, while the novices still relate to it according to the old ones, a commognitive conflict ensues. Commognitive conflicts are notoriously difficult to detect (Ben-Zvi & Sfard, 2007). However, their role in the processes of de-ritualization has so far rarely been inquired.

This gap leads us to our central research questions: Given an intervention specifically aimed at de-ritualizing a student's algebraic discourse, in what ways did this student's discourse advance and did it show signs of de-ritualization? In addition, we ask – were there commognitive conflicts in the teacher-learner interactions and if so, in what ways did they impact the de-ritualization process?

2 Method

This study was based on a 10-session one-on-one intervention conducted during the summer break with Fani, a 13-year-old student who had just completed 7th grade. The sessions took place at a prestigious university. Due to high achievement in elementary school, Fani had been placed in an advanced-level math track in 7th grade. However, toward the end of the school year, she began failing exams. Her continued placement in the advanced group depended on her performance on a test at the beginning of 8th grade and relied on a "summer task" consisting of a long set of exercises to prepare for this exam.

Fani was recruited following a call for volunteers to participate in an intensive one-on-one summer intervention for assisting students with mathematical difficulties. Only students completing 7th grade who had experienced a decline in math performance within the past year were considered eligible. Among the eligible candidates, interviews were conducted with the parents and mathematics teachers of ten students, from which two students were selected for initial written interviews. Fani was one of them, chosen because preliminary interviews with her, her school teacher, and her mother suggested that her mathematical difficulties had emerged only within the past year, and that her achievements in the subject had been high up till 7th grade. This suggested that her discourse might still be flexible, making her a promising candidate for a short-term intervention aimed at supporting a return to more explorative participation with mathematics.

The intervention was designed around the summer-task exercises. This served two purposes: first, these were best aligned with Fani's goals, given that her main motivation was to pass the tracking exam. Second, based on previous iterations of this study, we expected that designing alternative tasks, albeit possibly more appropriate for Fani's current discourse, but not sufficiently aid her to pass the school exam.

Overall, the data collected around Fani included: video and audio recordings of the mathematics lessons; iPad screen recordings; exams from school; recordings of interviews with Fani's mother and with her teacher. Out of this dataset we chose in this study to focus only on the pre and post-intervention interviews, since these could provide the clearest picture of Fani's progress in mathematical participation.

2.1 The intervention

The intervention consisted of 10 meetings of 2 hours, once a week. The first hour was dedicated to mathematics and the second hour to mathematical identity-enhancing activities, including walks around the campus, meetings with undergraduate students, and visiting laboratories. Due to space constraints, we will not treat the identity-enhancing part of the intervention.

The mathematical content of the meetings revolved around the summer-task curriculum but was carefully designed to accommodate as much as possible Fani's current discourse. This was done through iterative cycles of recordings, observations and analysis of the lessons with Fani, performed by the research group (authors and another mathematics education expert). Briefly, the topics of the lessons included: negative numbers and the four arithmetic operations on them; combining like terms; applying the distributive property; one step equations; two-step equations; multi-step linear equations with the distributive property; and word problems associated with linear equations.

2.2 Pre and post interviews

To examine the change in Fani's mathematical participation, the first author conducted with her two mathematical interviews - at the beginning (pre) and at the end of the intervention (post). In these interviews, Fani was presented with a set of algebra-related problems taken from Early Algebraic Discourse Profile (EADP) protocol (Elbaum-Cohen et al., 2023). The EADP protocol is structured as a think-aloud interview and includes tasks that aim to map the level of participation in the early algebraic discourse. It mostly includes tasks from the standard Israeli curriculum, aimed at surfacing the objectification of variables, unknowns, algebraic expressions and equations.

The interviews could provide a lens into the changes which may (or may have not) occurred in Fani's forms of participating in the algebraic discourse, as well as to the dynamics between her and the instructor during the intervention itself. Nevertheless, the interviews also formed a slightly different situation than the teaching ones since in them, the interviewer attempted to avoid interfering with Fani's modes of solutions. We therefore need to take caution in generalizing from them to the teaching-learning situations during the intervention.

2.3 Data analysis

The interviews were fully transcribed, including attention to non-verbal communication and snippets of the writing and visual mediators used by Fani. The main unit of analysis was the task cycle - a sequence of attempts to deal with a given task, beginning with an initial response and ending either with a presented solution or with giving up (Heyd-Metzuyanim, 2025). The mapping of the discourse was conducted in several stages. First, the transcripts were divided into task situations – distinct segments in which Fani acted in response to a prompt or demand, either oral or written (Lavie et al., 2019). Within each task situation, one or more task cycles were identified. In cases where the interviewer encouraged a renewed attempt, a new task cycle began within the same task situation. For each cycle, we recorded whether a narrative emerged at the end of the procedure and, if so, whether it was canonical - that is, consistent with an expert's expected or accepted response. We also noted teacher prompts. These included requests for substantiation, eliciting alternative procedures, and requests for clarification. Task cycles were classified as either ritual or explorative, based on Lavie et al's. (2019) dimensions of de-ritualization, as reviewed in the background. Finally, we analyzed Fani's discourse to identify underlying meta-rules - implicit guidelines governing her participation and routines. We then compared these meta-rules to the canonical ones and searched for indications of commognitive conflicts – namely places in which Lilach and Fani used the same words in different ways or assumed different rules for substantiating mathematical narratives.

3 Findings

The findings are organized in two sections. We first provide an overall view of Fani's pre- and post-intervention progress in answering the EADP tasks. We then move to a more close-up analysis of her participation in one particular task that exemplifies the obstacles that may have hindered the de-ritualization of her discourse.

3.1 Fani's overall progression

Table 1 provides an overview of the questions in which Fani's solutions changed from the pre-intervention to the post-intervention interview. The table highlights where her responses shifted from non-canonical to more canonical forms, indicating progress in her mathematical discourse.

Table 1. Change in Fani's answers across interviews

Note: numbers in parentheses indicate the points given to the solution, imitating a scoring system that would be typical in an Israeli classroom

Task#	Task Type	Pre	Post
1	Visual pattern recognition – the sequence 3, 5, 7,.. given in circles	Relied on incorrect proportional reasoning: the 5th drawing has 11 circles, therefore the 10th would have 22. **Canonical: 4th & 5th drawing Non-canonical; 10th, 42nd, general (nth) drawing (2/5)**	Began identifying and using the correct pattern by adding twos until the 10th item, but then lapsed to a mix of proportional and additive reasoning **Canonical: 4th, 5th & 10th drawing Non-canonical: 42nd, general (nth) drawing (3/5)**
2	Combining Like Terms - $5x + 3y - x + 2y =$	$= 5x + x - 3y + 2y = 6x - 5y$ Negative signs stay in place, only the symbols are moved around **Canonical: no (0/1)**	$= (5x - x) + (3y + 2y) = 4x + 5y$ **Canonical: yes (1/1)**
3	Distributive Property & Combining Like Terms $(x - 3) - 3(4 - x) =$	$= 4 \cdot 3 - 3 \cdot 4 + x - x = x$ No application of the distributive property; - x treated as "erasing the x". **Canonical: no (0/2)**	$= 4x - 12 - 12 - 3x$ Applies distributive law correctly except for one error; other routines are canonical **Canonical: (1/2)**
4	The students' council word problem. Should be translated to the equation: $\frac{1}{4}x + \frac{1}{3}x + 40 = x$	Solves arithmetically **Canonical: yes, but arithmetic (1/2)**	Solves arithmetically **Canonical: yes, but arithmetic (1/2)**
5	Simple Linear Equation $2x + 3 = 11$	Solves by working backwards: Writes: 11-3=8; 8=2x 8:2=4, 1x=4 **Canonical: yes (idiosyncratic procedure) (1/2)**	Solves canonically by applying the same operation to both sides of the equation. **Canonical: yes, including canonical procedure (2/2)**
6	Solving Linear Equation with Distribution $8(x - 1) - 4x = 0$	Claims that x=0 because "the whole thing equals zero" **Canonical: no (0/1)**	Solves canonically by applying operations symmetrically on both sides. **Canonical: yes (1/1)**
7	Solving Linear Equation with Distribution and integers $-5x - (3 - x) = 2x + 3$	Fails to apply distributive law, combine like terms, or manipulate both sides. Writes $-5x - x + 2x = 3 + 3$ **Canonical: no (0/2)**	Still makes error in distributing the negative sign before parentheses but handles all other routines correctly. Writes $-6x - 3 = 2x + 3$ /+3 $-6x = 2x + 6$ /$-2x$ **Canonical: partial (1/2)**
8	Word problem which should be translated to the equation 7x-54=x	Disengages (declares immediately "I don't know") **Canonical: no (0/2)**	Solves arithmetically by guessing and checking. **Canonical: yes, but arithmetic (1/2)**
	Total points	4/17	10/17

As table 1 shows, Fani's performance improved significantly, especially in her ability to carry out procedural routines. Overall, if one applies a scoring system that would be typical in school - giving points to canonical solutions and partial points to non-canonical solutions - Fani's performance would rise from 24% to 59% canonical responses.

Upon a closer look, however, one can observe that most of Fani's progress appeared in the accurate use of algebraic procedures. For example, she significantly improved in canonically applying procedural rules for combining like terms, including negative ones (tasks 2, 3, 5, 6, and 7). In contrast, word-problems that demanded translation of a real-life situation to algebraic symbols hardly improved (tasks 1, 4, 8).

Interestingly, her improvement occurred through arithmetic procedures (e.g. task 8), despite the algebra-focused intervention.

Notably, Fani's spontaneous algebraic discourse hardly improved between the pre and post interviews. She was unable to use letters to expect the regularity that she was finding in a pattern (task 1) and did not see equations as relevant for both the words problems presented to her (tasks 4 and 8).

3.2 Impasses in task-cycles: what may have hindered Fani's progress

Our micro-analysis of the tasks cycles in the two interviews revealed a pattern: whenever Fani's meta-rules were different than those followed by Lilach, Lilach's prompts attempting to de-ritualize Fani's participation had little effect. If anything, they would lead either to increased ritualization or to disengagement. We exemplify this process on one task that Fani engaged with in the post-interview – the "I thought of a number" task (no. 8 in table 1). The task was presented as follows:

> I thought of a certain number. If I multiply it by 7 and subtract 54 from the result, I will get the number I originally thought of. What is the number I thought of? Explain your solution.

Table 2 presents a partial transcript of the interaction between Fani and Lilach around this task.

Table 2. Transcript of Fani`s algebraic attempt to solve "I thought of a number" task

Line#	Speaker	What is said (what is done)
452	Fani	(writes) $7 \cdot 9 = 63$ $63 - 54 = 9$
453	Lilach	Now can you try solving it with a variable?
454	Fani	(writes) $7 \cdot x = 7x$ $7x - 54 = 7x - 54$
455	Lilach	What? What is it?
456	Fani	Okay, seven times X, that's seven X, seven X.
457	Lilach	Let's start with (explaining) what is X?
		…
460	Fani	An unknown number.
461	Lilach	So the unknown is the number she thought of? Okay.
462	Fani	Seven X minus… minus fifty-four equals seven X minus fifty-four.
463	Lilach	That sounds reasonable, I must say (smiles)
464	Fani	No, yeah, sounds reasonable…
465	Lilach	Seven X minus fifty-four does sound like it equals seven X minus fifty-four, but… what were you trying to write? What is seven X minus fifty-four? Explain it to me.
466	Fani	It's seven times the unknown number minus fifty-four.
467	Lilach	Okay, so why did you write seven X minus fifty-four?
468	Fani	Because she said if she multiplies that number by 7 and then subtracts fifty-four from the result, then she'll get the same number she thought of originally.
469	Lilach	Okay, so where did you write the part about "getting the number she thought of"?
470	Fani	She first thought of X
471	Lilach	Okay, so where is that written in the equation?
472	Fani	Here. (highlights the letter X in both lines) $7 \cdot x = 7x$ $7x - 54 = 7x - 54$
473	Lilach	(revoices Fani's claims, then asks) … So where did you write that part about "getting the number she thought of"?
474	Fani	What, like with this exercise?
475	Lilach	No, with our variable.
476	Fani	Here. (draws a circle around the last 7x, points to the X next to the 7) $7 \cdot x = 7x$ $7x - 54 = 7x - 54$

The episode opens with Lilach prompt, issued after Fani successfully solved the task arithmetically using "guess and check" procedures. Signaling her expectation that Fani shifts to a formal algebraic discourse, Lilach said: "Now can you try solving it with a variable?" [453]. This invitation, aimed at eliciting an algebraic formulation of the task, assumed a shared interpretation of the goal - constructing an equation that reflects the situation described of the word problem. However, Fani interpreted the prompt differently: rather than generating a relational equation, she symbolically translated arithmetic procedures she had previously carried out ("7·9=63" and, underneath it "63−54=9" [452]).

A closer examination of Fani's discourse revealed she was operating under the common but noncanonical meta-rule that an equal sign signifies "the answer is…" (Kieran, 1981; Knuth et al., 2006; McNeil & Alibali, 2005). This is evident when she writes "7·x = 7x" and explains, "seven times x, that's seven x" [456], implying that this is a valid procedural step. Within this meta-rule, "seven x" stands for the 63 – the result of 7·9. The same meta-rule explains the somewhat surprising equation Fani later produces: 7x – 54 = 7x – 54 [462].

Lilach, who viewed the equal sign as representing equivalence (in line with the canonical meta-rules), responded with mild amusement, remarking that "7x-54 does sound like 7x-54" [465]. Yet Fani did not appear to grasp either the irony or the alternative meta-rule guiding it. Attempting to justify her equation, she explained: "Because she said if she multiplies that number … then she'll get the same number she thought of originally" [468]. For Fani, the two sides of the equation served different purposes: the left-hand expression represented the operations performed on the unknown number (x), while the right-hand expression represented the outcome of those operations. Her arithmetic notes support this interpretation: "7·9−54=9" and "63−54=9." Thus, the left-hand "7x−54" symbolically mirrors the first numeric expression (7·9−54), while the right-hand one likely signifies the result (63−54). This situation, in which Lilach and Fani use the same keywords but are guided by different meta-rules, illustrates the commognitive conflict embedded in their interaction.

The surprising effect of the commognitive conflict in this episode is that for almost every communication (both uttered by Lilach and by Fani), there are reasonable interpretations that can be made according to the **different meta-rules** followed by the two participants. These are shown in table 3

Table 3. Fani and Lilach's differing interpretations during the probing of Fani's solution

What is said	Interpretation according to Lilach's meta-rules	Interpretation according to Fani's meta-rules
Lilach: Now can you try solving it with a variable? [453]	"Solve it with a variable" means constructing an equation and arriving at a solution through it	"Solve it with a variable" means expressing my arithmetic solution with variables.
Fani: (a) Seven X minus… minus fifty-four (b) equals (c) seven X minus fifty-four. [462]	Fani's statement is describing an identity between two algebraic expressions	(a) means the manipulation I did on the 9 $(7 \cdot 9 - 54)$, (b) equals means "will give me", (c) means the result of calculating $7 \cdot 9 -$

		54
Lilach: Seven X minus fifty-four does sound like it equals seven X minus fifty-four, but… what were you trying to write? What is seven X minus fifty-four? Explain it to me. [465]	The first 7x-54 is what Fani is expected to write. The question “what is 7x-54”? will lead to “the number I thought of” can be signified by X	Lilach is asking me to explain the meaning of each of the expressions. There’s nothing wrong with them. It’s simply a way to express the arithmetic calculations with symbols.
Lilach: So where did you write that part about “getting the number she thought of”? [473]	Fani has not yet expressed “the number she thought of”. If I direct her attention to this missing part, she will come up with the “x” of the right side of the equation.	Lilach is searching for how I expressed “the number she got”. “The number she got” is expressed in the right hand 7x-54 and the x signifies it is unknown.

Table 3 illustrates just part of the differing interpretations that could be given to the same utterances and prompts appearing in Lilach and Fani’s interaction. While Lilach’s prompts aimed to encourage Fani to author the equation that she expected to see according to the canonical meta-rules, Fani understood these through her arithmetic-based meta-rules, and interpreted them as requesting her to explain the representational choices she made to describe her arithmetic operations.

4 Discussion

This study examined whether and how an intervention, aimed at de-ritualizing a student’s algebraic discourse, led to changes in the student's discourse. Focusing on one case study -of a 7th grader named Fani, the findings indicate that while the intervention led to measurable gains in Fani’s ability to author more canonical narratives, especially in tasks focused on symbolic manipulation, her algebraic discourse remained largely ritual, with persistent non-canonical meta-rules continuing to shape her participation.

Fani’s improved performance across a range of tasks, especially those involving algebraic syntax and arithmetic reasoning, signals a shift toward more conventional school mathematics discourse. When assessed through traditional criteria, such progress might be interpreted as satisfactory. However, a closer commognitive analysis reveals that many of these gains occurred in tasks that afford ritualized engagement with procedures.

Moreover, our analysis of the post-intervention interview revealed that de-ritualizing Fani’s discourse may be a much more challenging task that we had originally anticipated. Prompts that are commonly thought to afford explorative participation, such as eliciting explanation, reflection, and substantiation failed to disrupt Fani's existing meta-rules. Instead of being perturbed or challenged by Lilach's questions, Fani interpreted them through the meta-rules familiar to her from the arithmetic discourse – a discourse in which she became quite fluent and successful throughout her elementary school years.

This finding underscores a central insight of the study: de-ritualization is not merely a function of the teacher's intention or the affordances of the task, but is critically dependent on the learner's meta-level rules in this discourse. Fani's persistent alignment with ritualized participation, despite consistent instructional scaffolding, highlights the depth and resilience of meta-rules as regulators of mathematical activity.

The implications for instruction are significant. For de-ritualization to occur, learners must not only be exposed to explorative prompts but must also interpret these prompts according to the same meta-rules followed by the teacher. This implication has much to say to teaching recommendations that focus on "talk moves" (e.g. Michaels & O'Connor, 2015) as the main vehicle for affording students' explorative opportunities to learn. It may mean that such talk moves are a necessary, yet not sufficient condition for de-ritualization to occur. We say "necessary" since, for example in the case of Fani, had Lilach not asked all the "press for reasoning" questions, Fani's non-canonical meta-rules would not have been exposed at all. Yet "insufficient" because, once exposed, the illusive nature of commognitive conflicts usually hinders their discovery "online", during the interaction itself.

Yet although commognitive conflicts are extremely difficult to detect, some of them may be more expected than others. For example, in our case, Fani's interpretation of the equal sign as an operational signal (an "answer marker") rather than a relational symbol denoting equivalence is a well-documented meta-rule of beginning algebra students (Kieran, 1981; Knuth et al., 2006; McNeil & Alibali, 2005). At least in cases of such common non-canonical meta-rules, teachers increased sensitivity to the entailments of these meta-rules and the commognitive conflicts that may emerge from them, can assist in early detection of them.

That said, we should stress that, even if commognitive conflicts are exposed, it is unclear in what ways they may be overcome. A limitation of the study is that the first author served as both researcher and interviewer, which may have influenced the interview interactions and the discourse produced. Future studies should therefore attend more closely to how meta-rules can be surfaced, challenged, and potentially transformed within instructional dialogue.

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