Looking back at MAVI 30 conference in Freiburg: Some recent comments from a former MAVI founder – a foreword and an epilogue

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1. Introduction

This is both a foreword and an epilogue, which should soon become clear. The author is delighted to have been commissioned to write both.

First of all, this is an *epilogue* because from September 17 to September 20, 2024, the 30th MAVI conference took place in the beautiful university city of Freiburg in the south of Germany at the *University of Education* (Pädagogische Hochschule Freiburg).

What is MAVI? MAVI is an acronym made up of the two words *Mathematical Views*. This acronym describes the keywords of interest originally used by a group of scientists within mathematics education. The group of originally Finnish and German scientists came together for the first time in 1994 for a joint conference in Duisburg (Germany) to survey the specific characteristics of primarily *mathematical views* among people involved in mathematics (e.g. research mathematicians, scientists researching the teaching and learning of mathematics, mathematics teachers, pupils in mathematics lessons, etc.) to investigate their consequences. What interests us about mathematical views will be explained in a moment.

To be more precise: The first MAVI conference took place from October 4 to 5, 1994 at the former University of Duisburg (Germany); today this – originally -- old university is part of the large University of Duisburg-Essen in the Ruhr area in the west of Germany. The first MAVI conference was initiated by the colleague Prof. Dr *Erkki Pehkonen* (University of Helsinki) and the author of this article, Prof. Dr *Günter Törner* (actually Emeritus at the Faculty of Mathematics at the University of Duisburg-Essen).

In short, it was a German-Finnish research group that had been formed at that time and met in Duisburg. According to the text of the first conference proceedings, which have been preserved, the aims were *to examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education. There is a vivid movement around research on mathematical beliefs in Europe*.

In short, our views on the huge field of mathematics are a priori (subjective) beliefs and convictions in order to address a central field of subject-didactic research topics.

Figure 1. *Erkki Pehkonen (right) and Günter Törner* (at the MAVI conference at Genova, 2009).

In March 1994, a first meeting was held at the University of Kassel, and we quote from the first conference proceedings:

> We established a working group of interested scientists called Mathematical World Views in order to build up a network of belief-researchers in Europe.

Note the wording: At that time, we took up the important concept of the "*mathematical worldview,"* which Alan Schoenfeld (1985) had likely used for the first time in our field.

In particular, the aim was to encourage young researchers to step out of their national context and meet with other scientists in an international context -initially within a European framework- and to initiate initial collaborations and final joint projects.

Bear in mind, the year was 1994. English was a rare language of communication in the didactics of mathematics in Germany, and research results were rarely published in English at that time, sorry to confess, and what was new back then. We had no objections to many doctoral candidates, in particular, taking part in a conference, and this has remained an unwritten rule for MAVI conferences to this day. We didn't want them to be overwhelmed by researchers who had already been established for many years.

We abbreviate our historical review and refer to the homepage of the active MAVI group[1].

But we should be fairer and take another fact into account. As can be seen from the information above, another founder of the MAVI group was Prof. Dr Erkki Pehkonen. Actually, all participants of this actual 30th MAVI conference were silently hoping that perhaps our friend and scientific colleague could attend this conference.

Now, as this text is being written, we have learned with regret that our dear colleague (see picture above) passed away on November 27, 2024, at the age of 83. We bow silently before his great scientific life's work and thank him for the many fruitful comments he gave to many of us.

– We will honour his memory. –

Mathematical worldviews were one of the first research issues. At that time, Byers' book (Byers, 2007), showing the many ambiguities with which every mathematician is struggling, did not yet exist. Meanwhile, however, Byers' assessments of the nature of mathematics in educational contexts are always eye-

[1] https://lnu.se/en/meet-linnaeus-university/collaborate-with-us/projects-and-networks/natverk/fakulteten-for-teknik/Mavi-Mathematical-Views/

opening (see Byers, 2007 & 2017). These facets or colours have a significant influence on discussions of how to orient the teaching of mathematics at all levels (see Ziegler & Loos, 2017; Loos & Ziegler, 2016).

We simply have to admit that academics qualified in didactics are rarely experts in mathematics and have often hardly familiarised themselves with the various facets of mathematics. Conversely, this may also apply to research mathematicians, particularly regarding their expertise in learning and teaching mathematics. That is why we emphasise the plural: Mathematical *views* and their impacts.

2. The dawn of mathematical belief research

The discussion of beliefs boomed from 1990 onwards at the major international conferences, e.g. the PME or PME-NA; there was often a section in which the presentations on beliefs were summarised. Beliefs had already been addressed many years earlier (see the discussion in Törner, 2002).

The author's own database (with more than 9.500 items) already contains the first (early) papers from 1968, and only three years later Green's book (Green, 1971), which is still sometimes stimulating today, was published.

At the beginning of MAVI in the 1990s, the work of Alba Thompson (1992) and the book by McLeod and Adams (1989), to which many scientists referred, were particularly topical, along with many other publications. They marked two important focal points of research in the field thirty years ago.

For the sake of completeness, we should also mention the early work of M. Frank (Frank, 1985) and J.J. Dionne (Dionne, 1984), insights which played a key role, but these papers are largely forgotten today.

No attentive classroom observer questioned the role of beliefs, but the terms belief or belief system remained vague, almost undefined for a long time – to be honest, still today.

If you look across the broad field of work on (mathematical) beliefs, you can roughly recognise two groups: Some, like McLeod at the time, integrate affections into their understanding, while others assume an emotion-free definition – in the sense of Thompson (Thompson, 1992) – without denying that emotions can occasionally be inextricably linked to beliefs. In A.G. Thompson's very early work from 1984, beliefs are interpreted as individuals' subjective perspectives; the word "view" alone is used more than 50 times.

The beliefs of mathematicians were seen as a key attitude for teaching, learning and understanding mathematics in schools. Pupils across various classes and school types constitute the primary clientele for teachers. However, they were not yet the subject of research in Germany at the time. Grigutsch's 1996 dissertation and his work with co-authors (Grigutsch et al., 1998) provided important insights previously unknown at the time.

The title of Grigutsch's groundbreaking dissertation—which is (unfortunately) available only in German—prominently features the term " mathematical worldview extensively." The author wishes to emphasise that his concept of "worldview" already closely aligns with the definition presented in his 2026 work (see Törner-Sriraman, 2026). This is not merely about an individual's subjective response to the frequently debated question of what mathematics actually is. No, Grigutsch's questionnaires also include items that address subjective responses to questions regarding the teaching and learning of mathematics.

It is worth recalling the initial results of Stefan Grigutsch's dissertation (Grigutsch, 1996), which empirically examined almost 1,000 pupils from different types and levels of school. The large questionnaire developed for this purpose in the working group at the time is still frequently used as a source by researchers today (Eichler et al., 2023).

3. The MAVI book on beliefs as hidden variables

What made the MAVI conferences productive was the fact that, over time, colleagues with prominent names appeared at the MAVI conferences, which were then organized annually and internationally announced. This gradually created a growing pool of scientists who became candidates to eventually join our book project.

With the support of the *Mathematical Research Institute Oberwolfach*[2] (MFO) (the institute is located at a lonely place in the Black Forest, Germany and is likely known to mathematicians worldwide as a well-known (and sought-after) conference venue; wherever new conference venues for mathematicians have been established in recent decades, Oberwolfach has served as the benchmark), we were able to obtain a one-week slot for a didactic conference in 1998 to discuss the book project Beliefs – Hidden Variables in Mathematics Education in more detail. The book (Leder et al., 2002) was finally printed in 2002 and was increasingly recognised

[2] https://www.leibniz-gemeinschaft.de/en/institutes/leibniz-institutes-all-lists/mathematisches-forschungsinstitut-oberwolfach

internationally. It can be seen as an important, broad snapshot of research on beliefs.

However, relevant research never stands still and so the author and his colleague Bettina Rösken, together with the American beliefs researcher Gerald Goldin (Rutgers University), wrote in 2008 an important article on beliefs research: Beliefs – no longer a hidden variable in mathematical teaching and learning processes (Goldin, et al., 2008).

4. Alan Schoenfeld's far-reaching mathematical research based on Beliefs

For the first author, and looking ahead in the sense of a foreword to future research directions, it was a highlight of the Freiburg conference to have the senior *Alan Schoenfeld* as a speaker. Since the early days of belief research, Alan has always been a major driving force in our research. In his actual lecture with the title *Thoughts on Beliefs (or) How my ideas about beliefs have evolved over the years*, he reiterated what were initially common, familiar experiences for us: Beliefs play a special role if you want to experience *problem solving*. Yes, e.g. Marta Frank had already told us that and who wouldn't have known it.

Early on, he emphasised the close connection between insights in *problem solving* and the influence of beliefs (see Schoenfeld, 1989). For him – Alan – however, it was important not to denounce only inadequate beliefs and to improve them appropriately. No, for Alan Schoenfeld, beliefs were important fossil indicators of a learner's learning and actions. Conversely, the various beliefs provide the observing scientist with important information about the students' thinking.

For Alan, this approach was like a decisive switch, seemingly moving away from the general trend in beliefs research and seeking answers to the fundamental question of educational science: How we think... (Schoenfeld, 2010). At first glance, this important question seemed to have nothing to do with beliefs, however...

What do we learn en passant, as it were, from Schoenfeld's work? Since there is still no generally accepted (formal) definition for beliefs, we make do with linguistic descriptions of what we think we can do with beliefs. We look for definitions of terms that appear to us, in each case subjectively, to be synonymous.

Schoenfeld lists the terms – his subjective favourites – in the 2010 book (p. 39), which are close to each other in terms of content and sometimes seem to capture beliefs more, sometimes less, specifically.

We cite (Schoenfeld, 2010, p. 39):

> I use the term orientation as an inclusive term encompassing a group of related terms such as dispositions, beliefs, values, tastes and preferences. How people see things (their worldviews and their attitudes and beliefs about people and objects they interact with) shapes the very way they interpret and react to them. In terms of sociocognitive mechanisms, people's orientations inuence what they perceive in various situations and how they frame those situations for themselves. They shape the prioritization of the goals that are established for dealing with those situations and the prioritization of the knowledge that is used in the service of those goals.

The same applies to the author's German: here, there is more than one term that could be equated with the author's own favoured definition of belief.

The same applies to linguistic diversity in the designation of belief systems. Here, the use of the term "worldview" has become almost canonical.

Readers of Alan's essays often noticed that he seemed to prefer the term orientation instead of the word beliefs. The word 'orientation' often implicitly refers to goals that one wants to achieve, which somehow influence one's actual actions.

The author believes he can also recognise a key to the question that is repeatedly asked at MAVI conferences: Are beliefs permanent, or do beliefs possibly change faster than we think? (see Liljedahl et al, 2012)

If we already associate beliefs with the conceptual content of the word orientation, which is accepted as a synonym, then we have to admit that goals can change from case to case. We see this as evidence of how important it would be if our joint work on belief definitions were to be intensified.

His many years of empirical observations finally led him to answers to his fundamental question: *a long road in research*.

5. Decision making – a possible construction site where we have to fall back on beliefs.

Schoenfeld formulated an important interim result as follows:

> I knew there were belief systems but not how they affected behavior in general, or how to model them. This led him to a limited research question, which he placed in front of his question above, namely the role of beliefs in decision

> making. More understandably, when making decisions, people look for favorable paths to the goal they want to achieve. This important (and not really surprising) observation shed new light on beliefs.

For reasons of space, we have to stop our virtual discussions on aspects of beliefs research here.

Schoenfeld managed to provide plausible answers by building a bridge from decision making to the question How we think posed above? We conclude here on the basis of Schoenfeld's lecture in Freiburg with the insight that in future we should concern ourselves even more intensively than before with the definitional foundations of beliefs.

Beliefs are by no means – as the mathematician would call them – monomorphic conceptual contents. Beliefs, as a family of conceptual expressions, play a central role in learning and teaching if we give them the chance not to be used canonically.

In this respect, the statement by Pajares quoted above is certainly forward-looking and we repeat it: Schoenfeld's research proves it to be correct.

It will not be possible for researchers to come to grips with… beliefs, however, without first deciding what they wish belief to mean and how this meaning will differ from that of similar constructs… the most fruitful concepts are those to which it is impossible to attach a well-defined meaning. (Pajares) (see Törner, 2002, p. 74 and 75)

This card to *cleaning up a messy construct*, – as Pajares puts it – does not seem to be played sufficiently also after 30 years.

Acknowledgements

The author would like to thank the MAVI Group's board for giving him the opportunity to deliver this overview presentation; as a result, he was able to update the manuscript, originally written in 2024, in 2026.

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