How teacher professional development based on Large-Scale Assessment resources can act on teachers' beliefs affecting their teaching practice

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Abstract: In this paper we present a model for teacher professional development, rooted in a national research project funded by the Italian government, that aims to develop theoretical and operational tools for exploiting the results of the Italian National Large-Scale Assessment (LSA), with respect to teachers' professional development. One of the project's main expected theoretical outcomes is the development of a model for pre-service and in-service mathematics teacher professional programs, based on the effective and conscious use of LSA. We first introduce the theoretical framework of the model, based on the notions of beliefs, the Mathematics Teacher's Specialised Knowledge model, and the concept of a Community of Inquiry. We then present, exemplify, and discuss the model we are designing for the training of pre-service and in-service mathematics teachers, aiming to support the development of beliefs that can enhance teaching practice by utilising LSA resources. Finally, we provide some data collected in the first experimental phase of the project.

Keywords: beliefs on large-scale assessment resources, pre-service teachers, in-service teachers, models in mathematics education, teachers' professional development.

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1 Rationale

Large-Scale Assessments (LSAs), which are designed to have a system-wide impact, have also been challenged for their possible pedagogical, educational and policy implications (Carnoy, 2015; Cochran-Smith, 2001). This led to large-scale standardised assessments becoming a subject of research in mathematics education in various directions (De Lange, 2007; Meinck et al., 2017; Suurtamm et al., 2016). The discussion in these research proposals revolves around the perspective with which LSAs are used, being part of a broader issue involving summative assessment in general. As highlighted in the literature, some evidence gained from LSAs can be used as a tool for teaching purposes. For example, according to Doig (2006), the tests, the data they allow to be collected, and their processing can yield a valuable tool for didactic, educational, and research purposes. In Italy, the National Institute for the Evaluation of the Education and Training System (INVALSI) is a public research institution, part of the National Evaluation of the Education and Training System (SNV), that promotes the improvement of educational levels through the deployment of tools to measure the students' learning outcomes and skills, and through the quality assessment of schools. The INVALSI tests are administered at a census nationally from grade 2 to grade 13; each year, the results of the national survey, elaborated on a valid statistical sample, are returned to the schools by a report regarding the average score of each school class and the comparison of the school's average scores with the average scores of other schools with similar background (numbers of students, socio-cultural context, etc.).

The theoretical framework of the INVALSI mathematics test is consistent with the Ministerial Curricular Guidelines (INVALSI, 2018) and with the main research results in mathematics education (Garuti et al., 2017). This aspect, together with the characteristics of the survey design and the data return modalities, ensures that the macro phenomena highlighted in these surveys can provide useful information and serve as interpretative tools for certain aspects of mathematics teaching and learning processes specific to the Italian context (Ferretti et al., 2024). By the term macro-phenomenon, we refer, for example, to what emerges from items with very low percentages of correct responses (below 40%, according to the difficulty benchmarks in Mullis et al., 2021) with incorrect answers interpretable through constructs from mathematics education research (Bolondi & Ferretti, 2021). In such cases, a nationwide critical issue becomes apparent (independent of external factors such as school type or geographical origin), which requires thorough investigation in order to understand its causes and to implement effective actions aimed at addressing and/or overcoming the issue (Ferretti & Bolondi, 2019). Thus, a macro-phenomenon is a complex reality, consisting of two components, one qualitative and one quantitative, which are complementary and interdependent (Johnson & Onwuegbuzie, 2004). The data collected by INVALSI represents the quantitative component of the macro-phenomenon identification. At the same time, the qualitative phase pertains to the interpretation provided by researchers through the theoretical lenses of mathematics education. Furthermore, research has shown that the study of macro-phenomena can be a valuable component of teacher professional development (Ferretti et al., 2020; Ferretti & Santi, 2023). Indeed, results from analysis of the macro-phenomena of standardised assessment, if appropriately and consciously integrated into teacher education, can improve teachers' knowledge and skills (Campbell & Levin, 2009). In particular, the analysis of LSA results, interpreted in the light of individual school and class contexts, can be useful for teachers to reflect on teaching-learning processes and identify the possible strengths and weaknesses of specific teaching choices, provided they are appropriately trained to use the resources described above proficiently. However, their effective use relies on teachers' awareness and ability to engage meaningfully with such resources. Many teachers are often not aware of the potential of LSA resources for their teaching practice (Dello Iacono & Spagnolo, 2024). As shown in the literature, teachers exhibit both positive and negative emotions towards LSA (Di Martino & Signorini, 2019), partly due to a lack of knowledge about their characteristics and potential. Teacher training that acts on teachers' beliefs concerning the role of LSA resources in teaching practice could help reduce such negative emotions, which arise from a lack of familiarity with these resources, and increase positive emotions by highlighting opportunities for their effective use in practice.

It is in this direction that the research to which this contribution belongs has been developed. Our study shows the first theoretical outcomes of the Italian National Research Project "Mathematics standardised assessment as a tool for teachers' professional development", which aims to develop theoretical and operative tools for exploiting the results of the mathematics national assessment tests with respect to teachers' professional development (Viola et al., 2024; Cibien et al., in press).

There is a long-standing debate on how these results can be effectively utilised at the classroom level and how to integrate them into both summative and formative assessments (Looney, 2011). The impact of standardised assessment is traditionally a top-down impact: survey results influence public opinion, and this moves policymakers who act on the structure of the system at different levels (system architecture, curriculum, teacher training, recruitment). Only at a final stage do surveys and their results have an impact on local teacher action. The innovative idea that induced the Italian Government to fund the research project this paper refers to is to provide tools for a bottom-up approach, in which the LSA resources, i.e., the INVALSI theoretical frameworks, tests, and results, are used immediately by teachers, specifically to implement formative assessment and individualised instruction.

Focusing on this purpose, this paper presents a theoretical model for the professional development of in-service and pre-service teachers, based on the use of

LSA resources, which constitutes the goal of the aforementioned national research project. In detail, the macro-phenomena that have emerged at the national level from the INVALSI tests represent the starting point of our study. Beginning with teachers' beliefs regarding these macro-phenomena, the model aims to influence teachers' beliefs about how such resources can affect their daily practice. The research questions we face in this context are:

RQ1: What are the characteristics of a teacher's professional development model that aims to act on teachers' beliefs on the role of LSA resources for everyday teaching practice, inducing professional development?

RQ2: Does this model for teacher professional development also foster the emergence of beliefs related to transversal dimensions?

Furthermore, an example will be presented illustrating how the phases of the model will be developed based on the analysis of a macro-phenomenon at the national level.

2 Theoretical framework

The theoretical framework of the model is composed of three elements: (1) the notion of *Community of Inquiry* (Jaworski, 2006); (2) the MTSK model - Mathematics Teacher's Specialised Knowledge (Carrillo-Yañez et al., 2018); (3) the notion of beliefs (Philipp, 2007). Following, we present these components and briefly discuss their mutual relations.

2.1 The notion of *Community of inquiry*

Within a Community of inquiry (Jaworski, 2006), the development of specialised knowledge may require tools and, in a broader sense, resources that mediate activity, contribute to interpersonal exchange between its actors, and bring out cultural and conceptual objects. On the other hand, underpinning the construction of teacher subjectivity is social interaction within a Community of practice (Wenger, 1998), which is characterised by its joint enterprise, the relationships of mutual engagement and the shared repertoire. Sociocultural perspectives in mathematics education have shown the role of socio-communicative practices in a cultural-historical context on both learning processes and identity construction. In our model, we extended these research findings to the professional development of mathematics teachers acting on their beliefs. From a sociocultural perspective, we advocate that mathematical knowledge and pedagogical knowledge are not a priori fixed entities that must be taken for granted; rather, they are continuously reflected and refracted in social and communicative activity. Jaworski (2014) highlights an important issue that characterises a Community of Practice (Wenger, 1998), namely inquiry, which involves critical thinking, investigation, doubt, and bringing new points of view. The transformation of a Community of practice into a Community of inquiry requires that participants critically examine their practices as they engage in them, become aware of their beliefs, question what they do as they do it, and explore new aspects of practice. Therefore, within this conception of Community of practice, we can think of inquiry as a “way of being”, in which teachers assume the role of inquiry as central to the way they think, act, and develop in practice and encourage their students to do the same (Jaworski, 2006).

The subjectivity that teachers realise in their professional development within a Community of Inquiry cannot be separated from their mathematical knowledge and their knowledge of teaching, and thus, from their beliefs about these knowledge components. In the context of the model of teacher training we are presenting, mathematical knowledge is anchored in the INVALSI tasks, based on the National Guidelines. In contrast, the knowledge for teaching encompasses all aspects related to interpreting the LSA results within the INVALSI theoretical framework and the findings in Mathematics Education research. The training course aims to establish a Community of Inquiry that fosters the emergence of beliefs, intended as lenses that shape participants' views on the role of LSA resources in their teaching practice, promoting a proactive approach to their dispositions toward action. In this sense, the *Community of Inquiry represents the socio-cultural context from which the beliefs emerge*; the components of the MTSK-model represent different facets of knowledge that contribute to shaping the participants’ beliefs.

In the following section, we present a model for the professional training of pre- and in-service teachers, based on the theoretical framework outlined in this section.

2.2 The notion of belief

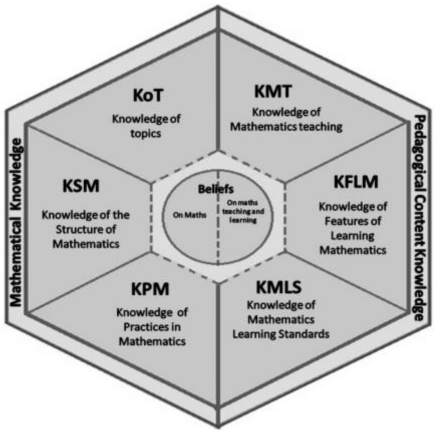
As pointed out by Zhang and Morselli (2016) and described in the literature, teachers’ beliefs emerge from their experiences (personal, professional, and formal mathematical knowledge), constituting complex belief systems rather than isolated entities. Furthermore, according to Schonfeld (2011), in the literature, there is often a mismatch between professed beliefs and the beliefs inferred from observed practice within a given context. Zhang and Morselli (2016) trace back the focus on context in research on beliefs to Skott (2009), who advocated a “social turn” in research on teacher beliefs, suggesting that it should focus on teachers’ contexts and the actual and virtual communities of practice they live in, not only on beliefs intended as explanatory principle for classroom practice.

In our model, following Philipp (2007), we conceive of beliefs as “lenses that affect one’s view of some aspect of the world or as dispositions toward action” (p. 259). This definition enables us to consider beliefs as individual, theory-driven, or experience-driven lenses that emerge from social interaction within a community of inquiry, specifically in the context of teacher professional development. Teachers’ beliefs are thus supposedly traceable in the teachers’ discursive practices (both written and spoken), as well as in their engagement during the group activities. In presenting the model, we focus on how different types of beliefs regarding the role of LSA resources evolve in relation to teachers’ professional development. We discuss here the ongoing analysis of the pilot study results related to a questionnaire administered to pre-service and in-service teachers, aiming to extract, among others, categories of teachers’ beliefs. We emphasise that these results will inform the researchers' understanding of how to apply the model's stages.

2.3 The Mathematics Teacher’s Specialised Knowledge (MTSK) model

The importance of knowledge required for teaching a specific discipline has been the focus of extensive research. As early as the mid-1980s, Shulman (1986) reflected on the areas of knowledge teachers should possess, in terms of *Pedagogical Content Knowledge*. In recent years, several researchers have explored various aspects related to knowledge for and in teaching. Some did not start with the contents of school curricula but focused on empirical approaches to identify and understand mathematical knowledge for teaching. An example in this sense is the one which led to the construction of the Mathematics Teacher’s Specialised Knowledge (MTSK) model (Carrillo-Yañez et al., 2018), exemplified in Figure 1.

Figure 1. MTSK Model (Carrillo-Yañez et al., 2018, p. 34).



The MTSK model coordinates two broad areas of knowledge, Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK) by placing teacher beliefs at the centre (Figure 1). MK refers to the knowledge a mathematics teacher possesses in terms of disciplinary knowledge within an educational context. In contrast, PCK refers to the knowledge related to mathematics content in terms of teaching and learning processes. Beliefs about mathematics and its learning and teaching are precisely at the centre of the model, emphasising the reciprocity between the beliefs and knowledge domains (Carrillo-Yañez et al., 2018). In the model, MK and PCK are divided into three subdomains.

The MK is composed of:

- *Knowledge of topics - KoT* (e.g., knowledge of definitions, properties, representations, procedures…);
- *Knowledge of the structure of mathematics - KSM* (this knowledge may include viewing elementary mathematics from a higher point of view, but also knowing how to switch and connect activities in different domains of mathematics, for example, between algebra and arithmetic);
- *Knowledge of practices in mathematics - KPM* (e.g., knowing how to prove, justify, define, make deductions and inductions, give examples and understand the role of counterexamples).

The three subdomains of PCK are:

- *Knowledge of mathematics teaching - KMT* (e.g., knowledge of the mathematics teaching theories, resources and materials and technologies, the strategies for introducing, representing content and concepts…);
- *Knowledge of the characteristics of mathematics learning - KFLM* (e.g. knowing how students might act, their errors and difficulties in specific topics,

and, in general, how students interact with mathematics while also taking into account affective aspects);
- *Knowledge of mathematics learning standards - KMLS* (knowledge of national curricula as well as international documents and not only for one's own school level but for all).

In particular, the MTSK model allowed us, in the preliminary phase of the research, to frame the teachers' responses to the questionnaire representing the starting pilot study of the project.

3 The model for pre-and in-service teachers' professional development

The training model presented here draws inspiration from the four-step model described in Ferretti et al. (2022). Still, it focuses on the participants' beliefs, referring them to the development of the teachers' specialised knowledge. It consists of six steps that we introduce and briefly discuss in the following.

1. *Activity Introduction*. During this phase, participants are asked to answer a questionnaire anonymously, in which they are asked to reflect on an INVALSI task chosen by trainer[1]. The aim of this phase is to make emerge the participants' initial beliefs on some crucial aspects related to the LSA resources: the mathematical content they believe the task is focusing on, the perceived difficulty of the task (Nicchiotti & Spagnolo, 2024); possible reasons of the choice of the distractors if the chosen task is a multiple choice task or possible mistakes; linguistic and/or semiotic aspects that could be challenging for students; an example of how they would use the task in their classroom practice if they think they would do so.
2. Whole group discussion led by the trainer. In this phase, the trainer shares the answers collected during the previous phase and conducts a whole-group discussion with the aim of eliciting the participants' beliefs. The beliefs are detected by the researcher via their (argumentative and semiotic) instantiations in the teachers' answers. During this phase, teachers begin to

[1] For a task example, please refer to the next section.

perceive themselves as members of a Community of inquiry focused on professional development.

3. Example analysis. In this phase, the trainer discusses with the entire group the example they have reflected on during the first phases, stimulating reflection on one or more subdomains of the specialised knowledge of the MTSK model. This phase fosters the teachers' identity as members of a Community of Inquiry by confronting their own perspective on the mathematical and pedagogical knowledge involved in the task with the one displayed by the researcher, rooted in their competence in mathematics education research.
4. Introduction to the Gestinv Database. During this phase, participants are introduced to Gestinv, a database containing structured information on Italian standardised assessments, which comprises 1718 test items spanning 10 years of INVALSI activity (Ferretti et al., 2020). The database allows to search for tasks based on their classification according to the mathematical content and the learning objectives pursued in it in reference to the National Guidelines, on the vertical cognitive processes outlined by the INVALSI theoretical framework, on the national rates of correct/incorrect/invalid answers, on the types of test questions (multiple choice, open questions, etc.), as well as following a guided cross search (with and/or logical connectors) involving all the parameters mentioned above (Ferretti et al., 2020). In this phase, they are asked to explore the database and use it concretely, looking for items related to what they have seen and discussed thus far with the researcher and reflecting on how they might use this instrument with their students. Thus, teachers can become aware of the potential of LSA resources as a tool for everyday teaching practice, and a concrete instrument is also provided for use in the next step.
5. Group activity. During this phase, the trainer assigns tasks to the teachers, who work in small groups. The task focuses on mathematical content chosen from the National Guidelines. It is appropriate for the school grade of the involved teachers and is also inherent to one or more learning difficulties identified in Literature[2]. The group activity aims to construct a multimedia product, an artefact, and design an activity for students, among other objectives. In presenting their final product, trainee teachers are asked to make explicit how they use LSA resources referring to: topic (mathematical

[2] For an example in this sense, please refer to the next section.

content related to definitions etc.), structure (relations between different topics involved in the same task), practices (strategies, arguments, examples, counterexamples, semiotic representations and registers involved); mathematics teaching (aspects related to teaching theories, strategies and semiotic resources for introducing and representing content and concepts involved, also in reference to the INVALSI theoretical framework), characteristics of mathematics learning (possible mistakes, misconceptions, difficulties in specific topics, semiotic representations and transformations between them), mathematical learning structures (National Guidelines, not only for one's school level but also for neighbouring levels). This phase requires teachers to explicitly describe how they might utilise the LSA resources in their teaching practice, implicitly referencing the six subdomains of specialised knowledge according to the MTSK model.

6. General discussion. During this phase, the subgroups present their productions to the whole group and the trainer. Each presentation is discussed within the Community of Inquiry to highlight beliefs, address doubts, difficulties, and unclear content regarding both MK and PCK, and to make explicit the subdomains of MTSK that have emerged throughout. In this way, it is possible to summarise the results of the Community of Inquiry's activity and to "institutionalise" the knowledge that emerges from it, making participants aware of their own professional development. In particular, during the collective reflection, the trainer discusses with the teachers the beliefs that emerged during the teacher professional development, highlighting how they are related to the domains of specialised knowledge according to the MTSK model. In this way, the aim is to foster the awareness' development of the evolution of their beliefs and how these influence their specialised knowledge; furthermore, the discussion allows the researcher to collect new data, useful for improving the ongoing characterization of beliefs about the role of LSA resources in teaching practice, which can be used in the next cycle, as explained in section 2.2.

3. 1 Methodology

We are currently still at the initial stage of the model. For the first phase (*Activity Introduction*), we designed a questionnaire divided into six different versions, depending on the group of participants involved in the trial. Indeed, in-service and pre-service primary school teachers, in-service and pre-service lower secondary school teachers and in-service and pre-service upper secondary school teachers are involved in the teacher professional development course. The questionnaire consists of several open-ended and closed-ended queries and was delivered via Google Forms to reach a sample of respondents as large and diverse as possible. It contains a number of questions specifically referring to precise INVALSI items chosen from the Gestinv Database because they are representative examples of macro phenomena at the national level; by this, we mean, for example, multiple-choice items for which the percentage of correct answers by students at the national level is very low and whose distractors (i.e. wrong answers) can be framed by constructs of mathematics education shown in the literature. The queries are formulated to investigate how teachers perceive the item in terms of assessment, whether they are aware of possible mistakes and their causes, how students might perceive the difficulty, and how the wording of the item affects its difficulty, as well as to what extent the item is considered consistent with the curriculum guidelines. The answers to this class of questions are also framed in the knowledge domains of the MTSK model; asking, for example, to consider which students' answers might be the most wrong chosen, it is possible to investigate what knowledge of the teachers can be placed in the PCK subdomain called *Knowledge of Features of Learning Mathematics* (KFLM).

In addition, there are also questions not strictly related to the INVALSI items (the so-called *transversal questions*), which aim is to explore knowledge and beliefs relating to aspects that are transversal to the discipline precisely, but closely linked to the participants' perception of the purpose and usefulness of the LSA resources provided by the INVALSI and the impact of these on their teaching practice or future teaching practice.

The pilot study questionnaire was completed by 39 pre-service primary school teachers, 22 in-service primary school teachers, 11 pre-service lower secondary school teachers, 12 in-service lower secondary school teachers, 20 pre-service upper secondary school teachers, and 14 in-service upper secondary school teachers, all from different schools and universities across various Italian regions.

In the following section, we will present some data collected via the questionnaire mentioned above. In this way, we want to show what we mean by the use of data related to national-level macro-phenomena to investigate the beliefs of Italian mathematics teachers regarding the standardised INVALSI assessments and some transversal aspects.

4 Data collection

In this section, we introduce some of the data collected from the first experimentation using the survey questionnaire. In particular, we report on what we analysed with regard to the transversal questions aimed at investigating the mathematics teachers' perception of the role of natural language in learning mathematics. In detail, teachers (from each school grade considered) were asked:

Q1 - In your opinion, what role does natural language play in students' learning of mathematics?

Q2 - Do you think that the student's language skills play a role in performance in mathematics? If yes, in what aspects?

Table 1 shows the percentages of the answers to Q1. It can be seen that, in total, almost 91% of the teachers (107 out of 118, considering both pre-and in-service) believe that natural language plays a fundamental role in the learning process of students. To understand the reasons for these statements, we compared the answers to Q1 with those to Q2. Among those who claim the relevance of language in response to the first question, the main aspects in which this manifests itself during students' mathematical performance concern mostly comprehension and explanation (Table 1). In particular, respondents refer to the comprehension of the text or the assigned task (e.g. from an extract of an in-service teacher from low secondary school: "*Understanding the text of a question is crucial for resolution.*") but also of the context of the item itself (e.g. from an extract of a pre-service teacher from upper secondary school: "*Comprehension of the text is the first obstacle encountered in solving a problematic situation and understanding its context.*").

On the other hand, the student's language skills are considered crucial for the explanation of solutions and reasoning (e.g. from an extract of a pre-service teacher from primary school: "*In the verbalisation of solutions to problems or operations.*"), as well as explanation in the sense of being able to express mathematical concepts in different ways, using different representations (e.g. from an extract of a pre-service teacher from upper secondary school: "*They could express mathematical concepts differently.*"), However, explanation is also understood as the ability to use discipline-specific language (e.g., from an extract of a pre-service teacher from upper secondary school: "*In the development of problems, in the ability to argue and in the use of specific language.*").

Table 1. Data collected on pre-service and in-service teachers' beliefs regarding the importance of natural language in students' learning of mathematics and student's language skills playing an important role in mathematics performance.

	Natural language plays an important role in students' learning of mathematics		**Aspects in which students' language skills are important in mathematics' performance (percentage of total)**	
School grade	**Yes**	**No**	**Comprehension**	**Explanation**
Pre-service teachers in grade 5	34	5	48.98%	8.51%
In-service teachers in grade 5	20	2	55.55%	14.81%
Pre-service teachers in grade 8	9	2	54.54%	9.09%
In-service teachers in grade 8	12	0	75%	0%
Pre-service teachers in grade 10	19	1	65%	10%
In-service teachers in grade 10	13	1	64.28%	0%

We can relate and compare the answers to the transversal questions in natural language with those collected from non-transversal questions, which are also presented in the questionnaire in relation to each INVALSI item proposed (regardless of the school grade involved). The question is: "On a scale of 1 to 4 (1=not at all; 4=completely), how influential do you think the wording of this item is in determining the difficulty of the item for students? Motivate your answer." (Q3) and was designed and proposed to investigate both the perception of the difficulty of the questions (and thus, identify which aspects of KFLM could emerge) and, at the same time, to highlight any teachers' beliefs regarding the relation between language and students' learning processes. Even among the answers to this question, we find several references to comprehension of the item, although not as explicitly as in the answers to the transversal questions. We also observe that, on the other hand, in these responses, the aspect of explanation is never considered, most probably because the proposed INVALSI items are all multiple-choice. Therefore, there is no need to show the reasoning followed.

We provide an example of the answers to this kind of question, considering item 15 of the 2011 INVALSI tests proposed in the questionnaire for upper secondary school in-service and pre-service teachers (Figure 2).

Figure 2. Task administered to Grade 10 Italian students by INVALSI in 2011 (www.gestinv.it) and present in our survey questionnaire

D15. Dividing a number by 0.2 is the same as multiplying it by

☐ A. $\frac{1}{5}$

☐ B. $\frac{1}{2}$

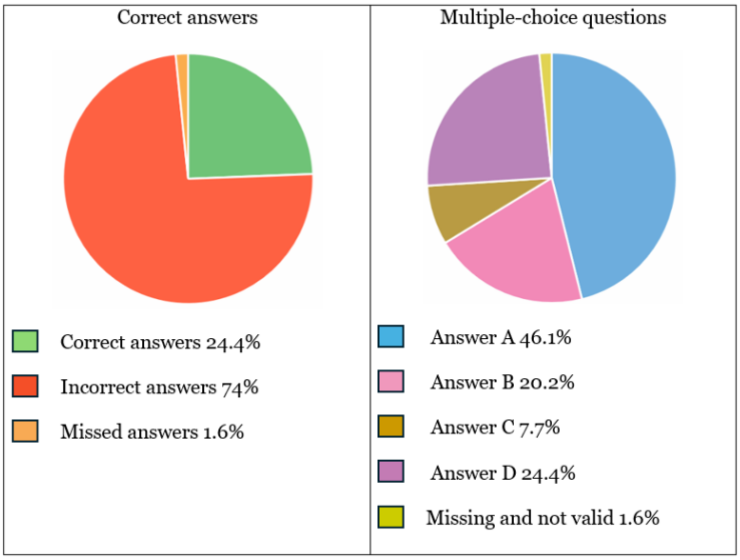
☐ C. 2

☐ D. 5

The purposes of the item, in accordance with the Italian National Guidelines, concern the ability to use different notations (semiotic representations) and to convert from one to the other (from fractions to decimals and/or vice versa) and to be able to carry out operations with rational numbers, both in writing them as fractions and in decimal representation. On the other hand, the competencies required to answer correctly are closely linked to the use of arithmetic calculation techniques and procedures.

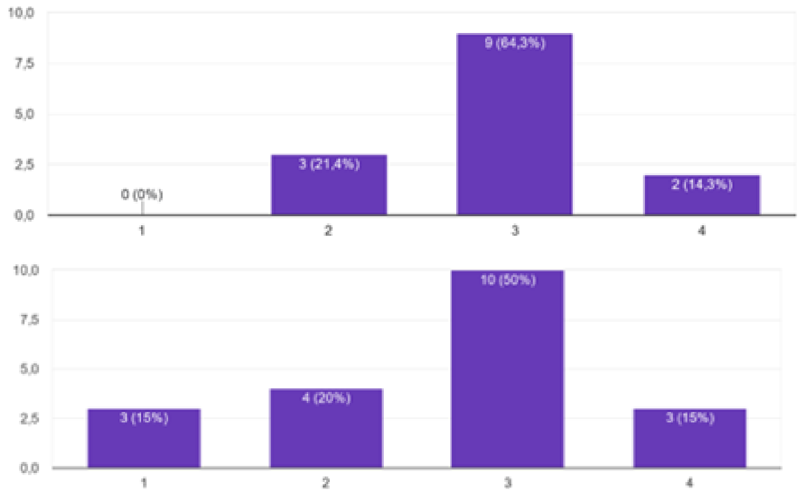
The choice to use this item is due to the fact that it is an emblematic example of a macro phenomenon at a national level, in the sense discussed above: the percentage of correct answers is very low (Figure 3), and it is possible to frame the choice of distractors using the research lenses in mathematics education. For example, it can be assumed that distractor A was chosen as a possible result of a misconception related to the operations of division and multiplication with rational numbers (Fandiño Pinilla, 2005). At the same time, the item is also interesting from a semiotic point of view since, to answer correctly, a treatment in the sense of Duval (2006) must be made between the decimal representation of the number and its representation as a fraction of integers. Thus, the choice of B and C distractors can be justified by considering a problem that arises during the treatment of one representation transitioning to another.

Figure 3. National survey of response rates to INVALSI task in Figure 2



From the data collected, we can see that 65% of pre-service teachers (13 out of 20) and 78% of in-service teachers (11 out of 14) answers to Q3 declares that the item's wording may have had a significant effect on its difficulty (Figure 4), i.e. have a score of more than 2 on the proposed Likert scale.

Figure 4. Percentages of in-service teachers for each Likert score on Q3 (above); percentages of pre-service teachers for each Likert score on Q3 (below)



The main motivation given by both groups of teachers to justify the claim that the wording affects the difficulty of the item is that the words "dividing" and "multiplying" express two opposite concepts, but are present in the same prompt, thus confusing the students. Teachers state that this wording may therefore create an obstacle to the comprehension of the task: "*Students may misunderstand the question and make a mistake dividing by multiplying*." (an extract from a pre-service teacher); "*Probably if they saw the written operation there would be fewer* *errors than understanding the question "in words"*." (an extract from an in-service teacher). In some cases, the reference to comprehension is not explicit (i.e. "*Each word in the text must be correctly translated into mathematical language. There are also words that lead to "opposite" thoughts (divide and multiply) and different representations of numbers (natural, fractions, decimals)*." from a pre-service teacher, or "*There is the difficulty of grasping that first says to divide and then multiply*." from an in-service teacher). Still, it is clear that what makes the task difficult is the fact that the assignment is not understood correctly by the students.

5 Discussion and conclusion

This paper presents some initial theoretical findings from an Italian national project aimed at promoting the professional development of in-service and pre-service mathematics teachers through the effective and informed use of LSA resources. These findings consist of a model for teacher training based on the notion of belief (Philipp, 2007), the MTSK-model (Carrillo-Yañez et al., 2018), and the notion of Community of Inquiry (Jaworski, 2006).

According to the model, the activity of the Community of Inquiry is carried out through interaction with LSA resources, promoting an evolution in teachers' specialised knowledge and beliefs regarding the role of LSA resources in teaching and learning mathematics. During the training activities, teachers can discuss issues related to the mathematical content, assessment methods and results, teaching strategies and methodologies, cognitive processes, students' mistakes and difficulties, and become aware of their beliefs regarding LSA resources and of their potential in effectively supporting their teaching practice. In developing our model, we adopted an evolutionary approach that stimulates teachers and educators to a deeper awareness of their own actions, motivations and goals (Jaworski & Goodchild, 2006), making explicit how their beliefs affect both subdomains of the MTSK model: the MK and the PCK.

In response to the first research question (**RQ1**), it can be stated that all the steps of the model we designed are intended to influence teachers' initial beliefs concerning LSA resources. Indeed, the results presented in the paper refer to the initial phase of the study, which specifically aimed at revealing a variety of such beliefs, in order to discuss them and how they affect teachers' perception of these resources in the next step of the model (*Whole group discussion led by the trainer*). Subsequently, during the phase of *Example analysis*, teachers start to perceive themselves as part of a *Community of Inquiry,* also experiencing a confrontation of their perspective on the mathematical and pedagogical knowledge involved in the task with the one displayed by the researcher/trainer and rooted in his/her competence in mathematics education research (e.g. by discussing aspects related to the teachers' Semiotic Interpretative Knowledge, according to Asenova et al., 2023a; b). Finally, after the *Introduction to the Gestinv Database* and the concrete experience of the *Group Activity* phase, in the sixth step of the model (*General discussion*), the results of the activity of the Community of Inquiry are summarised in order to "institutionalise" the knowledge that emerged from it. At the same time, the last phase enables teachers to discuss and confront their beliefs, becoming aware of their evolution over time and how these beliefs affect their teaching practice. It also allows the researcher to collect new data useful for improving the ongoing characterisation of the beliefs on the role of LSA in teacher practice and use them, adopting a grounded approach, to fine-tune the model during subsequent cycles.

To address the second research question (**RQ2**), we provided an example illustrating how, even during this first phase of the study, beliefs emerged that extended beyond disciplinary knowledge concerning LSA resources. Indeed, the data collected through the initial questionnaire shows that, for example, different beliefs emerge regarding the role of natural language in a context of mathematics teaching-learning: related to the comprehension of students, how linguistic aspects can influence the way they express themselves during performances in mathematics, how the formulation of some items can affect their difficulty and the students' comprehension and explanation of concepts.

At present, the first experiment has been carried out on a group of 118 teachers from different levels of education, both pre-service and in-service. One of the main limitations of this initial pilot study is the small sample size. Therefore, the main objective for the near future is to expand the sample of teachers in order to conduct quantitative research aimed at investigating what teachers' needs for professional development and beliefs about LSA resources emerge regarding LSA resources. Moreover, during the next phases of the national project, qualitative and quantitative analyses of the first results collected will lead to an increasingly fine-tuned delineation of the model, which will be implemented in numerous in-service teacher professional development courses and degree courses for pre-service teachers.

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Author contributions

All authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy.

Conflicts of Interest

The authors declare no conflicts of interest.

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